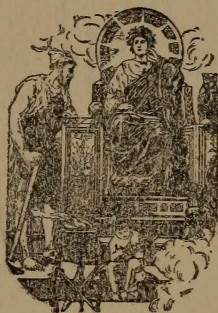


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THE
LONDON, EDINBURGH, AND DUBLIN
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CONDUCTED BY
JOHN JOLY, M.A. D.Sc. F.R.S. F.G.S.
AND
WILLIAM FRANCIS, F.L.S.

"Nec araneorum sane textus ideo melior quia ex se fila gignunt, nec noster
vilior quia ex alienis libamus ut apes." *Jusr. Lips. Polit. lib. i. cap. 1. Not.*

VOL. XVIII.—SIXTH SERIES.

JULY—DECEMBER 1909.

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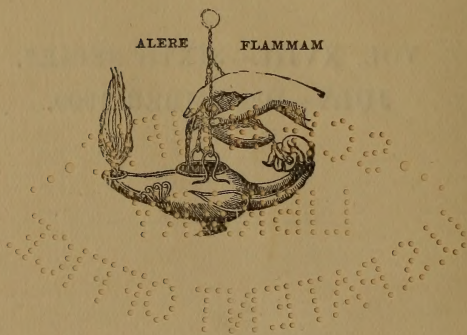
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“Meditationis est perscrutari occulta; contemplationis est admirari perspicua Admiratio generat quæstionem, quæstio investigationem, investigatio inventionem.”—*Hugo de S. Victore.*

—“Cur spirent venti, cur terra dehiscat,
Cur mare turgescat, pelago cur tantus amaror,
Cur caput obscura Phœbus ferrugine condant,
Quid toties diros cogat flagrare cometas,
Quid pariat nubes, veniant cur fulmina coelo,
Quo micet igne Iris, superos quis conciat orbes
Tam vario motu.”

J. B. Pinelli ad Mazonium.



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ERRATA.

Page 434, third line, *delete* M_n

„ 436, second line, for x read $\dot{x}$

„ 436, second line from bottom, *for reflexion read* deflexion.

INDEXED

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[SIXTH SERIES.]

JULY 1909.

I. *On the Instantaneous Propagation of Disturbance in a Dispersive Medium, exemplified by Waves on Water deep and shallow.* By Lord RAYLEIGH, O.M., F.R.S.*

THE solution, first obtained by Cauchy and Poisson, for the propagation in one dimension over deep water of the waves which issue from a concentrated initial disturbance presents peculiar features. One form of it may be written

$$\eta = \frac{1}{\pi x} \left\{ \frac{gt^2}{2x} - \frac{1}{3 \cdot 5} \left(\frac{gt^2}{2x} \right)^3 - \frac{1}{3 \cdot 5 \cdot 7 \cdot 9} \left(\frac{gt^2}{2x} \right)^5 - \dots \right\} \quad (1)$$

where η denotes the elevation of the surface at time t and place x , $d\eta/dt$ being initially zero throughout and η being initially zero except at $x=0$. g denotes, as usual, the acceleration of gravity. So far as general mechanical theory is concerned, it might be expected that under these initial conditions the elevation would at first vary as t^2 ; but that the effect should commence without delay at all points seems at first to conflict with ideas derived from the more familiar theories of sonorous and luminous undulations, according to which a finite time must elapse before any effect reaches places finitely distant from the source.

A plausible explanation of the discrepancy may be found in the reflexion that trains of simple waves move in deep water with velocities proportional to the square roots of the wave-lengths and thus capable of assuming infinite values, and that in accordance with Fourier's theorem the initial

* Communicated by the Author.

disturbance in question must actually be regarded as including trains of infinite wave-length and velocity. But in the sequel we shall find reason for questioning the sufficiency of this explanation.

In the deep water solution the speed σ is proportional to $k^{\frac{1}{2}}$, where $k=2\pi/\lambda$ and λ is the wave-length. We will now suppose more generally that in a dispersive medium $\sigma=k^m$. In the general Fourier solution

$$\eta = \frac{1}{\pi} \int_0^\infty \cos \sigma t \cdot \cos kx \cdot dk \cdot \dots \quad (2)$$

σ is to be given the above value, and for the sake of convergency we must introduce under the integral sign the factor e^{ky} where y is negative and ultimately is made to vanish. In Prof. Lamb's exposition* of the deep water problem the velocity-potential is employed as an intermediary, and into this the exponential factor enters essentially. All ambiguity is thus avoided; but on the other hand it would seem that it should not be necessary to go behind (2), at any rate if this expression for the elevation is correctly understood. And in the extended problem, where nothing more is known than that $\sigma \propto k^m$, we have no choice.

Expanding the cosine in (2), we get

$$\eta = \frac{1}{\pi} \int_0^\infty \left\{ 1 - \frac{k^m t^2}{1 \cdot 2} + \frac{k^{4m} t^4}{1 \cdot 2 \cdot 3 \cdot 4} - \dots \right\} \cos kx \, dk, \quad (3)$$

in which on the above understanding

$$\int_0^\infty k^{2mn} \cos kx \, dk = \frac{\Gamma(2mn+1) \cos (mn + \frac{1}{2})\pi}{x^{2mn+1}}. \quad (4)$$

When $n=0$, (4) vanishes; and in general we have a term in (3) proportional to t^2 , predominating when t is small.

We fall back at once upon the simple case of waves in deep water by making $m=\frac{1}{2}$. Then (4) vanishes, if n is even; and if n is odd,

$$\int_0^\infty k^n \cos kx \, dk = \frac{n!}{x^{n+1}} (-1)^{\frac{1}{2}n + \frac{1}{2}}, \quad \dots \quad (5)$$

from which (1) follows ($g=1$). In this case the series is convergent for all finite values of x and t .

It may be of interest to examine the application to aerial waves for which the velocity of propagation (σ/k) is constant. Here $m=1$, and (4) vanishes for all (integral) values of n .

* Proc. Lond. Math. Soc. ser. 2, vol. ii. p. 373 (1904); Hydrodynamics, § 236.

Hence all the coefficients in the expansion (3) of η in powers of t vanish, and the general conclusion that the elevation commences everywhere without delay fails, as we know it ought to do. At the same time the special character of the failing case becomes apparent. But we have not yet examined the convergency of the series.

Apart from signs and from the cosine factor in (4), the ratio of the term in t^{2n+2} to that in t^{2n} is

$$\frac{t^2}{(2n+1)(2n+2)x^{2m}} \frac{\Gamma(2mn+2m+1)}{\Gamma(2mn+1)} \dots \quad (6)$$

To find the limit of (6) as n becomes infinite, we may use the formula

$$\Gamma(n+1) = e^{-n} n^{n+\frac{1}{2}} \sqrt{(2\pi)}.$$

The second fraction in (6) has thus the limiting value

$$(2m)^{2m}(n+1)^{2m},$$

so that (6) becomes ultimately

$$-\frac{t^2}{4x^{2m}} (2m)^{2m}(n+1)^{2m-2} \dots \quad (7)$$

The series is thus certainly convergent if m be less than unity. If $m=1$, (7) reduces to t^2/x^2 , and convergence is not assured unless $x > t$. This marks the moment at which the wave propagated with velocity 1 reaches the point x .

If we were to make $m=2$, corresponding to the propagation of flexural disturbances along a bar, the cosines in (4) would all vanish, but the lack of convergency indicated by (7) prohibits the conclusion that the disturbance undergoes a finite delay in reaching the point x . From Fourier's special solution* we may see that there is in fact no such delay.

It would be of great interest to examine the influence on water waves due to a limitation of depth (h), but a complete solution for this case analogous to (1) seems hardly practicable. But without much difficulty we may obtain the first term in the series, viz., the term proportional to t^2 .

In general from (2)

$$\frac{d\eta}{dt} = -\frac{1}{\pi} \int_0^\infty \sigma \sin \sigma t \cos kx \, dk,$$

* 'Theory of Sound,' § 192.

which vanishes when $t=0$, and

$$\frac{d^2\eta}{dt^2} = -\frac{1}{\pi} \int_0^\infty \sigma^2 \cos \sigma t \cos kx \, dk,$$

so that for $t=0$

$$\frac{d^2\eta}{dt^2} = -\frac{1}{\pi} \int_0^\infty \sigma^2 \cos kx \, dk. \quad . \quad . \quad . \quad (8)$$

For waves on water of depth h

$$\sigma^2 = gk \tanh kh, \quad . \quad . \quad . \quad (9)$$

passing when k is great (λ small) into the simple deep water formula $\sigma^2 = gk$, and when k is small (λ great) into the shallow water formula $\sigma^2 = gh k^2$, indicating a uniform velocity of propagation*.

When $a < \pi$, we have†

$$\int_0^\infty \frac{e^{ax} - e^{-ax}}{e^{\pi x} + e^{-\pi x}} \sin cx \, dx = \frac{(e^{\frac{1}{2}c} - e^{-\frac{1}{2}c}) \sin \frac{1}{2}a}{e^c + e^{-c} + 2 \cos a}.$$

If we differentiate this with respect to c and then make $a = \pi$, to which there seems to be no objection on the understanding postulated,

$$\int_0^\infty \frac{e^{\pi x} - e^{-\pi x}}{e^{\pi x} + e^{-\pi x}} x \cos cx \, dx = -\frac{\frac{1}{2}(e^{\frac{1}{2}c} + e^{-\frac{1}{2}c})}{(e^{\frac{1}{2}c} - e^{-\frac{1}{2}c})^2}. \quad (10)$$

Adapting (10) to our present purpose, we have

$$\int_0^\infty \sigma^2 \cos kx \, dk = -\frac{g\pi^2}{2h^2} \frac{e^{\pi x/2h} + e^{-\pi x/2h}}{(e^{\pi x/2h} - e^{-\pi x/2h})^2}, \quad . \quad (11)$$

whence $d^2\eta/dt_0^2$ is given by (8). The first term in the expression for η is accordingly

$$\eta = \frac{g\pi t^2 \cosh(\pi x/2h)}{8h^2 \sinh^2(\pi x/2h)}. \quad . \quad . \quad . \quad (12)$$

We learn from (12) that, no more than in the case of deep water, is there any delay in the commencement of disturbance at a finite distance x from the source, and the extension is not without importance, seeing that in truth water cannot

* It may be remarked that a modified formula, viz.

$$\sigma^2 = gk(1 - e^{-kh}),$$

agreeing with (9) in the extreme cases, would be more amenable to calculation.

† De Morgan's 'Differential Calculus,' p. 669.

be very deep in relation to *all* the wave-lengths concerned, since among these infinite wave-lengths are included. In the present case all the wave-velocities of simple trains are finite, and thus the sudden propagation to a distance is independent of an interpretation earlier suggested.

If in (12) h is small compared with x , we may write

$$\eta = \frac{g\pi t^2}{4h^2} e^{-\pi x/2h}, \quad . \quad . \quad . \quad . \quad (13)$$

showing that when x is relatively great the elevation in its earlier stages, though finite, is on a greatly diminished scale.

As regards the general question, I do not think that the instantaneous propagation of disturbance should be considered paradoxical. It is to be remembered that in (9) the water is treated as incompressible, *i. e.* that the velocity of propagation of waves of *expansion* is regarded as infinite.

A more complete treatment of the case of finite depth on the basis of (2) and (9) would be instructive, even if limited to selected values of the ratio $x:h$. So far we have considered only the first stage. When h is small compared with x , an important part of the disturbance arrives under the law applicable to shallow water, *viz.*

$$\sigma = \sqrt{(gh)} \cdot k(1 - \frac{1}{6}k^2h^2). \quad . \quad . \quad . \quad (14)$$

Writing (2) in the form

$$\eta = \frac{1}{2\pi} \int_0^\infty \cos(\sigma t - kx) dk + \frac{1}{2\pi} \int_0^\infty \cos(\sigma t + kx) dk, \quad (15)$$

we see that the first integral acquires a specially enhanced value when x and t are so related that $\sigma t - kx$ is approximately zero for small values of k . The condition is of course

$$t = x/\sqrt{(gh)}, \quad . \quad . \quad . \quad . \quad (16)$$

and the important part of the integral will be

$$\eta = \frac{1}{2\pi} \int_0^\infty \cos(\frac{1}{6}xh^2k^3) dk = \frac{1}{6\pi} \cos \frac{\pi}{6} \cdot \Gamma(\frac{1}{3}) \cdot \left(\frac{xh^2}{6}\right)^{-\frac{1}{3}}, \quad (17)$$

large when h is relatively small, but still finite.

For larger values of t the most important part of the first integral of (15) occurs in the neighbourhood of such values of k as make $\sigma t - kx$ stationary. This happens when k has a value k_0 such that

$$x - t d\sigma/dk_0 = 0. \quad . \quad . \quad . \quad . \quad (18)$$

As Kelvin has shown*, the first integral of (15) takes approximately the form

$$\eta = \frac{\cos(\sigma_0 t - k_0 x - \frac{1}{4}\pi)}{\sqrt{\{-2\pi t d^2\sigma/dk_0^2\}}}, \quad . \quad . \quad . \quad (19)$$

while the second integral is relatively negligible. For example in the case of deep water waves, where $\sigma = \sqrt{(gk)}$, (18) gives

$$k_0 = gt^2/4x, \quad \sigma_0 = gt/2x,$$

so that

$$\sigma_0 t - k_0 x - \frac{1}{4}\pi = gt^2/4x - \frac{1}{4}\pi;$$

and finally

$$\eta = \frac{g^{\frac{1}{2}}t}{2\pi^{\frac{1}{2}}x^{\frac{3}{2}}} \cos\left\{\frac{gt^2}{4x} - \frac{1}{4}\pi\right\}. \quad . \quad . \quad . \quad (20)$$

If we attempt to fill up the gaps in our solution by applying quadratures to (2), we have to face the difficulty that, as written, the integral is not convergent. Some analytical transformation is called for. One way out of the difficulty might be to calculate the *difference* between the solutions for finite and infinite depth, for it would appear that the non-convergent part of the integral, corresponding to infinitely small wave-lengths, must be the same for both.

Terling Place, Witham,
May 4, 1909.

II. On the Relation of Airy's Integral to the Bessel Functions.

By J. W. NICHOLSON, M.A., D.Sc., Isaac Newton Student in the University of Cambridge†.

IN an earlier paper‡ the Author has indicated a relation between the integral tabulated by Airy in the course of an investigation of the intensity of light near a caustic, and the Bessel functions of high order. This relation is approximate, and is valid only when the argument of the functions is large. Its main utility depends upon the fact that Stokes has given extensive tables for the integral, which may be quickly transformed into tables of the Bessel functions of high order. But there exists also an exact mathematical connexion, which will now be developed. The integral may be expressed in terms of the Bessel functions of order $\frac{1}{3}$, and certain associated integrals possess the same property.

* Proc. Roy. Soc. vol. xlii. p. 80 (1887).

† Communicated by the Author.

‡ Phil. Mag. Aug. 1908.

When these functions of fractional order are tabulated for a small argument, the results may be readily applied to the rapid tabulation of all Bessel functions of very high order, if the order and argument do not differ widely.

Airy's integral may be defined by the formula

$$f_1(\rho) = \int_0^\infty dw \cdot \cos(w^3 + \rho w), \quad . \quad . \quad . \quad (1)$$

and it will be convenient to define also two associated integrals in the forms

$$f_2(\rho) = \int_0^\infty dw \cdot \sin(w^3 + \rho w) \quad . \quad . \quad . \quad (2)$$

and

$$f_3(\rho) = \int_0^\infty dw \cdot e^{-w^3 + \rho w} \quad . \quad . \quad . \quad (3)$$

Stokes has studied the properties of $f_1(\rho)$ in detail, and more particularly its asymptotic expansion when ρ is not small. He has also given a differential equation satisfied by the integral, but appears to have overlooked a transformation reducing it to a form which was already well known.

If

$$u = \int_0^\infty dw \cdot e^{-w^3 - \iota \rho w} = f_1 - \iota f_2,$$

it may be shown at once that

$$u'' - \frac{1}{3}\rho u = \frac{1}{3}\iota$$

where the accent denotes differentiation with respect to ρ .

Accordingly, isolating the real and imaginary portions,

$$f_1'' - \frac{1}{3}\rho f_1 = 0, \quad . \quad . \quad . \quad (4)$$

$$f_2'' - \frac{1}{3}\rho f_2 = -\frac{1}{3}. \quad . \quad . \quad . \quad (5)$$

Moreover,

$$\begin{aligned} f_3'' - \frac{1}{3}\rho f_3 &= \int_0^\infty (w^2 - \frac{1}{3}\rho) dw e^{-w^3 + \rho w} \\ &= -\frac{1}{3} \left[e^{-w^3 + \rho w} \right]_0^\infty = \frac{1}{3}, \end{aligned}$$

so that

$$(f_2 + f_3)'' - \frac{1}{3}\rho(f_2 + f_3) = 0; \quad . \quad . \quad . \quad (6)$$

and $f_2 + f_3$ therefore satisfies the same differential equation as f_1 . If f denote f_1 or $f_2 + f_3$,

$$f'' - \frac{1}{3}\rho f = 0.$$

Writing $f = \rho^{-\frac{1}{3}}F$,

then on reduction

$$F'' - \frac{F'}{2\rho} + \left(\frac{5}{16\rho^2} - \frac{1}{3}\rho \right) F = 0.$$

Taking a new independent variable y defined by $\rho^3 = y^2$, after further reduction,

$$\frac{d^2F}{dy^2} + \left(\frac{5}{36y^2} - \frac{4}{27} \right) F = 0.$$

This is identical with the normal form of the equation for the Bessel function $I_n(z)$, provided that

$$\frac{5}{36} = \frac{1}{4} - n^2 \quad \text{or} \quad n = \frac{1}{3}.$$

A solution of the original equation is therefore

$$f = y^{\frac{1}{3}} I_{\frac{1}{3}} \left(\frac{2y}{3\sqrt{3}} \right). \quad \dots \dots (7)$$

In this formula, I_n denotes the function

$$I_n(x) = \frac{x^n}{2^n \Gamma(n+1)} \left\{ 1 + \frac{x^2}{2^2 1! n+1} + \frac{x^4}{2^4 2! (n+1)(n+2)} + \dots \right\}, \quad (8)$$

where, if $J_n(x)$ be the usual Bessel function of order n ,

$$I_n(x) = i^{-n} J_n(ix). \quad \dots \dots (9)$$

If $I_{-n}(x)$ denote the corresponding function with the sign of n changed throughout, f_1 must be expressible in the form

$$f_1 = \rho^{\frac{1}{3}} \left\{ A I_{\frac{1}{3}} \left(\frac{2\rho}{3} \sqrt{\frac{\rho}{3}} \right) + B I_{-\frac{1}{3}} \left(\frac{2\rho}{3} \sqrt{\frac{\rho}{3}} \right) \right\}.$$

Similarly, writing $\rho = -\sigma$,

$$\begin{aligned} \int_0^\infty dw \cdot \cos(w^3 - \sigma w) \\ = \sigma^{\frac{1}{2}} \left\{ A J_{\frac{1}{3}} \left(\frac{2\rho}{3} \sqrt{\frac{\rho}{3}} \right) + B J_{-\frac{1}{3}} \left(\frac{2\rho}{3} \sqrt{\frac{\rho}{3}} \right) \right\}, \end{aligned}$$

A and B may be at once determined. When $\sigma = 0$,

expanding the Bessel functions, we obtain

$$B \cdot \left(\frac{2}{3}\right)^{-\frac{1}{3}} 3^{\frac{1}{6}} 2^{\frac{1}{3}} / \Gamma\left(\frac{2}{3}\right) = \int_0^\infty \cos w^3 dw = \frac{1}{2\sqrt{3}} \Gamma\left(\frac{1}{3}\right),$$

or

$$B = \frac{1}{6} \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{2}{3}\right) = \frac{\pi}{3\sqrt{3}}$$

by a property of the Gamma functions.

Again,

$$\begin{aligned} A \cdot \left(\frac{2}{3}\right)^{\frac{1}{3}} 3^{-\frac{1}{6}} 2^{-\frac{1}{3}} / \Gamma\left(\frac{4}{3}\right) \\ = \frac{d}{d\rho} \int_0^\infty dw \cos(w^3 - \rho w) \quad \text{with } \rho = 0; \\ = \int_0^\infty w \sin w^3 dw = \frac{1}{2\sqrt{3}} \Gamma\left(\frac{2}{3}\right), \end{aligned}$$

leading to

$$A = \frac{\pi}{3\sqrt{3}};$$

and finally, when σ is positive,

$$\begin{aligned} \int_0^\infty dw \cdot \cos(w^3 - \sigma w) \\ = \frac{\pi}{3} \sqrt{\frac{\sigma}{3}} \left\{ J_{\frac{1}{3}}\left(\frac{2\sigma}{3} \sqrt{\frac{\sigma}{3}}\right) + J_{-\frac{1}{3}}\left(\frac{2\sigma}{3} \sqrt{\frac{\sigma}{3}}\right) \right\}. \quad (10) \end{aligned}$$

Moreover, when ρ is positive,

$$\begin{aligned} \int_0^\infty dw \cos(w^3 + \rho w) \\ = \frac{\pi}{3} \sqrt{\frac{\rho}{3}} \left\{ I_{\frac{1}{3}}\left(\frac{2\rho}{3} \sqrt{\frac{\rho}{3}}\right) - I_{-\frac{1}{3}}\left(\frac{2\rho}{3} \sqrt{\frac{\rho}{3}}\right) \right\}. \quad (11) \end{aligned}$$

By the property of $f_2 + f_3$ indicated above, there exists also a relation of the form

$$\begin{aligned} \int_0^\infty dw e^{-w^3 - \sigma w} + \int_0^\infty dw \sin(w^3 - \sigma w) \\ = C\sigma^{\frac{1}{2}} J_{\frac{1}{3}}\left(\frac{2\sigma}{3} \sqrt{\frac{\sigma}{3}}\right) + D\sigma^{\frac{1}{2}} J_{-\frac{1}{3}}\left(\frac{2\sigma}{3} \sqrt{\frac{\sigma}{3}}\right), \end{aligned}$$

where

$$\begin{aligned} D &= 3^{-\frac{1}{3}} \Gamma\left(\frac{2}{3}\right) \int_0^{\infty} (\sin w^3 + e^{-w^3}) dw \\ &= 3^{-\frac{2}{3}} \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{2}{3}\right) \left(1 + \sin \frac{\pi}{6}\right) = \frac{\pi}{3}, \end{aligned}$$

and

$$\begin{aligned} C &= -3^{-\frac{1}{3}} \Gamma\left(\frac{1}{3}\right) \int_0^{\infty} w(\cos w^3 + e^{-w^3}) dw \\ &= -3^{-\frac{2}{3}} \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{2}{3}\right) \left(1 + \cos \frac{\pi}{3}\right) = -\frac{\pi}{3} \end{aligned}$$

so that, when σ is positive,

$$\begin{aligned} \int_0^{\infty} dw \cdot \{\sin(w^3 - \sigma w) + e^{-w^3 - \sigma w}\} \\ = \frac{\pi \sqrt{\sigma}}{3} \left\{ J_{-\frac{1}{3}}\left(\frac{2\sigma}{3} \sqrt{\frac{\sigma}{3}}\right) - J_{\frac{1}{3}}\left(\frac{2\sigma}{3} \sqrt{\frac{\sigma}{3}}\right) \right\}; \quad (12) \end{aligned}$$

with the associated equation, when ρ is positive,

$$\begin{aligned} \int_0^{\infty} dw \cdot \{\sin(w^3 + \rho w) + e^{-w^3 + \rho w}\} \\ = -\frac{\pi}{3} \sqrt{\rho} \left\{ I_{\frac{1}{3}}\left(\frac{2\rho}{3} \sqrt{\frac{\rho}{3}}\right) + I_{-\frac{1}{3}}\left(\frac{2\rho}{3} \sqrt{\frac{\rho}{3}}\right) \right\}. \quad (13) \end{aligned}$$

Writing

$$\frac{2\sigma}{3} \sqrt{\frac{\sigma}{3}} = 2\lambda^3$$

so that $\sigma = 3\lambda^2$, and combining (10) and (12), we obtain the curious result

$$J_{\frac{1}{3}}(2\lambda^3) = \frac{3^{\frac{1}{3}}}{2\pi\lambda} \int_0^{\infty} dw \{ 3^{\frac{1}{3}} \cos(w^3 - 3\lambda^2 w) - \sin(w^3 - 3\lambda^2 w) - e^{-w^3 - 3\lambda^2 w} \},$$

or finally

$$J_{\frac{1}{3}}(2\lambda^3) = \frac{3^{\frac{1}{3}}}{2\pi\lambda} \int_0^{\infty} dw \{ 2 \cos(w^3 - 3\lambda^2 w + \frac{1}{6}\pi) - e^{-w^3 - 3\lambda^2 w} \}. \quad (14)$$

In a similar manner,

$$J_{-\frac{1}{3}}(2\lambda^3) = \frac{3^{\frac{1}{3}}}{2\pi\lambda} \int_0^{\infty} dw \{ 2 \cos(w^3 - 3\lambda^2 w - \frac{1}{6}\pi) + e^{-w^3 - 3\lambda^2 w} \}, \quad (15)$$

$$I_{\frac{1}{3}}(2\lambda^3) = \frac{3^{\frac{1}{3}}}{2\pi\lambda} \int_0^{\infty} dw \{ 2 \cos(w^3 + 3\lambda^2 w + \frac{1}{6}\pi) - e^{-w^3 + 3\lambda^2 w} \}, \quad (16)$$

$$I_{-\frac{1}{3}}(2\lambda^3) = -\frac{3^{\frac{1}{3}}}{2\pi\lambda} \int_0^{\infty} dw \{ 2 \cos(w^3 + 3\lambda^2 w - \frac{1}{6}\pi) + e^{-w^3 + 3\lambda^2 w} \}. \quad (17)$$

The Evaluation of some Definite Integrals.

Some further integrals which present themselves in connexion with the Bessel functions of order $\frac{1}{3}$ will now be considered. The symbols $J_{\frac{1}{3}}$, $J_{-\frac{1}{3}}$, $I_{\frac{1}{3}}$, $I_{-\frac{1}{3}}$ will be used for the corresponding Bessel functions of argument $\frac{2\sigma}{3}\sqrt{\frac{\sigma}{3}}$.

Writing

$$u + v = \int_0^\infty dw \cdot e^{uw^3 - \sigma w},$$

then if accents denote differentiations with respect to σ ,

$$u'' - v'' = -\frac{1}{3}u - \frac{1}{3}v\sigma(u + v),$$

so that

$$u'' + \frac{1}{3}\sigma v = 0,$$

$$v'' + \frac{1}{3}\sigma u = -\frac{1}{3};$$

whence

$$(u + v)'' + \frac{1}{3}\sigma(u + v) = -\frac{1}{3},$$

$$(u - v)'' - \frac{1}{3}\sigma(u - v) = \frac{1}{3}.$$

Treating the second equation first, we note that a particular integral of

$$z'' - \frac{1}{3}\sigma z = \frac{1}{3}$$

is given by

$$z = \int_0^\infty dw \cdot e^{-w^3 + \sigma w}.$$

Adding the complementary function, it appears that

$$u - v - z = A\sigma^{\frac{1}{3}}I_{\frac{1}{3}} + B\sigma^{\frac{1}{3}}I_{-\frac{1}{3}}.$$

Similarly, if

$$y = \int_0^\infty dw \cdot e^{-w^3 - \sigma w},$$

Then

$$u + v + y = C\sigma^{\frac{1}{3}}J_{\frac{1}{3}} + D\sigma^{\frac{1}{3}}J_{-\frac{1}{3}}.$$

It is found at once that

$$B = -\frac{2\pi}{6\sqrt{3}}(\sqrt{3}-1) = -A,$$

$$D = \frac{\pi}{3\sqrt{3}}(\sqrt{3}+1) = -C,$$

whence

$$\int_0^\infty dw(\cos w^3 + \sin w^3 + e^{-w^3})e^{-\sigma w} \\ = \frac{\sigma^{\frac{1}{2}}\pi}{3\sqrt{3}}(\sqrt{3}+1)\{J_{-\frac{1}{3}}-J_{\frac{1}{3}}\} \quad . \quad . \quad (18)$$

and

$$\int_0^\infty dw(\cos w^3 - \sin w^3)e^{-\sigma w} - \int_0^\infty dw \cdot e^{-w^3 + \sigma w} \\ = -\frac{\pi\sigma^{\frac{1}{2}}}{3\sqrt{3}}(\sqrt{3}-1)\{I_{\frac{1}{3}}+I_{-\frac{1}{3}}\}. \quad . \quad . \quad (19)$$

By addition and subtraction, we may express the integrals

$$\left. \begin{aligned} \int_0^\infty dw \cdot (\cos w^3 \cdot e^{-\sigma w} - \sinh \sigma w \cdot e^{-w^3}) \\ \int_0^\infty dw(\sin w^3 e^{-\sigma w} + \cosh \sigma w \cdot e^{-w^3}) \end{aligned} \right\} \quad . \quad (20)$$

Again, another particular integral of

$$x'' + \frac{1}{3}\sigma x = -\frac{1}{3}$$

is

$$x = \int_0^\infty dw \sin(w^3 - \sigma w).$$

With the aid of this, we may express the two integrals,

$$\left. \begin{aligned} \int_0^\infty dw (e^{-\sigma w} - \sin \sigma w) \cos w^3 \\ \int_0^\infty dw (e^{-\sigma w} \cos \sigma w) \sin w^3 \end{aligned} \right\} \quad . \quad . \quad . \quad (21)$$

It may be shown that no other integrals of the linear-cubic type can be so expressed unless they are derived by the processes of addition and subtraction from those already given.

Asymptotic expansions.

In a previous paper,* the author has shown that when $n-z$ is not very great in comparison with $z^{\frac{1}{3}}$, which is itself

* Phil. Mag. August 1908.

not small,

$$J_n(z) = \frac{1}{\pi} \left(\frac{6}{z} \right)^{\frac{1}{3}} f_1(\rho) \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$J_{-n}(z) = \frac{1}{\pi} \left(\frac{6}{z} \right)^{\frac{1}{3}} \{ f_1(\rho) \cos n\pi + f_2(\rho) \sin n\pi \} \\ + \frac{1}{\pi} \left(\frac{6}{z} \right)^{\frac{1}{3}} e^{i\pi/6} \sin n\pi \left\{ e^{\frac{i\pi}{3}} f_1(\rho e^{-\frac{i\pi}{3}}) + e^{-\frac{i\pi}{3}} f_2(\rho e^{-\frac{i\pi}{3}}) \right\} \quad (23)$$

and if n be an integer,

$$Y_n(z) = - \left(\frac{6}{z} \right)^{\frac{1}{3}} \left\{ e^{\frac{i\pi}{3}} f_1(\rho e^{-\frac{i\pi}{3}}) + e^{-\frac{i\pi}{3}} f_2(\rho e^{-\frac{i\pi}{3}}) \right\}. \quad (24)$$

The functions f_1, f_2 have the same significance as before, f_1 being an Airy's integral of argument ρ .

$Y_n(z)$ is Hankel's function defined by

$$Y_n(z) = -\pi \left\{ \frac{J_{-n} - \cos n\pi J_n}{\sin n\pi} \right\}_{n=\text{integer}}. \quad . \quad . \quad . \quad (25)$$

or, in series form,

$$Y_n(z) = 2J_n(z) \{ \log \frac{1}{2} z + \cdot 557 \} - \left(\frac{2}{z} \right)^n \left\{ n-1! + \frac{n-2!}{1!} \left(\frac{z}{2} \right)^2 + \dots \right\} \\ - \left(\frac{z}{2} \right)^n \left\{ \frac{S_n}{n!} - \frac{1}{1! n+1!} \left(\frac{1}{1} + S_{n+1} \right) \left(\frac{z}{2} \right)^4 + \dots \right\}, \quad (26)$$

where

$$S_n = 1 + \frac{1}{2} + \dots + \frac{1}{n}. \quad . \quad . \quad . \quad . \quad (27)$$

An alternative expression for $J_{-n}(z)$ is

$$J_{-n}(z) = \frac{1}{\pi} \left(\frac{6}{z} \right)^{\frac{1}{3}} \{ f_1(\rho) \cos n\pi + f_2(\rho) \sin n\pi \} + \left(\frac{6}{z} \right)^{\frac{1}{3}} \frac{\sin n\pi}{\pi} \int_0^\infty dw e^{-w^3 + \rho w} \quad (28)$$

where the latter integral has already been denoted by $f_3(\rho)$.

In all these results ρ stands for

$$\frac{1}{\pi} \left(\frac{6}{z} \right)^{\frac{1}{3}} (n-z) \quad . \quad . \quad . \quad . \quad . \quad (29)$$

and is positive if n is greater than z .

These expressions are asymptotic, but may be verified to three places of decimals when $n=z=8$.

When n differs from z , it is necessary for ρ to be very small in comparison with $z^{2/3}$, the error then introduced being

of magnitude $(n-z)/z$. It follows that all the higher Bessel functions whose order and argument do not differ widely may be asymptotically expressed by means of similar functions of order $\frac{1}{3}$.

In the first place, z may be supposed greater than n . Thus ρ is negative, and

$$f_1(\rho) = \frac{\pi}{3} \sqrt{\frac{\sigma}{3}} \left\{ J_{\frac{1}{3}} + J_{-\frac{1}{3}} \right\}$$

where $\sigma = -\rho$, and the functions J have argument

$$\frac{2\sigma}{3} \sqrt{\frac{\sigma}{3}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Moreover,

$$f_2(\rho) + f_3(\rho) = \frac{\pi}{3} \sqrt{\sigma} \left\{ J_{-\frac{1}{3}} - J_{\frac{1}{3}} \right\},$$

and therefore on reduction

$$J_n(z) = \frac{1}{3} \sqrt{\left(\frac{2}{z} \cdot z - n\right)} \left\{ J_{\frac{1}{3}} + J_{-\frac{1}{3}} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

$$J_{-n}(z) = \frac{2}{3} \sqrt{\left(\frac{2}{z} \cdot z - n\right)} \left\{ J_{\frac{1}{3}} \cos(n\pi + \frac{1}{3}\pi) + J_{-\frac{1}{3}} \cos(n\pi - \frac{1}{3}\pi) \right\}. \quad (32)$$

Similarly, or direct from the definition, in the case of n integral,

$$Y_n(z) = \pi \sqrt{\left(\frac{2}{3z} \cdot z - n\right)} \left\{ J_{-\frac{1}{3}} - J_{\frac{1}{3}} \right\} \quad . \quad . \quad . \quad (33)$$

Another function of great importance in physical theory may be defined by

$$K_n(z) = \frac{\pi}{2 \sin n\pi} e^{-in\frac{\pi}{2}} \left\{ J_{-n}(z) - e^{in\pi} J_n(z) \right\} \quad . \quad . \quad (34)$$

Many other definitions have been proposed, but this, which is due to Macdonald*, is the most convenient for practical applications.

When n is an integer,

$$\begin{aligned} K_n(z) &= \frac{\pi}{2 \sin n\pi} e^{-in\frac{\pi}{2}} \left\{ J_{-n}(z) - \cos n\pi J_n(z) \right\} - \frac{1}{2} i\pi J_n(z) e^{-in\frac{\pi}{2}} \\ &= -\frac{1}{2} \left(Y_n(z) + i\pi J_n(z) \right) e^{-in\frac{\pi}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (35) \end{aligned}$$

evaluating the undetermined form.

$K_n(z)$ is the type of function occurring in all harmonic

* Proc. Lond. Math. Soc. vol. 32.

expressions of disturbances in wave theory originating from a body and passing off into space.

It may be at once shown that

$$K_n(z) = \frac{1}{2} \left(\frac{6}{z} \right)^{\frac{1}{3}} e^{-\frac{n\pi}{2}} \left\{ f_2 + f_3 - \iota f_1 \right\} \quad . \quad . \quad . \quad (36)$$

where the functions f have the usual argument, if z and n do not differ widely in comparison with $z^{\frac{1}{3}}$. Therefore, if the Bessel functions of order $\frac{1}{3}$ have the appropriate argument σ , whether n be integral or not,

$$K_n(z) = \frac{1}{2} \left(\frac{6}{z} \right)^{\frac{1}{3}} e^{-\frac{n\pi}{2}} \frac{\pi}{3} \sqrt{\frac{\sigma}{3}} \left\{ \sqrt{3} J_{-\frac{1}{3}} - \sqrt{3} J_{\frac{1}{3}} - \iota J_{\frac{1}{3}} - \iota J_{-\frac{1}{3}} \right\}$$

which may be written

$$K_n(z) = \left(\frac{6}{z} \right)^{\frac{1}{3}} \frac{\pi}{3} \sqrt{\frac{\sigma}{3}} \left\{ e^{-\frac{\iota\pi}{6}} J_{-\frac{1}{3}} - e^{\frac{\iota\pi}{6}} J_{\frac{1}{3}} \right\} \quad . \quad . \quad . \quad (37)$$

But by the selected definition, for any argument

$$K_{\frac{1}{3}} = \frac{\pi}{\sqrt{3}} e^{-\iota\pi/6} \left(J_{-\frac{1}{3}} - e^{\iota\pi/3} J_{\frac{1}{3}} \right)$$

and thus when n and z are large, and $n-z$ not very large in comparison with $z^{\frac{1}{3}}$,

$$K_n(z) = \frac{1}{3} \left(\frac{6}{z} \right)^{\frac{1}{3}} \left(z - n \right)^{\frac{1}{2}} K_{\frac{1}{3}} \left(\frac{2}{3} \cdot n - z \cdot \sqrt{\frac{2}{z} \cdot n - z} \right) \quad . \quad . \quad (38)$$

if z be greater than n .

The modification necessary in this result, as in (31-33), when z is less than n , may be obtained at once. The accuracy of all the formulæ is as stated above.

The following tables have been calculated from the results given by Airy, with the aid of the formula

$$J_n(z) = \frac{1}{\pi} \left(\frac{6}{z} \right)^{\frac{1}{3}} f_1(\rho).$$

Airy employs the integral in the form

$$I(m) = \int_0^\infty dw \cos \frac{\pi}{2} (w^3 - mw) \quad . \quad . \quad (39)$$

which is identical with

$$I(m) = \left(\frac{2}{\pi} \right)^{\frac{1}{3}} f_1 \left\{ -m \left(\frac{2}{\pi} \right)^{\frac{1}{3}} \right\} \quad . \quad . \quad . \quad (40)$$

so that

$$z^{\frac{1}{3}} J_n(z) = \left(\frac{3}{\pi^2} \right)^{\frac{1}{3}} I \left\{ z - n \cdot \left(\frac{3\pi}{z} \right)^{\frac{1}{3}} \right\} \quad . \quad . \quad . \quad (41)$$

TABLE I.— n greater than z .

$\frac{n-z}{z^{\frac{1}{3}}} \times 2.1123.$	$z^{\frac{1}{3}} J_n(z).$	$\frac{n-z}{z^{\frac{1}{3}}} \times 2.1123.$	$z^{\frac{1}{3}} J_n(z).$
·2	·38666	2.2	·04007
·4	·32849	2.4	·02987
·6	·27459	2.6	·02203
·8	·24786	2.8	·01609
1.0	·18344	3.0	·01163
1.2	·14684	3.2	·00833
1.4	·11601	3.4	·00591
1.6	·09050	3.6	·00416
1.8	·06977	3.8	·00290
2.0	·05317	4.0	·00200

TABLE II.— z greater than n .

$\frac{z-n}{z^{\frac{1}{3}}} \times 2.1123.$	$z^{\frac{1}{3}} J_n(z).$	$\frac{z-n}{z^{\frac{1}{3}}} \times 2.1123.$	$z^{\frac{1}{3}} J_n(z).$
·0	·44549	2.2	·23779
·2	·50789	2.4	·07882
·4	·56506	2.6	·08616
·6	·61474	2.8	·24364
·8	·65228	3.0	·37869
1.0	·67264	3.2	·47653
1.2	·67093	3.4	·52457
1.4	·64282	3.6	·51447
1.6	·58528	3.8	·44412
1.8	·49714	4.0	·31901
2.0	·37982	—	—

Tables for shorter intervals may be constructed by interpolation. The first table exhibits the rapid convergence of $J_n(z)$ towards zero when n increases beyond z , even when z is very great. This convergence soon becomes exponential. A formula suitable for this case has been given by the author*.

The limitations to be applied to these tables are well defined. When z and n are exactly equal, the error in the value of $J_n(z)$ is ·0007 when $z=8$, and ·0006 when $z=20$. It is, in fact, fairly constant so far as tables have hitherto been constructed†. It may be shown to affect only the fifth significant figure when $n=100$, from theoretical considerations. Other portions of the tables have a more restricted application.

* Brit. Assoc. Report, Dublin, 1908.

† To $n=24$. *Vide* Gray & Matthews' Treatise.

They involve an error of magnitude comparable with $(n-z)/z$ at most. Thus if $z=20$, $n=18$, even the first figure may be wrong. Their utility is confined to the very high values. For example, they will yield J_{1005} (1000) to two significant figures, and probably more, as $(n-z)/z$ is actually larger than the involved error.

Trinity College, Cambridge.

III. *Theories of Matter and Mass.* By LOUIS T. MORE, Ph.D.,
Professor of Physics, University of Cincinnati*.

IT is perhaps reasonable to associate the name of Newton with speculations regarding the nature of matter and the fundamental principles of mechanics, and those who attempt to dispense with the axiom that mass or inertia is an inherent and unexplainable attribute of matter, are apt to call their systems non-Newtonian mechanics. In fact, the problem is far older than Newton, and goes back as a persistent dualism to at least the time of the Greek philosophers, who recognized that cosmic theories could be built up on the axiom of the objective reality of matter, or on the theory that the external world is but a subjective idea mirrored by our sensations. Since Newton's formulation of the laws of gravitation and motion scientific thought has, for the most part, squarely placed itself on the side of the objective reality of matter, and associated with it an inalienable property, inertia or mass. To be sure, Newton himself, in letters and by queries in his treatises, ventures certain metaphysical guesses as to the causes of inertia and of the force of gravitation. But when he discusses mechanics scientifically he puts such speculations resolutely aside with the remark, *hypotheses non fingo*, undoubtedly feeling that science begins with the simple assumption of some such axioms, and has no concern with their causes.

With the discovery of the law of conservation of energy, another apparently invariable property of matter has become available, on which to base theory. As the scientist accepts without hesitation the reality and continuity of space and time, it is natural that a controversy should arise as to which of the two properties of matter, mass or energy, may serve best to make concrete to us the abstractions space and time†.

* Communicated by the Author.

† The only attempt I am acquainted with, to express time as a function of space, has been made by Bergson in his "Essai sur les données immédiates de la conscience."

As both attributes, mass and energy, are always associated with matter, and as matter apparently is revealed to us only through them, it is possible to take either one as the fundamental attribute, the "Ding an sich" of Kant. Thus the postulate of inertia is the basis of an atomic or corpuscular theory, while energy serves in the same capacity for the science of energetics. But the attempt to explain matter and energy has led to the substitution of vortex rings, strains or electric charges in an æther for the hard atom, and has refined the concept of energy to a mathematical symbol. These methods sooner or later lead to purely metaphysical discussions, expressed by mathematical equations in place of scholastic logic, in which a hypothetical universe is substituted for sensible matter. When we assert that matter is an attribute of the æther or a complex of centres of energy, we explain nothing as we merely endow a more tenuous hypothetical fluid with all the properties of matter under discussion. For example, to explain the genesis and growth of a crystal we transfer to each constituent molecule all the properties of the complete crystal; an evasion rather than a solution of the problem.

For a century or more science has tried to explain all phenomena as variations of mechanical force, acting according to Newton's law, between atoms and æthers. The attempt has failed, mainly for two reasons, the simple atom proved to be inadequate, and no æther could be imagined which did not contain irreconcilable contradictions when phenomena were to be coördinated. When the step was once taken of considering the atom as a complex system of subatoms, the passage to the converse problem, of explaining mass as a variable function of some more fundamental attribute of matter, has been rapid.

In recent theoretical articles matter is no longer an aggregation of concrete atoms or subatoms, places of more or less differentiation in an æther possessing inertia, but a manifestation of some more fundamental entity, such as a negative charge of electricity. The argument of these writers is essentially as follows: since it may be proved that a moving body of ponderable matter possesses more momentum when it is electrically charged than when uncharged, therefore we may account for not only the inertia and momentum of matter, but also for matter itself, by endowing *something* with an adequate electrical charge and velocity. This conclusion is, at least with our present knowledge, a *non-sequitur*; in the first place we have no experience of the *something*, and second, the requisite velocity may be an unattainable

one. The discussion of these theories is perhaps more readily accomplished by a review of a paper by Dr. G. N. Lewis*.

This paper is an ingenious effort to reconcile modern ideas and Newtonian mechanics. In it the writer attempts two distinct things: first, to establish a quasi-corpuscular theory of light, and second, to explain inertia, wholly or in part, as a function of velocity.

We may pass over the difficulties all corpuscular theories of light plunge us into when such phenomena as interference, polarization, diffraction, &c. are discussed, as they are not touched upon. The fallacy of his hypothesis lies in his cardinal assumption "That a beam of radiation possesses not only momentum and energy, but also mass, travelling with the velocity of light, and that a body absorbing radiation is acquiring this mass as it also acquires the momentum and energy of the radiation. Therefore a body which absorbs radiant energy increases in mass"†.

In the first place, the assumption made, that because energy is involved in radiation, and because a black body absorbing this energy is moved mechanically, therefore radiation is due to a mass in motion, is not a necessary one. Maxwell proved that such a mechanical action must occur if radiation were due to electromagnetic waves, and Nichols and Hull measured the effect as a property of such waves.

Secondly, the assumption is an arbitrary one, contrary to our experience. As an illustration, if a ball moving toward a man were stopped by his hand he would evidently absorb this energy, and yet no one would claim that the man had increased in mass. Still less would the claim be made if the ball were the mass of a beam of radiation which, according to Dr. Lewis, would have no "mass if it were at rest, or indeed if it were moving with a velocity even by the smallest fraction less than that of light."

The fallacy in the assumption is even more clearly seen in the mathematical deductions, from which I quote:

"The momentum of any part of a beam of radiation having the mass m will be given by the equation

$$M = mV. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The increase dM in the momentum of the body absorbing the radiation will, therefore, equal the increase dm in its

* Lewis, "On a Revision of the Fundamental Laws of Matter and Energy," *Phil. Mag.* vol. xvi. p. 705 (1908).

† *I. c.* p. 707.

mass, multiplied by the velocity of light,

$$dM = V dm. \quad . \quad . \quad . \quad . \quad . \quad (6)''$$

Substituting energy, E , for momentum we obtain

$$"dm = \frac{dE}{V^2} . \quad . \quad . \quad . \quad . \quad (7)''$$

By the pure mathematician any quantity of an equation may be considered either as a constant or as a variable, but such is not the case when the equation is an expression of a physical law. Quantities are then fixed as constants or as variables, according as our experience requires them to be. If in equation (5) the velocity of light is said to be a constant and the "mass of radiation" a variable, some necessity, or at least some plausibility, must be given to what is otherwise an arbitrary assumption contrary to our ideas and experience.

One might equally well transform the equation for potential energy

$$P = m(gh)$$

into the differential

$$dP = (gh)dm,$$

and declare that a change in the potential energy of a body is due to a variation in the mass of a body maintained in a constant position. From this axiom we may deduce the law that mass is a function of the position of a body, just as readily as Dr. Lewis proves it a function of momentum. The only difficulty is that experience teaches differently. Measurements of the velocity of light all show that it decreases when passing into absorbing media, and Nichols and Hull detected a change in position or motion of a body which absorbs light and not a change in its mass. Nor is it made clear how the quantity of m , which in (5) refers to a beam of radiation, later is the mass of the absorbing body.

Again, other questions arise in connexion with this theory. If a body, emitting radiant energy, loses 1.111×10^{-21} gramme for every erg expended, a simple calculation, from the estimate that the sun radiates 6×10^{30} calories (pounds per degree centigrade) in a year, gives a mass of 1.2×10^{20} grammes a year for its loss in mass. Such a quantity should have an influence on cosmic problems.

Another difficulty occurs of a more serious nature. The "something possessing mass" whose momentum is a beam of radiation has no mass while it is a part of the radiating body before it begins its journey of radiation. In the free æther

it attains the velocity of light and acquires the property, mass. Now the mass of a body, according to the summary of the writer at the end of the paper, becomes infinite if it has the velocity of light. To escape this dilemma either light must move with a velocity less than the velocity of light, or else these light-particles must be a new creation of matter, having nothing in common with our ideas of ponderable matter. Nor do their unusual properties end here. If the energy of the radiation is absorbed the particles again cease to be, vanish, and pass into the unknowable. They are ghosts which appear and disappear at the will of the necromancer. Surely we should pause before adopting so fantastic a view.

The second part of the article is an attempt to construct a new system of mechanics. In this system we may retain all the axioms and propositions of mechanics previously held excepting the one which states that the mass of a body is independent of its velocity. For this we should substitute that mass is proportional to content of energy, and therefore changes with velocity. This is not a new idea, or the school of energetics has laboured in vain. In fact any treatise on hydrodynamics arrives at the same conclusion, except that we usually call it *effective* or *apparent* mass. But the deductions from this axiom will repay study as they are typical of a new movement in physics.

On page 711 we find the equation

$$dE' = v dM \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where E' is the kinetic energy, M the momentum, and v the velocity of a moving body.

And as

$$\begin{aligned} M &= mv, \\ dM &= m dv + v dm. \end{aligned}$$

Since the mass as well as the velocity is to be reckoned a variable, we have by substitution

$$dE' = m v dv + v^2 dm. \quad . \quad . \quad . \quad . \quad (14)$$

Introducing the relation of mass to energy given in equation (7) we may write

$$dE' = V^2 dm$$

and combining this equation with (14) gives

$$V^2 dm = m v dv + v^2 dm.$$

Here again we have the unexplained duality of the mass m ,

which in (14) refers to the body, and in the following equation to a beam of radiation. It is also perplexing to understand what (7) has to do with this problem. In a simple discussion of the momentum of a moving body such an attribute as light or radiation is not necessarily involved, nor even the existence of an absorbing medium.

Granting this objection is not valid, we arrive by simple transformations to the following equation

$$m/m_0 = \frac{1}{(1-\beta^2)^{\frac{1}{2}}},$$

where m_0 is the mass of the body at rest, m its mass when in motion, and β the ratio v/V .

We may further obtain the expression for the kinetic energy acquired by a moving body in the forms

$$E' = mV^2\{1 - (1-\beta^2)^{\frac{1}{2}}\}, \quad . \quad . \quad . \quad (16)$$

or

$$E' = mV^2\{\frac{1}{2}\beta^2 + \frac{1}{8}\beta^4 + \dots\}. \quad . \quad . \quad . \quad (17)$$

For small values of v

$$E' = \frac{1}{2}mv^2,$$

and when $v \doteq V$

$$E' = mv^2.$$

The writer states that this difference for the energy "instead of demolishing our theory actually furnishes a remarkably satisfactory argument in its favour." It really does neither, for the ordinary hydrodynamic equation for energy, given later, expresses precisely the same results without a need for the assumption that mass varies with velocity.

If we strip the problem of perplexing details and then compare it with analogous examples taken from other branches of physics, the earlier ideas of inertia should be easier to compare with those of the present time. The problem may be put in the following form:—in all cases where a body is giving or receiving energy its mass is a variable. This interchange of energy may take place between two bodies or between a body and a fluid in which it is immersed. For the latter the fluid must be of such a nature as to be able to absorb and emit energy, and so be endowed with what Newton called inertia. It is evident that if the fluid cannot absorb energy there can be no loss or gain of energy to the body, and consequently no change in its mass.

Does this interchange of energy require a real change in the mass of a body, or does it produce a variation in other

attributes of matter such as velocity or change of position? The fallacy involved is really a question of definition. According to Newton* the inertia of a body is an inherent and inalienable property of it, independent of the influence of any other body or æther, and forms the connecting link between ourselves and the external world. The inertia referred to by Lewis and other modern writers is a relative property of matter, depending entirely upon the presence of other matter, and has nothing to do with the body *per se*. They generally take some form of energy as the fundamental attribute. But since the absolute nature of matter is entirely unknown to us, it seems reasonable to say that we have no criterion by which we can decide between the two, mass and energy.

Undoubtedly those who have maintained the variability of inertia of ponderable matter have been obliged to create some primitive fluid or æther and supply it with inertia in the Newtonian sense. Ponderable matter then becomes merely a place of discontinuity or variation in this primitive fluid, which is to be considered the only actual matter. Now to transfer the essential properties of a sensible body to another, which we have no faculty of detecting directly, is not an explanation, but an evasion of the problem.

In any mechanistic theory of phenomena, based on the mutual attractions between atoms, the relations between mechanics and the other branches of physics are expressed in three ways, by hydrodynamic, thermodynamic, or electrodynamic equations. Because of the contradictions which have occurred when all phenomena are explained by forces of attraction, the converse problem has been attempted of explaining the nature of atoms and their mutual forces, and we have as a result Boscovich's centres of force, vortices, charges of negative electricity, ætherial strains, the "something in a beam of radiation," and others. However diverse in appearance, these all have one property in common, they deal with a transfer of energy, and they either tacitly or openly postulate the existence of invariable inertia in something tangible or intangible.

An examination of any one of these will answer the purpose, and I have chosen one the least complicated by metaphysical abstractions.

If two spheres of radii a and b are immersed in a fluid,

* Princ. Math. Def. III. Hæc (materiæ vis) semper proportionalis est suo corpore, neque differt quicquam ab inertia massæ, nisi in modo concipiendi. Per inertiam materiæ fit, ut corpus omne de statu suo vel quiescendi vel movendi difficulter deterbetur.

and are moving toward each other along their line of centres with velocities u and u' , the momentum of each sphere is no longer its mass times velocity, but the action takes place as if the mass had been increased. The formula for the kinetic energy* is

$$2E' = Lu^2 + 2Mu u' + Nu'^2,$$

where

$$L = 2/3\pi\rho a^3 \left(1 + \frac{3a^3b^3}{c^3f_1^3} + \frac{3a^6b^6}{c^3f_1^3(c-f_2)^3f_3^3} + \dots \right),$$

$$M = 2\pi\rho \frac{a^3b^3}{c^3} \left(1 + \frac{a^3b^3}{f_1^3(c-f_2)^3} + \frac{a^6b^6}{f_1^3f_3^3(c-f_2')^3(c-f_4')^3} + \dots \right),$$

$$N = 2/3\pi\rho b^3 \left(1 + \frac{3a^3b^3}{c^3f_1^3} + \frac{3a^6b^6}{c^3f_1^3(c-f_2')^3f_3'^3} + \dots \right),$$

in which the f 's are functions of a , b , and c only.

If the spheres are small compared with the distance between them

$$L = 2/3\pi\rho a^3 \left(1 + \frac{3a^3b^3}{c^6} \right),$$

$$M = 2\pi\rho \frac{a^3b^3}{c^3},$$

$$N = 2/3\pi\rho b^3 \left(1 + \frac{3a^3b^3}{c^6} \right),$$

approximately.

And, lastly, if there is only one sphere moving in an infinite fluid

$$L = 2/3\pi\rho a^3 = 1/2m \quad . \quad . \quad . \quad . \quad . \quad (A)$$

$$M = N = 0.$$

The quantities L , M , and N are the apparent or virtual increases in the masses of the spheres, and in the simple case of one sphere moving in the fluid reduce to one-half the mass of the sphere. In other words, the introduction of a second moving sphere in an infinite liquid acts as a constraint, increasing the kinetic energy for a given velocity, and so *virtually* increases the inertia of the system. Equation (A) should be a satisfactory answer to Dr. Lewis's surprise that (17) shows an increase of the energy, E' , of a moving body.

If we compare these equations with (17) developed by Dr. Lewis we cannot but be struck with their similarity.

* Lamb, 'Hydrodynamics,' p. 139.

In each the kinetic energy of the body in motion is increased, in the one case by hydrodynamic and in the other by thermodynamic relations. And yet no one claims that to speak of this increase in mass is anything more than a figurative way of saying that the fluid pressure acts as if it had increased the true inertia of the moving body. Why then should we claim otherwise for a moving body radiating light? The same can be said of the apparent increase of inertia of an electrified body moving in an electromagnetic field.

While no one has used the hydrodynamic equations in the above form to explain the cause of inertia, yet it is essentially the foundation of Lord Kelvin's theory of vortices moving in a primitive fluid. It was a promising and ingenious attempt to explain the nature of matter, and it readily and sufficiently accounted for such fundamental attributes as impenetrability, diversity, conservatism, &c., but it failed because the effective inertia of the vortex ring was not invariable but dependent upon the position and influence of its neighbours.

Maxwell* certainly felt this to be the fatal defect in the theory of the vortex atom as in other similar theories, and has expressed his opinion with such perspicuity that I shall quote from his article at some length:—

“One of the first, if not the very first desideratum in a complete theory of matter is to explain—first mass, and second gravitation. To explain mass may seem an absurd achievement. We generally suppose that it is of the essence of matter to be the receptacle of momentum and energy, and even Thomson, in his definition of his primitive fluid, attributes to it the possession of mass. But according to Thomson, though the primitive fluid is the only true matter, yet that which we call matter is not the primitive fluid itself, but a mode of motion of that primitive fluid. It is the mode of motion which constitutes the vortex rings, and which furnishes us with examples of that permanence and continuity of existence which we are accustomed to attribute to matter itself. The primitive fluid, the only true matter, entirely eludes our perceptions when it is not endued with the mode of motion which converts certain portions of it into vortex rings, and thus renders it molecular.

“In Thomson's theory, therefore, the mass of bodies requires explanation. We have to explain the inertia of what is only a mode of motion, and inertia is a property of matter, not of modes of motion. It is true that a vortex ring at any given instant has a definite momentum and a definite energy,

* Maxwell, ‘The Atom,’ *Encyc. Brit.*

but to show that bodies built up of vortex rings would have such momentum and energy as we know them to have is, in the present state of the theory, a very difficult task."

Our aspect toward nature and natural law is constantly shifting, but the general principles underlying science remain much the same. No amount of mathematical manipulation will enable us to discover the fundamental property which makes the external world evident to us. Whatever it may be it must remain for us a mere postulate, call it what we will, inertia, mass, or energy. If we abstract this attribute from ponderable matter we assign it to an atom; if we subdivide the atom it is associated with a subatom; if this is but a discontinuity in a primitive fluid the same process of thought occurs. Not only is this pursuit of no real advantage to us, it is a hindrance, as it apparently confuses the purposes of science which should attempt to express the phenomena discovered by experience in general mathematical laws. The discussion of the postulates of science is extra-scientific and disturbs the boundaries between physics and metaphysics. The deductive methods of the pure mathematician are unsuited to experimental science, and however little we may value the discoveries of Bacon, yet he did announce the scientific method which made possible modern science. We should avoid many a pitfall if we kept in mind those things which have obstructed natural philosophy, the wrong use of logic, of theology, and of mathematics,—"*Mathematica, quæ philosophiam naturalem terminare, non generare aut procreare debet.*"

If I have seemed to put undue emphasis on this latest of the theories of matter and energy it is not because of its uniqueness, but because Dr. Lewis has presented his ideas without the complexity which usually obscures in such attempts the real issue.

University of Cincinnati,
January 1909.

IV. *The Ionization of Electrolytic Oxygen.*

By ROBERT TABOR LATTEY*.

IN 1897 Townsend† showed that the gases obtained by electrolysis of concentrated aqueous solutions of potassium hydroxide and of sulphuric acid contain charged particles which serve as nuclei for the formation of mist.

* Communicated by the Author.

† Proc. Camb. Phil. Soc. ix. 1897, pt. v.; Phil. Mag. [5] xliv. 1898, p. 125.

By measuring the total charge q in a given volume of gas and the number n of drops in the cloud he obtained the value 3×10^{-10} for q/n . If all the drops had charges of like sign q/n would represent the charge on each; experiments on the conductivity of the charged oxygen from sulphuric acid showed that it contained a mixture of positively and negatively charged particles in the ratio of about 4 : 1, so that the charge on each is greater than q/n in the proportion of 5 : 3. When this fact is taken into account the value of e would be about 5×10^{-10} E.S. units.

At Prof. Townsend's suggestion I have examined the oxygen obtained by electrolysis of a solution of potassium hydroxide of sp. gr. about 1.2, as it appeared probable that an accurate value might be obtained by using the method devised by Prof. H. A. Wilson for measuring the charge on each particle in the case of mists produced by adiabatic expansion*.

An electrolytic fog of this nature has two advantages over an expansion fog which ensure much greater accuracy of measurement. The radii of the particles are extremely small, and hence their rate of fall is slow (from 1 to 2 mm. in a minute) and easily observed; also the cloud is stable, while in a fog produced by expansion the velocity is great and it is certain that a more or less rapid evaporation must be taking place, so that it is impossible to know the size of the drop which corresponds to the velocity that is observed.

In my experiments observations on the rate of fall were always made over several successive millimetres, and in no case was any evidence of a change in the size of the drops observable during the course of an experiment.

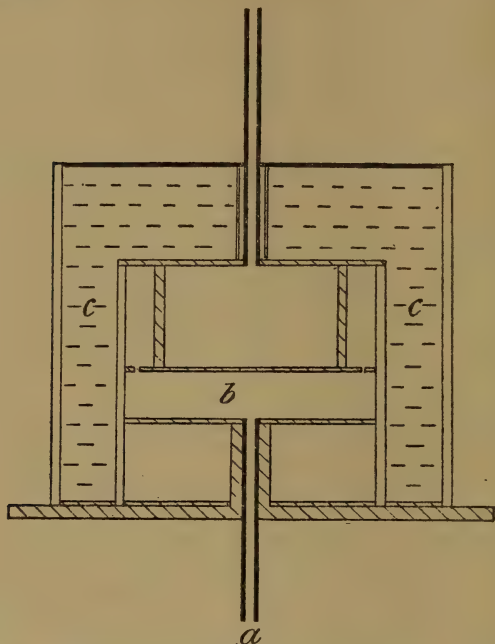
A current of about 10 amperes was used for electrolysis and electrodes of platinum foil about 2.5×3 cms. were found to give the best results. The oxygen from the anode was passed into an apparatus shown in vertical section in the figure.

The gas passed up the brass tube a into the observation vessel b . This had plane glass slides; its upper and lower boundaries were square brass plates which could be connected to the terminals of a battery of secondary cells. The upper plate had eight small holes near its edge by which gas might escape, and it was suspended from a slightly larger brass plate by four brass rods. This part of the apparatus was surrounded on five sides by a water jacket c having plane glass slides. In order to prevent electrolysis in this water it was insulated from the base plate by a layer of shellac.

On part of one wall of the chamber b a scale of millimetres

* Phil. Mag. [6] vol. v. 1903, p. 429.

was engraved ; a beam of light from a Nernst lamp passed through this and thence obliquely across the chamber. The upper boundary of the fog could then be distinguished through a telescope by means of the light scattered by the fog particles.



The shaded portions are of brass, the vertical walls of glass. Water is represented by broken horizontal lines.

In some of the earlier experiments a simpler apparatus without a water-jacket was used and difficulties, due to heat convection currents, were often experienced ; these results are given in the table on page 29 under the heading "first set" ; not only was the observation chamber different in these, but the electrolytic apparatus was also different, and hence the rate of fall of the fog under the influence of gravity was different from that obtained in the "second set" with the modified apparatus.

By this method the charge on a particle is deduced from the change of velocity produced by a known field. Since the charges on the particles vary in sign, the effect of a field is to accelerate some and to retard others in their fall ; this made the outline of the fog somewhat indistinct, but it was found that the boundary of the rapidly falling portion could

be always followed with fair accuracy and that occasional observations on the more slowly moving particles could be made.

If V' is the velocity with which a particle of radius α falls in a medium whose viscosity is η , then Stokes' theorem gives the equation

$$\frac{4}{3} \pi \alpha^3 g = 6 \pi \eta \alpha V'.$$

If an electric field whose gradient is X units per cm. is acting, and if the particle has a charge of E units, then the velocity V with which it falls is given by

$$\frac{4}{3} \pi \alpha^3 g + E X = 6 \pi \eta \alpha V.$$

Combining these equations we get

$$E = \frac{18 \pi \eta}{X} \sqrt{\frac{\eta V'}{2g}} (V' - V).$$

Taking the viscosity of oxygen at about 15°C. as 2.12×10^{-4} , this reduces to

$$E = 3.94 \times 10^{-6} V'^{\frac{1}{2}} (V' - V) \div X.$$

In the following table X is given in E.S. units per cm., the sign refers to the potential of the upper plate. V is in cms. per second:—

X.	V×10 ³ .	E×10 ¹⁰ .	
First Set.			
0.0	2.725		
-0.067	3.087		-11.17
+0.067	2.439		- 8.82
-0.133	2.500	+3.48	
+0.133	2.150		- 8.86
-0.200	3.030	-3.13	
+0.200	1.852		- 8.99
-0.267	3.788		- 8.21
Second set.			
0.00	1.764		
-0.0545	2.415		-19.76
-0.0606	2.110		- 9.45
+0.0667	1.961	±4.88	
-0.0909	2.325		- 9.28
+0.1182	2.174	+5.74	
-0.1182	3.096		-18.65
-0.1485	2.525		- 8.48
±0.1636	3.774		±20.34
Mean ±4.47 - 8.80 ±19.85			

In calculating the means the individual values have been weighted according to three factors:

- (i.) The error caused by 5 secs. in the time to fall 1 cm.,
- (ii.) The error of 1 volt per cm. in the value of X , and
- (iii.) The number of observations on which V depends.

It will be observed that the values of E group themselves about three values, indicating that the positively and negatively charged groups of particles are each subdivided into sets of particles carrying charges of differing magnitude. Which set of particles would be observed in any given experiment was apparently a matter of chance. Alterations in the method of illumination apparently caused one set to show up rather than another; but unfortunately the relationship between the conditions of illumination and the portion of the fog observed could not be elucidated. That the different sets of particles differed only in charge and not in magnitude was shown by the constancy of the rate of fall under the influence of gravity alone.

In the case of the fog nuclei produced by Röntgen rays, Prof. H. A. Wilson was able to observe three sets of particles which fell at different rates under the influence of an electric field, and he concluded that the charges on the more highly electrified particles were multiples of those on the particles on which most of his observations were made.

If we assume that this is also the case for the particles in an electrolytic fog, we obtain as the mean weighted submultiple 4.66×10^{-10} , and the final mean values of $E \times 10^{10}$ will be

$$4.66 \qquad 9.33 \quad \text{and} \quad 18.66.$$

Rutherford and Geiger* obtained 9.3×10^{-10} for the value of the charge on an α -particle, and as they consider that an α -particle probably has two atomic charges, they give 4.65×10^{-10} as the value of unit atomic charge in E.S. units.

The values obtained by observation of the rate of fall of an expansion fog in gases ionized by various methods vary somewhat, but are all of the same order of magnitude.

6.5×10^{-10}	J. J. Thomson.	Phil. Mag.	[5]	xlvi. 1898, p. 528;
6.8×10^{-10}	"	"	[5]	xlvi. 1899;
3.4×10^{-10}	"	"	[6]	v. 1903, p. 352;
3.1×10^{-10}	H. A. Wilson.	"	[6]	v. 1903, p. 429.
4.06×10^{-10}	Milikan & Begeman.	Phys. Rev.		1908, p. 197.

* Proc. Roy. Soc. lxxxi. A, 1908, p. 169.

Though these results are probably not so accurate as those obtainable in the case of an electrolytic fog or by Rutherford's method, yet they are of greater interest, since they give the charges on ions for which Ne can be found directly where N is the number of molecules in a cubic centimetre of gas. The most recent value of Ne is 1.23×10^{10} at 15°C . * Faraday's experiments show that 1 c.c. of hydrogen at 15°C . is liberated from acidulated water by 2.44×10^{10} E.S. units of electric current. Now 1 c.c. will contain $2N$ atoms, and hence we may conclude that Ne for hydrogen ions in solutions is $1/2 \times 2.44 \times 10^{10} = 1.22 \times 10^{10}$, a figure which is in substantial agreement with Ne for gaseous ions. Hence it has been concluded that e has the same value for gaseous as for electrolytic ions.

If we assume that $e = 4.66 \times 10^{-10}$, we obtain for N the value 2.62×10^{19} , which may be compared with that derived from other considerations :

2.61×10^{19} Planck (Theory of Heat Radiation) †.

3.0×10^{19} Perrin ‡.

The last value is of especial interest as it was obtained by the most direct method. The numerical distribution of particles at different depths in a colloidal solution was obtained by direct counting under a microscope ; the osmotic pressures calculated from these numbers gave the actual numbers of molecules corresponding to a known osmotic pressure.

Since the experiments here described did not yield results of the accuracy expected, owing to the various causes noted above and since it is sufficiently clear that the charge is the same as that on ions produced by other methods, they were not further pursued. It may possibly be found that the most accurate method of finding e is from the quantities $Ne = 1.23 \times 10^{10}$ and N as found by Perrin's method.

The author wishes to acknowledge his indebtedness to Prof. J. S. Townsend for his many suggestions and his sympathetic help in the course of these experiments.

University Museum, Oxford.

February, 1909.

* J. S. Townsend, Proc. Roy. Soc. vols. lxxx. & lxxxi. (1908); C. E. Haselfoot, Proc. Roy. Soc. vol. lxxxii. (1909).

† Wied. Ann. [4] iv. 1901, p. 504.

‡ Compt. Rend. cxlvii. 1908, p. 594.

V. *Note on the Theory of the Greenhouse.* By C. G. ABBOT, Director, Astrophysical Observatory, Smithsonian Institution*.

IN a paper of the above title† Professor R. W. Wood states that he has compared two “hot-boxes” one having a glass cover, the other a cover of rock salt, but otherwise similar. A glass plate was interposed in the path of the entering sun rays. He observed a maximum temperature of about 55° C. within each box when exposed to the sun. He concludes that the function of the cover is mainly to prevent the loss of heat by convection, rather than the escape of long wave rays, and asks: “Is it therefore necessary to pay much attention to trapped radiation in deducing the temperature of a planet as affected by its atmosphere?”

It may interest some to know that much higher temperatures can be reached within a “hot-box” than that observed by Professor Wood, if precautions are taken to diminish the loss of heat by convection from the warmed outer surface of the cover. On November 4, 1897, the thermometer recorded 118° C. within a circular wooden box 50 centimetres in diameter, 10 centimetres deep, insulated in feathers, covered with three superposed and separated sheets of plate glass and exposed normally to the sun rays in the yard of the Astrophysical Observatory at Washington. The temperature outside was 16° C.

Agreeing with Professor Wood that the main function of the cover of a “hot-box” or “hot-house” is to prevent loss of heat by convection, it is interesting to see if this could be predicted. Published experiments on the cooling of solids in dry air and in vacuum give the relative rates of loss by convection and radiation under known circumstances. Planck’s radiation formula for the “black body” enables computations to be made of the losses by radiation for different temperatures of source and sink. The transmission of glass, salt, and the water vapour of the atmosphere, and the effective temperature of the latter are approximately known. I have attempted to compute from such data the relative hindrance which salt and glass covers would interpose to the loss of heat by convection and radiation combined from a “black” surface at 55° C. For the dependence of the temperature of the earth’s surface on the atmosphere, some numerical data can be assigned also, and as shown below there is reason to think that “trapping” is more important perhaps than Professor Wood thinks.

* Communicated by the Author. Published by permission of the Secretary of the Smithsonian Institution.

† Phil. Mag. 6th series, vol. xvii. p. 319 (1909).

From an interesting paper of P. Compan * it may be seen that for a blackened copper ball 2 centimetres in diameter cooling from a temperature of 55° C. to nearly "black" surroundings at 0° C., the rate of loss of heat by convection in still dry air at atmospheric pressure is four-thirds as rapid as the simultaneous loss by radiation. In a breeze of 3 metres per second the convection loss becomes 3 times as rapid as in still air, or 4 times as rapid as the loss by radiation. The loss of heat by convection alone is approximately proportional to the difference of temperatures between the source and the sink.

If the covers had been absent in Professor Wood's experiments, the boxes would have been exchanging radiation principally with the water vapour of the lower atmosphere. Experiments of Langley, Rubens and Aschkinass, and others indicate † that less than 10 per cent. of the radiation from the earth's surface can penetrate the water vapour of the atmosphere above a coast station like Baltimore. Hence the water vapour of the atmosphere can be considered as practically a "black body" for rays of great wave-length. The effective temperature of the water-vapour layers with which a coverless "hot-box" would have been exchanging radiation may be estimated at 0° C. If glass is interposed the radiation is entirely cut off. If a rock-salt plate 1 centimetre thick is interposed between the body at 55° C. and surroundings at 0° C., the absorption of the plate is about 19 per cent. ‡ and the reflexion probably nearly 10 per cent. more, so that the transmission may be reckoned at about 70 per cent.

In combining the preceding results with those of Compan we will at first neglect the loss of heat by convection from the outside of the cover. We may assume the temperature of the air just outside the "hot-box" to be 15° C., and also that Newton's law of cooling is applicable to the convection loss. In still air with no cover the rate of loss of heat towards the front by convection and radiation combined is proportional to :

$$\frac{40}{55} \times 133 + 100 = 197.$$

With glass cover : $0 + 100 \times 0.00 = 0.$

With salt cover : $0 + 100 \times 0.70 = 70.$

* *Annales de Chimie et de Physique*, t. xxvi. pp. 488-574 (1902).

† See *Annals, Astrophysical Observatory, Smithsonian Institution*, vol. ii. pp. 167-172.

‡ See Kayser's *Handbuch d. Spectroscopie*, vol. iv. p. 485, and Planck's formula of radiation.

Thus of the heat which would have escaped toward the front from a coverless box at 55°C. in still moist air at 15°C. , the salt hinders $\frac{197-70}{197} = 65$ per cent. as much as the glass.

Remembering, however, that owing to its higher absorbing power for long wave rays the glass will be warmed more than the salt, the convection loss from the outside of the warmed cover will be greater for glass than for salt, so that the efficiency of a salt cover may be much more than 65 per cent. of that of one of glass. The convection loss from the front of the cover is a considerable factor, for in the "hot-box" tried at this observatory the front of the inner glass cover became too hot to handle and often cracked with the heat.

In view of these figures we may agree with Professor Wood that a salt cover * is nearly as efficient as a glass one for a "hot-box," although it would seem strange that he observed no difference at all. Perhaps in spite of the glass filter the cover-glass obstructed the entering sun rays more than salt. But is not the case quite different with a planet?

Let us take the mean temperature of the earth's surface at 14°C. , the mean effective temperature of the water-vapour layers to which it principally radiates as 0°C. , the temperature of space as -273°C. Then the rates of escape of heat from the surface by radiation, first with the water-vapour layer interposed, and, second, imagining the air to be completely transparent to earth rays, would be in the ratio of 19 to 100 according to Planck's formula. It is very difficult to estimate how fast the heat of the earth's surface escapes by convection, because neither the difference of temperature between the surface and the air nor the rate of motion of the air is well known. But if for the sake of discussion we suppose a temperature difference of 10°C. and a velocity of 3 metres per second, the rate of convection loss comes out only 0.54 as great as the rate at which heat would escape by radiation if the air was no hindrance. This assumed convection loss is 2.8 times as great, on the other hand, as the estimated rate of escape of heat by radiation to the water-vapour layers at 0°C. In other words, according to this estimate the convection is the main agent in removing heat from the earth's surface as things are, but would be only a small factor if the air was transparent to long-wave rays.

If these figures represent at all the order of magnitudes of the quantities there can be no doubt, I think, that the atmosphere is important as a trapping agent to increase the earth's surface temperature.

* A salt cover, however, is better than a perfectly transparent one.

A fair estimate of the actual increase of the earth's temperature due to "trapping" has been made. Imagine a perfectly "black" and rapidly rotating planet of the earth's dimensions, situated beyond the orbit of the earth at such a distance from the sun that the radiation absorbed by it would be equal to that available to be absorbed by the earth, allowing for the reflexion of clouds, et cetera. Such a planet would assume the temperature of -17°C. , whereas the real earth has a mean temperature of $+14^{\circ}\text{C.}$ * The difference, 31°C. , is attributable to three causes:—(1) The imperfect "blackness" of the earth. (2) The "blanket effect" of the atmosphere. (3) The warming of the earth's surface by radio-active substances and internal heat. Remembering that the earth is mainly water covered, it must be almost "perfectly black" for long-wave rays. I myself regard the conduction of internal heat and that of radio-active substances as negligible. This would leave the full 31° as due to the "blanket effect."

If there were no water on the earth, the emissive power of its surface for long-wave rays would be less. Also, on account of the absence of clouds, its absorption of solar rays would be greater. The two differences would perhaps more than counterbalance the loss of the "blanketing effect," so that the mean temperature of the earth without water would perhaps be rather higher than now, but much less uniform, ranging from above present temperatures by day to far below 0°C. by night.

VI. *On the Steady Flow of an Incompressible Viscous Fluid through a Circular Tube with Uniformly Converging Boundaries.* By A. H. GIBSON, M.Sc., Lecturer in Hydraulics in the Manchester University †.

THE general equations of motion for the three-dimensional flow of an incompressible viscous fluid are ‡:—

$$\left. \begin{aligned} -\frac{dp}{dx} &= -\mu \left\{ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right\} + \frac{\varpi}{g} \left\{ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t} \right\} \\ -\frac{dp}{dy} &= -\mu \left\{ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right\} + \frac{\varpi}{g} \left\{ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} + \frac{\partial v}{\partial t} \right\} \\ -\frac{dp}{dz} &= -\mu \left\{ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right\} + \frac{\varpi}{g} \left\{ u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} + \frac{\partial w}{\partial t} \right\} \end{aligned} \right\} \quad (1)$$

* Annals, Astrophysical Observatory, vol. ii. p. 175.

† Communicated by the Author.

‡ 'Hydraulics,' Gibson, p. 65.

where μ is the coefficient of viscosity; ϖ is the weight of unit volume; u , v , and w are the velocities of flow in directions x , y , and z respectively; and p is the pressure (reckoned positive if compressive).

Assuming that in a circular tube with uniformly converging boundaries motion takes place in uniformly converging lines, so that the pressure is uniform at all points on a spherical shell having the point of convergence as centre, we have

$$v = 0; \quad w = 0; \quad \frac{dp}{dy} = 0; \quad \frac{dp}{dz} = 0;$$

while if the motion is steady $\frac{\partial u}{\partial t} = 0$, so that in the latter case the state of flux along the tube is represented by the single equation

$$-\frac{dp}{dx} = -\mu \left\{ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right\} + \frac{\varpi}{g} u \frac{\partial u}{\partial x}. \quad (2)$$

If r be the distance of a particle from the centre of the tube at a point distant x from the point of convergence

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r}^*,$$

and the equation of motion becomes

$$-\frac{dp}{dx} = -\mu \left\{ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right\} + \frac{\varpi}{g} u \frac{\partial u}{\partial x}, \quad (3)$$

while if the angle of convergence of the sides is small, so

$$* \quad \frac{\partial u}{\partial y} = \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial y} = \frac{\partial u}{\partial r} \cdot \frac{\partial \sqrt{z^2 + y^2}}{\partial y} = \frac{\partial u}{\partial r} \cdot \frac{y}{\sqrt{z^2 + y^2}} = \frac{\partial u}{\partial r} \cdot \frac{y}{r}.$$

$$\begin{aligned} \therefore \frac{\partial^2 u}{\partial y^2} &= \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial r} \left(\frac{\partial u}{\partial r} \cdot \frac{y}{r} \right) \frac{\partial r}{\partial y} \\ &= \left\{ \frac{y}{r} \cdot \frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} \left(\frac{\partial}{\partial r} \left(\frac{y}{r} \right) \right) \right\} \frac{y}{r} \\ &= \frac{y^2}{r^2} \frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} \cdot \frac{r^2 - y^2}{r^3}. \end{aligned}$$

Similarly

$$\frac{\partial^2 u}{\partial z^2} = \frac{z^2}{r^2} \cdot \frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} \cdot \frac{r^2 - z^2}{r^3}.$$

$$\therefore \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r}.$$

that without sensible error $r = x\theta$, on converting to polar coordinates x and θ the equation becomes

$$-\frac{dp}{dx} = -\mu \left\{ \frac{\partial^2 u}{\partial x^2} + \frac{1}{x^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{x^2 \theta} \frac{\partial u}{\partial \theta} \right\} + \frac{\varpi}{g} u \frac{\partial u}{\partial x}^* \quad (4)$$

Since u is symmetrical with respect to θ , putting

$$u = A \left(\frac{\alpha^2 - \theta^2}{x^2} \right)$$

where 2α is the vertical angle of the conical pipe, and therefore assuming no slip at the boundaries, we get, on integration,

$$\begin{aligned} -p &= -\mu \left\{ \frac{2A(\theta^2 - \alpha^2)}{x^3} + \frac{2A}{3x^3} + \frac{2A}{3x^3} \right\} + \frac{\varpi u^2}{2g} + B \\ &= \mu \left\{ \frac{2u}{x} - \frac{4u}{3x} \cdot \frac{1}{(\alpha^2 - \theta^2)} \right\} + \frac{\varpi u^2}{2g} + B. \end{aligned}$$

When $p = p_0$, $u = u_0$ and $x = x_0$;

$$\therefore B = -p_0 - \mu \left\{ \frac{2u_0}{x_0} - \frac{4u_0}{3x_0} \cdot \frac{1}{(\alpha^2 - \theta^2)} \right\} - \frac{\varpi u_0^2}{2g}.$$

$\therefore$

$$\frac{p_0}{\varpi} + \frac{u_0^2}{2g} = \frac{p}{\varpi} + \frac{u^2}{2g} - \frac{\mu}{\varpi} \left\{ 2 - \frac{4}{3(\alpha^2 - \theta^2)} \right\} \left\{ \frac{u_0}{x_0} - \frac{u}{x} \right\}. \quad (5)$$

Again since the volume discharged per second is given by

$$\begin{aligned} Q &= \int_0^\alpha 2\pi x^2 \theta u \, d\theta \\ &= 2\pi A \int_0^\alpha (\alpha^2 \theta - \theta^3) \, d\theta \\ &= \frac{\pi A \alpha^4}{2}, \end{aligned}$$

$$\therefore A = \frac{2Q}{\pi \alpha^4},$$

$$* \quad \frac{\partial u}{\partial r} = \frac{\partial u}{\partial \theta} \cdot \frac{\partial \theta}{\partial r} = \frac{1}{x} \cdot \frac{\partial u}{\partial \theta}.$$

$$\begin{aligned} \therefore \frac{\partial^2 u}{\partial r^2} &= \frac{\partial}{\partial r} \cdot \frac{\partial u}{\partial r} = \frac{\partial}{\partial \theta} \left(\frac{1}{x} \cdot \frac{\partial u}{\partial \theta} \right) \cdot \frac{\partial \theta}{\partial r} \\ &= \frac{1}{x} \frac{\partial^2 u}{\partial \theta^2} \cdot \frac{1}{x} = \frac{1}{x^2} \frac{\partial^2 u}{\partial \theta^2}. \end{aligned}$$

while

$$u = A \cdot \frac{\alpha^2 - \theta^2}{\alpha^2}$$

$$= \frac{2Q(\alpha^2 - \theta^2)}{\pi\alpha^4 x^2} \cdot \cdot \cdot \cdot \cdot \cdot (6)$$

$$\therefore \frac{1}{\alpha^2 - \theta^2} = \frac{2Q}{\pi\alpha^4 x^2 u} = \frac{2Q}{\pi\alpha^4 x_0^2 u_0}.$$

Substituting this value in (5) we get

$$\frac{p_0}{\omega} + \frac{u_0^2}{2g} = \frac{p}{\omega} + \frac{u^2}{2g} - \frac{\mu}{\omega} \left\{ 2 \left(\frac{u_0}{x_0} - \frac{u}{x} \right) - \frac{8Q}{3\pi\alpha^4} \left(\frac{1}{x_0^3} - \frac{1}{x^3} \right) \right\} (7)$$

At the boundaries $u_0 = u = 0$, so that here

$$\frac{p_0}{\omega} - \frac{p}{\omega} = \frac{u}{\omega} \cdot \frac{8Q}{3\pi\alpha^4} \left\{ \frac{1}{x_0^3} - \frac{1}{x^3} \right\} \cdot \cdot \cdot \cdot (8)$$

$$= \frac{8\mu Q}{3\pi\omega} \left\{ \frac{x_0}{a_0^4} - \frac{x}{a^4} \right\},$$

giving the difference of head (expressed as the length of a column of the fluid) as measured by piezometers opening flush with the sides of the tube at points distant respectively x_0 and x from the point of convergence, and where the radii are respectively a_0 and a .

From (6) we have, at the centre where $\theta = 0$,

$$u_0 = \frac{2Q}{\pi\alpha^2 x^2},$$

while the mean velocity over the whole section

$$= \frac{Q}{\pi a^2}$$

$$= \frac{Q}{\pi\alpha^2 x^2}.$$

$$\therefore \frac{\text{Maximum velocity}}{\text{Mean velocity}} = 2.$$

VII. *The Determination of a Constant in Capillarity.* By
R. D. KLEEMAN, *D.Sc., B.A., Research Student of
Emmanuel College, Cambridge**.

WHEN the area of the surface of a liquid is increased the increase in the amount of energy in the surface-film of the liquid is equal to the sum of the external work done and the amount of heat absorbed from the surrounding bodies to keep the temperature constant during the process. The relation between these quantities can be immediately obtained by means of Helmholtz's free energy equation connecting the total energy with the free energy, which gives

$$E + T \frac{d\lambda}{dT} = \lambda, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where λ denotes the surface-tension and E the total energy corresponding to the temperature T . λ is equal to the external work done, and $-T \frac{d\lambda}{dT}$ is therefore equal to the amount of heat absorbed from the surrounding bodies. This formula was first given by Lord Kelvin.

Whittaker† has calculated the values of E , the total energy for different temperatures, for a number of liquids, using the surface-tension determinations of Ramsay and Shields‡. He found that the energy E per cm.^2 of the film of a liquid is proportional to the product of the internal latent heat of evaporation into the absolute temperature of the liquid, or

$$E = KTL, \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where L is the internal latent heat of evaporation corresponding to the temperature T , and K is a constant. The constant K was found to be different for the various liquids. The extent of the agreement of this empirical law with the experimental facts is shown by the following tables taken from Whittaker's paper.

* Communicated by the Author.

† Proc. Roy. Soc. A. vol. lxxxi. p. 21 (1908).

‡ Phil. Trans. A. vol. clxxxiv. p. 647 (1893).

Ether.

T.	E.	L.	$\frac{E \times 10^4}{TL \text{ or } K10^4}$
313	49.1	75.36	20.8
323	48.9	73.01	20.8
333	48.9	70.79	20.5
343	47.8	68.35	20.4
353	47.1	65.85	20.3
363	46.1	63.31	20.1
373	45.5	60.33	20.2
383	44.7	58.07	20.1
393	43.8	54.91	20.3
403	42.6	51.62	20.5
413	41.1	48.31	20.6
423	39.3	44.38	20.9

Methyl formate.

T.	E.	L.	$\frac{E \times 10^4}{TL \text{ or } K10^4}$
303	69.4	107.49	21.3
313	68.8	103.95	21.1
323	68.2	99.51	21.2
333	67.5	95.59	21.2
343	67.3	92.16	21.3
353	66.2	88.03	21.3
363	65.5	85.10	21.2
373	64.6	82.43	21.0
383	63.8	79.21	21.0
393	62.9	75.92	21.1
403	61.8	71.95	21.3
413	60.4	68.10	21.5
423	58.8	64.03	21.7

Benzene.

T.	E.	L.	$\frac{E \times 10^4}{LT \text{ or } K10^4}$
353	59.5	85.6	19.7
363	60.2	83.7	19.8
373	60.9	82.0	19.9
383	61.1	80.0	20.0
393	60.7	78.1	19.8
403	60.1	76.1	19.6
413	59.4	74.1	19.4
423	58.8	71.9	19.3
433	58.0	69.7	19.2
443	57.2	67.5	19.1
453	56.1	65.1	19.0
463	55.0	62.7	19.0
473	54.0	59.9	19.1
483	52.9	57.0	19.3
493	51.5	53.8	19.4
503	49.9	50.5	19.7
513	48.1	46.6	19.1

Carbon tetrachloride.

T.	E.	L.	$\frac{E \times 10^4}{LT \text{ or } K10^4}$
363	57.9	40.62	39.3
373	57.5	39.68	38.9
383	56.8	38.64	38.4
393	56.4	37.63	38.2
403	55.6	36.58	37.7
413	54.8	35.56	37.3
423	53.9	34.42	37.0
433	53.1	33.28	37.9
443	52.6	32.07	37.0
453	52.2	30.83	37.4
463	51.7	29.52	37.8
473	50.8	28.22	37.1
483	50.3	26.83	37.8
493	49.8	25.35	37.8
503	47.3	23.73	37.6

The object of this paper is to find a value for K of a liquid in terms of its absolute constants. Such a determination is evidently desirable.

Ramsay, following Eötvös, has shown that the surface-tension of a liquid at the absolute temperature T is connected with the absolute temperature and other quantities by

Chloro-benzene.

T.	E.	L.	$\frac{E \times 10}{TL}$ or K10 ⁴ .
423	59.8	65.81	21.4
433	59.9	64.12	21.6
443	60.0	63.02	21.5
453	60.0	61.46	21.5
463	59.3	59.97	21.3
473	58.6	58.31	21.3
483	58.1	56.81	21.2
493	57.4	55.29	21.1
503	56.7	53.83	20.9
513	55.9	52.43	20.8
523	55.1	50.81	20.7
533	53.8	49.09	20.6

the equation

$$\lambda v^{2/3} = A(T_c - T - a), \quad . \quad . \quad . \quad . \quad (3)$$

where v denotes the molecular volume of the liquid, and A is a constant which is practically the same for all liquids, being equal to about 2.1, T_c is the critical temperature, and a is another constant which is also approximately the same for all liquids, its mean value being about 5. Substituting for λ in equation (1) by means of equation (3), we obtain

$$E = A \frac{(T_c - T - a)}{v^{2/3}} + \frac{AT}{v^{2/3}} + \frac{2}{3} TA \frac{(T_c - T - a)}{v^{5/3}} \frac{dv}{dT}.$$

Let us suppose that λ refers to a temperature much below the critical temperature of the liquid, say near the absolute zero. The value of $\frac{dv}{dT}$ is then a very small fraction, and $T(T_c - T - a)$ small in comparison with $(T_c - T - a)$. The third term on the right-hand side of the last equation may at low temperatures be, therefore, neglected in comparison with the other terms, and in that case

$$E = \frac{A}{v^{2/3}} (T_c - a).$$

The molecular volume is given by $v = \frac{m}{\rho}$, where m is the molecular weight and ρ the density of the liquid. Substituting for v in the last equation we have

$$E = \frac{\rho^{2/3} A (T_c - a)}{m^{2/3}}.$$

And since $E = KLT$, we have

$$K = \frac{\rho^{2/3} A}{m^{2/3} LT} (T_c - a). \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The latent heat of evaporation L_1 of a liquid is given by the well-known thermodynamical relation

$$L_1 = (v_1 - v_2) T \frac{dp}{dT},$$

where v_1 and v_2 denote respectively the volumes of a gram of the saturated vapour and the liquid, and p denotes the pressure. For temperatures well removed from the critical temperature the vapour behaves approximately as a perfect gas, and this equation may then be written

$$L_1 = \frac{RT^2}{mp} \frac{dp}{dT},$$

where R denotes the gas constant. If the equation be written in the form

$$\frac{L_1 m}{T} = \frac{RT}{p} \frac{dp}{dT},$$

and the temperatures and pressures at the right-hand side be expressed in terms of the critical temperature and pressure, we obtain

$$\frac{L_1 m}{T} = \frac{R \alpha T_c}{\beta p_c} \frac{p_c}{T_c} \frac{d\beta}{d\alpha} = \frac{R \alpha}{\beta} \frac{d\beta}{d\alpha},$$

where $\alpha T_c = T$, and $\beta p_c = p$. Thus the relation between L_1 , T , and m for different liquids at corresponding states is given by $\frac{L_1 m}{T} = \text{constant}$. Since the temperatures of liquids

at their boiling-points are approximately corresponding temperatures, this relation holds approximately for liquids at their boiling-points; it is known in this form as Trouton's law. If the temperature of evaporation is so far removed from the critical temperature that the vapour behaves approximately as a perfect gas, the external work of evaporation $p(v_1 - v_2)$ is approximately equal to $\frac{RT}{m}$. Therefore

$$L + p(v_1 - v_2) = L + \frac{RT}{m} = L_1$$

and therefore $\frac{L_1 m}{T} = \frac{L m}{T} + R = \text{constant}$, or $\frac{L m}{T}$ is also

constant for different liquids at corresponding states. Writing $\frac{Lm}{T} = B$, where $B = \frac{R\alpha}{\beta} \frac{d\beta}{d\alpha} - R$, and substituting for L in equation (4) we obtain

$$K = \frac{\rho^{2/3} A (T_c - a) m^{1/3}}{T^2 B}.$$

Expressing ρ and T in terms of the critical density and temperature, we obtain

$$K = \frac{A \gamma^{2/3} \rho_c^{2/3} (T_c - a) m^{1/3}}{B \alpha^2 T_c^2},$$

where $\rho = \gamma \rho_c$.

Let us write the equation in the form

$$K = \frac{M m^{1/3} (T_c - a) \rho_c^{2/3}}{T_c^2},$$

where $M = \frac{A}{B} \frac{\gamma^{2/3}}{\alpha^2}$. Now it is known that K is a constant

for a given liquid, therefore M must be a constant for the liquid. Further, in obtaining K for different liquids we may suppose the liquids taken at corresponding states, so that the values of α, β, γ , are the same for each liquid. Then it follows that since M is a function of α, β , and γ only, it must be the same for each liquid.

Further, since a is equal to about 5 and T_c of the order 500, $\frac{a}{T_c^2}$ may be neglected in comparison with $\frac{1}{T_c}$. We have then, finally,

$$K = \frac{M m^{1/3} \rho_c^{2/3}}{T_c}, \quad . \quad . \quad . \quad . \quad (5)$$

where M is a constant which is the same for every liquid.

The following table contains the values of $10^4 \frac{M m^{1/3} \rho_c^{2/3}}{T_c}$ for a few liquids calculated from the data given in the second, third, and fourth columns of the table, the value of M being put equal to 5.57×10^{-1} in each case. The values of $10^4 K$, contained in the fifth column of the table, are the means of the values contained in the preceding tables. It will be seen that the values of K given by equation (5) agree fairly well with the values given by the equation $K = \frac{E}{L T}$. A better agreement can scarcely be expected since the critical constants cannot be determined with any great accuracy.

$M=5.57 \times 10^{-1}$.					
	T_c .	ρ_c .	m .	$K10^4$	$\frac{10^4 M m^{1/3} \rho_c^{2/3}}{T_c}$
Ether	466.6	.2604	74	20.5	20.4
Methyl formate	487	.3489	60	21.2	22.2
Carbon tetrachloride ...	556	.5576	153.8	38.1	36.4
Benzene	561.5	.3045	78.05	19.5	19.2
Chloro-benzene	633	.3654	112.5	21.2	21.7

The value of K given by equation (4) will not apply to polymerized liquids because the value of A is then not a constant, but varies with the nature of the liquid and its temperature. It is also very probable that the relation given by Whittaker does not apply to such liquids.

If the critical density and temperature of a liquid is known, and its capillary constants for temperatures differing by a small interval, the internal latent heat can be calculated for these temperatures by the equation

$$L = \left(\frac{\lambda}{T} - \frac{d\lambda}{dT} \right) \frac{T_c}{m^{1/3} \rho_c^{2/3}} \frac{1}{5.57 \times 10^{-1}},$$

deduced from the equations (1), (2), and (5). An inspection of Whittaker's results shows that K or $\frac{E}{LT}$ is very constant for different temperatures, and this equation would therefore give fairly accurate results.

If a large number of determinations of λ for different temperatures are not available, the values of λ and $\frac{d\lambda}{dT}$ in the above equation may be obtained from equation (2), the constants in the equation having first been determined by means of the values of λ for two different temperatures. This, however, will not give very accurate values for L .

Equation (5), by writing it in the form

$$\frac{E}{LT} = \frac{M m^{1/3} \rho_c^{2/3}}{T_c},$$

may be used to find either of the critical constants it contains if one of them is known. The right-hand side may be evaluated for any convenient temperature.

Cambridge, March 20, 1909.

VIII. *Theory of the Alternate Current Generator.* By THOMAS R. LYLE, M.A., Sc.D., *Professor of Natural Philosophy in the University of Melbourne* *.

IT has been usual hitherto to ascribe the distortion of the wave-form of the current given by an alternate current-generator to:—

1. "Lack of uniformity and pulsation of the magnetic field, causing a distortion of the induced E.M.F. at open circuit as well as under load."
2. "Pulsation of the reactance causing higher harmonics under load."
3. "Pulsation of the resistance causing higher harmonics under load also" †.

And, as far as I have been able to find out, another cause has been overlooked, namely, the mutual reactions between armature and field, which when the generator is loaded is at least as important as any of the foregoing.

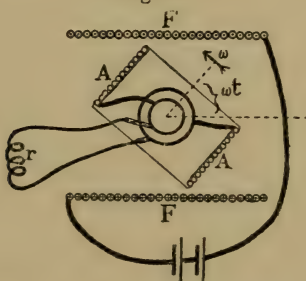
If such is the case it can only be explained by the fact that the theory of the simple alternator has not been completely worked out.

In the following paper this is done for an alternator with a uniform field by means of a new application of the vector method in which all the harmonics of a periodic function are dealt with simultaneously.

In the same is shown how to take account of hysteresis and eddy currents, and the theory of the action of dampers in reducing the heating in the field is also given.

The theory of the alternate current synchronous motor is also dealt with.

Fig. 1.



1. Let two coils be arranged as indicated in fig. 1, one of them, F, called the field coil, being fixed and having a battery of constant E.M.F. = η in its circuit, the other, A, called the

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† Steinmetz, 'Alternating Current Phenomena.'

armature coil, fitted in the usual way with slip rings for connexion to an external circuit and being rotated by power at a constant angular velocity ω round a fixed axis which is perpendicular to its own axis of figure and to the direction of the lines of force of F , and which passes through its own centre. It is required to determine completely the currents that flow in both A and F.

Let x and ξ be the currents at any instant in A and F respectively, and let the mutual inductance of A and F when their axes are coincident be m , and hence $m \cos \omega t$ at the time t . Also let r and l be the total resistance and self-inductance of the A circuit, and ρ , λ similar quantities for the F circuit.

Then, when the armature is being driven at constant angular velocity ω , and x and ξ are flowing, the total number of lines linked on A is

$$lx + m\xi \cos \omega t,$$

and the number linked on F is

$$\lambda\xi + mx \cos \omega t.$$

Hence

$$\left. \begin{aligned} rx + \frac{d}{dt} \{lx + m\xi \cos \omega t\} &= 0 \\ \rho\xi + \frac{d}{dt} \{\lambda\xi + mx \cos \omega t\} &= \eta \end{aligned} \right\} \quad \cdot \quad \cdot \quad \cdot \quad \text{(I.)}$$

where η is the applied steady E.M.F. in the F circuit.

2. If we assume as the solution of these equations

$$x = \frac{x_0}{2} + x_1 \sin (\omega t + c_1) + x_2 \sin (2\omega t + c_2) + x_3 \sin (3\omega t + c_3) + \&c.,$$

$$\xi = \frac{\xi_0}{2} + \xi_1 \sin (\omega t + \gamma_1) + \xi_2 \sin (2\omega t + \gamma_2) + \xi_3 \sin (3\omega t + \gamma_3) + \&c.,$$

we can see at once on substitution that $\rho\xi_0 = 2\eta$, and that $x_0 = 0$, and it will be shown afterwards, § 15, that when $x_0 = 0$ then $\xi_1, x_2, \xi_3, x_4, \xi_5$, &c., vanish, or in words, when $x_0 = 0$ only odd harmonics appear in x , and only even ones in ξ . Let us therefore take

$$\left. \begin{aligned} x &= x_1 \sin (\omega t + c_1) + x_3 \sin (3\omega t + c_3) + x_5 \sin (5\omega t + c_5) + \&c. \\ \xi &= \frac{\xi_0}{2} + \xi_2 \sin (2\omega t + \gamma_2) + \xi_4 \sin (4\omega t + \gamma_4) + \&c. \end{aligned} \right\} \text{(II.)}$$

Now any harmonic in either x or ξ , for instance $x_q \sin (q\omega t + c_q)$, being completely specified by x_q, c_q , and q ,

can, when its order q is known, be represented by the vector drawn from the origin in any reference plane to the point in that plane whose polar coordinates are x_q, c_q , twice the constant term in ξ being represented in the same plane by the vector to the point $\xi_0, \frac{\pi}{2}$

The form of solution (II.) assumed may now be written

$$\left. \begin{aligned} x &= a_1 + a_3 + a_5 + \&c. \\ \xi &= \frac{\alpha_0}{2} + \alpha_2 + \alpha_4 + \alpha_6 + \&c. \end{aligned} \right\} \dots \text{(III.)}$$

where $a_1, a_3, \&c., \alpha_0, \alpha_2, \alpha_4, \&c.$, are vectors whose orders are indicated by the subscribed numbers. Of these, one only, namely α_0 , is known, as it is drawn to the point whose polar co-ordinates are $\xi_0, \frac{\pi}{2}$, where $\xi_0 = \frac{2\eta}{\rho}$. The others have to be determined.

Note a.—In the sequel it will sometimes happen that a vector, say a_q , originally assumed of order q , will be used to represent an harmonic of a different order, say $q+1$. In such a case it will be written $(a_q)_{q+1}$; thus

$$a_q = x_q \sin(q\omega t + c_q),$$

but

$$(a_q)_{q+1} = x_q \sin\{(q+1)\omega t + c_q\}.$$

Note b.—The length of a vector α will be written as $\bar{\alpha}$ (*i. e.* with the bar); thus $\bar{a}_3 = x_3$, unless in cases where no ambiguity can arise, when α simply will be written for the length of the vector α .

3. If we agree to indicate by ι^θ the operation of rotating any vector to which it is prefixed through an angle θ in the positive direction, then

$$\iota^\pi \alpha = -\alpha \quad \text{or} \quad \iota^\pi = -1,$$

and

$$\iota^\theta \alpha = (\cos \theta + \iota^{\frac{\pi}{2}} \sin \theta) \alpha \quad \text{or} \quad \iota^\theta = \cos \theta + \iota^{\frac{\pi}{2}} \sin \theta.$$

Also, if $t = D\iota f$, $t\alpha$ is the vector obtained by increasing α in length D times and then rotating the increased vector through an angle f in the positive direction.

Plane vector operators such as t are well known to be subject to the same rules as ordinary algebraical symbols.

Again, the sum of two operators $a_1 \iota^{\theta_1}$, $a_2 \iota^{\theta_2}$, can be expressed as a single operator $A \iota^\psi$, say, that is

$$A \iota^\psi \alpha = a_1 \iota^{\theta_1} \alpha + a_2 \iota^{\theta_2} \alpha,$$

where α is any vector.

Using the expression for ι^θ given above,

$$\begin{aligned} A(\cos \psi + \iota^{\frac{\pi}{2}} \sin \psi) \alpha \\ = a_1(\cos \theta_1 + \iota^{\frac{\pi}{2}} \sin \theta_1) \alpha + a_2(\cos \theta_2 + \iota^{\frac{\pi}{2}} \sin \theta_2) \alpha, \end{aligned}$$

so that

$$A \cos \psi = a_1 \cos \theta_1 + a_2 \cos \theta_2;$$

$$A \sin \psi = a_1 \sin \theta_1 + a_2 \sin \theta_2.$$

Hence

$$A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos(\theta_1 - \theta_2),$$

and

$$\tan \psi = \frac{a_1 \sin \theta_1 + a_2 \sin \theta_2}{a_1 \cos \theta_1 + a_2 \cos \theta_2}.$$

Again, if

$$\alpha_p = \xi_p \sin(p\omega t + \gamma_p),$$

then

$$\frac{d}{dt}(\alpha_p) = p\omega \iota^{\frac{\pi}{2}} \alpha_p,$$

seeing that

$$\frac{d}{dt}(\alpha_p) = p\omega \xi_p \sin\left(p\omega t + \gamma_p + \frac{\pi}{2}\right).$$

Hence for x and ξ as expressed in § 2

$$\frac{dx}{dt} = \omega \iota^{\frac{\pi}{2}} \sum q a_q, \quad \frac{d\xi}{dt} = \omega \iota^{\frac{\pi}{2}} \sum p a_p.$$

4. By means of the formula

$$2 \sin a \cos b = \sin(a+b) + \sin(a-b)$$

it is easy to show that $2x \cos \omega t$, where x is the a series of odd order vectors in § 2, is represented by the series of even order vectors of which the one of the p th order is the vector sum of a_{p-1} and a_{p+1} , or that

$$2x \cos \omega t = (a_1)_0 + (a_1 + a_3)_2 + (a_3 + a_5)_4 + (a_5 + a_7)_6 + \&c.,$$

$(a_1)_0$ being the resolved part of a_1 along the y axis, that is along the direction of vectors of zero order (see Note a, § 2).

Similarly

$$2\xi \cos \omega t = (\alpha_0 + \alpha_2)_1 + (\alpha_2 + \alpha_4)_3 + (\alpha_4 + \alpha_6)_5 + \&c.$$

Again, by means of the formula

$$2 \sin a \sin b = \cos (a-b) - \cos (a+b),$$

it is easy to show that

$$\begin{aligned} 2\xi \sin \omega t &= \iota^{-\frac{\pi}{2}}(\alpha_0 - \alpha_2)_1 + \iota^{-\frac{\pi}{2}}(\alpha_2 - \alpha_4)_3 + \iota^{-\frac{\pi}{2}}(\alpha_4 - \alpha_6)_5 + \&c. \\ &= \iota^{-\frac{\pi}{2}} \Sigma (\alpha_{q-1} - \alpha_{q+1})_q, \end{aligned}$$

where q is odd with a similar result for the product $2x \sin \omega t$.

5. If we now substitute the vector expressions from §§ 2, 3, 4, in equations I. and equate separately to zero each set of vector terms of the same order we obtain the two series of vector equations

$$ra_q + q\omega \iota^{\frac{\pi}{2}} \left\{ la_q + \frac{m}{2} (\alpha_{q-1} + \alpha_{q+1}) \right\} = 0, \quad (\text{IV.})$$

$$\rho \alpha_p + p\omega \iota^{\frac{\pi}{2}} \left\{ \lambda \alpha_p + \frac{m}{2} (a_{p-1} + a_{p+1}) \right\} = 0, \quad (\text{V.})$$

together with $\xi_0 = 2\eta/\rho$, where q is any odd number and p any even number.

From IV. we deduce a series of equations of the type

$$\alpha_{q-1} + 2 \frac{q\omega l - r\iota^{\frac{\pi}{2}}}{q\omega m} a_q + \alpha_{q+1} = 0,$$

or

$$\alpha_{q-1} + t_q a_q + \alpha_{q+1} = 0,$$

where t_q is the operator $D_q \iota^{-f_q}$, in which

$$D_q \cos f_q = 2 \frac{l}{m}, \quad D_q \sin f_q = \frac{2r}{q\omega m},$$

that is

$$D^2 = \frac{4}{m^2} \left(l^2 + \frac{r^2}{q^2 \omega^2} \right), \quad \tan f_q = \frac{r}{q\omega l}.$$

Similarly from V. we deduce the series

$$a_{p-1} + \tau_p \alpha_p + a_{p+1} = 0$$

where τ_p is the operator $\Delta_p \iota^{-\phi_p}$, in which

$$\Delta_p \cos \phi_p = 2 \frac{\lambda}{m}, \quad \Delta_p \sin \phi_p = \frac{2\rho}{p\omega m},$$

or

$$\Delta_p^2 = \frac{4}{m^2} \left(\lambda^2 + \frac{\rho^2}{p^2 \omega^2} \right), \quad \tan \phi_p = \frac{\rho}{p\omega \lambda}.$$

Note that the vector equations in this paragraph are equations connecting the different vectors, considered purely as vectors, without any reference to the order of the harmonic they originally represented.

6. We have thus obtained the following infinite series of equations connecting the vectors used to represent x and ξ :—

$$\left. \begin{aligned} t_1 a_1 + \alpha_2 &= -\alpha_0 \\ a_1 + \tau_2 \alpha_2 + a_3 &= 0 \\ \alpha_2 + t_3 a_3 + \alpha_4 &= 0 \\ a_3 + \tau_4 \alpha_4 + a_5 &= 0 \\ \alpha_4 + t_5 a_5 + \alpha_6 &= 0 \\ &\&c., \&c. \end{aligned} \right\} \quad (\text{VI.})$$

And as it is well known that algebraic methods are applicable to plane vector operators of the type here made use of, we obtain the following infinite determinant vector solution for a_1 , namely,

$$\left| \begin{array}{cccccc} t_1 & 1 & 0 & 0 & 0 & 0 \dots \\ 1 & \tau_2 & 1 & 0 & 0 & 0 \dots \\ 0 & 1 & t_3 & 1 & 0 & 0 \dots \\ 0 & 0 & 1 & \tau_4 & 1 & 0 \dots \\ 0 & 0 & 0 & 1 & t_5 & 1 \dots \\ 0 & 0 & 0 & 0 & 1 & \tau_6 \dots \\ & & & & & \&c., \&c. \end{array} \right| a_1 = - \left| \begin{array}{cccccc} \tau_2 & 1 & 0 & 0 & 0 & 0 \dots \\ 1 & t_3 & 1 & 0 & 0 & 0 \dots \\ 0 & 1 & \tau_4 & 1 & 0 & 0 \dots \\ 0 & 0 & 1 & t_5 & 1 & 0 \dots \\ 0 & 0 & 0 & 1 & \tau_6 & 1 \dots \\ & & & & & \&c., \&c. \end{array} \right| \alpha_0$$

or

$$P_1 a_1 = -\Pi_2 \alpha_0$$

where P_1 is the infinite determinant operator whose leading term is t_1 , and Π_2 that whose leading term is τ_2 .

7. P_1 , Π_2 , P_3 , Π_4 , &c., being the determinants whose leading terms are t_1 , τ_2 , t_3 , τ_4 , &c. respectively, we find at once by expanding that

$$P_1 = t_1 \Pi_2 - P_3$$

$$\Pi_2 = \tau_2 P_3 - \Pi_4, \&c.,$$

hence

$$\begin{aligned} \frac{P_1}{\Pi_2} &= t_1 - \frac{P_3}{\Pi_2} = t_1 - \frac{1}{\Pi_2/P_3} \\ &= t_1 - \frac{1}{\tau_2} - \frac{1}{t_3} - \frac{1}{\tau_4} - \&c. \\ &= S_1 (\text{say}), \end{aligned}$$

so that

$$a_1 = -\frac{1}{S_1} a_0,$$

where S_1 is the infinite continued fraction operator whose leading term is t_1 .

Again, if

$$\Sigma_2 = \tau_2 - \frac{1}{t_3} - \frac{1}{\tau_4} - \frac{1}{t_5} - \&c.$$

$$S_3 = t_3 - \frac{1}{\tau_4} - \frac{1}{t_5} - \frac{1}{\tau_6} - \&c.$$

$$\Sigma_4 = \tau_4 - \frac{1}{t_5} - \&c.,$$

and so on, we find in a similar way, or by making use of equations VI., that

$$a_2 = -\frac{1}{\Sigma_2} a_1$$

$$a_3 = -\frac{1}{S_3} a_2 \&c., \&c.$$

Hence for the complete solution we have

$$a_0 = -S_1 a_1 = S_1 \Sigma_2 a_2 = -S_1 \Sigma_2 S_3 a_3 = \&c.,$$

which gives $a_1, a_3, a_5, \&c., a_2, a_4, a_6, \&c.$, in terms of the known vector a_0 provided the continued fraction operators $S_1, \Sigma_2, S_3, \&c.$, are determinate.

It is easily seen that they are determinate for when q becomes a large number

$$t_q = 2 \frac{l}{m} = t_{q+2}$$

$$\tau_{q+1} = 2 \frac{\lambda}{m} = \tau_{q+3} \text{ (see § 5) ;}$$

that is, $S_1, \Sigma_2, S_3, \Sigma_4, \&c.$ are recurring continued fraction operators, and the recurring elements when reached are simple numbers.

8. The form of solution obtained is one very easy of practical application. In computing the continued fraction operators just so many of their known t, τ elements (see § 5) need be taken account of as are necessary to give the required degree of approximation.

Moreover, in the case of a practical alternator, as the resistance of the field-magnet coils is negligible relative to

their reactance, all the τ elements are practically simple numbers independent of the field resistance, while for the t elements q is never a large number when t_{q+2} differs little from t_q . So that we could obtain S_q with considerable accuracy by assuming the recurring stage to be reached, and therefore

$$S_q = t_q - \frac{1}{\tau_{q+1} - \frac{1}{S_q}},$$

which gives the quadratic in S_q

$$S_q^2 - t_q S_q + \frac{t_q}{\tau_{q+1}} = 0,$$

from which S_q can be obtained by ordinary algebra.

[In solving this quadratic the two operators that come under the square-root symbol will have to be reduced by the addition theorem in § 3 to a single operator, at^b say, and the square root of this is $\sqrt{at^{3b}}$.]

If S_q obtained in either of these ways be $= s_q t^{-b_q}$, then as

$$\Sigma_{q-1} = \tau_{q-1} - \frac{1}{S_q} = \tau_{q-1} - \frac{1}{s_q} t^{b_q},$$

Σ_{q-1} can be obtained by the addition theorem, and so on for S_{q-2} , Σ_{q-3} , &c., up to S_1 . Let the results be written

$$S_1 = s_1 t^{-b_1}, \quad \Sigma_2 = \sigma_2 t^{-\beta_2}, \quad S_3 = s_3 t^{-b_3}, \text{ \&c.,}$$

and in general

$$S_q = s_q t^{-b_q}, \quad \Sigma_p = \sigma_p t^{-\beta_p}.$$

9. As α_0 is the vector of length $2\eta/\rho$ lying along the axis of y (phase $= \pi/2$), and as

$$a_1 = -\frac{1}{S_1}(\alpha_0)_1 = -\frac{1}{s_1} t^{b_1}(\alpha_0)_1$$

$$a_2 = -\frac{2\eta}{s_1 \rho} \sin\left(\omega t + \frac{\pi}{2} + b_1\right) = \frac{2\eta}{s_1 \rho} \sin\left(\omega t - \frac{\pi}{2} + b_1\right).$$

Again, as

$$\alpha_2 = -\frac{1}{\Sigma_2}(a_1)_2 = -\frac{1}{\sigma_2} t^{\beta_2}(a_1)_2,$$

$$\alpha_2 = \frac{2\eta}{s_1 \sigma_2 \rho} \sin\left(2\omega t + \frac{\pi}{2} + b_1 + \beta_2\right);$$

similarly

$$\begin{aligned} a_3 &= \frac{2\eta}{s_1\sigma_2s_3\rho} \sin\left(3\omega t - \frac{\pi}{2} + b_1 + \beta_2 + b_3\right); \\ \alpha_4 &= \frac{2\eta}{s_1\sigma_2s_3\sigma_4\rho} \sin\left(4\omega t + \frac{\pi}{2} + b_1 + \beta_2 + b_3 + \beta_4\right); \\ a_5 &= \frac{2\eta}{s_1\sigma_2s_3\sigma_4s_5\rho} \sin\left(5\omega t - \frac{\pi}{2} + b_1 + \beta_2 + b_3 + \beta_4 + b_5\right); \\ &\quad \&c., \quad \&c. \end{aligned}$$

And substituting these values in

$$x = a_1 + a_3 + a_5 + \&c., \quad \xi = \frac{\eta}{\rho} + \alpha_2 + \alpha_4 + \&c.,$$

we obtain the armature and field currents in the usual trigonometrical form of expression.

It is worth while drawing attention to the fact that the period of the alternating current induced in the field circuit is half that of the armature current, and that it contains all harmonics, both odd and even, relative to its own fundamental, and so its wave-form will in general be unsymmetrical with respect to the time-axis.

10. The total E.M.F. E generated in the armature circuit being equal to $-\frac{d}{dt}(m\xi \cos \omega t)$,

$$\begin{aligned} E &= -\frac{m}{2} \frac{d}{dt} \sum (\alpha_{q-1} + \alpha_{q+1})_q, \quad (\text{see } \S 4) \\ &= -\frac{\omega m}{2} \iota^{\frac{\pi}{2}} \sum q (\alpha_{q-1} + \alpha_{q+1})_q. \end{aligned}$$

Also as

$$\begin{aligned} \alpha_{q-1} + \iota_q a_q + \alpha_{q+1} &= 0 \\ E &= \frac{\omega m}{2} \iota^{\frac{\pi}{2}} \sum q \iota_q a_q. \end{aligned}$$

And in either of these formulæ the trigonometrical expressions in § 9 for the vectors can be substituted.

In the first, however, it must be noted that both α_{q-1} and α_{q+1} are to be taken of order q (odd). Thus the fundamental harmonic of E is

$$= \frac{\omega m \eta}{\rho} \left\{ \sin \omega t + \frac{1}{s_1\sigma_2} \sin (\omega t + b_1 + \beta_2) \right\}$$

from the first expression, or

$$= \frac{\omega m \eta}{\rho} \frac{D_1}{s_1} \sin (\omega t + \mathbf{b}_1 - f_1)$$

from the second, as $t_1 = D_1 t - f_1$.

Similarly the total alternating E.M.F. (H say) generated in the field circuit is given by either

$$H = -\frac{\omega m}{2} i^{\frac{\pi}{2}} \Sigma p (\mathbf{a}_{p-1} + \mathbf{a}_{p+1})_p$$

$$\text{or} \quad H = \frac{\omega m}{2} i^{\frac{\pi}{2}} \Sigma p \tau_p \alpha_p,$$

so that the fundamental harmonic of H is equal to either

$$\frac{2\omega m \eta}{\rho} \left[\frac{1}{s_1} \sin (2\omega t + \mathbf{b}_1 + \pi) + \frac{1}{s_1 \sigma_2 s_3} \sin (2\omega t + \mathbf{b}_1 + \beta_2 + \mathbf{b}_3 + \pi) \right]$$

$$\text{or} \quad \frac{2\omega m \eta}{\rho} \frac{\Delta_2}{s_1 s_2} \sin (2\omega t + \mathbf{b}_1 + \beta_2 - \phi_2 + \pi).$$

11. The mean value of the product

$$\sin (a\omega t + \theta) \sin (b\omega t + \phi)$$

being zero when a and b are unequal and $\frac{1}{2} \cos (\theta - \phi)$ when a and b are equal, we find that the mean value of x^2 where $x = \Sigma \mathbf{a}_q$ is

$$= \frac{1}{2} \Sigma \bar{\mathbf{a}}_q^2;$$

and the mean value of ξ^2 , where $\xi = \frac{\alpha}{2} + \Sigma \alpha_p$, is

$$= \frac{\bar{\alpha}_0^2}{4} + \frac{1}{2} \Sigma \bar{\alpha}_p^2.$$

Again, for the same reason, if α and β be any two vectors representing harmonics of the same order and if $S\alpha\beta$ be the product of the lengths of α and β into the sine of the angle from α to β measured in the positive direction, then the mean value of the product

$$i^{\frac{\pi}{2}} \alpha \text{ into } \beta$$

$$\text{is} \quad = \frac{1}{2} S\alpha\beta = -\frac{1}{2} S\beta\alpha.$$

Applying these principles to the determination of the mean value $\overline{Ex}$ of the product of E and x , that is of the

electrical power developed in the armature circuit, we find from the first expression for E in § 10 that

$$\begin{aligned}\overline{Ex} &= -\frac{\omega m}{4} \sum q S(\alpha_{q-1} + \alpha_{q+1}) \mathbf{a}_q \\ &= -\frac{\omega m}{4} \{S\alpha_0 \mathbf{a}_1 - S\mathbf{a}_1 \alpha_2 + 3S\alpha_2 \mathbf{a}_3 - 3S\mathbf{a}_3 \alpha_4 + 5S\alpha_4 \mathbf{a}_5 - 5S\mathbf{a}_5 \alpha_6 + \&c.\},\end{aligned}$$

and from the second expression for E that

$$\begin{aligned}\overline{Ex} &= \frac{\omega m}{4} \sum q S(t_q \mathbf{a}_q \cdot \mathbf{a}_q \cdot) = \frac{\omega m}{4} \sum q D_q S(\iota^{-fq} \mathbf{a}_q \cdot \mathbf{a}_q \cdot) \\ &= \frac{\omega m}{4} \sum \bar{\mathbf{a}}_q^2 D_q \sin f_q = \frac{1}{2} r \sum \bar{\mathbf{a}}_q\end{aligned}$$

$$\text{as } D_q \sin f_q = \frac{2r}{q\omega m} \quad (\text{see § 5})$$

= the heat developed in the armature circuit.

Similarly the total electrical power $(\overline{H + \eta})\xi$ developed in the field circuit is given by

$$\begin{aligned}\overline{(H + \eta)\xi} &= \frac{\eta \alpha_0}{2} - \frac{\omega m}{4} \sum p S(\mathbf{a}_{p-1} + \mathbf{a}_{p+1}) \alpha_p \\ &= \rho \frac{\bar{\alpha}_0^2}{4} - \frac{\omega m}{4} \{2S\mathbf{a}_1 \alpha_2 - 2S\alpha_2 \mathbf{a}_3 + 4S\mathbf{a}_3 \alpha_4 - 4S\alpha_4 \mathbf{a}_5 + \&c.\}\end{aligned}$$

or by

$$\overline{(H + \eta)\xi} = \frac{\eta \alpha_0}{2} + \frac{\omega m}{4} \sum p S(\tau_p \alpha_p \cdot \alpha_p \cdot)$$

which reduces to

$$\overline{(H + \eta)\xi} = \rho \frac{\bar{\alpha}_0^2}{4} + \frac{\rho}{2} (\bar{\alpha}_2^2 + \bar{\alpha}_4^2 + \bar{\alpha}_6^2 + \&c.)$$

= total heat developed in the field circuit

and is made up of two parts, the first $= \rho \frac{\bar{\alpha}_0^2}{4}$ due to the direct exciting current and the second $= \frac{\rho}{2} (\bar{\alpha}_2^2 + \bar{\alpha}_4^2 + \bar{\alpha}_6^2 + \&c.)$ due to the induced alternating field-current.

Adding the first expressions for $\overline{Ex}$ and $\overline{(H + \eta)\xi}$ and cancelling $\eta \bar{\xi}$ against $\rho \frac{\bar{\alpha}_0^2}{4}$ we find that

$$\overline{Ex} + \overline{H\xi} = -\frac{\omega m}{4} \{S\alpha_0 \mathbf{a}_1 + S\mathbf{a}_1 \alpha_2 + S\alpha_2 \mathbf{a}_3 + S\mathbf{a}_3 \alpha_4 + \&c.\}.$$

12. The torque exerted at any instant in driving the alternator is

$$\begin{aligned}
 &= -x\xi \frac{d}{d(\omega t)} \{m \cos \omega t\} = m x \xi \sin \omega t \\
 &= \frac{m}{2} \sum a_q \times i^{-\frac{\pi}{2}} \sum (\alpha_{q-1} - \alpha_{q+1})_q \quad (\text{see } \S 4) \\
 &= -\frac{m}{2} \sum a_q \times i^{\frac{\pi}{2}} \sum (\alpha_{q-1} - \alpha_{q+1})_q.
 \end{aligned}$$

Taking the mean value of this product we find that the mean driving torque T is given by

$$T = -\frac{m}{4} \{S\alpha_0 a_1 + S a_1 \alpha_2 + S \alpha_2 a_3 + S a_3 \alpha_4 + \&c.\},$$

which result, combined with the last one obtained in § 11, gives the power equation

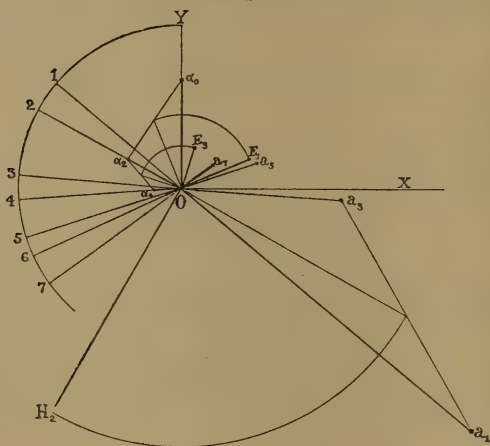
$$\omega T = \overline{Ex} + \overline{H\xi}$$

as it ought.

13. The solution obtained can be represented geometrically in an interesting way as follows:—

Take two lines OX, OY, fig. 2, at right angles. Measure

Fig. 2.



off from OY in the positive direction the angles YO1= β_1 , 1O2= β_2 , 2O3= β_3 , 3O4= β_4 , &c., where β_1 , β_2 , β_3 , β_4 , &c., are the angles determined in § 8.

In OY take $O\alpha_0 = 2\eta/\rho = 2 \times$ exciting current. Produce 1O through O to a_1 so that $Oa_1 = \frac{O\alpha_0}{s_1}$. In O2 take $O\alpha_2 = Oa_1/\sigma_2$. Produce 3O through O to a_3 so that $Oa_3 = \frac{O\alpha_2}{s_3}$, and so on where $s_1, \sigma_2, s_3, \sigma_4, \&c.$, are the quantities determined in § 8.

Then the vectors to $a_1, a_3, a_5, \&c.$, represent completely in amplitude and phase the different harmonics of the armature current, the subscribed numbers indicating the orders of the harmonics; and those to $\alpha_2, \alpha_4, \alpha_6, \&c.$, represent completely in the same way the different harmonics of the induced alternating field current.

Again (see § 10), if we rotate the vector drawn to the middle point of $\alpha_0\alpha_2$ backwards through a right angle, we obtain the vector OE_1 that represents $\frac{1}{m\omega}$ into the first harmonic of the total E.M.F. E generated in the armature; and if we rotate backwards through $\pi/2$ the vector to the middle point of $\alpha_2\alpha_4$ we obtain the vector OE_3 that represents $\frac{1}{3\omega m}$ into the third harmonic of E; and similarly for the other harmonics of E.

In the same way, by rotating backwards through $\pi/2$ the vector to the middle point of a_1a_3 , we obtain the vector OH_2 that represents $\frac{1}{2\omega m}$ into the fundamental harmonic of the E.M.F. H induced in the field circuit; and so on for the other harmonics of H.

Again, as $S\alpha\beta$ is = twice the area of the triangle whose sides are α and β and is positive if β follows α in rotation order in the diagram, the mean torque exerted on the generator by the driver (see § 12) is equal to $\frac{1}{2}m$ into the sum of the areas of the triangles $\alpha_0Oa_1, a_1O\alpha_2, \alpha_2Oa_3, a_3O\alpha_4, \&c.$, these triangles, in the case of any generator, being all taken as positive.

14. When, for any generator, the t, τ operators have been calculated for a particular load (see § 5), a geometrical solution can easily be obtained to a high degree of accuracy by aid of a ruler, scale, slide-rule, and protractor.

Thus if we neglect the harmonics $\alpha_8, a_9, \alpha_{10}, \&c.$, then, drawing any vector from the origin to represent a_7 we can construct for α_6 as $\alpha_6 = -t_7a_7$ (see § 6). From α_6 we can construct for $\tau_6\alpha_6$ and as $a_5 + \tau_6\alpha_6 + a_7 = 0$ the triangle of vectors

gives us $\mathbf{a}_5$. Proceeding in this way, we obtain in succession $\alpha_4, \mathbf{a}_3, \alpha_2, \mathbf{a}_1, \alpha_0$, which represent the harmonics of x and ξ correctly as regards relative phase and relative amplitude. But as α_0 must be equal to twice the exciting current we have a scale for our diagram, and hence obtain a complete solution.

The fact that for a practical alternator the τ operators may be taken as pure numbers (see §§ 8, 22), renders this method of solution both easy and expeditious.

15. If a source of constant E.M.F. be included in the armature circuit as well as in the field circuit of the simple alternator indicated in fig. 1, equations I. § 1 become

$$rx + \frac{d}{dt}(lx + m\xi \cos \omega t) = e$$

$$\rho\xi + \frac{d}{dt}(\lambda\xi + mx \cos \omega t) = \eta,$$

and both the armature and field currents will now contain harmonics of all orders, odd and even. In this case we assume that

$$x = \frac{\mathbf{a}_0}{2} + \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 + \&c.$$

$$\xi = \frac{\alpha_0}{2} + \alpha_1 + \alpha_2 + \alpha_3 + \&c.,$$

where $\mathbf{a}_0$ is the vector to the point whose polar coordinates are $2e/r$, $\pi/2$, and α_0 is, as before, the vector to the point $2\eta/\rho$, $\pi/2$. The other vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$, &c., $\alpha_1, \alpha_2, \alpha_3$, &c., have to be determined.

On substituting for x and ξ in the above equations it will be found that the odd order vectors in x and the even order ones in ξ are determined by the same equations (IV. § 6) as when $e=0$, and are completely independent of the even order vectors in x and the odd order ones in ξ , these latter depending only on e and vanishing with e .

This being so, $\mathbf{a}_1, \alpha_2, \mathbf{a}_3, \alpha_4$, &c., are given by the solution already obtained, and $\alpha_1, \mathbf{a}_2, \alpha_3, \mathbf{a}_4$, &c., will be given by a similar solution the equations expressing which may be written down from symmetry.

Thus the complete solution is given by

$$\alpha_0 = -S_1\mathbf{a}_1 = S_1\Sigma_2\alpha_2 = -S_1\Sigma_2S_3\mathbf{a}_3 = \&c.,$$

$$\mathbf{a}_0 = -\Sigma_1\alpha_1 = \Sigma_1S_2\mathbf{a}_2 = -\Sigma_1S_2\Sigma_3\alpha_3 = \&c.,$$

where α_0 is the vector to $2\eta/\rho$, $\pi/2$, as before,

and $\mathbf{a}_0$ is the vector to $2e/r$, $\pi/2$.

Note.—In the former case ($e=0$) the t operators were all of odd and the τ ones of even orders. In this case the operators of either class are of both orders.

The translation from the above vector solution to the ordinary sine form follows as in § 10.

16. In the preceding solutions the magnetic fluxes have been assumed to be in phase with the magnetizing current-turns, and so iron loss due to hysteresis and eddy currents has been neglected. To take account of the latter the interpretation of the well-known relation $B=\mu H$ connecting steady magnetizing force and induction produced has to be modified.

The induction produced by $H=H_1 \sin(\omega t + c_1)$ is known to be of the form

$$B = m_1 H_1 \sin(\omega t + c_1 - \delta_1) + \text{higher harmonics,}$$

and attending only to the fundamental harmonic in B , if H be represented as explained in § 2 by the vector h_1 , and B by the vector b_1 , then the above trigonometrical relation may be written

$$b_1 = \mu_1 h_1$$

where μ_1 is the operator $m_1 \epsilon^{-\delta_1}$.

In a former paper* by me was shown how these *permeability operators*, as they may be called, can be determined. They depend on the character of the iron and the thickness of the laminæ, on the amplitude and period of the fundamental harmonic of the induction oscillation they refer to, and to some extent on the wave form of the latter.

For the purposes of the following discussion we will assume that when the magnetizing force

$$H = h_1 + h_3 + h_5 + \&c.$$

produces the induction

$$B = b_1 + b_3 + b_5 + \&c.,$$

then $b_1 = \mu_1 h_1$, $b_3 = \mu_3 h_3$, $b_5 = \mu_5 h_5$, &c., where the μ 's are operators of the type given by

$$\mu_q = m_q \epsilon^{-\delta_q}.$$

[This assumption as regards all the harmonics of B but the fundamental is not strictly in accordance with what is known concerning the behaviour of laminated iron under periodic magnetizing forces, for b_3 , b_5 , &c., depend, at any

* "Variation of Magnetic Hysteresis with Frequency," Phil. Mag. Jan. 1905.

rate for high values of b_1 , more on b_1 than on $h_3, h_5, \&c.$ At the same time it is hoped that the following discussion may be of some value.]

In general if $H = \Sigma h_q$ produce $B = \Sigma b_q$, as the total iron loss per c.c. per cycle due to both hysteresis and eddy currents is

$$\frac{1}{4\pi} \int H dB = \frac{1}{4\pi} \int_0^T H \frac{dB}{dt} dt,$$

where T is the period, the total iron loss per c.c. per second is

$$= \frac{1}{4\pi} \cdot \text{Average value of product } H \frac{dB}{dt}$$

$$= \frac{1}{4\pi} \cdot \text{Av. product } \Sigma h_q \text{ into } i^2 \Sigma q \omega b_q$$

$$= \frac{\omega}{8\pi} \{Sb_1h_1 + 3Sb_3h_3 + 5Sb_5h_5 + \&c.\}$$

(See § 11)

$$= \frac{\omega}{8\pi} \Sigma q \bar{b}_q \bar{h}_q \sin \delta_q.$$

Again, it is well known that if the steady magnetizing current-turns nx act on a magnetic circuit composed of different materials, the flux F produced is given by

$$F = \frac{4\pi nx}{\Sigma \frac{L}{A\mu}},$$

where the L, A 's are the lengths and sectional areas of the different portions of the circuit, and the μ 's are the permeabilities of these portions for the particular flux densities in them.

If now the magnetizing current be an alternating one, that is if $x = x_1 \sin(\omega t + c_1) = a_1$ (a vector) the same equation will give the corresponding harmonic of the flux produced, but the μ 's are now the permeability operators, for the different portions of the circuit.

$\Sigma \frac{L}{A\mu}$ can, by the addition theorem in § 3, be reduced to a single operator so that if the flux f_1 (vector) be produced in any magnetic circuit by the current-turns na_1 we have always a relation of the form

$$f_1 = G_1 na_1$$

where G_1 is an operator of the form $g_1 e^{-\delta}$ which can be determined.

Hence, following the assumption already made, if the magnetizing current-turns

$$nx = n(a_1 + a_3 + a_5 + \&c.)$$

produce in a magnetic circuit the flux

$$F = f_1 + f_3 + f_5 + \&c.$$

then

$$f_1 = nG_1a_1, f_3 = nG_3a_3, \&c.$$

where the G's are operators of the type given by $G_q = g_q \iota^{-\delta_q}$.

In the above the back E.M.F., e say, in the magnetizing coils due to variation of flux is

$$e = n \frac{dF}{dt} = n\omega \iota^{\frac{\pi}{2}} \Sigma q f_q$$

and the power absorbed, that is the total iron loss per sec. in the magnetic circuit, is the mean value of ex , that is of the product

$$\Sigma a_q \text{ into } n\omega \iota^{\frac{\pi}{2}} \Sigma q f_q$$

which (see § 11)

$$= \frac{1}{2} n\omega \Sigma (q S f_q \cdot a_q.)$$

$$= \frac{1}{2} \omega \Sigma \left(q \frac{\bar{f}_q^2}{g_q} \sin \delta_q \right).$$

17. As an example let us determine the G operator for a magnetic circuit of uniform cross section = 100 cm.², made up of 40 cm. length of laminated iron and two air-gaps each 1 mm. when B maximum = 5000 and the frequency 30.

In the paper already quoted we find for a sample of No. 26 iron well insulated between the laminæ when $B_{\max} = 5000$ and frequency = 30 q.p. that

$$\mu = 2500 \iota^{-50^\circ} \quad (\text{nearly}).$$

Now

$$G = \frac{4\pi}{\Sigma \frac{L}{A\mu}}$$

and

$$\begin{aligned} \Sigma \frac{L}{A\mu} &= \frac{1}{100} \left\{ \frac{40}{2500} \iota^{50^\circ} + .2 \right\} \\ &= \frac{1}{10^5} \left\{ 16 \iota^{50^\circ} + 200 \right\} \\ &= \frac{2106}{10^4} \iota^{3^\circ 18'}. \end{aligned}$$

Hence

$$G = 5968 \iota^{-3^\circ 18'}.$$

18. Returning to the alternator, if n be the number of armature turns, ν the number of field turns, and

$$x = a_1 + a_3 + a_5 + \&c., \quad \xi = \frac{\alpha_0}{2} + \alpha_2 + \alpha_4 + \&c.,$$

the armature and field currents respectively, the magnetizing current-turns M_x producing flux across the air-gap and through the armature in a direction axial to its windings are given by

$$M_x = nx + \nu\xi \cos \omega t,$$

and the current-turns M_y producing flux across the air-gap and through the armature in a direction parallel to the planes of the windings, and *behind* that of M_x by 90° , are given by

$$M_y = \nu\xi \sin \omega t,$$

or in vector notation (see § 4)

$$M_x = \Sigma \left[n a_q + \frac{\nu}{2} (\alpha_{q-1} + \alpha_{q+1})_q \right]$$

$$M_y = \frac{\nu}{2} \iota^{-\frac{\pi}{2}} \Sigma (\alpha_{q-1} - \alpha_{q+1})_q,$$

which produce the armature fluxes A_x and A_y given by

$$A_x = \Sigma G_q \left[n a_q + \frac{\nu}{2} (\alpha_{q-1} + \alpha_{q+1})_q \right]$$

$$A_y = \frac{\nu}{2} \iota^{-\frac{\pi}{2}} \Sigma G_q (\alpha_{q-1} - \alpha_{q+1})_q,$$

where $G_q = g_q \iota^{-\delta q}$ and q any odd number.

[Note that the directions of A_x and A_y are fixed in the armature.]

Now if magnetic leakage be otherwise taken account of, the flux in the stator must be continuous with that in the rotor so that the flux F looped on the field-windings at any instant is given by

$$F = A_x \cos \omega t + A_y \sin \omega t,$$

which by means of the relations in § 4 can be reduced to

$$F = \Sigma \left[\nu G_p a_p + \frac{n}{2} (G_{p-1} a_{p-1} + G_{p+1} a_{p+1})_p \right]$$

where p is even and $2G_p = G_{p-1} + G_{p+1}$.

19. If l' be the self-inductance in the armature circuit either external to the armature or due to magnetic leakage in it, and if λ' be a similar quantity for the field circuit, the equations for the two circuits are

$$rx + l' \frac{dx}{dt} + n \frac{d}{dt} A_x = 0$$

$$\rho \xi + \lambda' \frac{d\xi}{dt} + \nu \frac{d}{dt} F = \eta$$

where r, ρ, η, x, ξ , have the same significations as in § 1.

Substituting in these equations from § 18, and then equating separately to zero each set of vector terms of the same order, we obtain the two series of vector equations,

$$ra_q + q\omega l' \iota^{\frac{\pi}{2}} a_q + nq\omega \iota^{\frac{\pi}{2}} G_q \left\{ na_q + \frac{\nu}{2} (\alpha_{q-1} + \alpha_{q+1}) \right\} = 0$$

$$\rho \alpha_p + p\omega \lambda' \iota^{\frac{\pi}{2}} \alpha_p + \nu p\omega \iota^{\frac{\pi}{2}} \left\{ \nu G_p \alpha_p + \frac{n}{2} (G_{p-1} a_{p-1} + G_{p+1} a_{p+1}) \right\} = 0$$

with $\alpha_0 = \frac{2\eta}{\rho}$, where q is odd and p even.

These reduce at once to the two series,

$$\alpha_{q-1} + t_q G_q a_q + \alpha_{q+1} = 0$$

$$G_{p-1} a_{p-1} + \tau_p \alpha_p + G_{p+1} a_{p+1} = 0$$

or, after putting a'_q for $G_q a_q$, to

$$\alpha_{q-1} + t_q a'_q + \alpha_{q+1} = 0$$

$$a'_{p-1} + \tau_p \alpha_p + a'_{p+1} = 0$$

equations of exactly the same form as those for the simple case but in which the t and τ operators are now given by

$$t_q = \frac{2}{n\nu G_q^2} \left\{ n^2 G_q + l' - \frac{r}{q\omega} \iota^{\frac{\pi}{2}} \right\}$$

$$\tau_p = \frac{2}{n\nu} \left\{ \nu^2 G_p + \lambda' - \frac{\rho}{p\omega} \iota^{\frac{\pi}{2}} \right\}.$$

These operators having been calculated from known data, the solution for a'_1, a'_3, a'_5 , &c., α_2, α_4 , &c., proceeds exactly as in the simple case, and as $a'_1 = G_1 a_1$, $a'_3 = G_3 a_3$, &c., a_1, a_3, a_5 , &c., can then be obtained.

20. In § 16 it was shown that the iron loss (*i. e.* energy dissipated per sec. in the iron) in a magnetic circuit is

$$\frac{1}{2}\omega\Sigma q\frac{\overline{f_q^2}}{g_q}\sin\delta_q;$$

hence the loss due to the flux A_x § 18 is

$$\frac{1}{2}\omega\Sigma qg_q\sin\delta_q\overline{na_q+\frac{\nu}{2}(\alpha_{q-1}+\alpha_{q+1})}^2$$

and that due to the flux A_y is

$$\frac{1}{2}\omega\Sigma qg_q\sin\delta_q\overline{\frac{\nu^2}{4}(\alpha_{q-1}-\alpha_{q+1})}^2$$

Adding these we find that the total iron loss in the generator is

$$\frac{1}{2}\omega\Sigma qg_q\sin\delta_q\left\{\overline{na_q+\frac{\nu}{2}(\alpha_{q-1}+\alpha_{q+1})}^2+\frac{\nu^2}{4}\overline{(\alpha_{q-1}-\alpha_{q+1})}^2\right\}.$$

Expanding and remembering that

$$\overline{\alpha+\beta}^2=\overline{\alpha}^2+\overline{\beta}^2+2\overline{\alpha\beta}\cos\hat{\alpha\beta},$$

and that

$$\alpha_{q-1}+\alpha_{q+1}=-t_qG_qa_q \quad (\text{see § 19})$$

we find that the total iron loss is equal to

$$\Sigma\left[\frac{1}{2}q\omega g_q\sin\delta_q\left\{\frac{\nu^2}{2}(\overline{\alpha_{q-1}^2}+\overline{\alpha_{q+1}^2})-\overline{n^2a_q^2}\right\}-\left\{r\sin^2\delta_q+q\omega l'\sin\delta_q\cos\delta_q\right\}\overline{a_q^2}\right]$$

where q is odd.

21. An approximate determination of the effect of iron loss on the performance of an alternator can be obtained by taking all the G operators for its magnetic circuit as equal to G_1 , that is, equal to the one for the fundamental harmonic of the armature flux.

Making this simplification in the equations of § 19, we find that $a_1, a_3, a_5, \&c., \alpha_0, \alpha_2, \&c.$, are connected by the two series of equations

$$\alpha_{q-1}+t_qa_q+\alpha_{q+1}=0$$

$$a_{p-1}+\tau_pa_p+a_{p+1}=0$$

with $\alpha_0 = 2\eta/\rho$, in which

$$t_q = \frac{2}{nvG} \left\{ n^2G + l' - \frac{r}{q\omega} i^{\frac{\pi}{2}} \right\} = \frac{2}{nv} \left\{ n^2 + \frac{l'}{g} i^{\delta} - \frac{r}{q\omega g} i^{\frac{\pi}{2} + \delta} \right\}$$

$$\tau_p = \frac{2}{nvG} \left\{ \nu^2G + \lambda' - \frac{\rho}{p\omega} i^{\frac{\pi}{2}} \right\} = \frac{2}{nv} \left\{ \nu^2 + \frac{\lambda'}{g} i^{\delta} - \frac{\rho}{p\omega g} i^{\frac{\pi}{2} + \delta} \right\}$$

as $G = g i^{-\delta}$.

Putting l for gn^2 , λ for $g\nu^2$, m for gnv , and—remembering that δ is a small angle (see § 17)—unity for $\cos \delta$, we find that

$$t_q = D_q i^{-f_q}, \quad \tau_p = \Delta_p i^{-\phi_p},$$

where

$$D_q^2 = \frac{4}{m^2} \left\{ (l + l')^2 + \frac{r^2}{q^2 \omega^2} + 2 \frac{lr}{q\omega} \sin \delta \right\}$$

$$D_q \sin f_q = \frac{2r}{q\omega m} - \frac{2l'}{m} \sin \delta$$

$$\Delta_p^2 = \frac{4}{m^2} \left\{ (\lambda + \lambda')^2 + \frac{\rho^2}{p^2 \omega^2} + 2 \frac{\lambda\rho}{p\omega} \sin \delta \right\}$$

$$\Delta_p \sin \phi_p = \frac{2\rho}{p\omega m} - \frac{2\lambda'}{m} \sin \delta.$$

The t and τ operators having been calculated from these formulæ, the rest of the solution for this case follows in every particular the course for the simple case fully explained in §§ 8, 9.

22. In order to illustrate the practical application of the foregoing theory, I will determine the performance of a small two-pole alternator when carrying a rather heavy non-inductive load.

The details of the alternator are as follows :—

$$\text{Armature} \begin{cases} \text{diameter 12 cm.} \\ \text{length 8 cm.} \\ \text{turns } n=100. \\ \text{resistance } \cdot 25 \text{ ohm.} \end{cases}$$

$$\text{Air-gap} = 1 \text{ mm.}$$

$$\text{Field} \begin{cases} \text{turns } \nu=400. \\ \text{resistance } \rho=3 \text{ ohms.} \\ \text{exciter, three storage-cells ; } r=6\cdot 6 \text{ volts.} \end{cases}$$

$$\text{Frequency } 100/\pi, \text{ i. e. } \omega=200.$$

$$\text{Magnetic leakage}=5 \text{ per cent.}$$

$$\text{Flux operator } G=5000 i^{-3^\circ}.$$

Let the external resistance in the armature circuit in the case in hand be 4.75 ohms so that $r=5$ ohms.

Hence (see § 21)

$$\begin{aligned} l &= 5 \cdot 10^7, & l' &= \cdot 05 l = \cdot 25 \cdot 10^7 \\ \lambda &= 8 \cdot 10^8, & \lambda' &= \cdot 05 \lambda = 4 \cdot 10^7 \\ m &= 2 \cdot 10^8, & r &= 5 \cdot 10^9, & \rho &= 3 \cdot 10^9 \\ \omega &= 200, & \delta &= 3^\circ & \text{and} \\ \bar{\alpha}_0 &= 2\eta/\rho = \cdot 44 \text{ (absolute).} \end{aligned}$$

Using these values for the constants and the formulæ in § 21 we obtain the t and τ operators which are given in the following table:—

$t_q = D_q t - f_q$			$\tau_p = \Delta_p t - \phi_p$		
q .	D_q .	f_q .	p .	Δ_p .	ϕ_p .
1	·592	24° 51'	2	8.4	0° 22'
3	·535	8 48	4	8.4	8'
5	·530	5 18	6	8.4	1'
7	·528	3 45	8	8.4	—1'
9	·527	2 54	10	8.4	—2'
11	·526	2 21	12	8.4	—3'
13	·526	1 57	14	8.4	—4'
15	·526	1 40	16	8.4	—5'
17	·526	1 28	18	8.4	—5'

[Note that the τ operators are all practically equal and simple numerical multipliers (see § 8).]

And from these, by the method explained in § 8 we obtain the S , Σ , continued fraction operators which are given in the following table:—

$S_q = s_q t - b_q$			$\Sigma_p = \sigma_p t - \beta_p$		
q .	s_q .	b_q .	p .	σ_p .	β_p .
1	·457	36° 32'	2	5.82	7° 46'
3	·366	15 32	4	5.63	5 8
5	·353	9 50	6	5.57	3 40
7	·350	7 9	8	5.55	2 54
9	·348	5 34	10	5.53	2 18
11	·346	4 30	12	5.51	1 51
13	·346	3 42	14	5.50	1 21

And from these as explained in § 9 we obtain the different harmonics of both the armature current and the induced field current. These also are given in tabular form:—

$x = \sum x_q \sin(q\omega t - \frac{\pi}{2} + c_q).$			$\xi = .22 + \sum \xi_p \sin(p\omega t + \frac{\pi}{2} + \gamma_p).$		
$q.$	$x_q.$	$c_q.$	$p.$	$\xi_p.$	$\gamma_p.$
1	.963	36° 32'	2	.166	44° 18'
3	.453	59 50	4	.080	64 58
5	.228	74 48	6	.041	78 28
7	.117	85 37	8	.021	88 31
9	.061	94 5	10	.011	96 23
11	.032	100 53	12	.006	102 44
13	.017	106 26	14	.003	107 47

The virtual armature current in amperes being equal to $10 \sqrt{\frac{1}{2} \sum x_q^2}$ is 7.75 amps., and the terminal virtual voltage being 4.75 times this is 36.8 volts. The no-load voltage for the same exciting current is 62.2.

The virtual value of the alternating current induced in the field circuit in amperes being equal to $10 \sqrt{\frac{1}{2} \sum \xi_p^2}$ is 1.34.

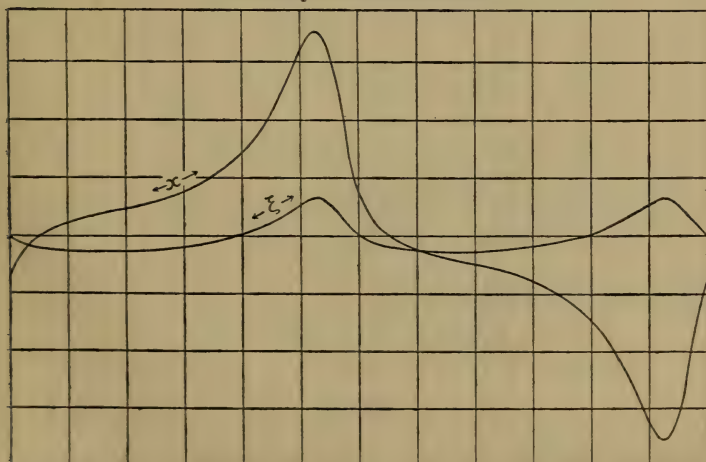
The copper losses are:—

In the armature.....	15 watts.
In the field due to exciting current...	12 „
In the field due to induced current...	5.4 „

The total iron loss calculated by means of the formula in § 20 is 20 watts.

The total losses are therefore 52.4 watts, and as the output is 285 watts the efficiency is 84 per cent.

Fig. 3.—Theoretical armature current (x), and induced field-current (ξ), in a fully loaded alternator.



In fig. 3 are plotted the wave forms of both the armature

and induced field-currents determined above, correctly as regards both their relative amplitudes and phases.

23. In designing the field of an alternator attention should be given to the fact that the conductor has to carry not only the exciting current, but also the induced field-current, which, as we have seen, may at full load attain a relatively large value. In addition it should not be forgotten that in the field-magnet cores there is the associated alternating flux which causes some additional heat.

It is well known that in a case of excessive heating in the field, reduction of the heating is effected by the employment of heavy closed copper conductors, called dampers, embracing the field-magnet poles.

To explain this action, let us consider a two-pole machine on each field pole of which is a damper.

Neglecting magnetic leakage and iron loss, if ζ be the current in each damper, the magnetic flux through the armature windings is

$$g\{nx + (\nu\xi + 2\zeta) \cos \omega t\},$$

and that through the field windings and the dampers is

$$g\{\nu\xi + 2\zeta + nx \cos \omega t\},$$

so that the equations connecting x , ξ , and ζ are

$$\left. \begin{aligned} rx + gn \frac{d}{dt} \{nx + (\nu\xi + 2\zeta) \cos \omega t\} &= 0, \\ \rho\xi + gv \frac{d}{dt} \{\nu\xi + 2\zeta + nx \cos \omega t\} &= \eta, \\ z\zeta + g \frac{d}{dt} \{\nu\xi + 2\zeta + nx \cos \omega t\} &= 0, \end{aligned} \right\} \quad \text{. . . (VII.)}$$

where z is the resistance of each damper and the other symbols have the same significations as in the previous sections of this paper.

Obviously there is no constant term in ζ , and considering only the variable terms (harmonics) in ξ , we see at once that

$$\nu z \zeta = \rho \xi,$$

from which it follows that

$$\nu z + 2\zeta = \nu\xi(1 + \kappa) = \nu\xi' \text{ say,}$$

where

$$\kappa = \frac{2\rho}{\nu^2 z},$$

and that

$$2z\zeta^2 = \kappa\rho\xi^2.$$

The first two of equations VII. may now be written

$$rx + gn \frac{d}{dt} \{nx + v\xi' \cos \omega t\} = 0,$$

$$\frac{\rho}{1+\kappa} \xi' + gv \frac{d}{dt} \{v\xi' + nx \cos \omega t\} = \frac{\eta}{1+\kappa},$$

η being divided by $1+\kappa$ as the constant term in ξ' is equal to the constant term in ξ (ξ having no constant term).

Now x and ξ' determined from these equations will be very approximately the same as x and ξ determined from the equations in § 1 for the alternator without dampers; for α_0 the given vector is the same for both, as are also all the t operators.

The τ operators differ in that for ρ , $\rho/(1+\kappa)$ is substituted, but in § 8 and in note § 22 it is shown that the τ operators for generators as ordinarily constructed are practically independent of the value of ρ the field resistance.

Hence we see that ξ' is the alternating field current if the dampers are absent, ξ its value when the dampers are attached and these currents are connected by the relation

$$\xi' = (1+\kappa)\xi.$$

In addition as $2z\xi^2 = \kappa\rho\xi'^2$,

$$\rho\xi^2 = \frac{2}{\kappa} z\xi^2 = \frac{1}{(1+\kappa)^2} \rho\xi'^2.$$

Hence if H' be the copper loss in the field coils due to induced alternating current when the generator is without dampers, H the same when dampers are attached, and h the loss in the dampers when attached,

$$H = \frac{H'}{(1+\kappa)^2}, \quad h = \frac{\kappa H'}{(1+\kappa)^2}, \quad H+h = \frac{H'}{1+\kappa}.$$

If we assume that the mean length of a field turn is equal to the length of a damper turn, it is easy to show that κ is the ratio of the volume of copper in the dampers to the volume of copper in the field windings when there is no resistance external to the windings in the field circuit. If there is resistance external to the windings in the field circuit, κ is greater than the above volume ratio.

The magnetic flux in the field coils being equal to

$$g\{v\xi' + nx \cos \omega t\}$$

is practically unaffected by the presence of dampers, so that the iron loss in the field-magnets remains the same.

24. If a source of alternating E.M.F. = E where

$$E = E_1 \sin(\omega t + h_1) + E_3 \sin(3\omega t + h_3) + \&c., \\ = e_1 + e_3 + e_5 + \&c. \text{ (vectors)}$$

be included in the armature circuit, [and if the armature rotate in synchronism with this E.M.F., we have the case of the synchronous A.C. motor.

In this case the armature and field currents x and ξ are connected by the equations (see § 1)

$$rx + \frac{d}{dt}(lx + m\xi \cos \omega t) = \Sigma E_q \sin(q\omega t + h_q)$$

$$\rho\xi + \frac{d}{dt}(\lambda\xi + mx \cos \omega t) = \eta.$$

Assuming as in § 2 that

$$x = a_1 + a_3 + a_5 + \&c.,$$

$$\xi = \frac{\alpha_0}{2} + \alpha_2 + \alpha_4 + \&c.,$$

and proceeding exactly as in § 5, we obtain the infinite series of equations

$$t_1 a_1 + \alpha_2 = -\alpha_0 - \kappa_1$$

$$a_1 + \tau_2 \alpha_2 + a_3 = 0$$

$$\alpha_2 + t_3 a_3 + \alpha_4 = -\kappa_3$$

$$a_3 + \tau_4 \alpha_4 + a_5 = 0$$

$$\alpha_4 + t_5 a_5 + \alpha_6 = -\kappa_5$$

$$\&c., \&c.,$$

in which

α_0 = the vector to the point $2\eta/\rho$, $\pi/2$, as before

$$\text{and} \quad \kappa_q = \frac{2}{q m \omega} t^{\frac{\pi}{2}} e_q,$$

where Σe_q is the applied E.M.F., and the t and τ operators have the same values as in § 5.

Solving for a_1 we find that

$$P_1 a_1 = -\Pi_2(\alpha_0 + \kappa_1)_1 - \Pi_4(\kappa_3)_1 - \Pi_6(\kappa_5)_1 - \&c.,$$

where P_1 , Π_2 , Π_4 , &c., are the infinite determinant operators whose leading terms are t_1 , τ_2 , τ_4 , τ_6 , &c., respectively as in § 6.

Reducing to the continued fraction operators of § 7 we obtain

$$a_1 = -\frac{1}{S_1}(\alpha_0 + \kappa)_1 - \frac{1}{S_1 \Sigma_2 S_3}(\kappa_3)_1 - \frac{1}{S_1 \Sigma_2 S_3 \Sigma_4 S_5}(\kappa_5)_1 - \&c.,$$

and using the equations

$$\alpha_2 = -t_1 a_1 - \alpha_0 - \kappa_1; a_3 = -\tau_2 \alpha_2 - a_1; \&c.,$$

the successive harmonics of the armature and field currents can be obtained.

25. If in the last example the E.M.F. inserted in the armature circuit be sinusoidal and equal to $E \sin(\omega t + h) = e$ (a vector), the solution will (see equations in last paragraph) obviously be identical with that for the simple generator given in §§ 5 *et seq.*, when in the latter $\alpha_0 + \kappa$ is substituted for α_0 where

$$\kappa = \frac{2}{\omega m} \iota^{\frac{\pi}{2}} e,$$

and in this case it is important to know the condition which determines whether the machine will run as a motor and develop mechanical power.

In § 12 the *driving* torque T was shown to be given by

$$T = -\frac{m}{4} \{S\alpha_0 a_1 + S a_1 \alpha_2 + S \alpha_2 a_3 + S a_3 \alpha_4 + \&c.\}$$

and T must be negative for a motor.

Now, as

$$a_1 = -\frac{1}{S_1}(\alpha_0 + \kappa)_1 = -\frac{\iota^{b_1}}{s_1}(\alpha_0 + \kappa)_1 \text{ (see § 8)}$$

$$\begin{aligned} S\alpha_0 a_1 &= -\frac{1}{s_1} \left\{ S\alpha_0 \cdot \iota^{b_1} \alpha_0 + \frac{2}{\omega m} S\alpha_0 \cdot \iota^{\frac{\pi}{2} + b_1} e \right\} \\ &= -\frac{1}{s_1} \left\{ \bar{\alpha}_0^2 \sin b_1 + \frac{2}{\omega m} \bar{\alpha}_0 \bar{e} \sin(b_1 + h) \right\}. \end{aligned}$$

Again, as

$$\alpha_2 = -\frac{1}{\Sigma_2} a_1 = -\frac{\iota^{\beta_2}}{\sigma_2} a_1,$$

$$S a_1 \alpha_2 = -\frac{\sin \beta_2}{\sigma_2} \bar{a}_1^2 = -\frac{\sin \beta_2}{s_1^2 \sigma_2} (\overline{\alpha_0 + \kappa})^2;$$

similarly

$$S \alpha_2 a_3 = -\frac{\sin b_3}{s_1^2 \sigma_2^2 s_3} (\overline{\alpha_0 + \kappa})^2, \&c., \&c.;$$

but

$$\begin{aligned} (\alpha_0 + \kappa)^2 &= \bar{\alpha}_0^2 + \bar{\kappa}^2 + 2\bar{\alpha}_0\bar{\kappa} \cos \widehat{\alpha_0\kappa} \\ &= \bar{\alpha}_0^2 + \frac{4}{\omega^2 m^2} \bar{e}^2 + \frac{4}{\omega m} \bar{\alpha}_0 \bar{e} \cos h; \end{aligned}$$

and if

$$B = \frac{\sin \beta_2}{s_1^2 \sigma_2} + \frac{\sin b_3}{s_1^2 \sigma_2^2 s_3} + \frac{\sin \beta_4}{s_1^2 \sigma_2^2 s_3^2 \sigma_4} + \&c.,$$

we find that

$$\frac{4T}{m} = \left\{ \frac{\sin b_1}{s_1} + B \right\} \bar{\alpha}_0^2 + \frac{2}{\omega m} \left\{ \frac{\sin (b_1 + h)}{s_1} + 2B \cos h \right\} \bar{\alpha}_0 \bar{e} + \frac{4}{\omega^2 m^2} B \bar{e}^2,$$

which must be negative if the machine runs as a motor.

Now, $s_1, \sigma_2, s_3, \sigma_4, \&c., \sin b_1, \sin \beta_2, \sin b_3$, are all essentially positive and therefore B is so. Also $\bar{\alpha}_0$ and $\bar{e}$ are essentially positive. So, in order that the machine may work as a motor, h , the phase angle of the applied E.M.F. must have such a value as to make the above expression for T negative.

The power supplied by the source e being the mean value of the product of e and x , that is of e and a_1 , that is of e and $-\frac{1}{s_1} b_1(\alpha_0 + \kappa)$ is

$$\begin{aligned} &= -\frac{1}{2s_1} \bar{\alpha}_0 \bar{e} \sin (h - b_1) + \frac{1}{2s_1} \bar{e} \bar{\kappa} \sin b_1 \quad (\text{see } \S 11) \\ &= -\frac{\bar{e}}{2s_1} \left\{ \bar{\alpha}_0 \sin (h - b_1) - \frac{2}{\omega m} \bar{e} \sin b_1 \right\}. \end{aligned}$$

It is interesting to note that the armature and alternating-field currents which flow when an E.M.F. = $E \sin (\omega t + h)$ is acting in the armature circuit, the angular velocity of the armature is ω , and the exciting current C , would be unchanged if the E.M.F. $E \sin (\omega t + h)$ be removed, the speed maintained, and the exciting field-current changed from

$$C \text{ to } \sqrt{C^2 + \frac{2}{\omega m} CE \cos h + \frac{1}{\omega^2 m^2} E^2}.$$

This follows immediately from the vector equations in § 31 connecting $a_1, a_2, a_3, \&c.,$ with $\alpha_0 + \kappa$ when $\kappa_3, \kappa_5, \kappa_7, \&c.,$ are zero.

26. The case of the synchronous motor with sinusoidal applied E.M.F., discussed in the last paragraph, can easily be represented geometrically.

The power supplied by the source is

$$\begin{aligned}
 &= \frac{1}{2} \cdot \frac{\omega m}{2} \overline{OE'} \cdot \bar{a}_1 \cos a_1 OE', \\
 &= \frac{\omega m}{4} \bar{a}_1 \bar{\kappa} \sin a_1 O\kappa, \\
 &= \frac{\omega m}{2} \times \text{area of triangle } a_1 O\kappa,
 \end{aligned}$$

and the power developed by the motor $= \omega T = \frac{\omega m}{2} \times$ sum of areas of the triangles $\alpha_0 Oa_1$, $a_1 O\alpha_2$, $\alpha_2 Oa_3$, &c., attending to signs.

Hence the efficiency is equal to

$$\frac{\alpha_0 Oa_1 + a_1 O\alpha_2 + \alpha_2 Oa_3 + \&c.}{a_1 O\kappa}.$$

By rotating the vector from O to the middle point of $\alpha_0 \alpha_2$ backwards through 90° and doubling we obtain OE_1 , which represents in amplitude and phase $2/m\omega$ times the first harmonic of the total E.M.F. of the motor (see § 13).

This vector can now be compared with OE' which represents $2/m\omega$ times the applied E.M.F.

Fig. 4 easily explains how, by increasing the exciting current of an A.C. motor, the phase of the armature current is advanced relative to that of the applied E.M.F.

IX. Magnetic Rotation in Iron Cathode Films. By L. R. INGERSOLL, Ph.D., Assistant Professor of Physics, University of Wisconsin.*

[Plate I.]

Part I. The Bolometric Method of Measuring Rotations.

Part II. Rotation dispersion Curves for the Faraday and Kerr Effects in the Infra-red Spectrum.

Part III. Dependence of the Kerr Rotation on the Refractive Index of the Overlying Medium.

INTRODUCTION.

THE purpose of the present work has been an experimental study of certain problems in the magneto-optics of the magnetic metals, viz. the Faraday and Kerr effects for a long range of spectrum; the thickness of the surface layer which

* Communicated by the Author. This work has been aided by a grant from the Rumford Fund of the American Academy of Arts and Sciences, for which the writer wishes to express his appreciation.

gives rise to the Kerr rotation ; and the relation between the Kerr rotation and the optical density of the surrounding medium. These questions have been studied for the simplest typical case,—that of iron in the form of thin films, the cathode film being chosen for reasons which will appear later.

The rotation of plane polarized light on transmission through thin metallic films in a magnetic field has been studied by Kundt*, Du Bois†, Lobach‡, Righi§, Harris||, Skinner and Tool¶, and others. This effect, discovered by Kundt, has since been measured for light of various wave-lengths by other investigators, and so-called rotation dispersion curves thus obtained for films of iron, nickel, and cobalt. Recently Harris made some very careful determinations of such curves for iron films deposited cathodically in atmospheres of hydrogen, nitrogen, and oxygen, while Skinner and Tool find that, independent of changes in atmosphere, two different sorts of films with different rotatory powers may be produced for some metals by the cathode discharge, and moreover that the rotation for any wave-length bears more or less relation to the absorption for that wave-length.

An effect quite similar to this of rotation by transmission through a magnetized film, and which, doubtless, owes its existence to quite similar if not identical causes, is the so-called Kerr effect, or rotation of the plane of polarization upon normal reflexion from a polished mirror held perpendicular to a strong magnetic field. The mirror may be of iron, cobalt, nickel, or magnetite, and indeed in the earlier experiments was formed by merely polishing one end of a pole-piece of the magnet.

The work of Kerr** was extended by Du Bois††, who determined the variation of this effect with wave-length for mirrors of the above-mentioned substances. Such rotation dispersion curves resemble those obtained for the case of transmission through thin films, in that they are anomalous, showing in general an increase of rotation with wave-length, as far as the limits of the visible spectrum, instead of a marked decrease, as is the case for practically all transparent

* Wied. *Ann.* xxiii. p. 228 (1884); xxvii. p. 191 (1886).

† Wied. *Ann.* xxxi. p. 941 (1887).

‡ Wied. *Ann.* xxxix. p. 347 (1890).

§ *Mem. R. Accad. Sc. d. Bologna*, 1886, p. 443.

|| Phys. Rev. xxiv. p. 337 (1907).

¶ Paper read before the American Physical Society, Chicago, Dec. 1907.

** Phil. Mag. [5] iii. p. 339 (1877).

†† Wied. *Ann.* xxxix. p. 25 (1890); Phil. Mag. [5] xxix. p. 253 (1890).

substances. Moreover, the rotation, while of about the same order of magnitude in the two cases, is opposite in sign, a point not satisfactorily, or at least simply, explained by the theory. Complicated phenomena, which have been studied by many other observers, arise for cases of oblique incidence or of magnetization parallel to the surface, but in the present work we shall consider only the more common case of nearly normal incidence, and of magnetization perpendicular to the surface.

A few years ago the writer* developed a method of measuring such magnetic rotation for wave-lengths of the infra-red spectrum, and supplemented the work of Du Bois by extending his curves to about $\lambda = 3.5 \mu$. The striking thing shown by these curves was that the rotation in each case reached a maximum for wave-length about 1μ , and decreased rapidly for longer wave-lengths. Moreover, in the case of nickel the rotation changed sign for wave-lengths longer than 1.4μ , while for magnetite the case was still more complicated.

It was desired at the time to proceed at once with measurements on thin films of the magnetic metals, but the determination of such small rotations was too difficult and uncertain a process with the apparatus as it then existed. Accordingly, the method of measurement was considerably modified, and thereby greatly improved, and the apparatus almost entirely rebuilt on a more extensive scale, with a more careful working out of details; this has made possible the handling of the sort of problems it was desired to take up in this investigation.

Aim of the present work.—This has been: First, to measure the transmission, or Faraday, rotation for iron films of various thickness for as wide a range of wave-lengths as possible ($\lambda = .6 \mu$ to 2.2μ), thus supplementing the work of Lobach, Harris, and others by extending their curves over a spectral region of greater extent. Second, to determine the reflexion rotation, or Kerr effect, for the same films and in the same magnetic field, with the view of correlating as closely as possible these two rotation effects due to transmission and reflexion respectively. Third, to study the dependence of the Kerr rotation on the thickness of the film, a problem suggested by Kundt, and left by him after some preliminary observations with the hope of more careful treatment later. Fourth, to study briefly the effects of oxidation on the forms of the rotation dispersion curves, and the general relation between absorbing or reflecting power and rotatory power. Fifth, to

* Phil. Mag. [6] lxi. p. 41 (1906).

investigate the effect on the Kerr rotation of overlaying the surface with liquids of various refractive indices.

These questions are discussed in Parts II. and III. of the present paper. Iron has been chosen as a working substance for these experiments as it gives the largest, and hence most accurately measurable effects, but it is intended in the near future to study part or all of these effects with films of cobalt and nickel as well. The cathode type of film has been used chiefly because it may be readily produced of any desired thickness and without a preliminary deposit of platinum, as would be required for electrolytic films and which would be very objectionable for some of the experiments. It is true that cathode films differ somewhat in optical properties from the massive metal, but the same may be said of the electrolytic film which is well known to be granular in structure. In some cases the cathode film appears to resemble fused metal more nearly than the electrolytic.

PART I.—THE BOLOMETRIC METHOD AND APPARATUS FOR MEASURING MAGNETIC ROTATIONS.

This method with the necessary apparatus for measuring rotations for the shorter infra-red wave-lengths has already been described by the writer*, both in its original form and after important modifications had been introduced; but so many changes have been made since then that a brief description of method and apparatus seems desirable here.

The essential difference, in the optical system, between this method and the customary process of measurement applicable to the visible spectrum, arises from the fact that any small rotation of the plane of polarization occurring between crossed nicols, or other polarizing agents, causes the greatest *change* of intensity when the latter are crossed at 45° , a principle recognized and used over half a century ago by Provostaye and Desains†. This, then, is evidently the most effective arrangement when a bolometer or other radiation measuring instrument is to be used instead of the human eye.

In the first arrangement adopted, piles of thin glass plates were used as polarizer and analyser. Light from a Nernst glower, after being polarized by reflexion, passed through the magnet and the rotating substance between its pole-pieces, and after reflexion from the analyser fell on the slit of a

* Phil. Mag. [6] lxi. p. 41 (1906); Phys. Rev. xxiii. p. 489 (1906). Hereafter referred to as Phil. Mag. *loc. cit.* and Phys. Rev. *loc. cit.*

† *Ann. Chim. Phys.* [3] xxvii. p. 232 (1849).

spectrometer, to reach finally, after dispersion, the strip of a bolometer. On exciting the magnet the rotation of the polarization plane would cause a change of intensity of the radiation passing the analyser and falling on the bolometer, which change, when divided by the actual intensity of transmitted radiation of any given wave-length, gave a ratio proportional to the rotation for this particular wave-length.

The Compensating Principle.—The foregoing arrangement was capable of yielding very fair results at a great expenditure of labour in observing, but difficulty arose from small and unavoidable changes of brilliancy of the glower which would appear as spurious rotations. After trying many ways of compensating for this, there was finally adopted the scheme of using as analyser a large double-image prism, placed so that each of its principal planes made an angle of 45° with the original plane of polarization. The two transmitted beams of equal intensity were carried through the spectrometer, and after dispersion were allowed to fall on the two strips of a specially constructed bolometer. Any rotation would thus diminish one beam and increase the other by an equal amount—thus producing a doubled effect—while any small change in brilliancy of the glower would affect each beam equally, and hence give rise to no error. This was immeasurably superior to the former arrangement, and more than made amends for the small loss of range of spectrum its use involved; for the thick calcite prisms (the polarizing plates were also replaced by calcite) would transmit little beyond $\lambda = 2.5 \mu$.

The very simple mathematics of this method have already been worked out in a former paper*, and we need take from this only the formula

$$\delta = 45/\pi \cdot dI/I,$$

which gives for any wave-length the rotation δ in degrees, when I is the intensity of radiation on either bolometer strip for this wave-length, and dI the relative change of intensity of the two beams.

Use of High and Low Dispersion.—One of the problems to be met in this work was that of securing, in spite of the many losses, a sufficiently intense spectrum such that the small changes of intensity amounting to perhaps less than 1/10 per cent. would still be readily measurable. Every effort had been made to secure this in the earlier form of apparatus by the use of large apertures and a brilliant source, hence as a still further increase was desired for the present

* Phys. Rev. *loc. cit.*

work, it was necessary to resort to a reduction of dispersion. But this would mean a sacrifice of certainty of wave-length, and would magnify the rather uncertain errors due to the width of the slit image and bolometer strip in the spectrum.

To avoid this two distinct sets of prisms and mirrors were used in connexion with the same spectrometer table, giving in the one case a very intense but correspondingly contracted spectrum, and in the other a spectrum of several fold greater dispersion, but correspondingly weaker in energy.

In practice, then, some film or substance giving a relatively large rotation was tested with each of these combinations, and the two curves compared with a view of determining the effect of the lower dispersion, if any, in distorting the curve. This correction could then be applied with assurance to curves obtained with films which gave such small rotations that only the lower dispersion arrangement could be used.

Tests by Mechanical Rotation.—While in as simple a form as possible, to attain the desired end, the foregoing method is necessarily complicated when compared with the extremely straightforward ways of observing the same phenomena in the visible spectrum. Hence it is desirable to have some way of checking the results, or at least of applying a thorough test. This was done by mounting the polarizer in a divided circle: small and accurately known rotations were then given it, causing of course corresponding rotations of the plane of polarization. These were measured just as if of magnetic origin and checked with the known rotation produced mechanically. The results of this test, as well as of the measurements spoken of in the last paragraph, will be given later.

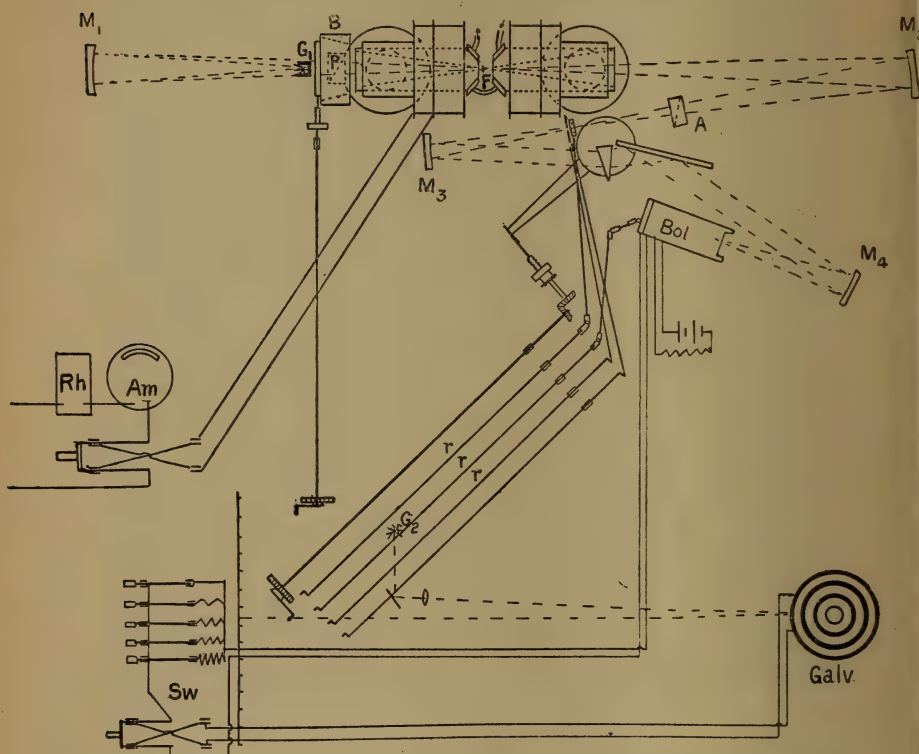
Apparatus.

In the matter of general arrangement of apparatus the former order was preserved, as will be seen from fig. 1. The whole was, however, mounted on massive slate slabs which rested on heavy wooden supports. The component pieces, with the exception of the optical parts, were constructed in the shops of this Department.

Electromagnet.—This had proved so satisfactory since its construction some years ago, that the only change necessary was the addition of cooling coils which carried circulating tap-water, and kept the ends of the pole-pieces at a nearly constant temperature. The method of excitation is worth brief mention. The winding, which consisted of about 6000 feet of No. 10 wire, was grouped in eight coils. The four pole

coils were most effective, and when excited with 16 amps. at 60 volts, gave a field of about 6000 units, which, while not as high as might be desired—on account of the large air-gap necessary—was very satisfactory for present purposes. Now

FIG. 1.—General arrangement of apparatus.



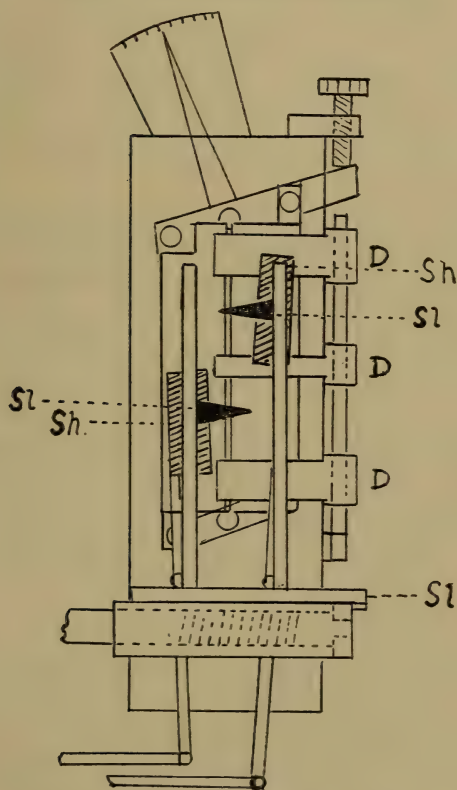
it was very desirable to have as small an external field as possible, particularly at the polarizer which had to be very near the magnet, and this was secured by a combination of series and parallel connexions of the remaining four coils, with suitable resistances, until a compass-needle remained unaffected at the position of the polarizer. This could be done without material sacrifice of the field due to the pole-coils.

Optical System.—Light from the D.C. Nernst-glow G_1 in its firebrick enclosure was converged by the mirror M_1 to an image between the pole-pieces, and to another image on the slit of the spectrometer by the mirror M_2 . A double image prism P of 4.5 cms. aperture served as polarizer, only

one of the two beams being used. It was mounted in a massive brass conical bearing B, and could be rotated by a micrometer tangent screw to produce an accurately known mechanical rotation of the plane of polarization, as mentioned above. The analyser, A, was a slightly larger double image prism of 5 cm. aperture.

Spectrometer.—The two images (of the glower) from the analyser, each about 1 cm. long, fell vertically over one another on the two parts into which the slit was divided by the diaphragms D, D (fig. 2); and shutters, Sh, allowed

FIG. 2.—The slit and its accessories.



each beam to be cut off separately. The sliding shade, Sl, was necessary for the following reason:—It was required that the initial intensities of the two spectra as they fell on the bolometer strips should be equal. This was accomplished by

the two thin wedge-shaped shades, which, when moved across the slits, darkened one image by cutting off a small part of the centre, and at the same time lightened the other until the two beams were of equal intensity. Because of slight inequalities in the bolometer strips this adjustment had to be repeated for each new setting of the spectrometer.

Each of the prisms for the spectrometer was of glass instead of rock-salt as formerly used. There was no need of rock-salt in this case, since there was at best a considerable thickness of glass to be traversed by the light, in connexion with the double image prisms; and, moreover, glass has the advantage of giving a somewhat more uniform dispersion than rock-salt. The prism of lower dispersive power had a refracting angle of 25° , and was of ordinary flint-glass with faces 9×9 cms. It was used with mirrors of 35 cms. and 30 cms. focus respectively. The other prism had a 45° angle and was of extra dense flint with faces 9×7.5 cms. The corresponding mirrors were of 50 cms. focal length each. This arrangement gave a spectrum from three to four times longer than the other.

Bolometer and Galvanometer.—The former might be said to be of the differential type, as both strips were exposed, being vertically above one another. Each strip was .5 mm. by 8 mm. The balancing coils and slide wire were all self-contained, and the instrument was remarkably steady and free from drift. The galvanometer, which was of the Thomson astatic type, had a system weighing about 2 mgs., and was worked at a sensibility of from 3 to 5×10^{-10} amp. per mm. for 6 ohms resistance. To free it from the influence of the magnet three large concentric cylinders of soft steel were used as magnetic shields, and in addition the coils themselves were embedded in the two halves of a mass of Swedish iron 3×5 cms. This combination gave a shielding effect of over 6000 times. Proportionality corrections were applied to all galvanometer deflexions.

It was necessary, in the process of measurement, to be able to reduce the sensitiveness of the bolometer-galvanometer combination in some known ratio. This was done by the use of four resistances which could be inserted separately in series with the galvanometer circuit, and which gave a graded reduction of sensibility of from 2.9 to 104.5 times.

Control and Operation.—The various parts of the apparatus were arranged to be controlled by the observer in front of the galvanometer scale. This was done by means of the five rods r, r, r , which operated respectively the spectrometer table, the sliding shade, the bolometer balancing arrangement,

and the two shutters. An auxiliary rod also turned the micrometer screw of the polarizer mounting.

Supposing everything in adjustment, the process of making observations was fairly simple. The spectrometer having been set at the wave-length for which it was desired to measure the rotation, the sliding shade was moved by means of its screw adjustment till both beams were of equal intensity, as shown by the galvanometer returning to zero. The magnet-switch was then thrown, and in a few seconds the reading taken. The current was then reversed and the new reading noted, and so on until a set of four or five readings at regular intervals of perhaps 15 seconds, had been taken for each direction of the current. The difference between the means of each set of readings would give the quantity dI , but it would also contain the small but still noticeable direct action of the magnet on the galvanometer. To eliminate this the galvanometer connexions were reversed by a suitable switch and the above process repeated, and a slightly different value for dI obtained. The mean of these two values gave the true one.

To measure the intensity I it was first necessary to reduce the sensibility by a considerable factor—usually 26·6 times—by pulling one of the switches, Sw, thus inserting a series resistance. Then by closing each shutter separately two nearly equal values of I , or rather of $I/26\cdot6$, would be given, the mean of which was used. As explained above, the ratio dI/I is proportional to the rotation occurring on reversal of the magnet.

By repeating this process for a dozen or twenty wave-lengths distributed over the spectral region, in which it was possible to work, say from $\lambda = 0\cdot6 \mu$ to $2\cdot2 \mu$, a very good rotation dispersion-curve could usually be obtained. As in the previous work, all the more careful measurements had to be made at night.

Large and Small Rotations.—In the determination of large rotations this method has very evident limitations when compared with visual schemes, but as the largest measurable angle, perhaps 30° , is many times what would be met with in any work with films, this consideration is of no importance. Of more interest is the question of the smallest measurable rotation. This is largely a matter of the amount of energy incident on the bolometer strips available for measurement, and hence depends on the part of the spectrum; also on the general conditions of steadiness of the apparatus. No especial pains have ever been taken to actually determine the smallest rotation which could be detected, but in practice angles of

less than $\cdot 003^\circ$ have frequently been measured, and it is believed that, with care, rotations of three or four, perhaps even of single, seconds could be detected.

PRELIMINARY MEASUREMENTS.

Calibration of Magnetic Field.—It was desired to test the transmission and reflexion rotations of films under exactly the same conditions of field strength. Since they could not be located in the same part of the field for the two tests, this necessitated a calibration of the field. To do this a small flip-coil was used having the same area as that of the cross-section of the beam as it passed through, or was reflected from, a film. From a large number of measurements curves were plotted showing the variation of field strength with position and with magnet current; and the current was accordingly adjusted in practice to give the same field strength whatever the position of the film.

Sensibility Reduction Factors.—As a very exact knowledge of the reduction effect of the series resistances on the galvanometer-bolometer sensibility is required, this was measured in three ways:—by a step by step process; by the use of rotating sectors; and by computations from the results of mechanical rotations of the polarizer. Four resistances giving reduction factors styled A, B, C, and D respectively, were used; and in the step by step way these factors were measured by comparing the effect on a certain galvanometer deflexion obtained, with full sensibility of the introduction of the resistance A. This was then similarly compared with B and the process repeated for C and D. Very careful galvanometer proportionality corrections were required for the success of this method, and at best there are certain theoretical objections to it.

In the second way, by the aid of a rotating sector which transmitted only $\frac{1}{25}$ of the incident light the reduction factor C was measured by the comparison of two nearly equal galvanometer deflexions. For the energy transmitted by the rotating sector and measured with the highest sensibility gave almost the same throw as the total energy measured with the resistance C in series. The values of A and B were likewise determined with the aid of suitable sectors.

By rotating the polarizer through small and accurately known angles, a third series of values was obtained, for these rotations were measured just as if they were of electromagnetic origin; and, by computing back, the necessary values to be assigned to the reduction factors were easily found. These values agreed very well with those determined by the last method, as may be seen from the following table.

TABLE I.
Values of Sensibility Reduction Factors.

	A.	B.	C.
(1) Values given by step by step method . .	2·95	9·45	27·25
(2) Measured with rotating sector	2·90	9·20	26·70
(3) Computed from rotations of the polarizer	2·92	9·15	26·45
Accepted values	2·90	9·20	26·60

D, which was rarely used, was found by comparison with C to be 104·5.

This table is of interest in that it gives an idea of the maximum accuracy which could be expected of this way of measuring magnetic rotations, or rather, of the largest error which is likely to enter from this source. It is true that sets (1) and (2) disagree by 2 per cent., and (1) and (3) by 3 per cent.; but, as stated above, method (1) is not as free from objection as the other two, whose results differ by less than 1 per cent., and this is estimated as about the probable error.

Correction for Errors due to Low Dispersion.—This was determined, as previously indicated, by comparison of the two curves made with the same rotating substance under the two different conditions of dispersion. As might be anticipated, the error in determining any particular rotation is dependent not only on the wave-length, that is, on the position of the point on the curve, but also on the actual form of the curve. Furthermore, the largest error is to be expected when we have the combination of an energy curve rapidly increasing—as the Nernst-glowler energy curve in the early infra-red—and a rotation dispersion curve rapidly falling, as is the case for glass and similar substances.

This is well shown by the rotation dispersion curves of fluorite in Pl. I. fig. 3. The rotations on the upper curve are 30 per cent. higher at 0.65μ , and the difference gradually lessens until they are equal at 2.0μ . An extrapolation to give the true form of the curve as determined with a spectrum of great dispersion would raise the upper curve by only about 1 per cent.

That the correction is much smaller, if not negligible, when the rotation increases with wave-length in the early infra-red is indicated in Pl. I. fig. 4. The curve is for rotation on reflexion from a polished steel mirror in a field of 5700 units. It is essentially identical, though plotted somewhat differently, with the curve as measured for the same mirror with the earlier form of apparatus*. The corrections for a third type of curve—a horizontal straight line—could be deduced at once from the figures in Table II., and are negligible. With

* Phil. Mag. *loc. cit.* p. 65.

these three types it will be possible to estimate the corrections with ample accuracy for any of the curves discussed in the present paper. Most of them, as a matter of fact, are of the latter two types.

Sources of Error. Accuracy of the Method.

The writer has previously discussed † a number of sources of error incidental to this work. In the present case an attempt, at least, has been made either to remove these or make them subjects of special experimentation, with a view of determining the magnitude of the errors they introduce. It would seem then that the accuracy with which magnetic rotations can be measured in the infra-red is largely dependent on the care with which the preliminary experiments have been carried out.

Fortunately it is possible, as previously stated, to get a check on this accuracy by making known mechanical rotations of the plane of polarization and measuring them just as if of electromagnetic origin. Table II. shows the results of these tests. The rotation of the polarizer of $1^{\circ}178$ —produced by six complete turns of the micrometer tangent-screw—was measured with the apparatus for a number of wave-lengths and with each of the two sets of prisms and mirrors. Lest it be very reasonably objected that the mechanical rotations have been already used to determine a series of values of the reduction factors A, B, C, and D,—and that if these values are used the table would necessarily show a correspondence,—the series (2), determined by the preferable or rotating sector process, has been used in this computation.

TABLE II.

Test of Accuracy of Method by Measurement of a known Rotation of $1^{\circ}178$.

Wave-length.	With 45° prism.	Wave-length.	With 25° prism.
$\cdot 69 \mu$	$*1.191^{\circ}$	$\cdot 59 \mu$	1.166°
$\cdot 75$	$*1.158$	$\cdot 64$	1.160
$\cdot 81$	1.170	$\cdot 72$	1.170
$\cdot 88$	1.163	$\cdot 85$	1.172
$\cdot 98$	1.174	1.04	1.168
1.23	1.169	1.31	1.170
1.61	1.170	1.64	1.171
1.84	1.169	1.97	1.173
2.01	1.189		

* The smaller amount of energy makes measurements in the shorter wave-lengths less reliable.

† Phil. Mag. *loc. cit.* p. 51.

It will be seen from the above table that the actual and computed rotations usually differ by 1 per cent. or less. This, then, may be taken as about the order of the outstanding probable error of rotation measurements in this work. It is about as great an accuracy as may be reasonably expected in almost any spectro-bolometric or radiation work, and is ample for the present investigation.

In the matter of certainty of wave-length, as great accuracy cannot be claimed as might be expected in radiation problems where a spectrum of large dispersion and a very narrow bolometer-strip can be used. The spectrometer was calibrated in the present case by double dispersion tests, using a rock-salt prism on the second spectrometer to fix the wave-length scale. Each of the two prisms was tested several times in this way. For the 25° prism the estimated probable error rises from a few thousandths of a micron in the visible and early infra-red to about 0.1μ at $\lambda = 1.1 \mu$, and is perhaps three or four times as much at 2.3μ . For the 45° prism the probable error hardly exceeds 0.2μ anywhere. It must be remembered, however, that the bolometer-strip covers a much greater width of spectrum than this; hence the determination of a rotation dispersion curve which had sharp bends or dips corresponding to sharp absorption-bands would be practically impossible.

PART II.—ROTATION DISPERSION CURVES FOR THE FARADAY AND KERR EFFECTS.

Deposition of Films.—The films were deposited cathodically on pieces of microscope cover-glass in an atmosphere of hydrogen, an induction-coil being used as a source of current and a small spark-gap introduced in the secondary circuit to make it practically one directional. A flat coil of pure iron wire, making a disk of 2.5 cms. diam., served as a cathode, and the best results were obtained with a vacuum of a fraction of a millimetre, and with the pieces of cover-glass just outside the cathode dark space and about 1 cm. from the cathode. It was possible to use thin microscope-cover glass as a backing for the films, in spite of its poor optical surfaces; for it will be noted that in all cases the film was placed where the beam of light converged to a focus. Hence the loss of light occasioned by the departure from planeness of the surfaces was immaterial, while the advantage of having only a small thickness of glass is evident.

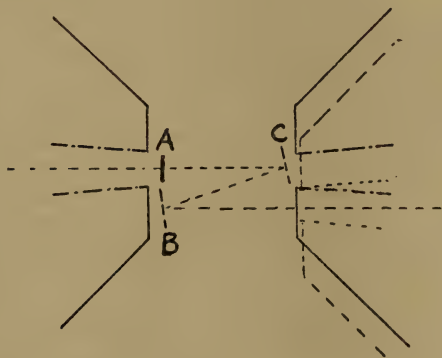
Considerable care was required to secure films which were sufficiently uniform over the required area of 4×15 mms., and free from oxidation. With the first induction-coil used—

one furnishing a high voltage and small current—it was almost impossible to avoid a certain amount of oxidation. This gave place to a smaller coil, furnishing a larger current and taking a much shorter time—half an hour or less—to deposit a film. Still later a 2000 volt generator set was used, but most of the films discussed in the present paper were deposited with the smaller induction-coil.

Oxidation, when present, was readily detected by the brownish colour imparted to the film. It was doubtless caused by the minute quantity of occluded air which would still remain even after repeated washings with hydrogen. When, however, the greatest care was taken in getting rid of this remaining air by frequent washings with dry hydrogen, between short periods of electrical discharge, films were obtained which were quite colourless, of a greyish appearance when thin, and looking like smoked glass when thick. This accords with the description of Houllevigue* of the appearance of iron films free of oxidation. The films were kept in an atmosphere of dry hydrogen when not being used, and did not deteriorate appreciably for many weeks. When left for some time in moist air they slowly oxidized to a yellow colour.

Measurements.—About forty films were deposited in all, in lots of two or three at a time usually, and perhaps half of these tested. Out of this number four have been selected which had suitably varying thicknesses and which showed almost no traces of oxidation; and the curves made from these are shown in Pl. I. figs. 6–9.

FIG. 5.



The rotations shown in the “transmission” curve were measured with the film in position A, fig. 5, the pole-pieces

* *Jour. d. Phys.* (4) iv. p. 406 (1905).

being in line. The field was approximately 5700 c.g.s. units with a magnet current of 16 amps.

For the "reflexion" curve the film was held in position B and a piece of plane silvered glass placed at C. The beam striking the silver surface would be reflected to the film and from this through the other pole-piece, which was thrown out of line to the dotted position to allow the passage of the beam: it would then follow practically the same path as in the first case. The reflexion at the silver surface did not affect the polarization of the beam and the departure from normal incidence on the film—about 8° —was well within the limits of 15° , for which, as shown by Righi*, the rotation is practically the same as for normal incidence. This arrangement, then, served the purpose of the customary optical system in which the normally reflected beam is turned aside by a plane glass plate at 45° ; such an arrangement would be of difficult, if not wholly impossible, application in the present problem. With a magnet current of 13.7 amps., the field in this position was identical with that in position A.

Corrections.—Following the practice of Kundt, the rotations given by the curves and tables of the present paper are for the reversal of the magnetic field, unless otherwise specified, and hence are really doubled values. They were all determined with the use of the 25° prism, and therefore, as has been noted in Part I., some correction may be necessary to the measured values. But it will be observed that the curves may be considered as partly straight lines and partly of the type of Pl. I. fig. 4; and in either case the corrections are negligible.

For the transmission curves, correction has been made for the rotation due to the glass on which the films were deposited. This was small—as the glass was only .18 to .22 mm. thick,—but by no means negligible, for in some cases it may be comparable with the rotation of the film itself. It was carefully determined for selected sample pieces (correction being made for the use of 25° prism as indicated in Pl. I. fig. 3), and in each case the correction applied was proportional to the thickness of the particular piece of glass on which the film was deposited. In Pl. I. figs. 6–9 the dotted curves give the rotations as measured while the solid line curve has been corrected for the glass rotation. The curve for the glass alone is shown in Pl. I. fig. 6.

To be certain that no other influence than the magnetized film was causing rotation, trials were occasionally made for rotation on reflexion from two silver surfaces, instead of one

* *Ann. Chim. Phys.* (6) ix. p. 132 (1886).

silver and one iron. Such rotations were always found to be extremely small. One has been plotted in Pl. I. fig. 6, and denoted by the letter S. It amounts to only about 0.02° .

It is undoubtedly true that the results as given for the thinner films are somewhat in error because of multiple internal reflexions, but it would be hopeless to try to allow for these until we can separate more clearly the two rotation effects and perhaps localize the reflexion rotation. This error is not considered of importance except in the case of the very thinnest films. It is also a fact, from the work of previous observers *, that transmitted light from a magnetized film is slightly elliptically polarized. This would not cause any error in the present work; the rotation measured would be that of the major axis of the resulting ellipse.

Absorption Measurements.—It was thought to be of interest to determine if the absorbing and reflecting powers of the films showed a variation with wave-length corresponding to the rotation. Accordingly measurements were made of the transmission of each film, as well as of the reflecting power (by comparison with silver) of the front and rear surfaces. The curves for several films are shown in Pl. I. fig. 10, where T , the transmission, is approximately equal to $e^{-\alpha t}$, α being the absorbing power and t the thickness. No attempt has been made to correct for the complicated system of internal reflexions: this is in itself an extremely difficult problem for films as thin as some of these. The values of T have been obtained by merely dividing the fraction of radiation transmitted by $1-R$, where R is the reflecting power of the air-film surface. While not possessing the accuracy of the rotation measurements they serve very well to indicate the selective or non-selective character of the absorption or reflexion, which is the only part in which we are particularly interested.

It will be noted that the transmission—and therefore also absorption—for the pure iron films 15, 22, and 24 is almost non-selective or independent of wave-length. The same is true for film 17 whose curve is not shown. This explains the greyish colours of these films, and, in agreement with the conclusions of other observers as already pointed out, is accepted as evidence of their freedom from oxidation.

The curve of reflecting power with wave-length for film 15 is incidentally shown in Pl. I. fig. 13. It shows a slight increase of reflecting power in the infra-red, as is the case with pure iron in the mass†. Film 17 shows a slightly greater increase

* See Harris, *Phys. Rev.* xxiv. p. 337 (1907.)

† Hagen and Rubens, *Ann. d. Phys.* viii. p. 1 (1902).

of reflecting power with increasing wave-length, while films 22 and 24 show a somewhat greater decrease, but on the whole the reflecting power for the pure films is also relatively non-selective.

Thickness of Films.—The absorption and reflexion data would afford a possible means for determining the relative thicknesses of the various films; but, as Houllevigue and Passa * have shown, it is more accurate to compute the thicknesses from the rotation produced, since this is proportional to the thickness. Accordingly the thicknesses of the four films already mentioned, 15, 17, 22, and 24 have been computed in this way, assuming Houllevigue and Passa's data and iron cathode films, as approximately $58\ \mu\mu$, $30\ \mu\mu$, $13\ \mu\mu$, for $8\ \mu\mu$ respectively †. Kundt's results for films by electrolysis would give thicknesses about one third smaller.

Very Thin Films.—Tests were made also on a number of very thin films, some of them less than $0.5\ \mu\mu$ in thickness, but the results are of questionable value, particularly those for rotation on reflexion. For in such thin films reflexion would take place not only at the first or air-film surface, but also at the film-glass boundary and at the glass-air or rear surface, and, because of the small loss on transmission, these three surfaces—or at least the first and last—would each return about the same amount of radiation. The latter part was generally excluded by covering the rear glass surface with an absorbing mixture of the same optical density as the glass, *i. e.* Canada balsam and lamp-black; but there would seem to be no way of separating the first two parts. In one case there was tried the experiment of depositing a thick film of platinum, and on this a thin film of iron. The rotation shown on reflexion was positive, as might be expected, due to the preponderance of the rotation of the doubly transmitted part of the beam. The only striking feature of the experiment was the enormously varying reflecting power shown by such a film, increasing from almost nothing in the visible to about 50 per cent. at $\lambda = 2\mu$.

In general, the results on very thin films indicated a somewhat larger reflexion rotation than the transmission. The latter rotation sometimes measured not more than $.003^\circ$, with the reflexion rotation perhaps twice as large and frequently appearing positive in the early infra-red.

Effect of Oxidation.—Considerable data on this point have

* *Comptes Rendus*, cxli. p. 29 (1905).

† This scale of thickness has been checked to within a few per cent. by subsequent measurements, by interference methods, of films deposited on optical glass. See "Addendum."

been collected, but the results on different films are somewhat at variance. This is in accordance with the observations of Kundt*, who found very complicated effects when the mirror for which he was measuring the Kerr rotation was oxidized. In general it was found, however, that films of a reddish or brownish colour—indicating more or less oxidation—gave curves similar to those shown in Pl. I. figs. 11, 12, and 13. The difference between these and the curves for the pure iron films is seen in the rapid decrease of rotation with wave-length after the early infra-red. This is particularly true for the case of rotation on transmission, and was very marked in a film deposited in air instead of hydrogen, and which was presumably completely oxidized. Furthermore, the curves for rotation on transmission and reflexion do not show the same similarity as in the case of pure iron, and indeed in two of the cases shown the reflexion rotation appears positive for the shorter wave-lengths.

The change in the character of the rotation curves is seen to be accompanied by a marked variation of the transmission with wave-length, as shown by the dotted curves of Pl. I. fig. 10. For the opaque partially oxidized film 29 of Pl. I. fig. 13 only the reflexion rotation and reflecting power could be measured, but each is seen to change markedly with change of wave-length. As to the effect of oxidation on the magnitude of the rotation, films 7 and 14 show smaller rotations than the pure iron films, although film 14 has about the same absorption as film 22. The reflexion rotation from film 29, however, is slightly greater for the early infra-red than that from film 15 or that due to a steel surface as shown in Pl. I. fig. 4.

Discussion of Results.—The first fact worthy of note which may be deduced from the curves is that the magnetic rotation, either on transmission or reflexion, of iron, instead of increasing indefinitely with wave-length, as might be concluded from observations in the visible spectrum, reaches a maximum value shortly outside the limit of this spectrum. This maximum rotation, which is reached for pure iron at about $\lambda = 1.5\mu$, is seen to be on the average a little over one and two-thirds times that for the wave-length of sodium light.

In the second place it will be noted that while the Faraday rotation decreases by five times in going from film 15 to film 24, the Kerr rotation is only halved. In other words the reflexion rotation increases less rapidly than the thickness of the film and soon reaches a maximum value. That this has been very nearly reached in film 15 is indicated by the

* Wied. *Ann.* xxvii. p. 199 (1886).

agreement in magnitude of the rotations with those obtained with the polished steel surface, as well as the agreement with the results of other observers of the Kerr effect—working visually—reduced for this same field strength, viz. 5700 units. Moreover some later tests made visually with the aid of an ordinary half shade polarimeter on an opaque unoxidized film gave a rotation for sodium light, almost identical with that of this film. The seat of the Kerr rotation is unquestionably a thin surface-layer, and the foregoing results allow one to say that a layer of less than perhaps $20\mu\mu$ thickness contributes considerably more than half of this rotation*. This is of the same order of magnitude as the thickness of the frequently assumed “transition layer” of optical theory.

A third point is the great similarity, if we confine our attention to the results for pure iron, of the curves for the two cases of rotation. While for the thinner films the Kerr rotation curve shows a more rapid decrease on the short wave-length side, the forms of the two curves for film 15 are almost identical—at least until beyond 1.8μ —as is shown by the following Table III. The rotation for the wave-length of sodium light is taken as unity in each case.

TABLE III.

Rotation dispersion of the Faraday and Kerr effects for a pure iron film (15).

Wave-length.	Transmission rotation.	Reflexion rotation.	Wave-length.	Transmission rotation.	Reflexion rotation.
589μ	1.00	-1.00	1.40	1.71	-1.70
.70	1.20	-1.16	1.50	1.73	-1.74
.80	1.33	-1.34	1.60	1.73	-1.74
.90	1.44	-1.42	1.70	1.73	-1.76
1.00	1.52	-1.50	1.80	1.71	-1.76
1.10	1.59	-1.56	1.90	1.70	-1.76
1.20	1.64	-1.62	2.00	1.68	-1.76
1.30	1.68	-1.66	2.10	1.65	-1.76
			2.20	1.62	-1.76

It would seem from the above that any explanation of the Kerr effect must indicate, more clearly than it does at present, its very close connexion with the Faraday rotation; but unfortunately for the generality of this statement the

* We might also say that practically the full value is given by a layer $60\mu\mu$ thick, which is about the same thickness as that found by Koenigsberger and Bender (*Ann. d. Phys.* xxvi. p. 763, 1908) for platinum and gold films which give the same phase change on reflexion as the massive metal.

results for partially oxidized films show a considerable difference between the two curves. Kundt explained the varying effects obtained when the steel mirror for which he was measuring the Kerr effect was slightly oxidized, by the mutually interfering action of the two beams, one reflected from the air-oxide surface, and the other reflected and rotated by the oxide-metal surface. In the present case, however, we are dealing with a homogeneous layer, not a film of oxide on the metal, and the discrepancies cannot be as easily explained away.

The resemblance between the reflexion rotation curve for film 29 of Pl. I. fig. 13 and that obtained with a steel mirror, Pl. I. fig. 4, would indicate that the latter, though polished brightly, still had more or less oxide on its surface, perhaps in the minute scratches left by polishing. In the previous work on the Kerr effect (*Phil. Mag. l. c.*) the rotation dispersion curves for polished surfaces of nickel and cobalt showed this decrease of rotation with wave-length after about 1μ , as well as steel. It remains to determine if cathodic surfaces of these metals show a similar effect. If, as shown above, the Kerr rotation takes place in an extremely thin surface film, it is not remarkable if polished surfaces fail to give results identical with those from the more perfect cathodic surfaces.

Some such connexion as is seen to exist between rotatory power and absorbing or reflecting power might reasonably be expected. Thus in film 29 a rotatory power decreasing with wave-length is accompanied by an increasing reflecting power. The readiest explanation of this—overlooking for the moment the somewhat different state of the iron—would be that the Kerr effect is a function of the penetration of the wave on reflexion, and this may on the whole be less for a greater reflecting power. Similarly for the pure films, the flat part of the rotation dispersion curve accompanies a non-selective absorption or reflexion.

But this does not account in either case for the rapid change of rotatory power with wave-length in the visible and early infra-red spectrum. This accords with the conclusions reached by the writer from a study of the Kerr rotation curves for mirrors of steel, cobalt, and nickel, that this spectral region,—viz. from perhaps $\lambda = .4\mu$ to 1μ —has much the same optical significance in the case of metals that a region of resonance absorption has for a dielectric. As is well known, the refractive indices of metals increase with wave-length in the visible spectrum. It remains to investigate their optical constants for the remainder of this region

and as far as possible in the infra-red, for it quite possible that the anomalous character of the dispersion may cease, or at least change—as it does for the rotation dispersion—within the region in which it is possible to study it. Experiments to this end are now in progress.

PART III.—DEPENDENCE OF THE KERR ROTATION ON THE REFRACTIVE INDEX OF THE OVERLYING MEDIUM.

In connexion with the foregoing experiments tests were made for several films of the rotation from the rear, or glass-metal, surface, and the unexpected result was found that the rotation in this case was more than half again as large as that for the ordinary air-metal surface. Thinking that some connexion might be established between the reflexion rotation and the optical density of the overlying medium, experiments were tried with films whose surfaces were covered with thin layers of various liquids, held in place and protected by a slip of microscope cover glass. The rotations measured in this way all proved larger than for the bare surface and by a factor not far different, in most cases, from the refractive index of the liquid.

It can be readily seen that the rotation as measured by the bolometric method for such cases as these, will not represent, without suitable corrections, the actual rotation which takes place at the liquid metal surface. For in the first place a part of the energy would be reflected from the surface of the glass cover slip and would be returned without undergoing any rotation. A smaller part would likewise be reflected from the glass-liquid surface, while the part which is reflected and negatively rotated by the true liquid-metal surface must also be affected by the positive rotation due to the double passage through glass and liquid. The rotation due to the latter need not be considered since the layer of liquid is so thin, but that due to the former must be corrected for. If we are dealing with rotation from the glass-metal surface of the film the corrections are of the same sort, but slightly simpler, since the liquid is lacking. To make the first correction, that is, for the reflected energy which does not penetrate to the metal surface, we must know the combined reflecting power R , of the air-glass, glass-liquid, and liquid-metal surfaces, and also R' of the air-glass and glass-liquid surfaces alone. The former is obtained by measurement, the latter by calculation from the known refractive indices. Then of every

* *Phil. Mag.*, *loc. cit.* p. 70.

R parts of energy which are reflected and go to make up the measured quantity I, R' parts are inactive, having suffered no rotation and hence should be subtracted from I in computing the actual rotation. This is equivalent to multiplying the rotation, measured in the ordinary way as dI/I , by $R/(R-R')$.

When there is added to this the doubled rotation due to the passage twice through the glass the corrected rotation is obtained. To take the first case in Table IV. as an example,

TABLE IV.

Reflexion rotations from the rear (glass-metal) surfaces
of pure iron films 15, 17, 22.

λ .	Measured rotation.	R.	R'.	α' .	Double glass rotation.	Corrected rotation.
Film 15.						
$\cdot 63\mu$	$-\cdot 262^\circ$	$\cdot 321$	$\cdot 040$	$-\cdot 299^\circ$	$\cdot 143^\circ$	$-\cdot 442^\circ$
$\cdot 74$	$-\cdot 348$	$\cdot 328$	$\cdot 040$	$-\cdot 396$	$\cdot 108$	$-\cdot 504$
$\cdot 94$	$-\cdot 480$	$\cdot 340$	$\cdot 039$	$-\cdot 543$	$\cdot 070$	$-\cdot 613$
1.28	$-\cdot 610$	$\cdot 335$	$\cdot 039$	$-\cdot 691$	$\cdot 037$	$-\cdot 728$
1.71	$-\cdot 662$	$\cdot 335$	$\cdot 038$	$-\cdot 747$	$\cdot 019$	$-\cdot 766$
2.14	$-\cdot 656$	$\cdot 336$	$\cdot 038$	$-\cdot 740$	$\cdot 009$	$-\cdot 749$
Film 17.						
$\cdot 65\mu$	$-\cdot 211^\circ$	$\cdot 257$	$\cdot 040$	$-\cdot 250^\circ$	$\cdot 133^\circ$	$-\cdot 383^\circ$
$\cdot 75$	$-\cdot 312$	$\cdot 259$	$\cdot 040$	$-\cdot 369$	$\cdot 100$	$-\cdot 469$
$\cdot 96$	$-\cdot 414$	$\cdot 247$	$\cdot 039$	$-\cdot 492$	$\cdot 066$	$-\cdot 558$
1.30	$-\cdot 517$	$\cdot 229$	$\cdot 039$	$-\cdot 624$	$\cdot 033$	$-\cdot 657$
1.74	$-\cdot 570$	$\cdot 215$	$\cdot 038$	$-\cdot 693$	$\cdot 016$	$-\cdot 709$
2.16	$-\cdot 567$	$\cdot 191$	$\cdot 038$	$-\cdot 708$	$\cdot 009$	$-\cdot 717$
Film 22.						
$\cdot 66\mu$	$-\cdot 084^\circ$	$\cdot 090$	$\cdot 040$	$-\cdot 152^\circ$	$\cdot 133^\circ$	$-\cdot 285^\circ$
$\cdot 75$	$-\cdot 181$	$\cdot 088$	$\cdot 040$	$-\cdot 332$	$\cdot 100$	$-\cdot 432$
$\cdot 96$	$-\cdot 314$	$\cdot 080$	$\cdot 039$	$-\cdot 613$	$\cdot 066$	$-\cdot 679$
1.30	$-\cdot 414$	$\cdot 081$	$\cdot 039$	$-\cdot 798$	$\cdot 033$	$-\cdot 831$
1.74	$-\cdot 459$	$\cdot 075$	$\cdot 038$	$-\cdot 930$	$\cdot 016$	$-\cdot 946$
2.16	$-\cdot 430$	$\cdot 070$	$\cdot 038$	$-\cdot 940$	$\cdot 009$	$-\cdot 949$

the measured rotation ($=45/\pi dI/I$) for $\lambda=\cdot 63\mu$ was found to be $-\cdot 262^\circ$. The reflecting power of the glass and glass-metal surfaces was $\cdot 321$ and that of the glass alone (there being no liquid in this case) was $\cdot 040$. This gives a correcting

factor of $\cdot 321/\cdot 281$ which brings the measured rotation up to $\alpha' - \cdot 299^\circ$. Subtracting from this the glass rotation of $+\cdot 143^\circ$ we get the corrected rotation of $-\cdot 442^\circ$. The process is entirely similar for the case when the liquid is used, save that the glass reflecting power is made slightly larger—perhaps $\cdot 045$ —to take account of the additional, or glass-liquid, surface. The cover slips used were of practically the same thickness as the glass on which the films themselves were deposited and hence the glass rotation was the same in either case.

The corrected rotations from the above table have been plotted in the curves of Pl. I. fig. 14. As in all the cases considered these are the doubled rotations occurring on reversal of magnetization. It will be noted that the curves have the same general form as those of the Kerr effect in Pl. I. figs. 6–8, but that the rotations are all considerably larger. The remarkable feature, however, is that the magnitude of the rotations is nearly the same for the three films of very different thicknesses, and that the thinnest film shows the largest rotation.

In the following Table V. (p. 98) the results are shown of a number of experiments with liquids of optical densities ranging from 1.33 to 1.72.

Discussion of Results.—In Voigt's * recent “Magneto- und Electrooptik,” which contains the most complete treatment of the Kerr phenomenon that has yet appeared, there is reached on p. 316 an expression for the rotation on normal reflexion and for polar magnetization—the case we are considering—which may be written

$$\text{Rotation} = x = \left| \frac{Q(k-i)}{n(1+k^2)} \right|,$$

where Q is a complex which does not contain n , and n and k are respectively the refractive index and absorption index of the metal. The equations have not been worked out for the case of a surrounding medium other than vacuum or air, and until this has been done it is useless to attempt any satisfactory justification of the above results on theoretical grounds. But it is of interest to note that if we grant two not unreasonable assumptions †—viz. that the refractive index

* W. Voigt, *Magneto- und Electrooptik*, Leipzig, 1908.

† Experiments now in progress in this laboratory show that the reflecting powers of metals, when overlaid with various liquids, are in agreement with the well-known formulæ for metallic reflexion, if assumptions similar to these are made. In this light they seem extremely reasonable.

TABLE V.

Showing effect on the Kerr rotation of overlaying iron surface with liquids of various refractive indices. α is the rotation due to the metal surface in air, and β to the same surface overlaid with the liquid.

(1) With Potassium mercuric iodide solution. $N_D 1.72$.

Film.	Wave-length.	Rotation α .	Rotation β .	β/α .	Mean.
15	$.84\mu$	$-.357^\circ$	$-.651^\circ$	1.82	1.75
	1.11	$-.420$	$-.748$	1.78	
	1.74	$-.462$	$-.763$	1.65	
16 (similar to 15).	$.84\mu$	$-.355^\circ$	$-.659^\circ$	1.85	1.79
	1.11	$-.435$	$-.780$	1.79	
	1.74	$-.480$	$-.830$	1.73	
29 (opaque)	$.67\mu$	$-.388^\circ$	$-.704^\circ$	1.81	1.71
	$.82$	$-.403$	$-.691$	1.71	
	1.07	$-.378$	$-.632$	1.67	
	1.69	$-.245$	$-.399$	1.63	

(2) With Bromnaphthalin. $N_D 1.66$.

15	$.84\mu$	$-.357^\circ$	$-.630^\circ$	1.76	1.67
	1.10	$-.420$	$-.698$	1.66	
	1.74	$-.462$	$-.737$	1.59	

(3) With Canada balsam. $N_D 1.55$.

15	$.84\mu$	$-.357^\circ$	$-.593^\circ$	1.66	1.64
	1.10	$-.420$	$-.686$	1.63	
	1.74	$-.462$	$-.753$	1.63	
Polished steel.	$.84\mu$	$-.385^\circ$	$-.504^\circ$	1.31	1.35
	1.41	$-.340$	$-.452$	1.33	
	1.96	$-.261$	$-.365$	1.40	

(4) With Glycerine. $N_D 1.45$.

15	$.84\mu$	$-.357^\circ$	$-.556^\circ$	1.56	1.50
	1.11	$-.420$	$-.630$	1.50	
	1.74	$-.462$	$-.670$	1.45	
29	$.67\mu$	$-.388^\circ$	$-.596^\circ$	1.53	1.50
	$.82$	$-.403$	$-.617$	1.53	
	1.07	$-.378$	$-.557$	1.47	
	1.69	$-.245$	$-.364$	1.48	
Polished steel.	$.84\mu$	$-.385^\circ$	$-.534^\circ$	1.39	1.42
	1.41	$-.340$	$-.484$	1.42	
	1.96	$-.261$	$-.379$	1.45	

(5) With Water. $N_D 1.33$.

16	$.84\mu$	$-.355^\circ$	$-.438^\circ$	1.23	1.20
	1.11	$-.435$	$-.523$	1.20	
	1.74	$-.480$	$-.567$	1.18	

n may be taken as relative to that of the surrounding medium, while the absorption index k is constant—we reach at once the conclusion from the above equation that the Kerr rotation should increase directly as the refractive index of the overlying medium.

That this is at least the general trend of the facts is shown by Table V. The agreement is by no means good, but, for the experiments with cathode films, the mean values of the ratios—if we allow that a mean has any significance when there is such a decided “drift” of the results—differ generally by only a few per cent. from the refractive indices. There is no evident explanation of the rapid decrease of this ratio with increasing wave-length; it is much greater than the decrease of index of the liquid would require it to be. The results with steel mirrors are seen to differ widely from those with cathode films, particularly in case (3) with Canada balsam as the liquid. But a polished metal surface, with all its microscopic scratches, is not to be compared for ultimate smoothness of structure with a cathode mirror on a glass surface. It may be indeed that the viscous Canada balsam did not penetrate to the bottom of crevices of the steel surface.

Passing to the case of rotation from the rear surfaces of films we should expect to find such rotations larger than those from the first surfaces by a fraction equal to the refractive index of the glass, in this case about 1.49. But if the curves of Pl. I. fig. 14 are compared with the corresponding ones of Pl. I. figs. 6–8, it will be seen that the rotations have increased from over 1.7 in film 15 to nearly 3 times in film 22. Two explanations are offered for this: that the state of aggregation and physical properties of the metal are modified in the layer next the glass; or, more probably, that the matter is complicated by internal reflexions in the case of the thinner films.

This would possibly explain these results and at the same time not throw discredit on the work of Part II. where the reflected rotation is measured from the thinner films, because the reflecting power of a plain or air-iron surface is much greater than that of the glass-iron. Hence, since by Stokes' law these reflecting powers also hold for the internal reflexions, the amount of energy returned on the first internal reflexion is much less in the cases of Part II. than in the present experiments.

An attempt was made to verify qualitatively these conclusions as to the variation of the Kerr effect with the overlying medium, by means of visual observations with a

half shade polarimeter, using sodium light. An opaque film was deposited on plate glass and the Kerr rotation measured with, and without, the use of Canada balsam. The observations were very difficult and not very satisfactory in the latter case, but, when corrected for the doubled rotation of the glass cover slip, came out about half larger.

SUMMARY.

1. The writer's bolometric method of measuring magnetic rotations has been developed until, under favourable circumstances, rotations of less than $.001^\circ$ can be detected, and angles somewhat larger measured with an accuracy approximating 1 per cent. It is applicable to a range of spectrum from the D lines to $\lambda = 2.2\mu$.

2. The dispersion of magnetic rotation for cases of reflexion from and transmission through, iron cathode films, has been measured. For the pure iron films these curves are entirely similar and show a gradual increase of rotating power until wave-length 1.5μ is reached, where the rotation reaches a maximum value over one and two-thirds times as great as that for the D lines.

3. After a certain thickness is reached the reflexion rotation ceases to increase further with thickness of film. A layer of iron, of thickness less than $20\mu\mu$, or about that of the assumed "transition layer" of optical theory, gives rise to considerably more than half of this rotation.

4. Partially oxidized films do not show the similarity between the reflexion and transmission rotation dispersion curves that pure films do. The rotation for oxidized films reaches a maximum in the early infra-red and falls off rapidly with further increase in wave-length. This change in form of the rotation curve with oxidation is accompanied by a corresponding change in the transmitting or reflecting powers as a function of wave-length.

5. The Kerr rotation is nearly proportional to the refractive index of the overlying medium, which, with certain assumptions, may be shown to be in accordance with Voigt's theory.

Physical Laboratory,
University of Wisconsin,
Oct. 31, 1908.

ADDENDUM.

Since the above was written, Skinner and Tool's paper on the optical properties of magnetic films has appeared (*Phil. Mag.* Dec. 1908). It is a valuable contribution to the literature of the subject, showing very clearly as it does the

different types of films which may be produced by the cathode deposit: the writer, however, is obliged to differ from them in some of their conclusions.

A comparison of their work with the pre-ent shows at once that what are here referred to as "oxidized" films would seem to be identical with their "dark" films, and the "pure iron" films of the present work, while not produced in just the same way, have the general appearance and non-selective reflecting and absorbing powers which they describe for their "metallic" type. The chief objection to calling the latter two virtually identical is the abnormally high rotating power (per unit thickness) and extinction coefficients which they assign to these "metallic" films. These are some three times larger than for electrolytic iron, while their determinations of these quantities for electrolytic films are considerably higher than those of other observers.

A possible explanation of this lies in their thickness measurements; for their method, it is believed, results in a considerable underestimate of the thickness of the film and hence overestimate of its rotating power and extinction coefficient. Their method—an interference one—involves the assumption that the phase changes on reflexion at glass and iron surfaces are the same, and this they were able to justify for very thin films. But Kœnigsberger and Bender (*Ann. d. Phys.* xxvi. p. 763, 1908) find for the cases of gold and platinum at least, that while this is true for extremely thin films it is by no means the case for thicker ones.

To test this for iron, a number of films were deposited on optical glass and their thicknesses measured for sodium light after the manner of Skinner and Tool. The thicknesses were then measured by the method of Patterson (*Phil. Mag.* ii. p. 652, 1902), whereby the effect of any possible phase change is eliminated by a second cathode deposit, and in every case this gave considerably larger results. The true thickness was from a fraction larger, to more than twice as great as that measured in the first way, and this is more than sufficient to explain the abnormal values above mentioned.

Since the magnetic rotation for sodium light as well as the thickness was measured for these latter films, data were obtained for checking the estimates of the thickness of the films used in Part II. of the present work. These were checked to within a few per cent., or within the error of measurement.

In regard to the yellow-brown or "dark" films, Skinner and Tool consider that the metal is here in a different state

of aggregation and deny the possibility of any chemical combination of the metal with a gas. The writer, on the other hand, has assumed in common with Houllevigue, that such films are the result of partial oxidation. This view seemed to be supported by the fact, already mentioned, that films deposited in oxygen (air) showed the same decrease of rotation with increasing wave-length after about $\lambda = 1\mu$. To throw more light on this point, a thick film, giving a rotation (Na light) of $+2^{\circ}5$ before oxidation, was oxidized by moist air and ozone and its rotation then measured bolometrically. When corrected for the glass, the results gave $+0.14^{\circ}$ for $\lambda = 1.08\mu$, and $+0.006^{\circ}$ for $\lambda = 1.75\mu$, the successive measurements differing by only a few thousandths of a degree. It will be noted that the rotation is still in the same sense but many times smaller than before, and that it shows this decrease with increasing wave-length after $\lambda = 1\mu$. These same characteristics are exhibited, in a smaller degree, by the yellow-brown films.

In conclusion, it seems to the writer that it is unnecessary at present to explain either of the types of films obtained by the cathode deposit as an especially anomalous state of the metal. For the "metallic" type, as shown by Skinner and Tool, in general resembles, more or less, electrolytically deposited films; and in the case of the "dark" films, while it is admittedly difficult to account for the presence of oxygen in the depositing chamber when one notes the care taken by these experimenters, these films show at least some of the characteristics of oxides.

X. On the Amount of Radium present in Sea-Water.

By A. S. EVE, M.A., D.Sc., McGill University, Montreal*.

IN this paper measurements will be expressed in terms of billionths (10^{-12}) gram of radium per kilogram of water tested.

Three observers have made determinations of the amount of radium in sea-salt or sea-water.

Strutt† found 7.5×10^{-14} gram of radium per gram of sea-salt, and this is equivalent to 2.3×10^{-12} gram radium per kilogram of sea-water. This, the first determination, was stated to be approximate only.

The writer‡ examined Inagua sea-salt, and a sample of

* Communicated by the Author.

† Proc. Roy. Soc. A. vol. lxxxviii. p. 151.

‡ Phil. Mag. Feb. 1907.

sea-water collected from mid-Atlantic, and found 0·3 and 0·6, respectively, expressed in the above units.

A large number of samples have been examined by Joly, and in his Presidential Address to the British Association, 1908, he stated that the average value found by him was 16.

These three observers then give results in the ratios of 4, 1, 27.

There are several points of special interest in Joly's paper *. In the first place he found that the addition of HCl tended sometimes to increase the amount of radium determined by his experimental method, apparently through clearing away precipitates, possibly of sulphates. Next, a second determination invariably afforded a higher reading than the first. Thirdly, two samples of sea-water which had stood for two months in bottles, before testing, gave low values.

I venture to give a brief summary of the results published by Joly.

Samples of Sea-water collected near land.

	Without HCl.	With HCl.
a. Valencia Harbour	35·6	40·0
b. Few miles W. of I. of Man	3·8	38·6
c. 65 miles W. of Valencia	12·6	31·4
d. 1·5 miles S. of Crow Head, Co. Kerry, ...	15·2	22·6
e. 20 miles W. of Bantry Bay	26·8	39·3

Samples from Ocean between Madeira and Bay of Biscay.

	Without HCl.	With HCl.
f_1	2·9	14·4
f_2	..	15·0, 21·3
g_1	..	19·3, 27·3
g_2	..	4·4, 8·0
h_1	..	2·0, 14·6
h_2	..	8·4
i_1	3·5	26·0
i_2	..	30·4
j_1	14·6	..

Arabian Sea.

k.	..	24·3, 31·4 27·8
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The range is from about 8 to 40, and the average value of all Professor Joly's determinations is 16.

* Phil. Mag. vol. xv. pp. 385-393 (March 1908).

As my published values were so much smaller than the above, it seemed desirable to make some more measurements and ascertain the reason for the difference.

In 1908, on a voyage from Liverpool to Montreal, six samples were collected, by myself, in a canvas bucket over the side of the vessel and placed in six clean new glass bottles. The first two samples were collected whilst the water was warm and the ocean currents were from the south, but the second pair were obtained when the sea was cold and the currents were from the north; while the last two were from the Gulf of St. Lawrence, and therefore nearer shore. The six samples then together seem to constitute a fair average sample of the waters of the North Atlantic.

The sea-water was taken from the bottles and placed in six new glass flasks and, except with sample 1, the bottles were well rinsed with HCl and distilled water, which were afterwards added to the sea-water in the respective flasks. A seventh flask was also filled with distilled water and 60 c.c. HCl, in order to see if there was any radium present in the flasks, water, or acid used.

Sample.	Date of Collection.	Lat.	Long.	Placed in flask.	No. of grams.	Distilled water.	HCl added.
1	30 August	56° 27'	26° 32'	29 Sept.	1272	500	0
2	31 „	56° 6'	33° 3'	18 Jan.	1135	600	60
3	1 Sept.	55° 5'	41° 32'	8 Dec.	1130	?	30
4	2 „	53° 33'	49° 32'	2 Oct.	1180	600	40
5	3 „	51° 34'	56° 26'*	29 Dec.	1149	149	60
6	4 „	Off Gaspe Co.*		10 Dec.	1215	?	50?

* In the Gulf of St. Lawrence.

A glass flask, silvered inside, was fitted as an electroscope, and the method followed was that described in a previous paper †. The natural leak was small and constant.

After the sea-water and acid had been sealed for about a month in the flasks they were boiled for some minutes. All the gases driven off, including the emanation, were collected over *water* and introduced through a drying-tube into the electroscope. As the method is a comparative one, there appears to be little objection to collection over water.

† Eve & McIntosh, Phil. Mag. Aug. 1907.

Nevertheless it might be objected that the radium emanation is absorbed by bubbling through water to a variable and unknown extent. Therefore re-determinations were made by collecting over mercury. There was no certain observable difference between the two methods. It is often simpler and it seems perfectly safe to collect over water.

To calibrate the electroscope I prepared solutions of radium bromide, using those standard solutions whose history is given in the *American Journal of Science*, vol. xxii. July 1906. These same standard solutions were those also used by Rutherford and Boltwood in their determinations of the ratio of uranium to radium in pitchblende. As Joly used this ratio for calibration, our standards are practically the same. Moreover Strutt, Joly, Eve and McIntosh all find values for the radium contents of rocks which are in satisfactory agreement, so that the question of relative standards does not arise.

One c.c. of the standard solution, containing 1.57×10^{-9} gram of radium, was taken with a new and clean pipette and placed in a flask with 99 c.c. of distilled water and HCl. Another new and clean pipette was used to remove 1 c.c. of the well-shaken mixture and to transfer it to a second flask containing 99 c.c. of distilled water and HCl. More distilled water was added to each flask, and, after sealing, the contents were in due course tested.

Gram radium.	Divisions per minute.	
	Collected over water.	Over mercury.
1.57×10^{-9}	8.43	8.43
1.57×10^{-11}	.080	.083

The agreement is better than can be expected and is partly accidental. One division per minute of the electroscope is a measure of the emanation from 1.90×10^{-10} gram of radium.

In testing the specimens of sea-water it was found that the readings by day were larger than those by night. Mr. F. W. Bates has recently proved that the conduction of sulphur is greatly increased by direct sunlight, and also to a less extent affected by daylight. My readings were, therefore, taken between 5 P.M. and 9 A.M., during which time the aluminium-leaf moved about 40 divisions of the scale in the eyepiece of the observing microscope. When samples were tested at intervals of less than a month the necessary corrections were made for the incomplete growth of the radium emanation in the flasks. The flask containing distilled water and HCl alone gave the same reading as the "natural leak."

The results obtained by me were :—

Sample.	Collected over water.	Over mercury.	Mean.
1	1.0 ; 0.89 ; 0.73	0.66	0.82
2	1.03	0.38	0.71
3	0.75 ; 1.74 ; 1.83	1.67	1.50
4	0.38 ; 0.28	0.75	0.47
5	0.71 ; 0.47	0.51	0.56
6	1.83 ; 1.60	1.25 ; 1.34	1.50
		Mean	0.94

Range : about 0.5 to 1.5.

The high reading for number 3 may perhaps be due to the fact that the HCl and distilled water were left for 6 days in the collecting-bottle. Number 6 was collected but a few miles from land. It will be seen that my result agrees very well with my previous value 0.6, but it does not agree with the value 16 found by Professor Joly. I publish this result, therefore, with some diffidence, but after taking every precaution in my power to eliminate error. At first sight it may seem a matter of small importance whether there is 9×10^{-16} or 1.6×10^{-14} gram of radium per gram of sea-water. That is not the case. The amount of radium in the ocean affects the question of the ionization of the atmosphere over the ocean with all the consequent problems of cloud formation, atmospheric electricity, and propagation of electric waves by wireless telegraphy. It also enters into the problem of penetrating radiation. Moreover, it has a direct connexion with the interesting results of Joly as to the amount of radium present in deep-sea deposits, and possibly with the amount determined by Strutt in sedimentary rocks such as chalk or limestone.

A measurement was also made of the radium present in the water of the River St. Lawrence. This water is supplied to the City of Montreal without any filtration or treatment, and my sample was taken from a tap in the Chemistry Building. No great reliance can, therefore, be placed on the result, 0.25. Joly found 4.2 for the water of the River Nile. Thus we both found only four times as much radium in the ocean as in the rivers named, although our relative values are as 17 to 1. This seems to show that the radium, and presumably the accompanying uranium, find their way somewhat rapidly to the deposit on the bed of the ocean, for the ratio of other salts in sea- and river-waters respectively show a vastly greater ratio. Further measurements of the amount of radium in river-water are needed, but the quantity is small and difficult to measure.

If we take the amount of radium in sea-water as 10^{-15} gram per gram of water, and in the soil as 4×10^{-12} , a simple calculation* shows that the penetrating radiation from the soil should be about 1600 times as great as from the ocean; so far as the penetrating radiation may be attributed to the presence of radium.

If we follow a calculation of Becker†, we find the radium contents of the ocean to be equivalent to the radium found in a very small thickness of the deposits at the bottom of the ocean. Let us suppose the ocean contains 9×10^{-15} , the deep-sea deposits 25×10^{-12} (Joly), and the sedimentary rocks 1.1×10^{-12} (Strutt), in each case in terms of grams of radium per gram of material. Take the mean depth of the ocean as 3.5 kilometres and the specific gravity of the deposit or soil as 2.5. Then the amount of radium in 3.5 km. depth of ocean, per sq. cm., is about equal to that, also per sq. cm., in 114 cms. of sedimentary rock, or 5 cms. of deep-sea deposit.

Summary.

1. Radium emanation may be collected over water or over mercury with about equal accuracy when testing for the quantity present in a given solution.

2. The result of testing six samples of sea-water from the North Atlantic leads to the average value of 9×10^{-16} gram of radium per gram of sea-water. This is about one-seventeenth of the value found by Joly.

Montreal, April 1909.

XI. Transformations of X-Rays. By CHARLES A. SADLER, M.Sc., Oliver Lodge Fellow, University of Liverpool‡.

WHEN a primary beam of Röntgen radiation falls upon any substance, secondary Röntgen rays are emitted, the character of which depends both upon the nature of the substance and upon the particular kind of primary beam used. With reference to the latter it has been found that variations in the intensity of the primary beam produce no perceptible change in the character of the resulting secondary, the sole controlling factor appearing to be the degree of "hardness" of the primary §.

* 'Terrestrial Magnetism,' March 1909.

† Bull. Geol. Soc. of America, vol. xix. p 113 (1908).

‡ Communicated by the Physical Society: read April 23, 1909.

§ Barkla, Phil. Mag. June 1906, pp. 812-828.

It has been shown * that if the radiating substance be an element of low atomic weight—as hydrogen, oxygen, or carbon—it emits a radiation similar in penetrating power to the primary producing it; its penetrating power varying with that of the producing primary. This type of radiation, which will be referred to as a “scattered” radiation, may be considered as produced by an acceleration of one or more electrons in the atom of the radiating substance, due to the action of forces in the primary pulse.

From an element of greater atomic weight than that of calcium (40)—and possibly from other elements of lower atomic weight under very penetrating primary beams—the emitted radiation has been shown to consist of scattered radiation, and superposed upon this, a type of radiation which is characteristic of the radiating element. The penetrating power of this radiation is a constant quantity peculiar to the substance and is independent of the penetrating power of the primary producing it. Moreover, this radiation appears to be entirely homogeneous in character, and experiments point to the conclusion that the radiating electrons producing this type of radiation are no longer appreciably under the influence of the forces in the primary pulse.

This homogeneous radiation is only produced when the penetrating power of the primary is greater than that of the homogeneous radiation characteristic of the radiator.

From the group of elements Cr–Ag the ionization produced by the homogeneous portion of the secondary radiation, emitted when any of its members is subjected to a sufficiently penetrating primary, is many times greater than that produced by the scattered portion—in the case of copper as radiator this ratio is as high as 150 : 1.

The homogeneous radiation from chromium is very “soft” much softer indeed than any ordinary primary beam, and from chromium down to silver and probably beyond, the penetrating power of the characteristic radiation increases with increase of atomic weight; the radiation from silver being many times more penetrating than that from chromium.

One of the chief difficulties experienced in the investigation of X-ray phenomena has been the heterogeneity of the primary beams hitherto available. Even where devices are adopted to ensure that the current through the X-ray bulb used as a source of primary rays is uni-directional and of nearly constant strength, the primary so obtained consists of a mixture of constituents of different penetrating power; so

* Barkla & Sadler, *Phil. Mag.* Oct. 1908 pp. 550–584.

there remains the difficulty of ascertaining which particular constituents of the composite beam are principally concerned in producing the phenomena under investigation.

It was thought that useful information concerning the nature of X-rays might be obtained if the homogeneous rays previously mentioned were used to excite tertiary radiation in different substances.

Sagnac has shown that the tertiary X-rays from metals excited by secondary X-rays are more easily absorbed than the exciting rays.

Previous experiments * had shown that if two substances A and B be taken, each of which is found to emit a homogeneous radiation when a suitable primary beam falls upon it, the homogeneous radiation from A being more penetrating than that from B, then if a homogeneous beam from A be passed through a thin plate of B, tertiary radiation is excited in B by the radiation from A, while if the process is reversed it is found that the radiation from B excites *no* tertiary radiation in A.

These phenomena were examined in greater detail in the experiments described below.

It was found that no trace of a homogeneous tertiary radiation could be detected from aluminium when subjected to any of the homogeneous radiations from the group of metals Cr-Ag, and the amount of scattered radiation produced was extremely small when compared with the secondary incident beam.

Advantage was taken of these facts, and the primary and secondary beams were passed through tubes of thick aluminium of rectangular cross-section. This enabled the apparatus to be arranged compactly with comparatively short distances between the anticathode of the X-ray bulb and the secondary radiator, and between the secondary and tertiary radiators respectively, a condition essential to secure that the ionization produced by the tertiary rays in a suitable ionization chamber should be sufficiently intense to ensure accurate readings in a reasonably short time, and that the direct tertiary radiation should produce an ionization large compared with that produced by stray secondary and tertiary rays from the surrounding air and neighbouring screens.

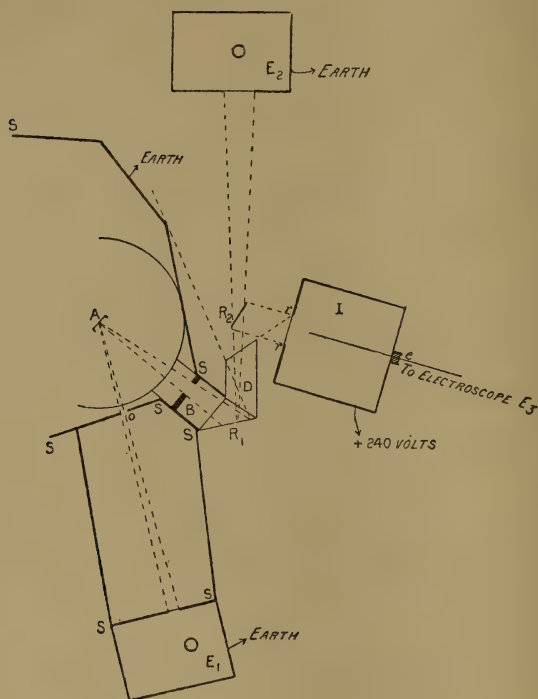
The general arrangement of the apparatus is indicated in Plan by fig. 1.

A rectangular brass tube B lined with aluminium .2 cm. thick was fitted into an aperture in the lead screen SS surrounding the X-ray bulb emitting the primary rays, and the

* Barkla & Sadler, Phil. Mag. Oct. 1908, pp. 550-584.

bulb was so placed relatively to this tube that the axis of the tube passed through the centre of the anticathode A normally.

Fig. 1.



The rays from A passing through this tube impinged on the radiator R_1 consisting of a rectangular plate of the metal, the secondary rays from which were to be studied. A portion of the secondary rays so produced passed down the rectangular brass tube D lined as before with .2 cm. aluminium, and a plate of any substance placed in this secondary beam provided a source of tertiary rays.

The intensity of the primary beam was measured by allowing a narrow pencil of rays proceeding from A through a small aperture O in the lead screen SS to enter an electro-scope E_1 of the ordinary Wilson type through an opening covered with tissue-paper and aluminium-foil in the wall of the electro-scope.

The deflexions of the gold leaf were observed by means of a microscope fitted with a scale in the eyepiece.

The intensity of the secondary beam was measured by means

of a similar electroscope E_2 placed in the path of the secondary beam passing down the tube D (R_2 removed) as indicated in the Plan; the size of the aperture in the wall of this electroscope being 3×2 cms.

It was found that by carefully shielding the electroscopes E_1 and E_2 from draughts and sudden changes of temperature very reliable readings could be obtained, the motion of the gold-leaf being absolutely dead-beat.

Preliminary experiments also showed that with a given radiator R_1 in position (R_2 being removed) the ratio of the deflexions in E_1 and E_2 when the bulb had reached a steady state rarely varied by more than 1 per cent. during a series of readings.

The ordinary Wilson type of electroscope was found to be quite unsuitable as a means of measuring the tertiary radiation; the type of electroscope described by R. T. Beatty in his paper on "Secondary Röntgen Radiation in Air" * was finally adopted. Briefly described, this consisted of a brass case having two sliding quadrants insulated from the case and charged to potentials of +240 and -240 volts respectively; an insulated gold-leaf hung vertically between them and was connected to the wire e , which projected into and was insulated from the ionization-chamber I, which itself was insulated and charged to +240 volts. An adjustable compensating-chamber insulated and charged to -240 volts eliminated the normal ionization in the chamber I. The sensitiveness of the electroscope was adjustable by means of the sliding quadrants.

With the order of sensitiveness required in all the experiments described, the motion of the gold-leaf was dead-beat, and a series of readings of the ratio of deflexions in electroscopes E_1 and E_2 showed that with deflexions up to 20 scale-divisions in E_2 this could be obtained with an accuracy of 2 per cent. with certainty.

A calibration of electroscopes E_1 and E_2 showed that the value of a deflexion of a scale-division was the same within 1 per cent. throughout the range of scale employed.

The sensitiveness of E_3 was adjusted and then maintained constant throughout a given series of readings, and in all readings a constant deflexion of E_3 was used, thus eliminating the effect of the slight change in value of a scale-division which was found to exist in different parts of the scale.

Preliminary experiments showed that the secondary radiation from air together with other stray effects were very

* R. T. Beatty, *Phil. Mag.* Nov. 1907, pp. 604-614.

small compared with the readings obtained when metallic radiators were employed.

The portion of the radiator R_1 exposed to the primary beam was limited to that opposite to the tube D by means of a suitable stop placed in the tube B as indicated in the diagram.

Nature of the Tertiary Rays.

It was perhaps reasonable to expect that the tertiary radiation emitted by any member of the group Cr-Ag when subjected to a more penetrating homogeneous beam from the same group would be identical in character with that emitted as secondary radiation by the same substance when excited by a suitable primary.

Direct experiments were carried out to test how far this was true, and for this purpose pure iron was chosen as the tertiary radiator.

The radiator consisted of a small rectangle 3 cms. high by 2 cms. broad, formed of pure iron wire interlaced so as to expose as large a surface as possible to the exciting beam. This was placed in the position indicated by R_2 , the centre of the radiator was at the intersection of the axis of the tube D with the normal from the centre of the aperture to the ionization-chamber I, the plane of the radiator making equal angles with these directions.

As a secondary radiator in the position R_1 a plate of pure copper was used.

The aperture YY of the ionization-chamber I in these experiments was 3 cms. high by 2 cms. broad, and the distance from the centre of the aperture to the centre of R_2 , 4 cms.

Owing to the obliquity of some of the tertiary rays, it was evident that the absorption coefficients, obtained by studying the absorption by thin plates of different substances placed parallel to the aperture YY in the path of the beam, would be greater than would have been the case had it been possible to utilize a *pencil* of tertiary rays.

A control experiment conducted with a fairly soft primary beam falling upon the same iron radiator similarly situated before an aperture of the same size as that in the screen YY in an electroscope of the Wilson type placed in the secondary beam from the iron, gave an increase in the value of the absorption coefficient by an aluminium plate .00297 cm. thick of 6 per cent. over the value found when a narrow pencil of secondary radiation was used.

The absorption coefficients of the tertiary beam from iron were then determined by thin sheets of aluminium, iron, copper, and zinc. The values so obtained are compared with those obtained when the same absorbers were used with the secondary rays in the control experiment in the following table.

TABLE I.

Absorber.	Value of $\frac{\lambda}{\rho}$ for Secondary Rays.	Value of $\frac{\lambda}{\rho}$ for Tertiary Rays.
Al (.00297 cm.) ...	93.8	94.2
Fe (.00315 cm.) ...	69.1	69.1
Cu (.00298 cm.) ...	101.0	102.5
Zn (.00132 cm.) ...	119.2	120.0

It will thus be seen that within the limits of experimental errors the penetrating power of the tertiary beam is identical with that of a similar secondary beam from the same substance.

The Homogeneity of the characteristic Tertiary Radiation.

The tertiary beam from iron excited by the secondary homogeneous beam from copper was now cut down by thin aluminium sheets to test its homogeneity.

It was to be expected, even were the beam perfectly homogeneous, that after cutting down by a few plates the beam would appear slightly more penetrating, for the more oblique rays would suffer extinction to a greater extent than those passing through the absorber in a perpendicular direction.

It was not found possible to test for homogeneity to an exhaustive limit owing to the smallness of the readings in the later stages, but the results obtained show that for all practical purposes the beam may be regarded as homogeneous.

The results are tabulated below :—

TABLE II.

Iron as Tertiary Radiator ; Copper as Secondary Radiator.

Amount previously absorbed by Aluminium.	Subsequent absorption by a sheet of Al (.00297) cm. thick.
None.	55.6 per cent.
55.6 per cent.	55.7 "
80.3 "	55.5 "
94.0 "	55.0 "

Previous experiments have shown that associated with the homogeneous secondary radiation from a metal of the group Cr—Ag is a small proportion of scattered radiation, the relative ionizations produced by the homogeneous and scattered portions being about 150 : 1.

If the secondary beam from iron be absorbed by say .0104 cm. aluminium, then while the homogeneous portion will be absorbed to the extent of about 90 per cent. the scattered portion will only be absorbed by about 30 per cent., giving an absorption of the whole beam of 89.6 per cent. (the absorption being measured by the relative diminution in ionization) ; so that the relative ionizations produced by the residual homogeneous and scattered portions respectively will now be as 21.4 : 1, and a sheet of .0104 aluminium would now only absorb about 87.4 per cent. of the whole beam, and this increase in penetrating power would become more and more pronounced as further absorptions took place.

In a corresponding case of the tertiary beam from iron excited by copper radiation, the amount of scattered copper radiation in the beam if present at all will not be present to so great an extent since the copper radiation will not penetrate to anything like the same depth in the iron as an ordinary primary radiation, though on the other hand the ionization produced by beams of scattered primary radiation and of scattered copper radiation conveying equal amounts of energy per second through unit volume of air, will not be equal ; the scattered copper radiation being more easily absorbed will produce the greater ionization.

No direct evidence has yet been obtained that when the normal tertiary radiation from iron is excited by homogeneous radiation from copper, any of the copper radiation itself is

scattered by the iron; its presence in the beam would be difficult to detect for a small percentage of copper radiation in the beam would produce a very minute change in its absorption coefficients. The figures given in Table II. therefore do not preclude the possibility of a small percentage of the ionization being due to scattered copper radiation.

*Independence of the Penetrating Power of the
Exciting Radiation.*

Experiments were made to test whether the penetrating power of the tertiary radiation emitted by a substance depended in any way upon the penetrating power of the exciting secondary beams. In the first test, iron was chosen as the tertiary radiator, copper and arsenic as the secondary radiators. The radiation from arsenic being about twice as penetrating as that from copper.

In the second test chromium was taken as the tertiary radiator, and iron, copper, and arsenic as secondary radiators; the radiations from each of these substances being more penetrating than that from chromium, the radiation from arsenic being about four times as penetrating as that from iron.

The results are tabulated below :—

TABLE III.
Iron as Tertiary Radiator.

Secondary Radiator.	Absorption coefficient of the Secondary Radiation by Aluminium.	Percentage Absorption of the Tertiary Radiation by (.00297 cm.) Al.
Copper	128.9	55.6
Arsenic	60.7	55.4

TABLE IV.
Chromium as Tertiary Radiator.

Secondary Radiator.	Absorption coefficient of the Secondary Radiation by Aluminium.	Percentage absorption of the Tertiary Radiation by (.00297 cm.) Al.
Iron	239	75.1
Copper	128.9	75.3
Arsenic	60.7	75.1

From these results it will be seen that the penetrating power of the tertiary radiations is uninfluenced, so far as can be measured, by variations in the penetrating power of the exciting secondary beams; they further show that if any secondary radiation is scattered by the tertiary radiator, the ionization it produces is small compared with that produced by the tertiary radiation.

This result is analogous with that obtained with secondary beams*.

It has thus been shown that the characteristic tertiary radiation from iron is identical with the characteristic secondary radiation from iron in its penetrating power, its homogeneity, and in its independence of variations in the penetrating power of the exciting radiation.

Similar results were obtained with copper as a tertiary radiator.

Connexion between the Secondary and Tertiary Radiators.

A series of experiments was now undertaken in which the several members of the group of metals Cr-Ag were used as secondary radiators, the tertiary radiators being also chosen from among the earlier members of the same group.

By referring to column 1, Table V. it will be seen that the absorption coefficients by aluminium of the radiations from the metals of this group indicate a wide range of penetrating powers, so that it was reasonable to expect that for each tertiary radiator there would be at least some secondary radiations sufficiently penetrating to cause it to emit its characteristic radiation.

The method of experimenting was as follows:—The secondary radiators successively placed in the position R_1 (see fig. 1) were subjected to the primary rays proceeding from the anticathode A, R_2 being temporarily removed. The ratio of the deflexions of the gold-leaves in the electroscopes E_2 and E_1 obtained from the microscope readings was determined in the case of each radiator; let r_1 be the value of this ratio and r_1' be the value of the ratio with no metallic radiator in the position R_1 , the air in its neighbourhood acting as a source of secondary radiation.

Next, the particular tertiary radiator under examination was placed in the position R_2 (as previously defined), and the secondary radiators were again in turn subjected to the primary rays. Ionization was now produced in the chamber I and deflexions of the gold-leaf in electroscope E_3

* Barkla & Sadler, Phil. Mag. Oct. 1908, pp. 550-584.

were obtained. Let the value of the ratio of the deflexions of the gold-leaves in E_3 and E_1 be r_2 , and when the tertiary radiator was removed and the air alone in its neighbourhood acted as a source of tertiary rays, let the corresponding value of the ratio be r_2' .

The tertiary radiator R_2 was then replaced and the ratio r_2 again found, and then finally removing R_2 the readings for the ratio r_1 were repeated.

Working with an X-ray bulb, having an auxiliary spark-gap, it was found that in the steady state the two sets of values for r_1 and likewise those for r_2 showed agreement to within 2 per cent.

In no case was the air-effect r_1' more than 1 per cent. of the value of r_1 , in most cases less than $\frac{1}{4}$ of 1 per cent.; and in no case was the air-effect r_2' more than 2 per cent. of the value of r_2 when the characteristic homogeneous radiation was excited.

In those cases where the penetrating power of the secondary beam did not exceed that of the characteristic radiation from the tertiary radiator employed, the air-effect r_2' became of considerable importance, being as high as $\frac{1}{4}$ of r_2 in some cases. The author hopes to obtain more reliable data in these particular cases in the course of further experiments.

It will be seen later, however, that an accurate knowledge of these particular data is not essential in the present investigation.

It was estimated that if in all other cases $\frac{1}{2}$ of the air-effect was taken in addition to the normal leakage, and this subtracted in each case from the direct effect when the metallic radiator was in position, the resulting ratios finally obtained would be accurate to at least 2 per cent. If a denote the ratio of the normal leakage in any given time in the electroscope E_2 to the deflexion in E_1 in the same time, with a primary beam as during the actual experiment, then since the normal leakage is compensated for in E_3 , the ratio of the ionizations in the electroscopes E_3 and E_2 due to the homogeneous radiations

$$= \frac{r_2 - \frac{1}{2}r_2'}{r_1 - [\frac{1}{2}r_1' + a]} = R \text{ (say).}$$

If the absorption coefficient of the homogeneous radiation from a given tertiary radiator by Al be denoted by λ_2 , and that of the exciting secondary radiation by λ_1 , then as long as $\lambda_1 =$ or is $> \lambda_2$ the value of R is quite small and approximately constant. As λ_1 decreases through the value λ_2 we get a rapid increase in the value of R , and for a comparatively

small subsequent increase in penetrating power of the secondary beam, values of R 30 to 40 times as big as the previous steady values are obtained; this increase corresponding to the excitation of the characteristic tertiary radiation.

Let us consider the tertiary radiation emitted normally from a given radiator of area S upon which a uniform parallel beam of secondary rays is incident normally.

Let us define a quantity k , such that the fraction of the energy of the secondary beam passing normally through a thin layer δx of the tertiary radiator which is transformed into tertiary radiation is $k\delta x$. Thus, if I be a measure of the energy passing normally per second through unit area of the tertiary radiator at a depth x below the surface, the energy transformed per second in a layer $\delta x = Ik\delta x$.

Now Barkla* has shown that when this homogeneous type of radiation is excited, it is practically evenly distributed in all directions: consequently, the energy passing per second as tertiary radiation from the layer δx through the tissue-paper-covered window of an electroscope of the Wilson type (I being constant over the whole area of the radiator)

$$= \frac{\omega}{4\pi} SIk\delta x e^{-\lambda_2 x}, \quad (1)$$

where λ_2 is the absorption coefficient of the tertiary radiation by the material of which the tertiary radiator is composed, and ω is the mean solid angle subtended by the aperture of the electroscope at all parts of the radiating area.

But if I_0 is a measure of the energy incident normally per second on unit area of the tertiary radiator at the surface,

$$I = I_0 e^{-\lambda_1 x},$$

where λ_1 is the absorption coefficient of the secondary beam by the material of which the tertiary radiator is composed, and the whole energy passing into the electroscope per second from the tertiary radiator

$$= \frac{\omega}{4\pi} SI_0 k \int_0^\infty e^{-(\lambda_1 + \lambda_2)x} . dx \quad (2)$$

$$= \frac{\omega}{4\pi} SI_0 k \frac{1}{\lambda_1 + \lambda_2} . \quad (3)$$

If now we remove the tertiary radiator, and place the electroscope previously used in the path of the secondary beam, with the centre of its tissue-paper-covered window occupying the position previously occupied by the centre of

* Barkla, Phil. Mag. Feb. 1908, pp. 288-296.

the radiator, and with its plane perpendicular to the axis of the secondary beam, the energy passing per second through the window, if its area is A , is $I_0 A$.

Therefore the ratio of the energy in the tertiary beam passing per second into the electroscope in the first position to the energy of the secondary beam passing per second into the electroscope in its second position

$$= \frac{\omega S}{4\pi A} \cdot \frac{k}{\lambda_1 + \lambda_2} \cdot \dots \dots \dots (4)$$

There is considerable evidence that the ionization produced in a given volume of air by a beam of Röntgen radiation is approximately proportional to the absorption of that radiation by the air. It has been found * also that the ratio of the absorption coefficients of homogeneous beams of different penetrating powers by any two substances, *e.g.* carbon and aluminium, in which no radiation of the homogeneous type is excited by the beam under consideration, is a constant. Also it has been found that the absorption of a beam of Röntgen rays by any substance depends only upon the quantity of matter present and not upon its state of aggregation.

It is assumed on the basis of these results, that the ionizations produced in a given volume of air by homogeneous beams of different penetrating powers are proportional to the absorption coefficients of these beams by carbon or aluminium.

Making this assumption, we have the ratio of the ionizations produced in the electroscope by the tertiary and secondary beams

$$= \frac{\omega S}{4\pi A} \cdot \frac{k}{\lambda_1 + \lambda_2} \cdot \frac{\beta}{\alpha} \cdot = \frac{\mathfrak{S}_2}{\mathfrak{S}_1} \text{ (say) } \dots \dots (5)$$

where α and β are the absorption coefficients by aluminium of the secondary and tertiary beams respectively. From (5) we find that k

$$= \frac{\mathfrak{S}_2}{\mathfrak{S}_1} \cdot \frac{\alpha}{\beta} \cdot \frac{A}{S} \cdot \frac{4\pi}{\omega} \cdot (\lambda_1 + \lambda_2).$$

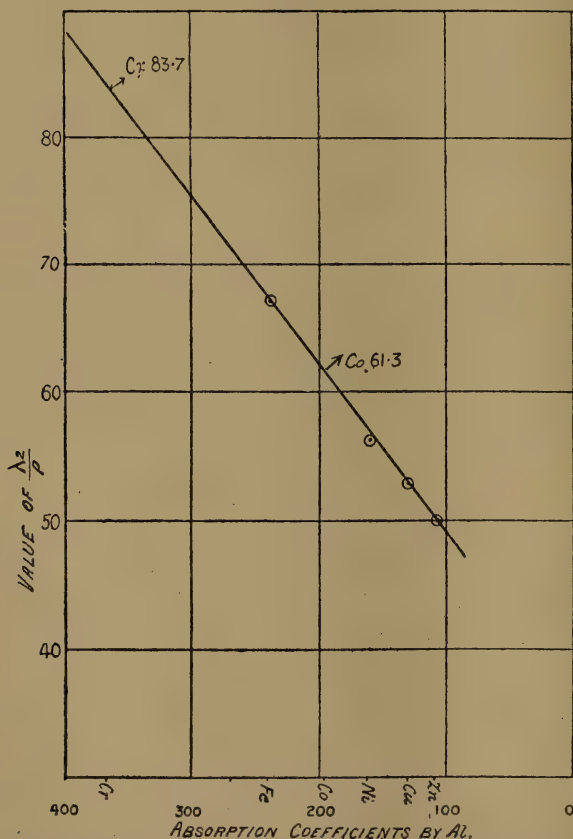
In the above calculations we have considered the special case of normal incidence and emergence, but, since we are dealing with radiators sufficiently thick to absorb the whole of the incident radiation, it is easy to show that the results (4), (5), (6) are quite general for oblique incidence and emergence, where the incident and emergent beams make equal angles with the normal to the radiating surface.

* Barkla & Sadler, Phil. Mag. May 1909.

In order to deduce values of k from the experimental data, it is necessary to know the values of α , β , λ , and λ_2 for each of the tertiary radiators employed. α and β are readily obtained for the whole series and likewise λ_1 and λ_2 for Zn, Cu, Ni, and Fe. In the case of Cr and Co, being unable to obtain thin plates of these substances, the values for λ_1 and λ_2 could not be obtained directly but were deduced by the following method.

If the values of $\frac{\lambda_2}{\rho}$ (where λ_2 is the absorption coefficient by a substance of its own characteristic radiation and ρ its density) for the substances Fe, Ni, Cu, and Zn are plotted

Fig. 2.



as ordinates, against the absorption coefficients for the radiations from these substances by Al as abscissæ, it is found that the points obtained lie on a straight line, as shown in fig. 2.

The values of $\frac{\lambda_2}{\rho}$ for Co and Cr deduced from this graph are Co (61.3), Cr (83.7).

Moreover, a regular relationship is found to exist between the ratios of λ_1 for the radiations from consecutive members of the group Cr-Se for each of the absorbers Zn, Cu, Ni, and Fe. Taking the values of the absorption coefficients for any three of these absorbers for the radiations from the group Cr-Se, values for the coefficients for the fourth absorber could be deduced, starting with a value of λ_2 as a basis and using these observed relationships. In no case was the discrepancy between the experimental and deduced values greater than 2 per cent.

In a similar manner the remaining values of the absorption coefficients for Co and Cr were deduced, taking the values of λ_2 obtained above as a basis for calculation in each case.

TABLE V.

RADIATORS.	Values of λ used in calculation of k .							Values of λ^1 used in calculation of the ratio $\frac{k}{\lambda^1}$.					
	ABSORBERS.												
	Al.	Cr.	Fe.	Co.	Ni.	Cu.	Zn.	Cr.	Fe.	Co.	Ni.	Cu.	Zn.
Chromium...	367	544											
Iron	239	2500	514					2150					
Cobalt	193.2	2076	521	545				1785	102				
Nickel.....	159.5	1730	2440	584	482			1490	2090	134.0			
Copper	128.9	1478	2080	2560	537	474		1279	1798	2194	150		
Zinc	106.3	227	1715	2172	2275	497	361	1072	1485	1875	1936	120.1	
Arsenic	60.7	755	1040	1333	1422	1575	1464	666	915	1161	1232	1356	1260
Selenium ...	51.0	652	903	1159	1211	1340	1258	577	800	1014	1055	1157	1080
Silver.....	6.75					214	195.1					189.2	170.0

It will be seen later that it is desirable to know what fraction of the absorption of the exiting radiation by the material of the radiator is directly concerned in the transformations of energy which are taking place during the

passage of the secondary beam through the substance of the tertiary radiator.

It has been found* that if the values of λ_1 for the radiations from the group Cr to Ag for any absorber, such as silver, in which no homogeneous radiation is excited by the radiations from any member of the group, be plotted as ordinates against the corresponding values of λ_1 for aluminium, in which also no homogeneous radiation is excited, as abscissæ; the points so obtained lie on straight lines, and these straight lines pass through a common point.

Similarly, if we plot as ordinates the values of λ_1 for any of the early members of the group Cr-Ag as absorber, in which the homogeneous type of radiation *is* excited by the radiation from those members of the group of higher atomic weight; it has been found, that up to the point corresponding to the atomic weight of the absorber, the relation is also linear, and if the straight line be produced, it passes through the common point previously mentioned. The values of λ_1 for the radiation from the radiators of higher atomic weight are abnormally increased, the increase being attendant upon the excitation of tertiary radiation.

This increase is discussed in the paper on "The Absorption of X Rays" previously referred to.

In the actual experiments to determine the values of R with the various combinations of secondary and tertiary radiators, it was found necessary to alter the position of the quadrants of the tertiary electroscope from time to time, owing to a slight shift of zero, due to small progressive variations in the applied potentials. In this way small changes in the sensitiveness of the instrument were introduced. The sensitiveness could, however, be maintained constant during a period of three or four hours during any given day.

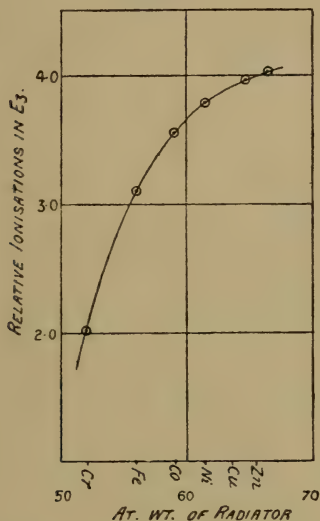
In order to obtain true relative values for the ionizations, the secondary radiation from arsenic was made to fall upon each of the tertiary radiators placed successively in the position R_2 , the sensitiveness of the electroscope remaining constant during this series of observations. Values of R were obtained in this way for Zn, Cu, Ni, and Fe as tertiary radiators, the surfaces in each case being brightly polished, of the same size, and thick enough to totally absorb the incident beam. These values of R plotted as ordinates against the atomic weight of the radiators as abscissæ, were found to lie on a smooth curve (see fig. 3).

In the case of Cr and Co, being unable to obtain them pure in the form of flat plates, radiators were made in the

* Barkla & Sadler, *Phil. Mag.* May 1909.

following manner. A flat plate of aluminium, of the same size as the required radiator, was covered with a thin layer of

Fig. 3.



adhesive (previously tested and found to emit no detectable radiation of the homogeneous type), and then the metals, finely powdered, were pressed into this layer, the surface being covered with powdered metal. A sample radiator prepared with pure iron filings in this way emitted a tertiary radiation indistinguishable (by absorption tests) from that produced from a plate of the metal. This, however, is quite to be expected since practically the whole of the tertiary radiation from iron is obtained from a layer .003 cm. thick.

But the value of R for a prepared plate of iron filings is a little less than that for a flat polished plate of iron of equal size placed in an identical position, owing to the irregularities of the surface reducing the effective area. Three equal prepared plates of Co, Fe, and Cr and a polished plate of iron were therefore used, the powders being as nearly as possible equally fine, and the relative values of R obtained. From a comparison of the two values of R for iron, values of R for polished plates of Co and Cr could be deduced, and these were found when plotted to lie almost exactly upon the curve in fig. 3.

From the set of values thus obtained the true relative values of R for the complete series of experiments previously described were deduced and are given below in Table VI.

TABLE VI.

	Values of R , calculated from the <i>total</i> Ionization in the Ionization-chamber I.						Values of R' , calculated from the <i>normal tertiary</i> Ionization alone in the Ionization-chamber I.					
TERTIARY RADIATORS.												
SECONDARY RADIATORS.	Cr.	Fe.	Co.	Ni.	Cu.	Zn.	Cr.	Fe.	Co.	Ni.	Cu.	Zn.
Cr	·076	·107	·109	·143	—	—	—	—	—	—	—	—
Fe	1·76	·076	·091	·118	·126	·124	1·68	—	—	—	—	—
Co	1·81	·42	·097	·118	·100	·106	1·73	·327	—	—	—	—
Ni	1·95	2·64	·466	·101	·084	·087	1·87	2·45	·366	—	—	—
Cu	1·92	2·64	3·00	·582	·093	·110	1·84	2·55	2·90	·472	—	—
Zn	2·02	2·88	3·37	3·53	·528	·115	1·94	2·79	3·27	3·42	·488	—
Ga ?	...	...	...	...	...	...	...	...	...	...	...	...
Ge ?	...	...	...	...	...	...	...	...	...	...	...	...
As	2·02	3·10	3·57	3·79	3·97	4·03	1·94	3·01	3·47	3·68	3·86	3·92
Se	1·86	2·87	3·11	3·47	3·61	3·86	1·78	2·78	3·01	3·36	3·50	3·75
...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...	...	...	...
Ag	—	—	—	—	1·35	1·40	—	—	—	—	1·24	1·29

However, it is found that when zinc is used as a tertiary radiator and the secondary beams from iron to zinc fall upon it successively, a small value R is obtained, even though none of the characteristic homogeneous radiation is excited by these secondary beams. The ionization in the chamber I in these cases is due, partly to the scattering of the secondary radiation, partly to the production of tertiary radiation by the small percentage of scattered primary radiation in the secondary beam employed, and possibly in part to the production of a feeble type of very easily absorbed radiation much softer than the normal homogeneous radiation from zinc.

All evidence obtained up to the present points to the persistence of these residual effects even when the exciting secondary beam is sufficiently penetrating to produce the characteristic zinc radiation, and since in the calculation of k we only require a measure of the homogeneous tertiary radiation excited by the homogeneous secondary beam, these residual effects have been estimated and subtracted in each case.

The values of R finally obtained, which we may denote by R' , are given in Table VI.

The next step was to calculate in some one particular case an actual value of k from the data obtained by using a suitable pair of secondary and tertiary radiators. A flat polished plate of pure copper (3.04 cm. $\times$ 3.03 cm.) was mounted on a fine aluminium stem and placed in the position R_2 . The ionization-chamber connected to the tertiary electroscope E_3 was removed and replaced by the electroscope E_2 . The distance from the centre of the radiator to the centre of the tissue-paper-covered window of E_2 was 4.7 cm., the area of the window being the same as in previous experiments. The plane of the radiator being vertical with the normal to its surface bisecting the angle between the axis of the secondary beam and the normal to the aperture at its centre, of the electroscope E_2 .

From the relative positions of the tertiary radiator and the window of E_2 (carefully determined) a value of ω was calculated and found to be approximately .222.

The ratio of the ionizations in the electroscopes E_2 and E_1 was then determined, allowance being made for the air-effect in the manner previously explained, and the mean of several readings, none of which differed by more than 2 per cent. from the mean, was found to be .115.

The tertiary radiator was then removed and the electroscope E_2 was placed with its window perpendicular to the axis of the secondary beam, the centre of the window being in the position previously occupied by the centre of the tertiary radiator. Since the area of the window of E_2 was almost exactly equal in shape and size to the area of the tertiary radiator projected in a plane perpendicular to the axis of the secondary beam, the ionization will correspond to the mean intensity over the area of the tertiary radiator in the previous part of the experiment. The mean value of the ratio of the deflexions in E_2 and E_1 was now equal to 11.48.

Substituting the values of α , β , λ_1 and λ_2 from Table V., we find

$$k = \frac{.115}{11.48} \times \frac{60.7}{128.9} \times \frac{6.0}{9.2} \times \frac{4\pi}{.222} \times (1575 + 488) = 361.$$

The value of λ_2 was increased by 3 per cent. to allow for the obliquity; the value of k was, however, affected by less than 1 per cent.

In the series of experiments previously mentioned, in the course of which a copper radiator was subjected to secondary beams from the group of metals Cr-Ag, we obtained values of R' with arsenic as radiator, and in these cases

$$k = \frac{R'}{\mu} \cdot \frac{\alpha}{\beta} \cdot \frac{A}{S'} \cdot \frac{4\pi}{\omega} (\lambda_1 + \lambda_2),$$

where μ is the ratio of the sensibility of the electroscope E_3 to that of E_2 ; S' is the area of the tertiary radiator and ω' the mean solid angle subtended by the radiator R_2 at the window of the ionization-chamber I; and α , β , λ_1 , and λ_2 as defined above and corrected for the obliquity of the rays. This correction was determined experimentally.

Equating these values of k , we find

$$\frac{4\pi A}{\mu\omega'S'} = \cdot 093,$$

and we may therefore write

$$k = \cdot 093 \times R' \times \frac{\alpha}{\beta} \times (\lambda_1 + \lambda_2).$$

The value of k calculated from the data given in Tables V. and VI. are given below.

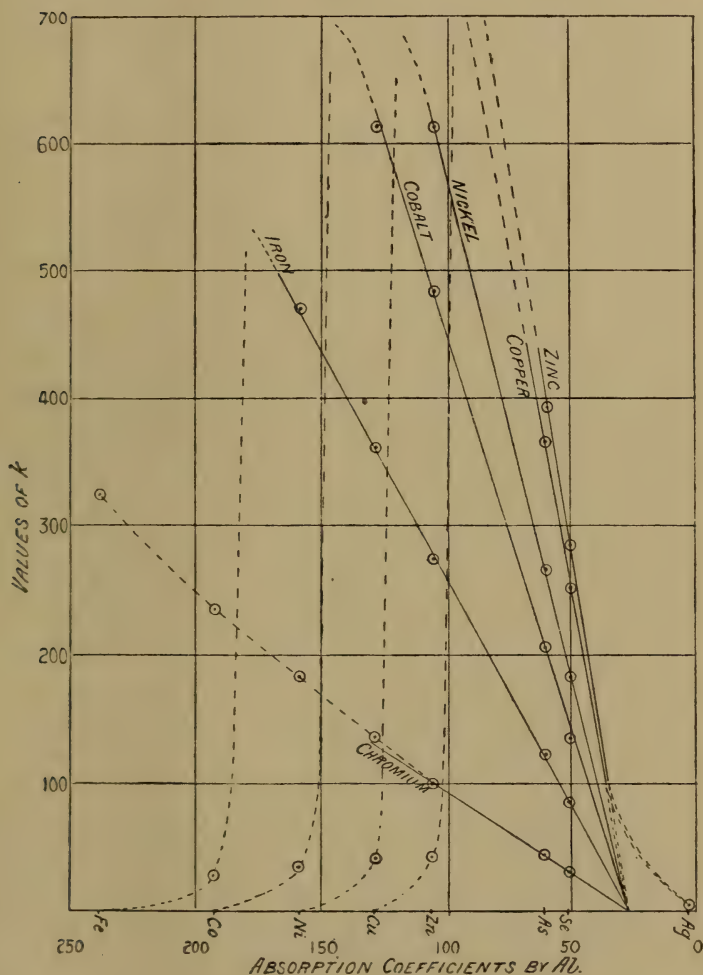
TABLE VII.

SECONDARY RADIATORS.	Values of k .						Values of $\frac{k}{\lambda'}$.					
	TERTIARY RADIATORS.											
	Cr.	Fe.	Co.	Ni.	Cu.	Zn.	Cr.	Fe.	Co.	Ni.	Cu.	Zn.
Iron	378						·176					
Cobalt	275	31·5					·154	·309				
Nickel	216	526	38·4				·145	·252	·287			
Copper	162	403	670	43·2			·127	·225	·305	·288		
Zinc	117·5	307	528	657	43·0		·109	·207	·282	·340	·358	
Arsenic	50·5	137	227	288	390	412	·0757	·150	·195	·233	·288	·327
Selenium ...	36·0	96·0	151·0	197	267	300	·0624	·120	·149	·187	·231	·278
Silver					5·35	5·30					·0282	·0313

NOTE.—In the case of each tertiary radiator, when the secondary beam is only just more penetrating than the radiation characteristic of the tertiary radiator, the value of k is small and cannot be determined nearly as accurately as the later values; nor can the value of λ' corresponding to the initial small value of k be determined with accuracy. Consequently the initial values of $\frac{k}{\lambda'}$ are somewhat uncertain, and it is impossible to determine from them whether $\frac{k}{\lambda'}$ for any given tertiary radiator rises to a maximum, and then decreases in value, or whether it continuously decreases from the initial value.

If the values of k for each tertiary radiator are plotted as ordinates against the absorption coefficients by aluminium of the exciting secondary beams as abscissæ, we obtain the graphs shown in fig. 4.

Fig. 4.



If λ_2 be the value of the absorption coefficient by Al of the characteristic homogeneous radiation from any substance A and λ_1 that of the exciting radiation (also homogeneous),

an examination of the graphs in fig. 4 leads to the following conclusions:—

(1) If λ_1 is equal to or greater than λ_2 , the intensity of the homogeneous radiation emitted by A is inappreciable.

(2) If λ_1 is slightly less than λ_2 the intensity is measurable but still small.

(3) For a further slight decrease in λ_1 a very rapid increase in the intensity of the radiation from A takes place, and for a value of λ_1 not greatly less than λ_2 the intensity reaches a maximum value.

(4) Beyond this point, over a considerable range, as λ_1 decreases the intensity decreases as a linear function of λ_1 . For this range the relationship may be represented by $k=a(\lambda_1-b)$, where a and b are constants. For the group of tertiary radiators employed, b has the same value for each member of the group, while a increases with the increase of the atomic weight of the tertiary radiator.

But the absorption by Al of the radiation from the group of metals Cr–Ag is proportional to the absorption by air of these radiations, as previously mentioned; and it has been shown that the intensity of the homogeneous radiation excited in a thin sheet of copper by a primary beam is proportional to the ionization produced in air by the same beam*: we may, therefore, over this range, write $k=a(ci-b)$, where i is a measure of the ionization produced in 1 c. c. of air by the exciting beam, and c the ratio of the absorption by 1 c. c. of air to the ionization produced in it in consequence of that absorption, a and b having the same values as before. (It will be seen from the Zn and Cu curves that there is a departure from this linear relationship for very penetrating beams.)

When a secondary homogeneous Röntgen beam falls normally upon a tertiary radiator, the amount of energy absorbed per sec. per unit area of its surface in a layer of thickness δx at a distance x below the surface is $I\lambda_1\delta x$, where I is the energy crossing unit area of the surface per second at the depth x below the surface and λ_1 is the absorption coefficient by the substance of the tertiary radiator of the secondary beam. If, however, we confine our attention to the absorption which is directly involved in the process of emission of tertiary rays, we must substitute λ' for λ_1 , where λ' is the increase in the value of the absorption coefficient, and the amount of energy so absorbed per second in this layer is $I\lambda'\delta x$. [The values of λ' are given in Table V.]

But the amount of energy emitted as tertiary radiation

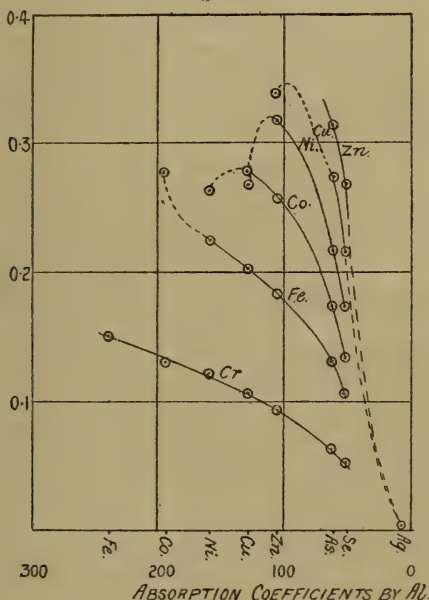
* Barkla & Sadler, *Phil. Mag.* Oct. 1908, pp. 550–584.

per second from this layer in consequence of such absorption is, as we have seen, $Ik\delta x$.

Therefore $\frac{k}{\lambda'}$ is a measure of that fraction of this special absorption of energy which is re-emitted as energy of tertiary radiation.

The values of $\frac{k}{\lambda'}$ are given in Table VII., and the curves obtained by plotting these values as ordinates against the values of λ_1 by Al of the exciting radiation as abscissæ are shown in fig. 5.

Fig. 5.



For any given tertiary radiator the value of $\frac{k}{\lambda'}$ attains to a maximum for a value of λ_1 in the neighbourhood of that which gives a maximum value of k . As the exciting radiation becomes more penetrating this fraction diminishes slowly at first and then more rapidly.

Taking the case of copper as a tertiary radiator for example, the maximum value of the fraction which corresponds to a value of λ_1 about 89 is nearly $2/5$, when λ_1 decreases to 51 the fraction has fallen to about $1/4$, and for a value of $\lambda_1 = 6.96$ to about $1/40$.

At the present stage of the inquiry the cause of this rapid
Phil. Mag. S. 6. Vol. 18. No. 103. July 1909. K

decrease in the value of $\frac{k}{\lambda}$, can merely be a matter for conjecture, but it is hoped that the data obtained in subsequent experiments will throw more light upon the phenomena*.

It will be observed that in determining these relationships the atomic weight of nickel is nowhere assumed, but the results obtained all point to the conclusion that the behaviour of nickel, whether as radiator or absorber of X-rays, is such that we should expect its atomic weight to lie between those of cobalt and copper.

The evidence for this, obtained during the course of these experiments, may be briefly summarized as follows:—

(a) The secondary homogeneous radiation from cobalt excites *no* homogeneous tertiary radiation in nickel.

(b) The secondary homogeneous radiation from nickel excites *no* homogeneous tertiary radiation in nickel.

(c) The secondary homogeneous radiation from nickel *does* excite homogeneous tertiary radiation in cobalt, and to an extent to be expected for the radiation from a substance of atomic weight about midway between those of cobalt and copper.

But it has been shown for the group of metals Cr–Ag that

(1) The penetrating power of the radiation characteristic of any member of the group increases with its atomic weight †.

(2) The radiation characteristic of a substance is only excited by a more penetrating radiation ‡.

If the atomic weight of nickel were greater than that of cobalt, then by (1) we should expect to find its characteristic radiation to be more penetrating than that of cobalt, and consequently by (2) that the radiation from nickel *should* excite in cobalt its characteristic radiation, while the radiation from cobalt should *not* excite the radiation characteristic of nickel. If, on the other hand, the atomic weight of nickel were less than that of cobalt, then the results would be the reverse of the former.

It has been suggested that the anomalous behaviour of

* *Note.* Experiments now in progress indicate that when the exciting beam is more penetrating than the radiation characteristic of the tertiary radiator, part of the energy absorbed reappears as an easily absorbed corpuscular radiation, and this effect becomes more pronounced as the exciting beam becomes more penetrating.

† Barkla & Sadler, *Phil. Mag.* Sept. 1907, pp. 812–828; also *Phil. Mag.* May 1909.

‡ Barkla & Sadler, *Phil. Mag.* Oct. 1908; also present paper.

nickel may be due to oxidation of the cobalt or the nickel. The author has found, however, that if iron be used as a tertiary radiator, and the secondary radiations from the group of metals Cr-Ag successively fall upon the iron in the manner described earlier in the paper, the results obtained are exactly similar whether the radiator is coated with a layer of rust or brightly polished, the relative values of k being the same in the two cases.

The result (*b*) would not have been obtained if the nickel experimented with had contained an impurity which emitted a characteristic type of radiation "harder" or "softer" than that characteristic of nickel. For the harder component of the secondary beam (whether due to the nickel or the impurity) would have excited the softer component radiation in the tertiary radiator, and this would have been detected if the impurity had been present to any appreciable extent.

Summary of Results.

(1) The characteristic Röntgen radiation from a substance is found to be identical in character whether it is emitted as a secondary or a tertiary radiation. In each case (*a*) the radiation is homogeneous, (*b*) the absorption coefficients of the radiation by other substances are the same, and independent of the penetrating power of the exciting radiation by which these homogeneous beams are produced.

(2) Using the homogeneous secondary beams emitted by the members of the group of metals Cr-Ag, excited by suitable primary beams, the emission of tertiary radiation by the earlier members of the group Cr-Ag when excited by these secondary beams was found to be governed by the following laws:—(*a*) With a given substance as radiator, its characteristic radiation is only excited by those secondary beams which are more penetrating than the tertiary radiation characteristic of the substance. (*b*) When the secondary beam is only just more penetrating than the tertiary the intensity of the latter is small, but as the secondary beam becomes more penetrating a very rapid increase in the intensity of the tertiary radiation to a maximum takes place. (*c*) As the secondary beam becomes more penetrating still, the intensity of the tertiary radiation decays as a linear function of the ionization produced in a given volume of air by the secondary beam.

(3) It has been found that when the secondary homogeneous beams from the group of metals Cr-Ag are absorbed by thin sheets of metals from the same group, a big increase in the absorption takes place when the secondary beams

become more penetrating than the radiation characteristic of the absorber, this increase in the absorption being intimately connected with the emission of tertiary radiation by the absorber in these circumstances (see paper on "The Absorption of X-Rays," Barkla & Sadler, Phil. Mag. May 1909).

The fraction of this increase in the absorption of the energy of the secondary beam, which is re-emitted as tertiary radiation, is not constant, but decreases as the secondary beam becomes more penetrating, slowly at first, and then more rapidly when a very penetrating secondary beam is used.

In conclusion I wish to thank Dr. Barkla for the interest he has shown throughout this research, and especially for his kindly criticism and advice during the writing of this paper.

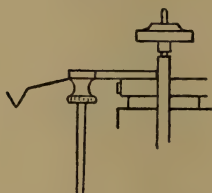
George Holt Physics Laboratory,
University of Liverpool,
24th March, 1909.

XII. On a Method of Determining the Sensibility of a Balance.

By J. H. POYNTING, *Sc.D., F.R.S.*, and G. W. TODD, *M.Sc.**

IN the method, as we have arranged it, a small frame (fig. 1, end view) is fixed at the centre of the beam of a 16-inch Oertling balance. This carries two Vs about 2 cm.

Fig. 1.



End view of V frame fixed to balance-beam.

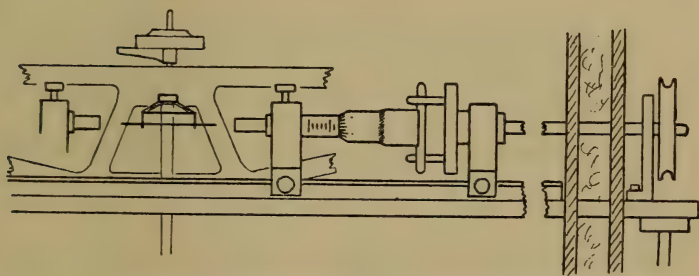
apart, and in the Vs lies a straight wire or fibre about $3\frac{1}{2}$ cm. long, parallel to the beam and level with the central knife-edge. This wire takes the place of the ordinary rider, and

* Communicated by the Physical Society: read June 25, 1909.

we shall call it "the rider." Its weight is determined before use as accurately as possible by weighing on an assay balance. The sensibility is determined by moving the rider either to right or left a measured distance. If this distance is d , if the half length of beam is b , and if the weight of the rider is R , the movement is equivalent to an addition of weight to one pan, Rd/b .

In order to move the rider a definite distance a stout horizontal rod (fig. 2) passes through the balance-case from side to side without contact with the case, and is supported at its ends outside, and independent of, the case. It is parallel to the beam and a little lower than the V frame. On the rod are fixed horizontally two Brown & Sharp micrometer-screws divided to $\cdot 01$ mm. and allowing an estimate of $\cdot 001$ mm.

Fig. 2.



Arrangement of right-hand micrometer-screw.

Their axes are in one line coinciding with the axis of the rider, and they are fixed so that one can bear against one end and the other against the other end of the rider. Their ends are plane and the ends of the rider are bluntly pointed. Each micrometer screw-head has a cross piece fixed on it, and a fork which can be rotated about an axis in the continuation of the axis of the screw by a pulley outside the case, can engage with the cross piece, and so advance or withdraw the screw. The pulley is worked by an endless string passing to a pulley at the side of the observer, who is about 2 metres in front of the balance. The micrometer divisions are illuminated and each micrometer is viewed by its own telescope. The position of the balance-beam is read by a double-suspension mirror, telescope and scale. The scale is divided to millimetres and is about 3 metres from the

mirror. The double-suspension mirror is fully described in the Phil. Trans. A, 1891, p. 572. It is of course not essential to the method, but was chosen because of the great magnification of the deflexion which it gives.

Let us suppose that the value of the scale-divisions of the deflexion is to be determined by a movement of the rider from right to left. The two micrometer-screws are withdrawn so that neither is in contact with the rider, that on the left so far that the rider will not touch it in its subsequent travel. The beam is lowered and allowed to swing. Then the right-hand screw is advanced till it bears against the end of the rider and pushes it some small distance. The contact is seen to have occurred by the interference with freedom of swing, as watched in the telescope. The micrometer is then read. Let its reading be m_1 . It is then withdrawn a little so as to leave the rider free, and the centre of swing C_1 is determined in the usual way from three successive turning points. Then the micrometer is advanced again so as to push the rod a little further, and its reading m_2 is taken. It is then withdrawn and the new centre of swing C_2 is taken. If $m_1 - m_2 = d$, $C_1 - C_2$ divisions deflexion are due to an addition of Rd/b to the left pan.

The right-hand micrometer may then be withdrawn and the left-hand micrometer may be brought into action in a similar manner, and so on, the two screws being used alternately.

The balance-case was fixed on a shelf and was enclosed in a tin-foiled wood box with wool loosely packed between box and case. The case and box were provided with plate-glass windows to view the mirror and the micrometer divisions. The following abstract of some determinations of sensibility will serve to show what accuracy may be attained :—

I. Rider German silver wire, 7.35 mgm.

Half length of beam, 20.272 cm.

10 determinations alternately left and right.

Mean travel of rider 2.4850 mm.

Mean deflexion 21.26 divisions.

Mean value for 20 divisions ... 0.0848 mgm.

The separate determinations range between

20 divisions = 0.0877

and 20 „ = 0.0824.

II. The same rider.

10 determinations alternately left and right.
 Mean travel of rider ... 5.2713 mm.
 Mean deflexion 45.47 divisions.
 Mean value for 40 divisions ... 0.1681 mgm.
 The separate determinations range between
 40 divisions = 0.1722
 and 40 ,, = 0.1632.

III. Rider German silver wire, 189.05 mgm.

7 determinations alternately left and right.
 Mean travel of rider 0.1764 mm.
 Mean deflexion 38.70 divisions.
 Mean value for 40 divisions ... 0.1691 mgm.
 The separate determinations range between
 40 divisions = 0.1709
 and 40 ,, = 0.1654.

IV. The same rider.

7 determinations alternately left and right.
 Mean travel of rider 0.3004 mm.
 Mean deflexion 64.84 divisions.
 Mean value for 60 divisions ... 0.2578 mgm.
 The separate determinations range between
 60 divisions = 0.2612
 and 60 ,, = 0.2518.

XIII. *The Balance as a Sensitive Barometer.*

*By G. W. TODD, M.Sc.**

IT occurred to the author while testing with Professor Poynting the accuracy of a new method of determining the sensibility of a balance, described on p. 132, in which a thin rod or fibre is used instead of the usual rider, that a balance might with proper precautions be converted into a very delicate barometer.

A difference in volume on the opposite sides of a balance will give rise to motions of the pointer when the density of the surrounding air changes. Small oscillations of the pointer, due chiefly to convection currents, become negligible if this difference in volume is sufficiently large. Alterations in the density of the air are produced by changes in pressure, in temperature, and in the percentage of aqueous vapour in

* Communicated by the Physical Society : read June 25, 1909.

it. By eliminating the effects of any two of these the variations in the other may be obtained. Thus the balance might be used either as a barometer, thermometer, or hygrometer.

Experiments were carried out with the balance mentioned in the previous paper. A sealed copper sphere, about 760 c.c., was suspended from one arm and counterpoised with brass weights placed in the opposite scale-pan. The movements of the pointer were observed by means of a small "doubly suspended" mirror which reflected a distant scale into a telescope. One division in this telescope corresponded to a rise or fall in the atmospheric pressure of the order of .002 mm. of mercury.

If W , V are the weight and volume of the sphere and W' v similar quantities for the counterpoise, we have, assuming that the difference in volumes of the opposite sides of the balance itself is nil and that the air is dry,

$$W - VDp \frac{273}{760} \frac{1}{A} = W' - vDp \frac{273}{760} \frac{1}{A},$$

where p = atm. press. in mm., D = density of air at 0° C. and 760 mm., A = abs. temp.

$$\therefore W - W' = (V - v)Dp \frac{273}{760} \frac{1}{A}.$$

If the air is moist and the pressure of aqueous vapour is f mm.,

$$W - W' = (V - v) \frac{273}{760} \frac{1}{A} \{(p - f)D + fd\},$$

where d = dens. of wat. vap. at 0° C. and 760 mm.

Suppose there is a change ∂W when there are changes ∂p , ∂f , ∂A , then

$$\partial W = (V - v) \frac{273}{760} \left[\partial p \frac{D}{A} + \partial A \frac{(D - d)f - Dp}{A^2} + \partial f \frac{(d - D)}{A} \right].$$

With the balance-case dry,

$$\partial W = (V - v) \frac{273}{760} \left[\partial p \frac{D}{A} - \partial A \frac{Dp}{A^2} \right]$$

or

$$\partial p = \frac{760A}{(V - v)273D} \partial W + \frac{p}{A} \partial A.$$

In the micro-barograph, which is given later (fig. 1),

$$\partial W = 4.6 \times 10^{-6} \partial S,$$

where S is the scale-reading in the telescope. This ratio was the mean of several obtained by moving the rider a known distance and noting the deflexion in the telescope, taking into account the rate at which S was altering at the time. The quantity $(V-v)$ differed by less than 1 c.c. from 760 c.c., so that

$$\partial p = \frac{4.6 \times 10^{-6} A}{273D} \partial S + \frac{p}{A} \partial A.$$

With the temperature and pressure variations acting together the whole scale would often pass across the field of the telescope in less than half an hour, and it had to be brought into view again by moving the rider in the required direction by means of the micrometer-screws. Observations of S and A were taken alternately every thirty seconds, and the barograph was obtained by plotting the terms on the right of the above equation and adding.

The barographs showed many oscillations in pressure of the order of .02 mm., the period of oscillation being from ten to fifteen minutes, a period quite different from that of the balance. The question arose whether these oscillations were real oscillations of pressure, or whether they were caused by oscillations in temperature not shown on the mercury thermometer suspended in the balance in consequence of the capacity for heat of its bulb.

To decide this question twelve thermojunctions were made with fine wires of copper and constantan, each wire being about six centimetres long. Six of these junctions were embedded in wax and placed in a box of brass filings, while the other six were left exposed to the air. The covered junctions having their capacity for heat made very large by the surrounding brass filings, would "lag" behind the exposed junctions and would not be affected by small oscillations in temperature. The ends of this thermopile were connected to a low resistance galvanometer. One division on the scale was approximately equal to a difference of .009° C. between the exposed and covered junctions. When the thermopile was exposed in the room the scale-reading oscillated aimlessly to and fro, but when placed inside the balance no oscillations could be detected. The spot of light travelled slowly and uniformly in one direction, showing

that the temperature in the balance was slowly rising and that the covered junctions were "lagging."

After this the temperature of the air inside the balance was measured by thermojunctions to avoid errors due to lagging. One set of junctions was kept at a fairly constant temperature in a Dewar tube containing water, the temperature being indicated on a Beckmann thermometer.

An idea of the sensitiveness of such a balance-barometer may be obtained from the table given below and the corresponding micro-barograph. The observations given extended over half an hour. The Beckmann thermometer in the Dewar vessel indicated the same temperature throughout.

TABLE.

1° C. difference between the junctions in the balance and those in the Dewar vessel corresponded to 28·8 divisions on the galvanometer-scale.

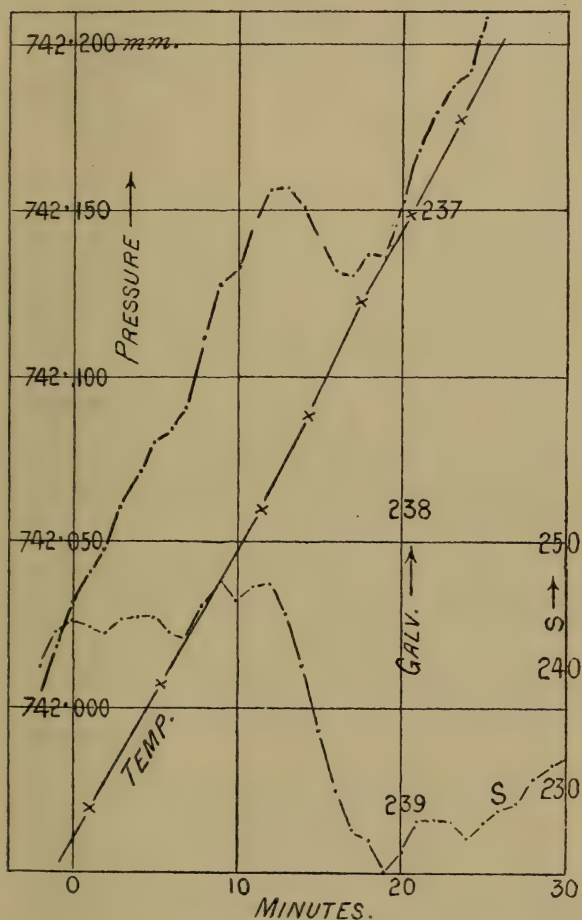
Galv. zero = 272·2. Room temp. = 13°·8 C. Rider at 13·152.

Time.	Telescope.	Galv.	Time.	Telescope.	Galv.
P.M. 4—0	243·7	239·1	P.M. 4—16	229·2	237·3
1	243·3		17	226·3	
2	242·6	238·9	18	225·6	
3	243·6		19	223·1	237·0
4	243·7	238·6	20	224·7	
5	243·9		21	227·0	
6	242·8		22	227·0	236·7
7	242·2	238·3	23	226·1	
8	244·9		24	225·8	
9	246·7		25	226·1	236·3
10	245·0	238·0	26	227·7	
11	246·2		27	228·5	
12	246·3	237·7	28	230·9	236·0
13	244·0		29	232·0	
14	240·1		30	232·6	
15	234·6		31	232·1	

The second column in the table gives the telescope-readings of the balance-pointer, and the third gives the

galvanometer readings from which the temperature inside the balance is obtained. In the barograph the nearly

Fig. 1.



straight curve represents the temperature, the lower irregular curve the actual observations of the position of the balance-beam, and the upper curve the positions reduced to a constant temperature.

XIV. *On the Distribution of Thorium in the Earth's Surface Materials.* By J. JOLY, F.R.S.*

SUBSEQUENT to the completion of the experiments described in my former paper on the subject of the distribution of thorium (Phil. Mag. May 1909, p. 760), I received through the kindness of Professor Sydney Young a sample of thorianite. At a more recent date Professor Wyndham Dunstan was so good as to send me some of the same mineral: part of a sample which he himself had analysed. This sample contains 79·36 per cent. of thorium oxide (ThO_2) and 14·32 per cent. of uranium oxide (U_3O_8).

A standard solution prepared from the first mentioned sample I shall refer to as solution A, and one prepared from the second sample, solution B. Both solutions are prepared by dissolving one decigram of the mineral in a few cubic centimetres of pure nitric acid and diluting with distilled water up to 100 c.cs. Each c.c. of solution B contains, therefore, $6·977 \times 10^{-4}$ gram of elemental thorium. It is probable that solution A does not differ much in thorium content (Dunstan, Proc. R. S. 1905, A, p. 261).

A considerable number of experiments were carried out with these solutions; and the constant of the electroscope was finally determined by means of solution B, as $2·2 \times 10^{-5}$ gram. That is to say, a gain in the rate of discharge of one scale-division per hour indicates $2·2 \times 10^{-5}$ gram of thorium in the solution being dealt with. The following results were obtained by taking in each case one c.c. of the standard solution and diluting up to one litre; or, again, by adding the standard solution to a rock solution:—

		Gain in Scale- Divisions per hour.
Solution A.	In water	30
	Same, tested 7 days later	30
	„ In water strongly acid	29
	Same, tested 3 days later	31
	„ In a rock solution, 800 ccs.	29·1
	Same, tested 3 days later.....	31·7
Solution B.	Same, tested 4 days after first test ...	29
	In water	32

Many additional experiments, varied in certain particulars, agreed in confirming the approximation of 29 to 32 scale-divisions per hour as the gain arising from the addition of 1 c.c. of the standard solution to the contents of the boiling-flask. The possibility that in certain rock solutions wherein but little thorium had been detected, and in others where there was an evident want of limpidity, some cause might

* Communicated by the Author.

exist to retain the short-lived emanation, was investigated by adding to such rock-solutions known quantities of thorium. Thus the rock solution referred to in the above table is that of St. Gothard, No. 162 South, which had yielded very little emanation, as may be seen by reference to the table of results on the St. Gothard rocks given further on. Again, the following experiments on rock solutions (prepared from 20 grams of rock) may be cited. One half a cubic centimetre of standard solution was used in the case of each.

		Gain in Scale- Divisions per hour.
Basalt, 800 ccs. (acid), before addition of sol. A.....		8
Same, after " " 		23
Granite, 1400 ccs. (acid), before addition of sol. A...		9.5
Same, after " " 		23.5

It will be seen that the basalt solution shows the normal increased rate of discharge for the added thorium, that is at the rate of 30 scale-divisions for one c.c. The granite gives the increased rate of discharge as 28 divisions for 1 c.c. The basalt solution was not limpid; the granite solution was muddy and very bulky.

In preparing test solutions, a small quantity of acid should be added to the water before the addition of the standard solution. I have found that in neutral solutions there is sometimes a partial loss of emanating power which may increase on successive experiments. This has not been noticed on acid solutions. Successive tests applied to rock solutions reveal rather a small increase of emanating power than otherwise.

The determinations given in my last paper were, as then stated, based on the emanating power of commercial thorium nitrate, and I drew attention to the fact that the calibration of the electroscope by a thorium mineral in radioactive equilibrium must probably have the effect of lowering the deduced results. As stated above, the constant of the electroscope referred to thorianite is 2.2×10^{-5} , whereas that based on commercial thorium nitrate was 2.8×10^{-5} . Some structural changes in the apparatus do not, however, permit the former results to be revised exactly according to the above figures; but a direct comparison of the standard nitrate solution with the standard thorianite solution B has shown that the emanating powers are closely in the proportion of 2 to 3. The reduction, by one third, of the values formerly given, will bring these results into close agreement with the thorianite standard of emanating power. The revised readings are given below with some few additional determinations. The result on the Moffat Dale shale has been

replaced by one on a mixed sample from six localities which since preparation had been stored some weeks. There is, I think, in some cases an increase of emanating power in the acid solutions which are kept for some weeks after the fusion with alkaline carbonates. All the tests given below are on old solutions.

	Thorium in grams per gram.
Lava, Vesuvius, 1906, 10 grams	2.6×10^{-5}
" " 1895-99, 10 grams	2.1 "
" " 1832, 10 grams	2.3 "
" " 1794, 10 grams	0.6 "
" " 1631, 10 grams	1.7 "
" Etna, 1855, 10 grams	1.2 "
" Pantellaria, 10 grams	2.2 "
" St Helena, 10 grams	0.6 "
Basalt, Giant's Causeway, 20 grams	0.9 "
Gneiss, Simplon Tunnel, 10 grams	1.1 "
Diabase, Fifeshire, 10 grams	less than 0.2 "
Shale, Moffat Dale, 10 grams	1.0 "
Granite, Co. Wicklow, 20 grams	1.1 "
Carboniferous Limestone, Armagh, 22.5 grams ...	less than 0.2 "
Marsupites Chalk, 32 grams	less than 0.6 "
Red Clay, N. Pacific, 2.4 grams	less than 0.9 "
" Central Pacific, 10 grams	0.4 "
Radiolarian Ooze, Central Pacific, 6.3 grams	less than 0.4 "
Manganese Nodule, S. Pacific, 12 grams	less than 0.2 "
Meteorite fell at Dundrum, 25.6 grams	0.09 "
Sea-Water, Coast of Dublin, 1400 ccs.	less than 1.4×10^{-8}
" S. Atlantic, 1790 ccs.	less than 1.1 "
" " 1500 ccs.	less than 1.4 "
" Indian Ocean, 2800 ccs.	0.9 "

In giving these results in the *Phil. Mag.* for May I regret that I was unaware of M. Blanc's important contributions to the same subject. M. Blanc has been so kind as to send me an account of his results which appears in this present number.

The determination of thorium in the meteorite which fell at Dundrum, Co. Tipperary, seems quite certain although the quantity present is small. Two prolonged experiments gave an identical result of a steady gain of little over one scale-division per hour. This meteorite is partly nickel-iron, but for the greater part, about 75 per cent., consists of non-metallic minerals.

The Rocks of the St. Gothard Tunnel.

The experiments which I had previously made upon the radium content of the rocks of the St. Gothard tunnel (Address to Section C, British Association, 1908) left me in possession of solutions of 51 rocks. These rocks had been selected so as to be as far as possible typical of the principal

Rocks of the St. Gothard Tunnel.

No. (Stapff).	Distance from N. entrance: Metres.	Description.	Thorium in grams per gram $\times 10^{-5}$.	Radium in grams per gram $\times 10^{-12}$.	
<i>Finsteraarhorn Massif.</i>					
1.....	0	Gneissic granite.....	7.55 grams.	1.8	5.2
6.....	304	Gneissic mica-schist, talcose ..	9.09	1.7	4.8
8.....	540	Gneissic granite	8.76	1.4	3.8
14.....	885	" "	8.93	2.7	5.5
16.....	957	" "	10.6	0.9	5.7
19 a ...	1099.5	Grey gneiss.....	12.0	3.2	6.1
23.....	1279.5	Grey gneissic granite ..	8.38	1.5	8.3
26.....	1520	Coarse gneissic granite	8.98	2.0	7.9
27.....	1600	" " "	10.0	0.9	9.9
28.....	1700	" " "	8.43	2.7	8.1
29.....	1900	" " "	8.27	1.5	14.1
<i>Ursernmulde.</i>					
33 a ...	2116.8	Ursern gneiss.....	11.14 grams.	1.4	3.2
36 b ...	2278.9	Quartz-schist	9.6	1.1	5.4
42.....	2582	Black lustrous slate	8.0	0.2	6.8
44.....	2605.1	Grey cipolin	9.47	0.4	2.4
47.....	2660.6	Quartzitic cipolin	11.36	0.4	14.3
55.....	2765.8	Black lustrous slate	10.66	1.0	4.2
61.....	2930.0	Ursern gneiss.....	9.1	1.3	2.1
71.....	3443.6	Sericite-schist	8.0	1.7	4.2
75.....	3756.1	Black lustrous schist	7.84	2.4	5.9
85.....	4038.8	Quartz mica-schist	8.16	<0.3	3.8
90.....	4290.0	Ursern mica-gneiss	8.38	0.5	1.7
<i>St. Gothard Massif.</i>					
92.....	4525.8	Gurschen mica-gneiss	8.53 grams.	0.9	3.2
95.....	4832.7	Hornblende rock	9.05	0.2	3.0
99.....	5250.0	Schistose serpentine	7.91	2.1	3.1
107 a ...	5734.2	Quartzose mica-gneiss ..	9.02	1.5	2.6
112.....	6075.4	" "	10.0	0.5	2.6
114.....	6204.4	Quartz-felspar mica-gneiss ...	8.47	1.3	5.7
116.....	6662.4	Gneiss.....	8.5	2.7	3.6
119.....	6426.9	Hornblende gneiss	9.02	0.8	8.4
124.....	6959.9	Gneiss.....	8.34	1.1	3.5
129 b ...	7366.6	Fine-grained gneiss	9.51	0.7	4.6
135.....	7695.3	Fine mica-gneiss	9.17	1.3	3.2
174.....	6741.1 from S. entrance.	Quartz-felspar mica-gneiss ...	8.67	1.3	2.1
167.....	6081.2	Mica-gneiss.....	10.0	0.9	1.3
166.....	5994.3	"	8.9	2.3	1.8
162.....	5507.2	Mica-schist gneiss	9.58	0.1	2.5
158.....	5267.3	" "	9.0	1.5	2.2
154.....	5002.4	Hornblende mica-gneiss	8.76	1.7	4.8
145.....	4501.9	Sella-gneiss	8.96	1.3	2.6
142.....	4078.2	"	8.4	0.7	2.7
132.....	3353.5	Quartz-gneiss	8.25	<0.3	8.9
130.....	3272.4	Mica-gneiss	8.73	1.5	9.2
<i>Tessinmulde.</i>					
121 d ...	3049.2	Hornblende-schist.....	8.64 grams.	<0.3	2.7
111 a ...	2792.8	Calcareous mica-schist.....	9.02	0.5	2.7
109.....	2682.9	Hornblende mica-schist	7.99	0.6	1.9
95.....	2008.1	Amphibole garnet mica-schist.	8.38	1.0	4.3
79.....	1528.3	Quartz-schist	9.5	<0.3	6.5
70.....	1014.5	Amphibole mica-schist	9.17	0.5	2.1
61.....	632.2	Quartz-mica-schist	8.8	0.5	2.7
15.....	80.0	Dolomite	8.66	0.4	4.1

rocks passed through by the tunnel. The solutions had been prepared with much care and stringent precautions against accidental contamination with radioactive materials. After each radium test the alkaline and acid solutions were put by in corked bottles. The quantities of rock in solution are, indeed, small; in most cases rather less than 10 grams. Notwithstanding this, in all but a very few instances, perfectly definite readings of the increased discharge rate due to the contained thorium have been obtained. The alkaline solutions have not in every case been dealt with. About one third of the entire number of alkaline solutions were investigated for thorium, but with negative results. This is to be expected from the limited sensibility of the method of observation and from the insolubility of both the carbonate and oxide of thorium, a property which must result in the retention of the greater part of the thorium in the residue of the melt after this is leached with water. In one case only was any appreciable quantity of thorium found in the alkaline solution. In this case a very abnormal amount of radium had also been detected in the alkaline solution.

The numbering of the rocks and their position in the tunnel are taken from Stapff's great work on the St. Gothard Tunnel (*Annexe Spécial aux Rapports du Conseil Fédéral Suisse sur la Marche de L'Entreprise du St.-Gothard*). The naming of the rocks is based on Stapff's nomenclature. I add the radium determinations for comparison. They have not before been published in detail. (Table, p. 143.)

The means of the determinations of the several geological divisions into which Stapff subdivides the course of the tunnel are as follows:—

	Thorium in grams per gram $\times 10^{-5}$.	Radium in grams per gram $\times 10^{-12}$.	Uranium in grams per gram $\times 10^{-5}$.
Granite and gneiss of Finsteraarhorn Massif .	1·85	7·7	2·26
Altered sediments, Ursernmulde	0·97	4·9	1·44
Schists, etc. of St. Gothard Massif	1·18	3·9	1·15
Altered sediments, Tessinmulde	0·51	3·4	1·00

The uranium is computed on Boltwood's measurement of the equilibrium amount of radium in one gram of uranium : $3\cdot4 \times 10^{-7}$ gram.

In my address, referred to above, I drew attention to a correspondence between the radium content of the St. Gothard

rocks and the distribution of temperature in the rocks as observed during the progress of the tunnel ; suggesting that the observed gradients may, in fact, be referable to radioactivity, wholly or in part. It will be seen that the radioactive energy due to thorium must also be predominant in that region of the tunnel in which the gradients were found to be steepest ; that is in the Finsteraar granite. To some source of heat in this granite, Stapff attributed the remarkable gradients obtaining in the first 2000 metres of the tunnel. We see now that this granite is not only richest in radium, but in thorium ; for however variable the results from one specimen to another may be, the added testimony of the 22 radioactive determinations in these rocks, and of the 80 measurements elsewhere throughout the tunnel, can hardly be deceptive. As observed in my former paper on the distribution of thorium, the thermal value of this element is not determinable without a knowledge of its rate of transformation or an experimental determination of its thermal out-put when in equilibrium with its transformation products*.

The quantities of thorium observed in the above experiments appear in the mean to come out as roughly proportional to the quantities of uranium. In individual experiments, however, a comparison of the radium and thorium content shows that very often such a relation is widely departed from. The net result is to ascribe to these rocks quantities of uranium and thorium which are not very different.

As in the case of the collective radium measurements now in our possession, so in the case of thorium according to the foregoing observations, sedimentary materials appear to reveal the denudative loss of radioactive materials.

That thorium is a generally distributed constituent of the earth's crust and enters into the composition of deep-seated materials seems established by its universal presence throughout so vast and complex a region as that traversed by the Alpine tunnel, as well as by its presence in plutonic rocks and lavas of many ages and from many parts of the world, and in the waters and sediments of the ocean.

Trinity College, May 22.

* I desire, here, to correct a mistake into which I fell when referring in my previous paper to the views of Professor Soddy. A misinterpretation of a passage in his paper of *Phil. Mag.* Oct. 1908, led me to believe that the writer maintained the probability of equality between the rates of break up of uranium and thorium. I have Professor Soddy's authority for stating that he had not intended to express that view.

XV. *On the Distribution of Thorium in the Earth's Surface Materials.* By G. A. BLANC, Ph.D. (Rome)*.

THE very interesting results communicated by Professor J. Joly in the May number of this Magazine induce me to recall the results of some researches which I have made on the same subject.

In a first note published in February 1908 (*Rendic. R. Accad. d. Lincei*, xvii. 1st sem. p. 101, 1908, and also *Phys. Zeit.* ix. p. 294, 1908) I gave the results of an investigation made by me in order to determine the amount of thorium contained in unit mass of the soil in Rome. The method consisted in comparing the amounts of excited activity of the thorium type which is produced on a negatively charged wire placed under a vessel covering a given area of soil with that produced when the vessel is placed over a layer of the same earth with which a known quantity of thorium hydroxide in radioactive equilibrium has been intimately mixed.

The result was that the Roman soil must contain ca. 1.45×10^{-5} gr. of thorium per gram. It is evident that this value must be considered as a minimum, the experiments having been made with thorium hydroxide, *i. e.* with the compound which has the strongest emanating power, while we do not know the real state of combination in which thorium is contained in the soil.

In this same note I announced that I had undertaken an investigation with the object of determining, in a more precise and satisfactory manner, the proportion of thorium contained in a certain number of rocks of different nature and origin.

The method followed in this second investigation (*Rendic. R. Accad. d. Lincei*, xviii. 1st sem. p. 241, 1909) consisted in dissolving, after fusion with alkaline carbonates, about 200 gr. of each sample of rock, and collecting the insoluble hydroxides (after having freed the solutions from radium by separating the insoluble sulphates). The hydroxides so obtained showed a noticeable activity when tested by the electroscopic method; this activity increased for a few weeks, until a maximum was reached.

This activity could not be due to radium, for the solutions from which the hydroxides were obtained no longer gave any appreciable trace of radium emanation after the insoluble sulphates had been separated; the test was made according

* Communicated by J. Joly, F.R.S.

to the method used by Strutt in his investigations on the amount of radium contained in rocks.

It remained to be seen whether part of the activity could not be due to actinium. Although the presence of traces of actinium cannot be excluded in an absolute way for the moment (careful tests are being carried out presently in order to settle definitely the question), it is certain that the greatest part of the activity shown by my hydroxides is due to thorium. In fact, by using a very sensitive electroscope built for researches on the ionization in closed vessels, I succeeded in obtaining quite distinct phenomena of excited activity from the hydroxides, the rate of decay being that of thorium-excited activity, without any sign of an initial rapid fall which could be attributed to the presence of actinium.

After the activity of the hydroxides had been determined, a given amount of thorium hydroxide in radioactive equilibrium was added to them, and the activity newly determined. In this way the amount of thorium hydroxide necessary to produce the activity originally shown by the hydroxides could be calculated, and therefore also the amount of thorium that must be present in each sample of rock.

The results so far obtained, to which has been added the one obtained in a quite different way for the soil of Rome, are the following:—

Vegetal earth (Rome)	1.45×10^{-5} gr. p. gr.
Syenite (La Balma, Biella)	8.28×10^{-5} „
Syenite (Bagni, Biella).....	6.30×10^{-5} „
Granite (Baveno, Lake Major)...	3.14×10^{-5} „
Granite (Voges, France)	2.07×10^{-5} „

Finally, in a quite recent paper (*Rendic. R. Accad. d. Lincei*, xviii. 1st sem. p. 289, 1909), although, of course, the small number of specimens of rocks examined by me does not allow us to draw any definitive conclusions, I have calculated the average production of heat due to the products of the thorium family contained in the rocks examined by me, finding that it is about twice the average production of heat due to the products of the uranium-radium family contained in the igneous rocks examined by Strutt. Further, as it is easy to calculate, the average amount of γ rays emitted by the products of the thorium family contained in the specimens tested by me is over six times greater than the average amount of γ rays emitted by the uranium-radium

family products contained in the igneous rocks tested by Strutt.

The research described above is being now extended to a large number of specimens of surface materials of the earth's crust, but the results so far obtained, as well as those now published by Professor J. Joly, allow us to conclude that thorium has certainly a considerable importance as regards terrestrial radioactivity.

Rome, Istituto Fisico della R. Università,
May 1909.

XVI. *On an Improved Form of Magnetometer and Accessories for the Testing of Magnetic Materials at Different Temperatures.* By JAMES G. GRAY, D.Sc., F.R.S.E., Lecturer on Physics in the University of Glasgow, and ALEXANDER D. ROSS, M.A., B.Sc., F.R.S.E., Assistant to the Professor of Natural Philosophy in the University of Glasgow*.

[Plate II.]

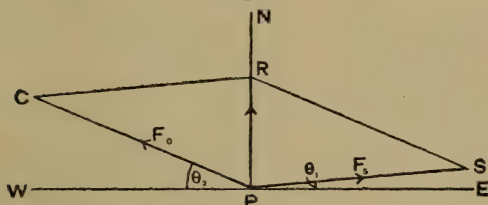
IN the usual form of magnetometer the magnetizing solenoid is placed with its axis in the magnetic east and west line passing through the magnetometer-needle. The effect of the current is balanced at the needle by means of a compensating coil connected up in the circuit. This latter coil has its axis coincident, or nearly so, with that of the solenoid. When a feebly magnetic specimen is under examination the solenoid, and consequently the compensating coil, must of necessity be brought up close to the needle. If large magnetizing currents are employed, any small shift of the coils from their correct positions may be sufficient to seriously impair the balance. In consequence of this the operation of adjusting the position of the compensating coil (the solenoid is usually clamped once for all in a convenient position) is a difficult one, especially as the slight inevitable movement of the coil which results from clamping it in position generally results in the balance being interfered with.

Even if this adjustment be accomplished with the requisite accuracy for the undisturbed position of the magnetometer-needle, it does not necessarily follow that the compensation is complete for the needle in its deflected position. In

* Communicated by Professor A. Gray, F.R.S. : read before the Royal Society of Edinburgh on February 1, 1909. Reprinted from the Proceedings of the Royal Society of Edinburgh.

practice, the axes of the solenoid and compensating coils are in general slightly inclined to one another and to the east and west line passing through the needle. The effect of this is to increase the directive force on the needle for one direction of the current and to diminish it for the other. That this is the case will be seen from fig. 1, in which the want

Fig. 1.



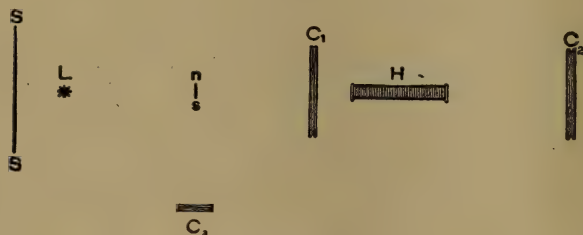
of alignment of the coil and solenoid has been greatly exaggerated. The magnetometer-needle is situated at the point P, and it has been assumed that the solenoid and coil are so placed that they produce fields at P in the directions PS and PC respectively. If the intensity of the field due to the solenoid be denoted by F_s , and that due to the coil by F_c , then since the coils balance for the undisturbed position of the needle it follows that $F_s \cos \theta_1 = F_c \cos \theta_2$. There are left, however, the components of the intensities in the north and south direction, and it is evident from the figure that if H is the horizontal component of the earth's magnetic field at P, the total directive force at the needle is $H + (F_s \sin \theta_1 + F_c \sin \theta_2)$. If the current is reversed in the circuit the directions of F_s and F_c change, and the directive force at the needle becomes $H - (F_s \sin \theta_1 + F_c \sin \theta_2)$.

The presence of the effect referred to may be made apparent by placing a permanent magnet close to the magnetometer, and thus deflecting the needle. On reversing a current in the circuit, a change in the deflexion will in general be observed. The magnitude of the errors introduced may be determined in this way for various parts of the scale and allowed for in the results, or the coils may be rotated until the effect disappears. If the former method is adopted, the labour of computing the results is much increased, and, further, it is difficult to make a proper correction, since the allowance to be made is a function both of the angle of deflexion and the strength of the current. The second method can only be used if the coils are capable of being rotated on their stands, and the adjustments would be difficult and troublesome to carry out.

The necessity for attending to this source of inaccuracy was first pointed out by Erhard*, who investigated the magnitude of the errors which were involved by neglecting it. In the case of a magnetometer of the usual type examined by him, it was found that with a magnetizing field of 128.3 c.g.s. units in the solenoid there was a change of 6.8 per cent. in the directive force on the needle on reversal of the current. Erhard advised that the magnitudes of the errors introduced should be determined for various parts of the scale and allowed for in the results.

While carrying out a research on certain feebly magnetic alloys, the authors found that the elimination of the aforementioned sources of error caused very considerable delay in the progress of the work. An attempt was therefore made to design a form of magnetometer which would overcome these disadvantages which are common to instruments of the usual type. In planning the apparatus the following requirements were kept constantly in view: (1) the magnetometer must be capable of accurate and rapid adjustment; (2) there must be no resultant Erhard effect; (3) the instrument must be suited for testing specimens at all temperatures from that of liquid hydrogen to the critical temperature; (4) it must be alike efficient for testing strongly magnetic and feebly magnetic specimens; (5) the magnetizing solenoid must be capable of furnishing fields up to at least 400 c.g.s. units; (6) the instrument must be rigid, all parts being fitted on one bed-plate, and the coils must be capable of being clamped without danger of destroying the compensation in so doing.

Fig. 2.



The general principle of the instrument which has been evolved will be seen from fig. 2. *ns* represents the magnetometer-needle provided with a concave mirror, by means of which and a source of light *L*, its movements are observed

* "Eine Fehlerquelle bei magnetometrischen Messungen," *Ann. der Phys.*, 1902, p. 724.

on the scale *SS*. *H* is the magnetizing solenoid placed due east or west of the magnetometer-needle and clamped in a convenient position. C_1 and C_2 are compensating coils placed with their axes approximately in coincidence with that of the solenoid. In adjusting the apparatus the effect of the current in *H* on the needle *ns* is first approximately annulled by means of C_1 , which is then clamped in position. The final adjustment of the compensation, so far as the undisturbed position of the needle is concerned, is carried out by means of C_2 , which on account of its great distance from the needle contributes only a small fraction of the balancing field, and thus provides an adjustment of great refinement. The position of C_2 necessary for balance having been obtained, it is clamped in position; obviously, since the distance of C_2 from the needle is great, any slight movement caused by doing so produces no perceptible effect on the compensation.

If the axes of C_1 , *H*, and C_2 were coincident and passed through the magnetometer-needle, the adjustment would now be complete. If, however, the needle *ns* is deflected by means of a permanent magnet, and a large current is reversed in the circuit, in general an alteration in the scale-reading on *SS* will be observed. A coil C_3 , placed with its axis in the magnetic north and south line passing through the needle, is now included in the circuit. By properly adjusting the direction of the current in C_3 , and altering the distance of C_3 from the needle, the compensation can be made perfect for all positions of *ns**. In a magnetometer where *ns*, C_1 , *H*, and C_2 are all carried on stands moving in one channel in the bed-plate, there should be little departure of the axes of the coils from coincidence. Accordingly the resultant magnetic field, due to the coils and solenoid, in the north and south direction will be small. The coil C_3 is therefore made of little power, and a small change in its position brings about only a very slight alteration in its effect upon the needle. It can therefore be clamped without any risk of upsetting the balance. The manner of making the adjustments will be fully explained later.

The instrument with its fittings is shown in plan in fig. 3 (p. 155). The bed-plate is in the form of a cross, and is built of well-seasoned mahogany planks 22 cm. broad and 2.5 cm. thick. The length over all is 350 cm., and the breadth from end to end of the arms 135 cm. The cross-piece is at a distance of 100 cm. from one end of the main length. Like

* A side coil has been used by Dr. G. E. Allan in his magnetometric work for giving compensation throughout the scale, but it does not permit of the adjustment here described.

the main portion of the bed-plate, it is formed from one piece of wood, the two lengths being set accurately at right angles and half checked into one another. The junction is made rigid by means of glue and brass screws. A channel 11.5 cm. broad is formed over the entire length of the cross-piece by means of two mahogany strips which are square in section and fixed parallel to the edges of the arms. A similar channel runs down the main length of the bed-plate, being discontinued where it is crossed by the channel already mentioned. The wooden strips forming these channels are permanently fixed by glueing and by brass screws driven in from the under side of the base-board. After they have been constructed they are made of perfectly uniform width by sand-papering, the width being tested from time to time during the process by means of a wooden gauge.

A is a mahogany box consisting of bottom, sides, and top, the ends which face east and west being left open. In the bottom is a slot running parallel to the cross-piece of the bed-plate. A brass screw projecting upwards from the base-board of the magnetometer passes through this slot and is provided with a brass washer and locking-nut. By this means the box can be moved through a small distance in the north and south direction, and securely clamped in position. On the upper surface of the box is fastened a plate of glass on which stands the magnetometer proper. This part of the instrument is also constructed of mahogany. A wooden pillar 20 cm. in height has a narrow hole drilled longitudinally down through it. This hole terminates in a small cell with a glass window in front. The cell is just large enough to contain the mirror of the magnetometer—a concave mirror, 1 cm. in diameter, having a focal length of 50 cm. The mirror has attached to its back a small piece of magnetized watch-spring about 8 mm. in length. The needle and mirror are suspended by a fine quartz fibre from a screw at the top of the upright pillar of the magnetometer. By means of this screw, the axis of which is vertical, all torsion can be removed from the fibre when the needle is hanging in its equilibrium position; and by giving the screw an observed number of complete turns a determination of the torsional rigidity of the fibre can be made. The pillar of the magnetometer is attached to a circular base provided with three small brass levelling-screws. The position of these feet on the glass top of the box-stand is defined by the hole, slot, and plane method.

AA (fig. 5) is the magnetizing solenoid. Two brass tubes 45 cm. in length are connected at their ends by brass rings

so as to form a water-jacket BB measuring 4 cm. in internal and 6 cm. in external diameter. On the outside of this is wound 868 turns of No. 15 s.w.g. copper wire in four layers (only one layer is shown in the figure). The wire is double silk-covered, and each layer is varnished over after winding. The terminals of the coil are mounted on an ebonite block at one end of the solenoid. D and C are the inlet and outlet tubes of the water-jacket. Although the water-jacket is somewhat narrow it is found to be effective in keeping the helix of wire cool, even though the interior is raised to a temperature of over 1000° C. by means of an electric furnace. The water-jacket is made small in capacity in order to keep down the mean radius of the solenoid, and hence maintain the end effect of the solenoid small. The field at the centre of a coil of length $2l$ and radius a is less than that given by $0.4\pi nC$ in the ratio $(l^2 - 2a^2)/l^2$, where n is the number of turns in the coil per unit length and C is the magnetizing current in amperes. In the case of the solenoid now described the reduction in the field from the value $0.4\pi nC$ due to the finite length of the coil is only 1.14 per cent. The solenoid is carried on a mahogany base-board provided with two vertical supports terminating in V-shaped grooves to receive the coil. The position of the solenoid carrier in the channel of the magnetometer board may be fixed by means of a brass clamp (shown in fig. 3). This friction clamp is furnished with two screws which press mahogany blocks against the outer surface of the wooden strips forming the channel of the magnetometer bed-plate.

C_1 and C_2 (figs. 2 and 3) are circular coils of 15 cm. radius erected on wooden stands provided with brass clamps as in the case of the solenoid. Each coil is wound in three sections, the terminals of which are screwed into the base of the stand. The sections in the case of C_1 contain 5, 7, and 9 turns of wire respectively, and in the case of C_2 6, 8, and 10 turns. These sections may be used singly or in combination, and accordingly there is a wide range of variability in the powers of the coils. C_3 is a coil of similar construction, but has a radius of only 6 cm., and is built in two sections of 1 and 3 turns of wire respectively.

D is a coil having a radius of 12 cm., and its function is to prevent loss of time due to the needle vibrating about its position of equilibrium. It is connected up in series with a single cell and a reversing key; and by properly tapping the key a series of impulses is communicated to the needle, which is thus quickly brought to rest.

L is a further sliding stand carrying the object screen.

This consists of a vertical wire placed in front of a window of obscured glass fitted in a metal box containing an electric lamp. By altering the position of this stand, the image of the cross-wire formed by the mirror of the magnetometer can be produced at any distance from 110 cm. upwards. From 150 cm. to 200 cm. is in most cases a suitable value. At this distance it is received on an engine-divided glass scale of the usual type.

E is a deflector stand on which a small permanent magnet may be mounted in the "B" position of Gauss. The construction of the stand is similar to that of the stand which carries the magnetometer proper. On the top of it is fixed a rectangular block of wood provided with a groove for receiving the magnet.

The bed-plate of the magnetometer is mounted on six pairs of mahogany feet, which are fastened to a rigid table by means of brass screws.

The process of setting up the apparatus is as follows. The centre of the magnetometer-needle has first to be placed on the axis of the solenoid. To accomplish this, coil C_1 (fig. 3) is removed, and the solenoid H is moved along the bed-plate towards A until its inner end is almost in contact with the back of the magnetometer casing. The stand A is then moved in its channel until the needle is brought exactly on to the axis of the helix, and is then permanently fastened in this position by means of the clamping screw already mentioned. The table carrying the magnetometer is now placed so that the long channel of the bed-plate lies due east and west, the adjustments being carried out and tested by means of the well-known method described in Gray's 'Absolute Measurements in Electricity and Magnetism.' The method is as follows:—A wire is stretched out vertically beneath the needle, and accurately parallel to the short channel of the bench. On passing a current through this wire a deflexion of the needle is produced. If the current is reversed in direction the deflexion will have the same numerical value as before, provided that the wire lies exactly magnetic north and south. The table is so placed that this condition is fulfilled, and its feet are then clamped to the floor by means of L-shaped brass brackets. The scale is erected on a separate table in order that the movements of the observer may not set up oscillations of the needle. The coils C_1 , H, and C_2 are now connected up in series with the storage-battery, ammeter, and variable resistances, &c., care being taken that the direction of the current in C_1 and C_2 is opposite to that in H. The permanent adjustments of the instrument are now complete.

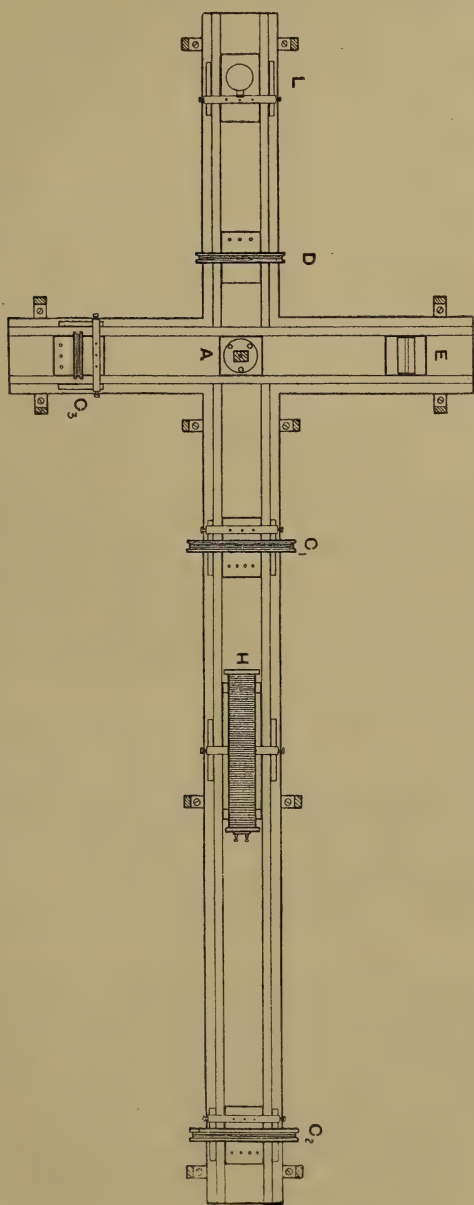


Fig. 3.

Plan of Magnetometer.

When a specimen has to be tested the solenoid H is moved to a convenient distance from the magnetometer-needle and firmly clamped. The coil C_2 is placed at the far end of the magnetometer table, and a current two or three times greater than the maximum to be used in the subsequent test is sent through the complete circuit. Coil C_1 is then moved until it just falls short of balancing the effect of the solenoid on the needle. It is then securely clamped. Coil C_2 is next slowly moved up towards the magnetometer-needle until the deflexion of the latter is brought exactly to zero; C_2 is now clamped, and the accuracy of the compensation verified by suddenly reversing the current in the coils. No measurable change in the scale-reading should result. The current having been interrupted, a small permanent magnet is next placed east and west on the stand E, and the stand moved along the cross channel in the magnetometer bed-plate until the spot rests near one extremity of the scale. The current is again made and reversed, and if any appreciable deflexion of the spot on the scale is observed coil C_3 is included in the circuit, the current being so directed through it that the deviations of the needle from its equilibrium position are diminished. The coil is gradually moved closer to the magnetometer until the Erhard effect is completely wiped out, and is then clamped in position. The compensation now holds for all parts of the scale, and the apparatus is ready for carrying out magnetic tests.

The several sections in which the three compensating coils are built allow the adjustment to be completely made with the coils in several different positions. This is a great advantage, as it always affords a means of escape from any arrangements of the coils which might prove awkward when specimens are in the solenoid.

With reference to the adjustment of the coils described above, it should be noted (1) that the method is systematic, and that there is no possibility of failure to secure a balance—all the adjustments are carried out in a perfectly definite manner; (2) that the method is delicate, for, owing to the great distance of coil C_2 from the magnetometer-needle, it may generally be moved several millimetres without causing any appreciable error in the compensation; (3) that the method is capable of furnishing a high degree of accuracy; with the near end of the solenoid at a distance of only 12 cm. from the magnetometer-needle it is quite possible to arrange that the change in the scale-reading brought about by reversing a current of 15 amperes in the circuit is only a fraction of 1 mm. with the scale at a distance of 175 cm.

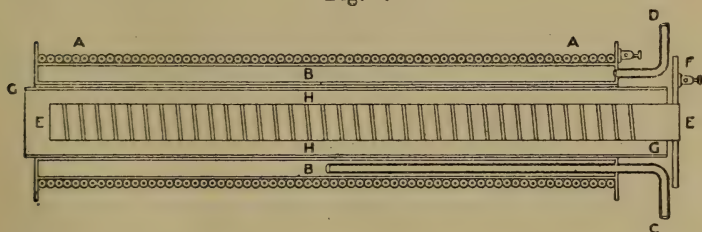
from the needle ; (4) that the operations can be carried out with great rapidity ; unless the solenoid is very close up to the magnetometer, the changing over of the apparatus from one degree of sensibility to another can be carried out within the space of two minutes.

The magnitude of the directive force at the needle is easily determined by passing a measured current through one of the balancing coils and noting the deflexion of the magnetometer-needle produced. The value of the directive force is then easily calculated.

Fig. 4 (Pl. II.) is a photograph of the apparatus when adjusted for the examination of a strongly magnetic specimen ; fig. 4 A shows the arrangement when a feebly magnetic specimen is being dealt with. When the solenoid has to be placed very close to the magnetometer-needle to allow of a very feebly magnetic specimen being examined, the coil C_1 is placed on the opposite side of the needle to the solenoid. For general use, however, it is convenient to have the solenoid and coil on the same side. It is worthy of remark, in passing, that even if C_1 is placed as close up as possible to the end of the solenoid, it cannot alter the field at the centre of the specimen by so much as $\frac{1}{4}$ per cent.

When used for testing specimens at temperatures higher than that of the room, an electric furnace of a type similar to that devised by Dr. G. E. Allan *, is placed within the helix. In fig. 5 it is shown in position. A tube E of

Fig. 5.



unglazed porcelain of about the same length as the solenoid having an internal diameter of 23.5 mm. and a thickness of about 2 mm., is wound non-inductively with fine platinum wire ; the ends of this wire are brought out to two terminals mounted on a slate frame at F. The tube is enclosed in a tube G of Jena glass, which fits as a cartridge within the magnetizing solenoid. The space HH between the glass and porcelain tubes is packed with dry kaolin clay, which performs the double duty of supporting the furnace and

* Phil. Mag. 1904, vol. vii. p. 46.

preventing the coils of the platinum wire from changing their positions when expanded by heat. A cylinder of electrolytic sheet-copper is placed within the tube E, and assists in maintaining a very uniform temperature over the space occupied by the specimens.

In the figure the platinum wire is shown equally spaced over the porcelain tube. In reality this is far from being the case. The proper winding of the tube is an exceedingly troublesome operation, and can only be accomplished by repeated trial.

The temperature of the furnace is measured by means of the ordinary thermo-element or a platinum resistance thermometer. The two wooden stands used for the pyrometer are shown in position in figs. 4 and 4 A. As will be seen at once, the several slots in the horizontal carrier for fitting on the tops of the stands permit of these latter being placed clear of the sliding bases of the compensating coils.

Fig. 6.



For tests at the temperature of liquid air the arrangement shown in fig. 6 is employed. The specimen A is enclosed in a glass tube BCD, of which the end B is closed and the end D is open and bent up. Cork bungs F, F are fitted on the tube so as to bring the axis of the specimen into coincidence with that of the solenoid. A third bung F or a pad of cotton-wool is used to prevent access of warm air into the interior of the solenoid, and a covering of cotton-wool on the portion CD prevents it from warming up and conducting heat to the specimen. Instead of closing the glass tube at B, a cork may be used to stop up the opening. The cork, however, if dry, is liable to loosen and permit the liquid air to leak out, or if it is at all damp it expands and fractures the tube.

Where tests have to be made as the specimen slowly warms up from the temperature of liquid air, a Dewar tube is used, with its mouth closed by a cork which has two bent tubes passed through it—one for pouring in the liquid air, and the other for the bringing out of the leads from one or more thermo-elements in contact with the specimen.

The dimensions given above for the internal diameter of the solenoid will be found sufficient for receiving a double vacuum Dewar tube for tests at -252°C . on specimens immersed in liquid hydrogen.

A slightly modified form of the stand supporting the solenoid permits of the latter being carried in an east and west position on one of the arms of the cross-piece of the magnetometer. The apparatus is therefore available for use with specimens in either the "A" or "B" position of Gauss; the methods described in Gray's 'Absolute Measurements in Electricity and Magnetism' for the determination of the effective lengths of the specimens thus become available.

The considerable height of the magnetometer-needle above the level of the magnetometer base-board (18 cm.) would also permit the apparatus to be readily adapted for testing by the "one-pole" method*.

Several instruments of the above type have been built in the Physical Institute of the University of Glasgow, and are giving every satisfaction.

XVII. *The Porous Plug Experiment.* By W. A. DOUGLAS RUDGE, M.A., Professor of Physics, Grey University College, Bloemfontein†.

WHEN a gas escapes through a narrow orifice so that its kinetic energy is nearly destroyed, it either falls or rises in temperature. This alteration in temperature is known as the Joule-Thomson effect. Few experimental determinations have been made of this effect since the original experiment of Joule and Thomson, but the matter has been so fully discussed that any reference to the previous work is unnecessary.

The author has made many experiments on the thermal change occurring during the expansion of a liquid gas, using for the purpose liquid carbonic acid contained in the small bulbs used for aerating water. If one of these is punctured below the surface of water contained in a calorimeter, and the rate of evolution of the gas controlled, a reduction in temperature will take place.

If we suppose that the gas remains liquid during the expansion, which might be the case if the rate of escape was slow, then the pressure would remain fairly constant, and the gas would thus be driven through the plug under a

* See Ewing's 'Magnetic Induction in Iron and other Metals,' p. 39.

† Communicated by the Author.

pressure whose value can be found from the tension of the gas at the temperature.

The total heat-change occurring in a calorimeter during the escape of the gas will be:—

- (1) The heat absorbed during evaporation.
- (2) " " " change in temperature of the gas.
- (3) The heat absorbed by the Joule-Thomson effect.
- (2) is a small quantity in the experiments described.

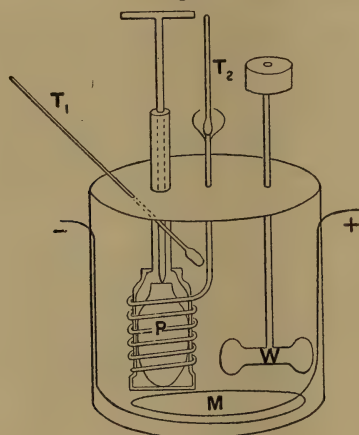
If H =amount of heat absorbed from the calorimeter, L =latent heat of the gas, S =specific heat, M =mass, and K =change of temperature due to the passage of the gas through the plug

$$H = M, L + (K \times M \times S),$$

$$\therefore \frac{H - M, L}{M, S} = K,$$

and if P =the difference in pressure on the two sides of the plug $\frac{K}{P} = \theta$ =change in temperature per atmosphere difference of pressure, the assumption being made that the specific heat remains constant.

Fig. 1.



P. Plug chamber. M. Manganin coil.
W. Turbine. T_1 and T_2 . Thermometers.

The apparatus (fig. 1) used consisted of a small oval-shaped calorimeter made of brass in which the plug was placed. The plug itself was a small brass vessel with a gas-tight cap

at the lower end, and having at the upper end a stuffing-box through which passed a hard steel screw for the purpose of perforating the bulb when an experiment had to be made. The vessel was of such a size that the bulb nearly filled it, and any free space could be plugged with cotton-wool if necessary. At the bottom of the vessel a fine brass tube 1 mm. in diameter was attached and carried round the chamber in the form of a spiral, the length of the tube being about 1 metre. The gas liberated from the bulb had to pass through the spiral before escaping into the air, and any heat absorbed by the gas during its expansion would be supplied by the liquid contained in the calorimeter. A spiral of "manganin" wire was placed in the bottom of the calorimeter so that any suitable temperature could be reached and maintained by an electric current. A small turbine wheel driven by an electromotor served to stir the liquid; the temperatures were read by a thermometer graduated into $\frac{1}{2}^{\circ}$. A second thermometer was placed in the escaping gas in order to ascertain whether the gas differed in temperature from the liquid in the calorimeter. This was not the case. The calorimeter was suspended inside a water-jacketed outer vessel, the temperature of which was adjusted by coils of wire through which a current was passing.

The mode of conducting the experiment was as follows :—

The bulb containing the gas was weighed and then placed in the plug-chamber. At first the bulb was packed round with cotton-wool, but experience showed that this was not necessary as the small hole pierced in the bulb, and the fine brass tube, checked the flow of the gas sufficiently.

In the first series of experiments the temperature of the liquid in the calorimeter was raised to about 20° C., and a cooling curve constructed over a range of temperature of 5° . A preliminary observation was taken of the alteration in temperature produced during the escape of the gas from a bulb so that radiation losses might be corrected in the usual way. The actual determination was now made with another bulb. The temperature of the calorimeter and its contents being adjusted to the required value, the screw was forced in and the gas allowed to escape slowly, the rate of escape of the gas being regulated by the position of the screw. The thermometer was read every minute and a curve plotted of the results. The time occupied by an experiment varied from 5 to 20 minutes, so that even at the shorter time the kinetic energy possessed by the escaped gas was small. The change of temperature was generally about 2° .

After the gas had escaped the bulb was weighed and the

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amount of gas originally filling it found. This averaged about 4.5 grammes. The internal volume of the bulb was about 5.35 c.c.

The heat absorbed was found as usual by multiplying the water equivalent of the calorimeter and its contents by the change in temperature, the water equivalent being found by sending a current of known strength through the manganin coil and calculating its value from $H = C^2RT$. The liquid in the calorimeter was generally water, paraffin was occasionally used, but as different samples showed variations in the specific heat it was considered best to adhere to water.

A typical experiment gave the following figures:—

Weight of bulb full of gas	17.31 grms.
" " empty	12.91 "
" gas	4.40 "
Temperature before escape of gas	22.25 °C.
Temperature after escape	20.35 "
Fall in temperature	1.9 "
Correction .05, total fall	1.95 "
Duration of experiment	25 minutes.
Water equivalent	134 units.
Heat absorbed	261.3 "
Latent heat of 4.4 grs. of gas	159.6 "
Heat absorbed by passing through the plug.	101.7 "
*Cooling of the gas necessary to account for the absorption of heat	110° C.
Pressure of gas at commencement of ex- periment	57.9 atmospheres.
Cooling effect per atmosphere	1.9 nearly.

The examples in the following table are selected because they are all about 20° C.

The numbers in the last column are fairly constant, and about 100 experiments have been made which give practically the same range in the numerical differences.

* The calculated value of the cooling was obtained by assuming the value of the specific heat of the gas to be .21, then

$$H = M \times S \times (\theta - t)$$

$$\therefore \text{Fall in temperature } (\theta - t) = \frac{H}{M \times S}.$$

TABLE I.

Temperature.	Weight of gas.	Heat absorbed from cal.	Latent heat.	Heat absorbed by plug.	Calculated cooling.	Pressure.	Cooling per atmosphere.
18.5	4.8	283.5	182.4	102.6	102	54.2	1.8
21.3	4.4	262	159.6	102.6	112	57.9	1.93
21.85	4.39	260	146	114	123	58.9	2.08
23	4.4	244	140	104	112	60.4	1.84
23.3	3.68	200	114	86	111	60.9	1.82

The latent heat of the whole mass of the gas was calculated by the usual formula

$$L = \frac{dk}{dt} T (V_1 - V),$$

the values of V , V_1 , and $\frac{dk}{dt}$ being taken from Amagat's table*,—the values of the pressures were also taken from the same source.

It may be objected that the pressure varied during the experiment, and this of course is true; but dealing with such small amounts of liquid gas, it would be extremely difficult to measure the alterations in pressure occurring during an experiment. If, however, we assume that the gas is boiling steadily, as is probably the case, the pressure inside the bulb will remain fairly constant until about $\frac{8}{9}$ of the gas has evaporated, and then the remainder of the gas would be in the gaseous condition, and during its escape the pressure would fall from the steady value down to that of the atmosphere. With a larger bulk of gas it would not be difficult to stop the experiment before all the gas had evaporated.

There is also a small change in the temperature of the gas, and this will influence results slightly.

The mean result deduced from these experiments is that carbonic acid in passing through the plug is cooled by

$$1^{\circ}.894 \text{ C.}$$

per [atmosphere] difference of pressure, at a temperature of

* Preston's 'Heat,' p. 394.

about 21° C., and under a difference of pressure of 57 atmospheres.

A second series of experiments was undertaken in which the gas was heated to a point above its critical temperature, so that the latent heat of evaporation did not require consideration. The same apparatus was used, the necessary temperature being reached by sending an electric current through the coil. The usual temperature was from 36° C. to 40° C. The experiment was carried out as follows:—A constant current measured by a Siemen's milliampere metre provided with shunts, was sent through the manganin coil, a carbon rheostat being included in the circuit so as to keep the current at a steady value, and this could be done to about 0.5 per cent. Observations were taken of time and temperature from about 32° to 40° and curves plotted:

(1) When the calorimeter containing the bulb was heated and the gas not allowed to escape;

(2) When the gas escaped during the heating.

The rate of rise in temperature was greater in (1) than in (2), and by comparison of the curves the fall in temperature during the escape of the gas was found.

The following table gives the result of one experiment.

TABLE II.

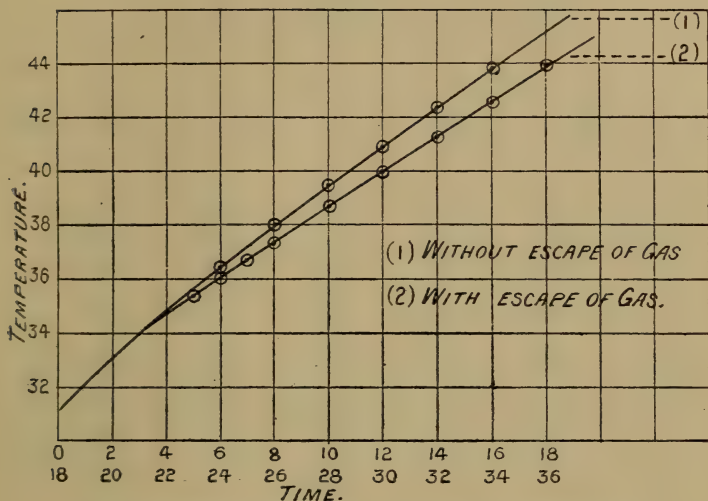
(1) When the gas not allowed to escape.		(2) During the escape of the gas.	
Time.	Temperature.	Time.	Temperature.
0	31.15	18	31.3
2	33.1	20	33.05
4	34.8	* G. 22	34.75
6	36.4	23	35.35
8	38.0	24	35.95
10	39.5	25	36.7
12	41.0	26	37.3
14	42.4	27	37.9
16	43.8	28	38.5
		29	39.15
		30	39.85
		32	41.2
		34	42.6
		36	43

* G indicates when the gas is liberated.

These two curves (fig. 2) become parallel after the evolution of gas has ceased, and the difference between two points on

the same ordinate during the escape of the gas gives the reduction in temperature caused by the escape, and this

Fig. 2.



(1) Without escape of gas.

(2) With escape of gas.

multiplied by the water equivalent of the apparatus gives the heat absorbed in the process.

In this particular case the cooling amounted to $0^{\circ}.926$ C., and the heat absorbed in the process to 215 units nearly. Since the gas during escape was at an average temperature of 37° C. it was above the critical temperature, and the pressure was calculated by finding the volume occupied by the gas, and dividing this by the capacity of the bulb.

The volume of the gas at 37° C. and 760 mm. was

$$\frac{4.56 \times 22320 \times 310}{44 \times 273} = 2630 \text{ cc.},$$

and since the capacity of the bulb was 5.35 c.c. the pressure was

$$\frac{2630}{5.35} = 492 \text{ atmospheres.}$$

Amagat's curves show that at this high pressure Boyle's law holds for carbonic acid.

As the gas escaped the pressure fell, and it was assumed

that the fall was uniform so that the average pressure was

$$\frac{492 + 1}{2} = 246.5 \text{ atmospheres,}$$

hence the cooling effect per atmosphere was

$$\frac{215}{4.56 \times .21 \times 246.5} = 0.911 \text{ C.}$$

Other experiments gave similar results, six typical ones being tabulated below.

Weight of gas.	Fall in Temp.	Heat absorbed.	Calculated fall *	Pressure of gas.	Cooling per atmosphere.
4.56	.962	224	223	492	0.911 C.
4.14	.926	215	247	445	1.15
4.13	.910	212	244	444	1.09
4.57	1.04	242	252	492	1.025
4.68	1.03	240	245	504	.967
4.59	1.04	242	251	494	1.015

* Calculated as on page 162.

These results are fairly consistent, and do not differ very much from the value originally given by Joule and Thomson. The temperature, however, did not remain constant during the experiment.

As there was some uncertainty as to the exact value to be taken for the fall in temperature, an attempt was made to conduct the experiment under such conditions that the temperature remained constant during the escape of the gas. This was done by so regulating the flow of the gas that the temperature remained constantly at 34° C. After many trials the best rate for the gas to escape and also the most suitable current to heat the calorimeter were determined, and it was found possible to keep the temperature constant within about $\frac{1}{20}$ ° C.

The amount of heat absorbed was found from the rate of rise of temperature just before and after the evolution of the gas. The results were not very different from those obtained in the previous experiments, but as the flow of gas was not free the variations in pressure were probably very considerable, and this may account for the variations in the value, and also for the higher numerical result.

TABLE III.
Constant Temperature Experiments.

Time.	Temperature.
0	31·6
5 m.	34·42
10 m.	33·23
15 m.	G 34·0
20 m. 15 s.	34·0
24 m. 10 s.	G ₁ 34·0
29 m. 10 s.	34·65

The average rate of rise of temperature during the escape of the gas was 0·142 per minute.

G and G₁ mark the commencement and cessation of flow of the gas.

TABLE IV.
Constant Temperature Experiments.

Weight of gas.	Time of escape.	Heat absorbed.	Calculated fall of T.	Pressure of gas.	Cooling per atmosphere.
4·52	7m. 45s.	260	267·5	473	1·128
4·52	7m. 55s.	296	312	473	1·31
4·67	9m. 30s.	308	314	487	1·29
4·53	5m. 20s.	249	262	474	1·103

The numbers in the last column do not agree very well, but the values are very much of the same order as those of the other experiments, and the variations indicate the order of the differences met with in about 50 experiments.

No very great accuracy can be claimed for these experiments, especially for those conducted below the critical temperature, as there is some uncertainty as to the correct value to be taken for the latent heat, and also the value of the specific heat must be continually changing under the great changes in pressure; but the method seems promising if carried out with more refined apparatus than is at present at the author's disposal. A piece of apparatus is now under

construction for carrying out the experiment at 0° C. There should be no difficulty in using large quantities of liquid gases as a means of obtaining constant high pressures provided a strong enough vessel is used.

The author has great diffidence in expressing any opinion as to the value of these experiments, but as very few determinations of the Joule-Thomson effect have been published, he thinks that these may be of some interest.

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XVIII. *On a Bifilar Vibration Galvanometer.*

By W. DUDELL, Fellow of the Physical Society.*

IN a paper read before this Society in May 1907, Mr. Albert Campbell drew attention to the great advantages of vibration galvanometers in the measurement of mutual inductances, and he further described a new type of vibration galvanometer which he had designed.

During the last few years the use of vibration galvanometers for the measurement of capacities, self and mutual inductions has greatly extended, and in the near future they may be expected to supersede the telephone and possibly the secohmmeter, for these purposes. As there are very few published data about the sensibility, etc., of these instruments I think it may be of interest to describe what I believe to be a new type and a series of tests made on it.

Vibration galvanometers like ordinary galvanometers may be broadly divided into two classes,—those in which the moving part consists of a piece of iron or steel and the current to be measured flows round fixed coils as in the case of the Thomson galvanometer,—those in which the current to be measured flows round a moving coil placed in a fixed magnetic field on the siphon recorder principle.

The vibration galvanometers of Max Wien and Rubens belong to the first class, while Mr. Campbell's moving coil vibration galvanometer belongs to the second, and so does the new bifilar instrument described below.

Generally speaking, when one requires to build a sensitive instrument having a short periodic time it is necessary to reduce as far as possible the mass of the moving parts in order to combine high sensibility with short period.

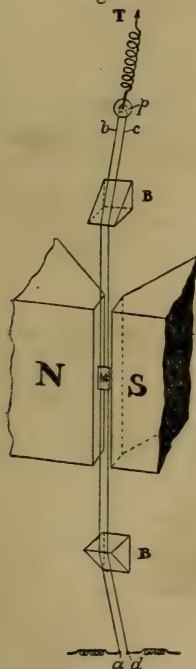
Further, in the case of a vibration galvanometer, it is also necessary to keep the damping forces as small as possible, as

* Communicated by the Physical Society : read May 14, 1909.

the sensibility to alternating currents depends very greatly on the magnification one can obtain by bringing the instrument into tune or resonance with the alternating current to be measured. These considerations have led me to construct a vibration galvanometer in which the mass of the moving parts is reduced to a minimum, the moving coil being reduced to the two wires forming its two sides, similar to a bifilar oscillograph, but with this difference:—Whereas the bifilar oscillograph is designed so as to make the damping aperiodic, the bifilar vibration galvanometer is designed so as to keep the damping as small as possible.

The design of the instrument* is shown in fig. 1, in which

Fig. 1.



a, b, c, d, is a fine bronze wire passing over a pulley *p*, and stretched tight by means of a spring, the tension on the spring being capable of variation by a milled head. The wires carry a mirror *M* in the centre and are placed in a strong magnetic field between the poles *N* and *S* of a magnet. The wires pass over two bridge pieces *B, B*, which limit the length of the wires which is free to vibrate. These two bridge pieces can be moved nearer together or further apart by means of a right and left handed screw as required. The current to be measured passes up one wire and down the other, causing one wire to tend to move forward and the other back in the magnetic field and so tilts the mirror *M* through a small angle.

The periodic time of the wires depends on their mass, length, and tension, as well as upon the moment of inertia of the mirror. In a completed instrument the moment of inertia of the mirror and the mass of the wires are fixed, but their length and tension can be altered in order

to adjust the periodic time. Fig. 2 shows the relationship between the free length of the wires and the frequency of the free vibrations of the instrument for different tensions, and fig. 3 gives the relationship between the frequency and the tension for a series of different lengths. It will be seen from the curves that the total range of frequency obtainable with the instrument is very large, namely, from about 90~ per second up

* Messrs. Nalder Bros. are manufacturing the instruments.

to 1900, though the wires are rather too loose below 100~ per second.

Fig. 2.

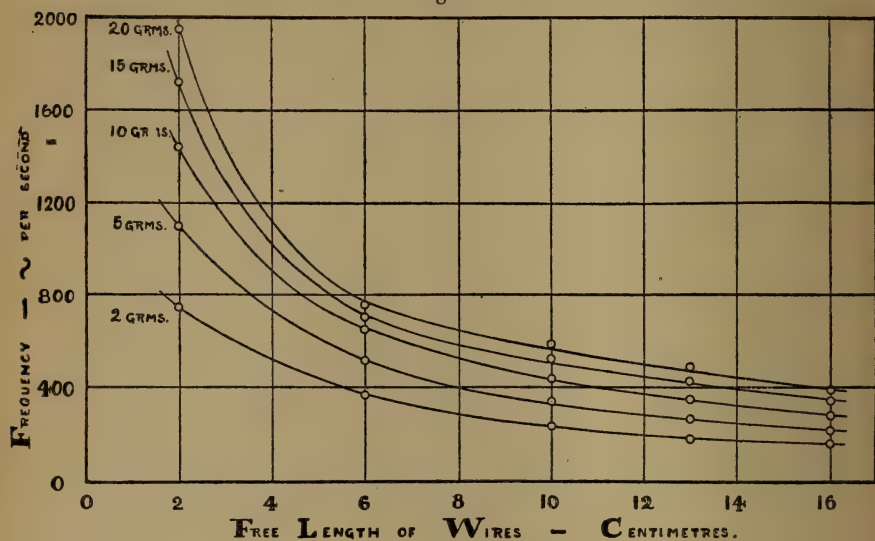
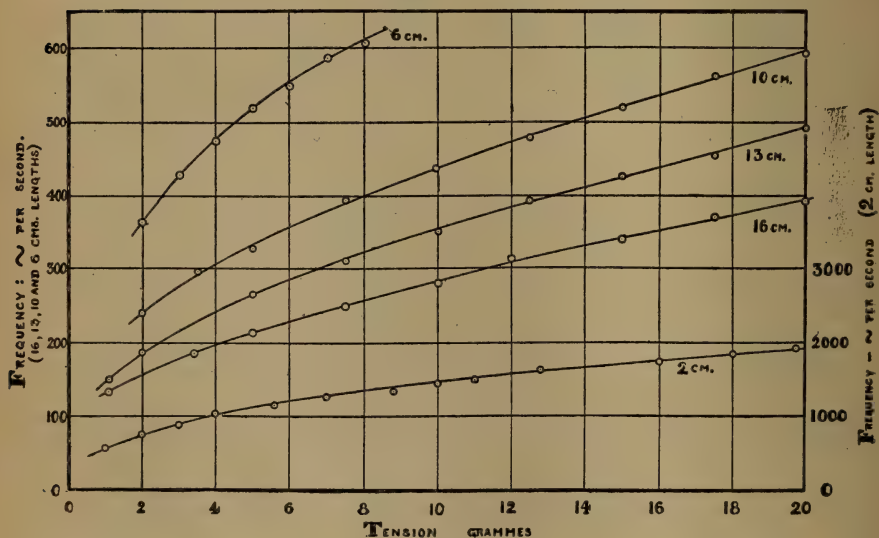


Fig. 3.



As the damping in this instrument is very small the resonance is very sharp. Figs. 4 and 5 show the amplitude

Fig. 4.

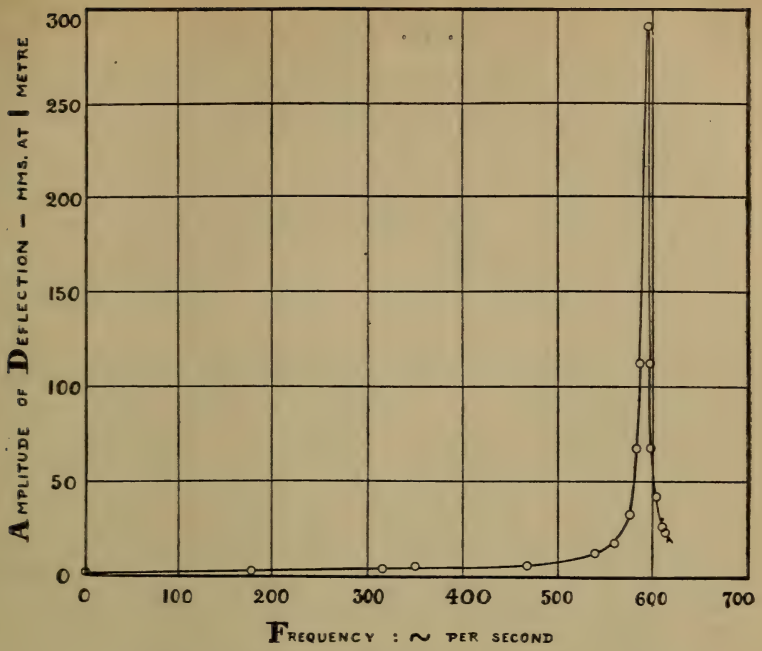
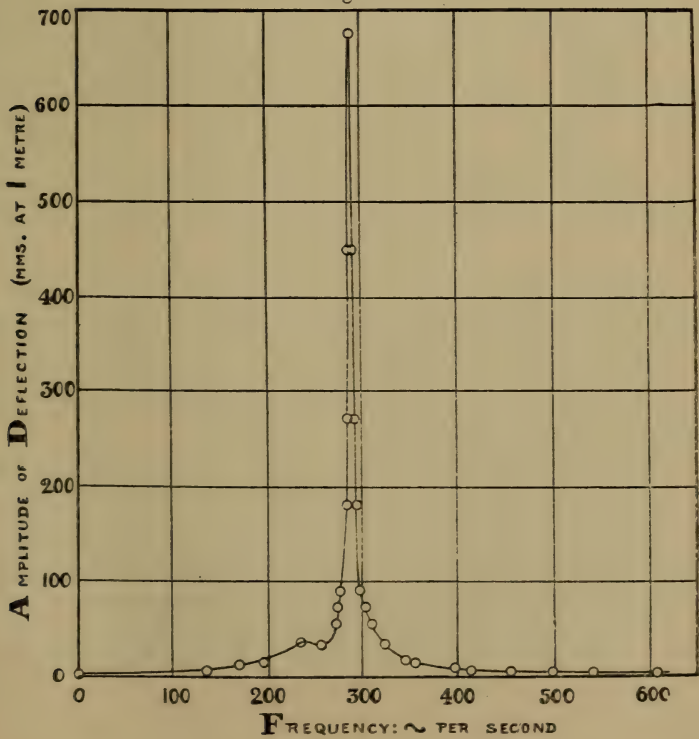


Fig. 5.



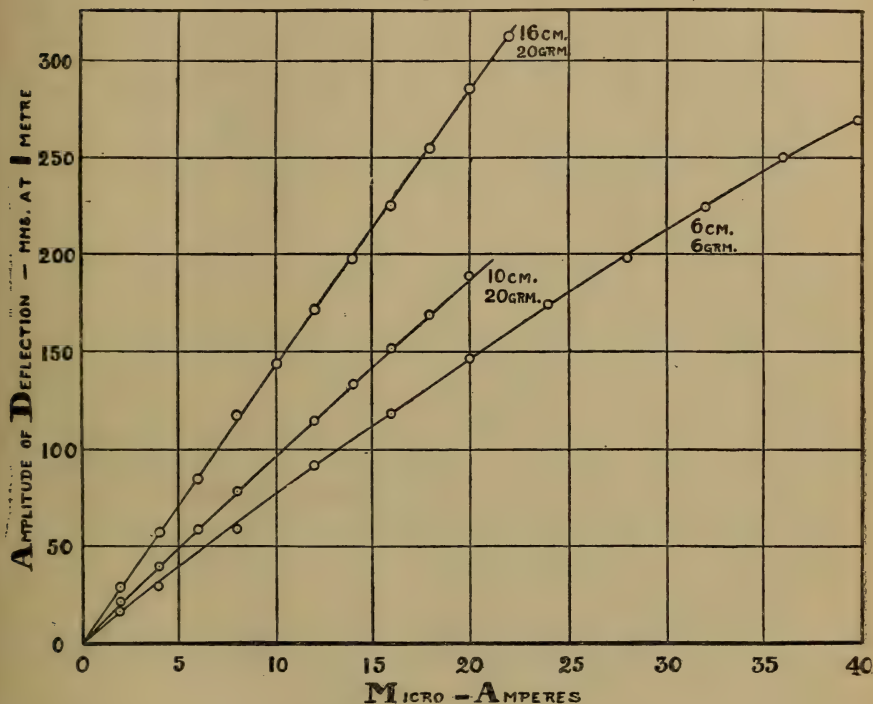
of the deflexion where an alternating current, having a constant R.M.S. value, is passed through the instrument, the frequency of the alternating current being varied. In fig. 4 the instrument was set so as to be in resonance at a frequency of 595~ per second, and in fig. 5 at a frequency of 290~ per second. The small irregularity in the curve fig. 5 at a frequency of 240~ per second was not an error of observation, but was, I think, due to the instrument resonating one of the higher harmonics of the wave form of the alternator.

To measure the sensibility of the instrument the following method was used:—A current generally 0.1 ampere from a small high-frequency alternator, having a very nearly sinusoidal wave-form, was passed through a small non-inductive resistance and an accurate dynamometer. The vibration galvanometer in series with a high non-inductive resistance was connected as a shunt to the terminals of the small resistance in the main circuit. The current of 0.1 ampere flowing through the main resistance produced a small known difference of potential applied to the galvanometer circuit from which the current through the galvanometer could be calculated. The vibration galvanometer it must be remembered, has when in operation a back E.M.F., so that it is very important to keep the resistance in series with the instrument very high to prevent the back E.M.F. from falsifying the calculation; 25,000 ohms was used for most of the tests. This method of testing the sensibility of the vibration galvanometer really calibrates the vibration galvanometer in terms of a standard dynamometer and known resistances. The vibration galvanometer is practically insensitive to anything except the fundamental of the wave-form of the alternating current for which it is tuned, whereas the dynamometer reads the mean squared current which is equal to the sum of the squares of the amplitude of all the harmonics including the fundamental. The two instruments are therefore not strictly comparable unless the amplitude of all the upper harmonics is zero, that is to say, that the current used has a pure sine wave-form.

With an instrument giving a very sharp resonance there is some difficulty in determining the exact value of the maximum amplitude, a fact which prevents such consistent results being obtained as would otherwise be the case. When the instrument is tuned so as to be in resonance with the alternating current to be measured the amplitude of the vibration is practically proportional to the R.M.S. current, as is shown by the tests recorded in fig. 6, hence it is

permissible to quote the sensibility in millimetres of amplitude at a metre per R.M.S. microampere. I have therefore adopted this method.

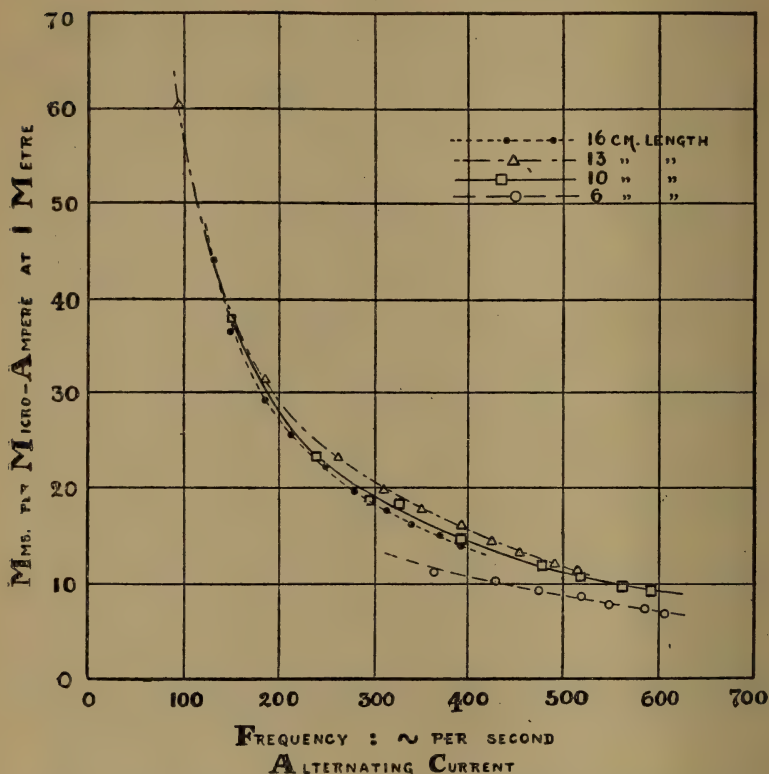
Fig. 6.



In fig. 7 (p. 174) the relationship between the sensibility of the instrument in millimetres of amplitude per R.M.S. micro-ampere, and the frequency of the alternating current is given. The different curves refer to different free lengths of the wires. One interesting point of this figure is that where it is possible to tune this vibration galvanometer to a given frequency, using various combinations of length and tension, the sensibility so obtained is very nearly the same as long as the wires are longer than the pole pieces. The practical result of this is that if at any given length of wire or tension, one can tune the instrument to suit the frequency of the alternating current, then no further adjustments need be made in the hope of finding a better combination of length and tension for the purpose.

With each adjustment of the instrument in fig. 7 to suit the various frequencies, the sensibility of the instrument was

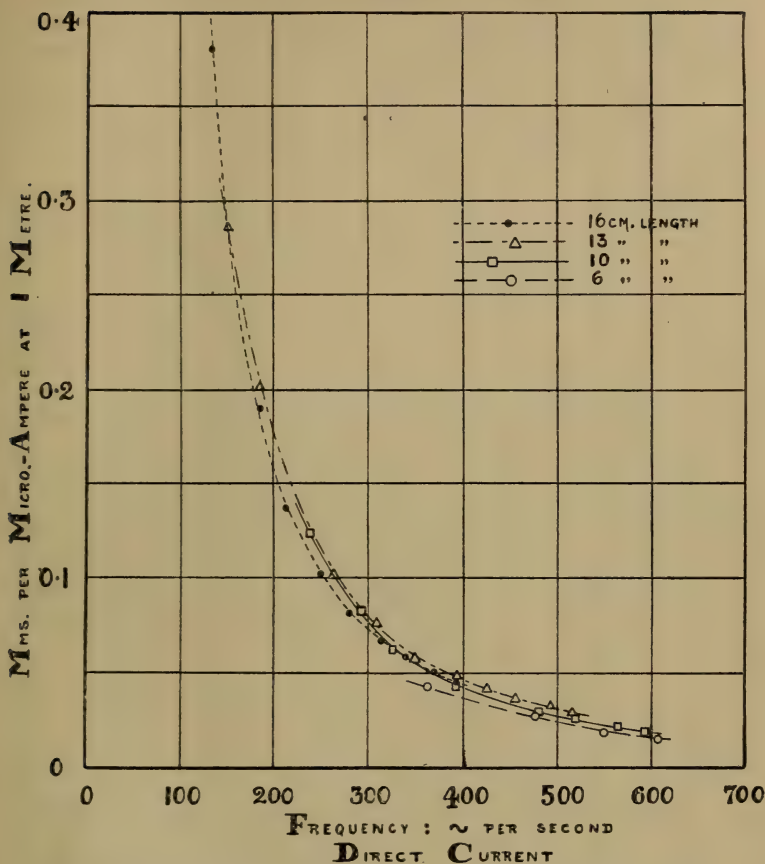
Fig. 7.



tested with direct current as an ordinary galvanometer (fig. 8).

It will be noted that the sensibility to alternating current decreases very nearly inversely as the frequency for which the instrument is adjusted, whereas for direct current the sensibility decreases approximately inversely as the square of the frequency for which the instrument is adjusted, which is what usually takes place with direct current galvanometers. By dividing corresponding ordinates in figs. 7 and 8 a new curve is obtained (fig. 9), which I have called the magnification, which shows for a given adjustment of the instrument, how much more sensitive it is to an alternating current of the proper frequency than to a direct current. The highest value obtained in the curves is about 460. Had the vibration

Fig. 8.

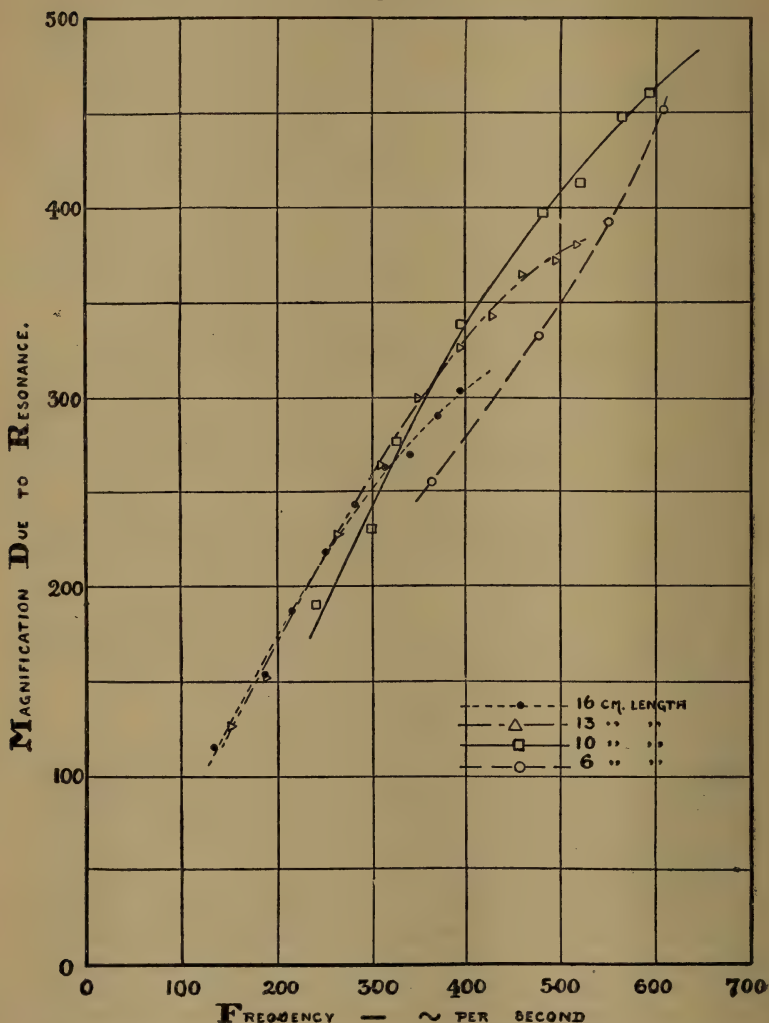


galvanometer been aperiodic and of sufficiently short period to follow the alternating current then the alternating current sensibility would have been only $2\sqrt{2}=2.82$ times the direct current sensibility.

The practical applications of the vibration galvanometer nearly all involve using the instrument in some form of bridge or null method for determining when a small difference of potential vanishes, that is to say the instrument is generally used as a sensitive detector for small alternating voltages. The resistance of the instrument used in these tests is 136 ohms, but owing to the back E.M.F. of the instrument its sensibility as a voltmeter must not be calculated on the basis of this figure. This resistance could be easily reduced by employing a more conducting material than hard phosphor

bronze for the wires. There would be no objection to doing this except that the upper limit of the frequency to which the instrument could be tuned, without risk of breaking the wires, would be some somewhat reduced.

Fig. 9.



In view of the importance of the fact that the back E.M.F. reduces the sensibility of the instrument, I have made some tests to determine its value, using the same connexions as before:—

Let E = the R.M.S. value of the E.M.F. impressed on the circuit consisting of the vibration galvanometer and its added resistance ;

d = the corresponding amplitude of deflexion in millimetres ;

r = the total resistance of the circuit ;

e = the back E.M.F. (R.M.S. value per millimetre of deflexion) ;

c = the true value in microamperes of the current through the vibration galvanometer ;

k = the true sensibility in millimetres per micro-ampere ;

then $d = ck$.

If the mechanical friction of the instrument is small, then, when it is tuned to resonance, the motion of the wires will be nearly 90° out of phase with the force, *i.e.* the current ; and the E.M.F. induced in the wires due to their cutting the magnetic field will also be 90° out of phase with their motion, so that the back E.M.F. will be approximately at 180° to the current. The E.M.F. sending the current through the vibration galvanometer may therefore be approximately taken as $E - ed$, and the current

$$c = \frac{E - ed}{r}, \text{ whence}$$

$$r + ek = k \frac{E}{d}.$$

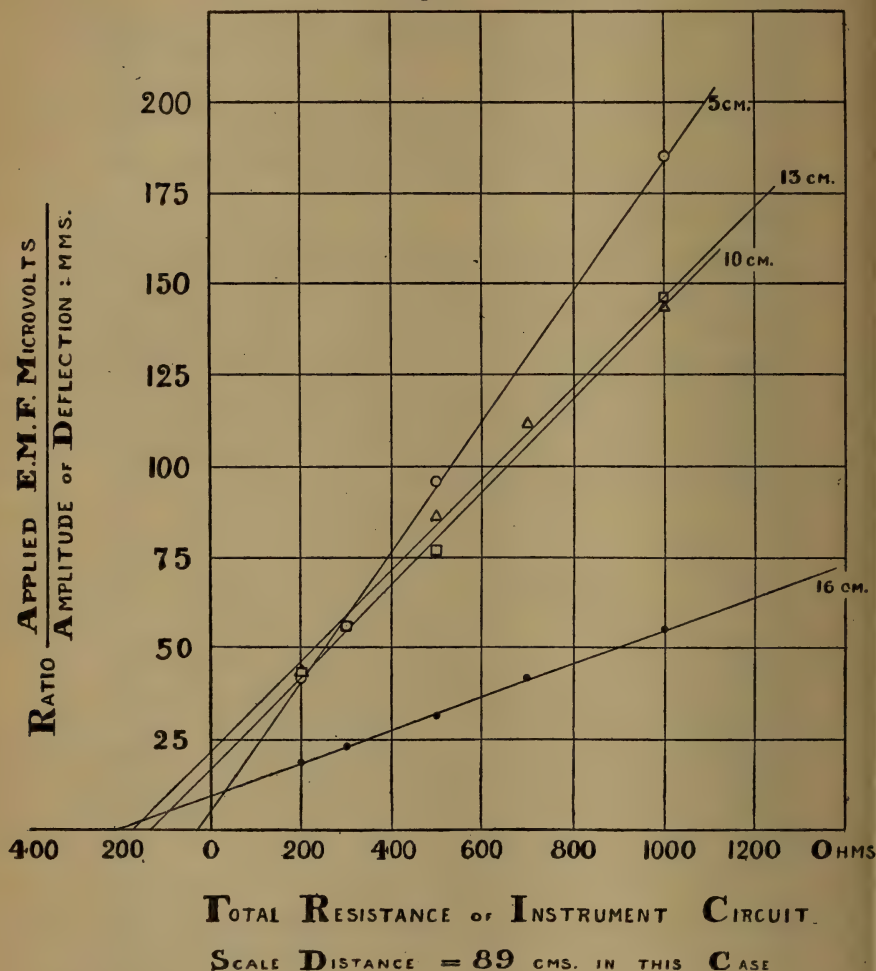
If therefore $\frac{E}{d}$ be plotted against r the resistance of the vibration galvanometer circuit, the graphs obtained should be straight lines which should not pass through the zero but should cut the zero line at a point ek which gives the apparent resistance of the instrument due to its back E.M.F.

This test has been carried out and the graphs are given in fig. 10. From these graphs the following results were obtained :—

Free length in cm.	Frequency.	Intercept in ohms.	Back E.M.F. in microvolts per millimetre.
5	538	30	4.8
10	532	130	14.8
13	530	165	18.4
16	228	205	8.4

The very large amount that the apparent resistance of the instrument is increased by its back E.M.F., over 200 ohms in one case, shows how very important it is to try to keep

Fig. 10.



this back E.M.F. as low as possible, and how little can be gained by reducing the real resistance of the wires themselves.

The fact that the back E.M.F. of the instrument is practically proportional to the deflexion and hence to the current

through it, and so appears, as far as the outside circuit is concerned, like an addition to the resistance of the instrument, indicates that no arrangement of condensers is likely to improve matters as one might at first expect.

It will be seen that the intercept in the above table decreases for a given frequency with the length of the wires in use, and as the current sensibility does not change at all rapidly with the active length of the wires there is some advantage in using the wires as short as possible when measuring small P.D.'s in a low resistance circuit.

I have not had an opportunity of testing the back E.M.F. of other vibration galvanometers. In the case of the moving coil vibration galvanometer I should expect that it would be considerably larger than in the bifilar owing to the coil having a number of turns. I trust that the discussion on the paper may bring out some information on this point.

All the sensibilities so far given for the vibration galvanometer correspond to the use of a permanent magnet for the field. The instrument has, however, been tried in an oscillograph electromagnet, and the current sensibility was found to be increased threefold.

This corresponds at a 100 frequency to a sensibility of not less than 160 millimetres per microampere; as a small fraction of a millimetre movement of the spot is noticeable we may reckon that at a 100 frequency with this instrument we can detect a current as low as 2 or 3×10^{-9} ampere.

The advantages of the bifilar galvanometer may be summarized as follows:—Simplicity—ease in tuning—wide range of frequency for which it can be tuned—high sensibility—negligible self-induction—comparatively small back E.M.F.

Its main defect is the small size of mirror that it is necessary to use on the instrument. With a carefully adjusted optical arrangement, and using a small 4 volt metal filament lamp, one can work with comfort at a scale distance of a metre in a room which is not too well lighted.

In conclusion, I wish to express my indebtedness to my assistant Mr. Neale, for the painstaking way in which he has made the experiments recorded in this paper.

XIX. *Note on König's Theory of the Ripple Formation in Kundt's Tube Experiment.* By J. ROBINSON, B.Sc., Pemberton Fellow of the Armstrong College, University of Durham*.

EXPERIMENTAL evidence† has been given by different investigators in support of König's theory of the ripple formation in a Kundt's tube. A point will here be brought forward which at all events is not in opposition to this theory.

König‡ showed that when two spheres of the same diameter are in a tube in which stationary sound-waves are set up, there is a force of repulsion between them in the direction of the axis of the tube of the magnitude,

$$-\frac{3}{2} \frac{\pi \sigma R^6 w_0}{r_0^4} \cos \theta (3 - 5 \cos^2 \theta),$$

where σ = density of the gas in the tube,

R = radius of each sphere,

r_0 = distance between the centres of the spheres,

θ = angle between the line joining the centres of the spheres and the axis of the tube,

$w_0 = 2\pi\nu a_0$, where a_0 is the amplitude of vibration of one of the spheres, and ν is the frequency of the sound-wave.

Now this force of repulsion between the spheres depends on w_0 , which varies along the tube when stationary waves are set up. If we consider that the amplitude is a_0 at an antinode, it is zero at a node, and at a distance x from the antinode it is

$$a = a_0 \cos \frac{\pi}{2} \cdot \frac{x}{l},$$

where $2l$ is the distance between two consecutive nodes.

Therefore in the expression for the force exerted by one sphere on the other, we must substitute w for w_0 where

$$w = w_0 \cos \frac{\pi}{2} \cdot \frac{x}{l},$$

w_0 being the value of $2\pi\nu a$ at the antinode, and w its value at a distance x from it.

* Communicated by Prof. H. Stroud, M.A., D.Sc.

† W. König, *Wied. Ann.* xlii. pp. 353, 549 (1891); S. R. Cook, *Phil. Mag.* May 1902, p. 471; J. Robinson, *Phys. Zeit.* 1908, p. 809.

‡ W. König, *loc. cit.*

A consequence of this would be that for equilibrium the arrangement of the particles is not the same at all parts of the tube when a stationary sound-wave is set up. We may find how the distance between the ripples varies at different parts of the tube in the following way.

For the equilibrium of a particle in one ripple, the forces of repulsion acting on it from all the particles in the ripples to one side of it, must equal those from the particles to the other side of it. Now as the forces considered vary inversely as the fourth power of the distance between the particles, we need only consider consecutive ripples, and we get an approximate result as follows:—

We consider three consecutive ripples, a , b , c (fig. 1), and for the equilibrium of the particles in the middle one b , we calculate the total force of repulsion exerted by the particles in each of the ripples a and c on a particle in b , and equate these forces.

Fig. 1.

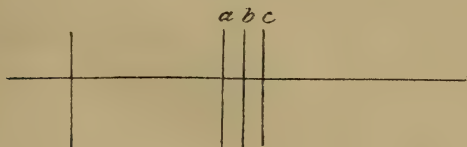
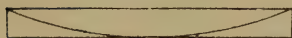


Fig. 2.



The cross-section of a ripple perpendicular to the axis of the tube is a segment of a circle (fig. 2). We get an approximation to the result if we consider the particles in this cross-section as uniformly distributed over a rectangle whose length is the same as that of the chord. This simplifies some integrations which follow.

Now consider the total force of repulsion exerted by one whole ripple $A B$ of length $2b$ on one particle at O , at the centre of the next ripple (fig. 3, p. 182). Let the distance apart of the ripples $= a$. In the ripple $A B$, suppose there are n particles per unit length, all the dust particles in the tube being supposed to be of the same size, *i. e.*, of radius R . These particles are of course supposed to be small.

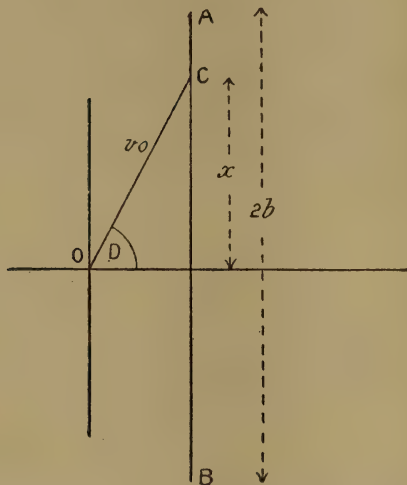
The force exerted by one particle at C on the particle at O

$$= \frac{-\frac{3}{2} \cdot \pi \sigma R^6 w^2}{r_0^4} \cdot \cos \theta \cdot (3 - 5 \cos^2 \theta).$$

Now in the length dx of the ripple there are $n dx$ particles, and the force exerted by these on O

$$= \frac{-\frac{3}{2} \cdot n \pi \sigma R^6 w^2}{r_0^4} \cos \theta (3 - 5 \cos^2 \theta) dx.$$

Fig. 3.



Therefore the total force exerted by the whole ripple on O,

$$\begin{aligned} = F &= - \int_{-b}^{+b} \frac{\frac{3}{2} n \pi \sigma R^6 w^2}{r_0^4} \cdot \cos \theta (3 - 5 \cos^2 \theta) dx \\ &= - 3 n \pi \sigma R^6 w^2 \int_0^b \frac{\cos \theta (3 - 5 \cos^2 \theta)}{r_0^4} dx. \end{aligned}$$

Now

$$\begin{aligned} r_0 &= a \sec \theta, \\ x &= a \tan \theta, \\ dx &= a \sec^2 \theta \cdot d\theta; \end{aligned}$$

and therefore

$$\begin{aligned} \int_0^b \frac{\cos \theta (3 - 5 \cos^2 \theta)}{r_0^4} dx &= \int_0^{\tan^{-1} \frac{b}{a}} \frac{\cos^3 \theta (3 - 5 \cos^2 \theta)}{a^3} d\theta \\ &= \frac{1}{a^3} \int_0^{\tan^{-1} \frac{b}{a}} (3 \cos^3 \theta - 5 \cos^5 \theta) d\theta. \end{aligned}$$

Now

$$\int 5 \cos^5 \theta \cdot d\theta = \cos^4 \theta \sin \theta + 4 \int \cos^3 \theta \cdot d\theta,$$

$$\text{and} \quad 3 \int \cos^3 \theta \cdot d\theta = \cos^2 \theta \cdot \sin \theta + 2 \sin \theta,$$

$$\therefore \frac{1}{a^3} \int \cos^3 \theta (3 - 5 \cos^2 \theta) d\theta = -\frac{1}{3a^3} \sin \theta (2 + \cos^2 \theta + 3 \cos^4 \theta)$$

$$\text{and} \quad F = n\pi\sigma R^6 w^2 \left[\frac{\sin \theta}{a^3} (2 + \cos^2 \theta + 3 \cos^4 \theta) \right]_0^{\tan^{-1} \frac{b}{a}}.$$

Now if the ripples are wide, as they are when wide tubes are used, we can have conditions so that b is large compared with a , and then $\tan^{-1} \frac{b}{a}$ is nearly a right angle. Then we can neglect $\cos^2 \theta$ and $\cos^4 \theta$, and we find

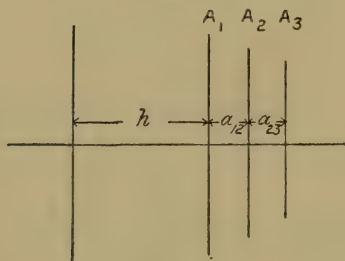
$$F = n\pi\sigma R^6 w^2 \left[\frac{2 \sin \theta}{a^3} \right]_0^{\tan^{-1} \frac{b}{a}},$$

which we can put approximately

$$\frac{2n\pi\sigma R^6 w^2}{a^3}.$$

We will now consider three consecutive ripples A_1, A_2, A_3

Fig. 4.



(fig. 4) so arranged that A_1 is distant h from the antinode, A_2 is distant a_{12} from A_1 , and A_3, a_{23} from A_2 .

The values of n and w at A_1 are n_1 and w_1 ,

" " " " A_2 " n_2 " w_2 .

Then the forces exerted by the ripples A_1 and A_3 on a

particle at the centre of A_2 are F_1 and F_3 respectively, where

$$F_1 = \frac{2n_1\pi\sigma R^6 w_1^2}{a_{12}^3}$$

$$F_3 = \frac{2n_3\pi\sigma R^6 w_3^2}{a_{13}^3}.$$

For equilibrium of this particle in A we have $F_1 = F_3$

and therefore

$$\frac{n_1 w_1^2}{a_{12}^3} = \frac{n_3 w_3^2}{a_{23}^3},$$

i. e.

$$\left(\frac{a_{23}}{a_{12}}\right)^3 = \frac{n_3 w_3^2}{n_1 w_1^2},$$

which gives the ratio of the distances between successive ripples.

Now

$$w_1 = w_0 \cos \frac{\pi}{2} \cdot \frac{h}{l}$$

$$w_3 = w_0 \cos \frac{\pi}{2} \cdot \frac{h + a_1 + a_2}{l}$$

$$= w_0 \cos \frac{\pi}{2} \cdot \frac{h + 2a}{l}$$

(where a is the mean of a_1 and a_2),

$$\therefore \left(\frac{a_{23}}{a_{12}}\right)^3 = \frac{n_3 \cos^2 \left(\frac{\pi}{2} \cdot \frac{h + 2a}{l}\right)}{n_1 \cos^2 \frac{\pi}{2} \cdot \frac{h}{l}}.$$

If we consider $n_1 = n_3$, *i. e.*, that the distribution of powder along the ripples is the same for all of them, then

$$\left(\frac{a_{23}}{a_{12}}\right)^3 = \frac{\cos^2 \left(\frac{\pi}{2} \cdot \frac{h + 2a}{l}\right)}{\cos^2 \frac{\pi}{2} \cdot \frac{h}{l}}.$$

And we see that a_{23} is smaller than a_{12} , *i. e.*, that the farther we get from the antinode the smaller is the distance between the ripples.

We can find the ratio of the distance apart of the r th and $r+1$ th ripples, and of that between the 1st and 2nd ripples

(the 1st being supposed to be at the antinode) as follows, if we consider

$$n_1 = n_2 = n_3 = \dots = n_r$$

$$\left(\frac{a_{23}}{a_{12}}\right)^3 = \left(\frac{w_3}{w_1}\right)^2$$

$$\left(\frac{a_{34}}{a_{23}}\right)^3 = \left(\frac{w_4}{w_2}\right)^2$$

$$\left(\frac{a_{45}}{a_{34}}\right)^3 = \left(\frac{w_5}{w_3}\right)^2$$

$$\left(\frac{a_{56}}{a_{45}}\right)^3 = \left(\frac{w_6}{w_4}\right)^2$$

$$\dots = \dots$$

$$\dots = \dots$$

$$\left(\frac{a_{r,r+1}}{a_{r-1,r}}\right)^3 = \left(\frac{w_{r+1}}{w_{r-1}}\right)^2.$$

Forming the product of all the terms on the left, and of those on the right, and equating, we get

$$\left(\frac{a_{r,r+1}}{a_{1,2}}\right)^3 = \left(\frac{w_r w_{r+1}}{w_1 w_2}\right)^2.$$

Now w_{r+1} is approximately $= w_r$,

and w_2 is „ $= w_1$.

Therefore we can write

$$\left(\frac{a_{r,r+1}}{a_{1,2}}\right)^3 = \left(\frac{w_r}{w_1}\right)^4.$$

If w_r is distant k from the antinode, we have

$$\left(\frac{a_{r,r+1}}{a_{12}}\right)^3 = \left(\frac{w_0 \cos \frac{\pi}{2} \frac{k}{l}}{w_0}\right)^4 = \cos^4 \frac{\pi}{2} \cdot \frac{k}{l}.$$

This gives approximately the law of variation of the distances between the ripples on the above assumptions.

From this we see that this distance gets smaller as we go from the antinode to the node. This statement will not be affected if we consider variations in the value of n , for if n varies, it can only diminish as we go from the antinode to the node. This is readily seen when we remember that the width of the ripples, *i. e.* the chord of the segment in fig. 2, gets smaller as we go away from the antinode. Then going

back to the equation

$$\left(\frac{a_{23}}{a_{12}}\right)^3 = \frac{n_3 w_3^2}{n_1 w_1^2},$$

we have

$$\left(\frac{a_{23}}{a_{12}}\right)^3 \cdot \frac{n_1}{n_3} = \left(\frac{w_3}{w_1}\right)^2 < 1$$

and n_3 can only be less than n_1 .

Therefore we have

$$\frac{n_1}{n_3} < \left(\frac{a_{12}}{a_{23}}\right)^3.$$

Now

$$\frac{n_1}{n_3} \text{ is } > 1,$$

$$\therefore \left(\frac{a_{12}}{a_{23}}\right)^3 > 1,$$

which means that

$$a_{12} > a_{23}.$$

Some experimental results in connexion with this point have been obtained. The distance between two nodes was divided into three parts, left, middle, and right, and the mean distance between the ripples in each part measured by means of a travelling microscope. It would be desirable to make such measurements whilst the tone is being emitted, but as I used the ordinary method of producing the figures * by means of stroking a glass rod, this was not done for the results here given. The results are given for different powders and for different frequencies, and in every case it is seen that the distance between the ripples is greater at the antinodes than near the nodes, which is quite in agreement with the above investigation.

Distance between two nodes.	Frequency.	Distance between consecutive ripples.											
		Fine iron powder.			Coarse iron powder.			Emery powder			Sand.		
		Left.	Mid.	Right.	L.	M.	R.	L.	M.	R.	L.	M.	R.
3.63 cm.	4545	mm. 0.90	mm. 0.94	mm. 0.86	mm. 1.11	mm. 1.50	mm. 1.31	mm. 1.43	mm. 1.91	mm. 1.57	mm. 1.23	mm. 1.72	mm. 1.06
5.03 cm.	3208	1.01	1.25	1.07	1.64	2.33	1.64	1.68	1.76	1.48	1.34	1.82	1.22

* *Phys. Zeit.* 1908, p. 808.

No objection can be made to these results on the ground of varying intensity of sound, as the measurements of each of the parts, left, middle, and right, are for the same set of ripples, and thus no doubt remains that the distance between the ripples diminishes in going from the antinode to the node.

My thanks are due to Prof. Geh. Rat Riecke for his kind advice and encouragement during the progress of this work, which was carried out in the Physical Laboratory of the University of Göttingen.

The Physical Laboratory,
University of Göttingen,
April 22, 1909.

V 1

XX. *The Selective Reflexion of Monochromatic Light by Mercury Vapour.* By R. W. WOOD, *Professor of Experimental Physics, Johns Hopkins University**.

[Plate III.]

THE present paper forms the first of a series now in preparation upon the optical properties of mercury vapour, which has yielded results quite as interesting as those which have been obtained with the vapour of metallic sodium, though of a somewhat different nature.

The vapour of mercury has a strong and narrow absorption line in the ultra-violet at wave-length $2536\cdot7$, not unlike one of the D lines of sodium in character. I have already published some observations of this very remarkable asymmetrical band in the *Astrophysical Journal* †.

There are a number of other absorption lines and bands, but it is with the one above mentioned that we are chiefly concerned in the present paper. There is a fainter line at $2539\cdot3$, which fuses with the stronger line as soon as the vapour acquires any considerable density.

Planck's theory of absorption is based upon the supposition that the energy, taken from the oncoming waves, is re-emitted laterally by the resonators. Though it is my opinion that this re-emission only occurs in exceptional cases, we do find it in some instances. As I have shown in a previous paper, sodium vapour, when illuminated with sodium light, emits the absorbed wave-lengths without change. The emission is diffuse, however, that is, it is scattered in all directions. Radiation of this nature I have called resonance radiation, to

* Communicated by the Author.

† "Change in the appearance and apparent position of an absorption band caused by the presence of an inert gas."

distinguish it from fluorescence, in which case there is a change of wave-length.

I predicted some years ago that if the molecular resonators were packed closely enough together, the secondary wavelets which they emit, having a definite phase relation, would unite into a single wave, and the scattered light would disappear, regular reflexion of the absorbed light taking its place. Repeated efforts to discover the phenomenon with sodium vapour yielded negative results, since it was impossible to obtain the vapour at great density with a sharply defined surface, on account of its corrosive action upon the transparent walls of the containing vessel.

In the course of a long series of experiments, which I have been making upon the fluorescence, dispersion, magnetic rotation, &c. of mercury vapour, it occurred to me that this substance was well suited to the study of the phenomenon, if it existed, for it is not difficult to obtain the vapour at a pressure of 20 or 30 atmospheres, in bulbs of fused quartz.

To separate the images formed by reflexion from the inner and outer surfaces of the bulb, the necks were made of very thick-walled tubing, so that the walls of the bulb were

Fig. 1.



prismatic as shown in fig. 1. These bulbs were made especially for the work by Heraeus of Hanau, and were found most satisfactory.

A good-sized globule of mercury was placed in a bulb which was then highly exhausted and sealed.

It seemed best to begin by using light of exactly the right frequency, that is of a wave-length identical with that of the absorption line. The light from the mercury arc in a quartz tube shows a strong line of exactly the right frequency. The bulb was mounted in a small tube of thin steel provided with an oval aperture in the side: the ends were closed with disks of asbestos board, one of which supported the neck of the bulb, as shown in fig. 1. The steel tube was heated by two Bunsen burners, usually to a full red heat, and the mercury arc placed as close as possible to the aperture and a little to one side, so that its image appeared reflected in the tapering neck of the bulb, as shown in the figure. The flame

of the burner must play over the aperture to prevent condensation of mercury drops at the point where reflexion occurs.

By properly choosing the direction in which the bulb was viewed, the reflexion from the outer surface disappeared, and the slit of a small quartz spectrograph was directed towards the bright image of the arc reflected from the inner surface of the wall.

A number of photographs of the spectrum of the reflected light were taken, the first with the bulb cold, the succeeding ones at gradually increasing temperatures. It was found that the relative intensity of the 2536·7 line in the spectrum of the reflected light increased rapidly as the temperature of the bulb increased. A series of these photographs is reproduced on Pl. III. fig. 1. The time of exposure was less in the case of the red-hot bulb, in spite of which the line 2536 is very strong while the other lines do not appear at all.

Previous work having shown, however, that mercury vapour emits scattered resonance radiation, when stimulated by the light of the 2536 line, it was next necessary to prove that it was not a diffuse radiation which had caused the increased intensity of the line. A very nice method was found of proving this. If our eyes were sensitive to the ultra-violet light, we should see, in the case of the diffuse emission, the entire surface of the bulb glowing with light, while if the radiation was regularly (*i. e.* specularly) reflected by the vapour, we should see merely the small image of the arc increase in brilliancy.

The steel tube was mounted vertically in such a position that the two images due to reflexion from the inner and outer surfaces of the front wall of the bulb appeared one above the other. Inasmuch as the mercury arc consisted of a narrow vertical column of light, these images appeared as narrow vertical lines of light, and could be used in place of the slit of the spectrograph. The arc in this case was placed at a distance of about a metre from the bulb.

The slit tube of the spectrograph was removed, and the instrument placed in such a position that the two reflected images occupied the position previously occupied by the slit. On exposing a plate we obtain two spectra one above the other, the one that of the light reflected from the outer surface, the other that of the light reflected from the inner surface.

Two photographs were taken, one with the bulb cold, the other with the bulb red-hot. The latter showed that the image of the arc as seen in the 2536 light had been increased

tremendously in brilliancy by the presence of the mercury vapour. The two photographs are reproduced on Pl. III. fig. 2, the spectrum image formed by the light of wave-length 2536 being indicated. The light reflected from the inner surface is not as intense as that reflected from the outer, consequently one of the spectra is weaker than the other in each case. The 2536 line is, however, many times brighter in the spectrum of the image formed by the inner surface of the bulb when hot.

This experiment shows that the mercury vapour reflects light of this particular wave-length in much the same way as would a coating of silver on the inside of the bulb.

Experiments were next undertaken to ascertain how nearly the frequency of the light must agree with that of the absorption band in order that metallic reflexion should take place.

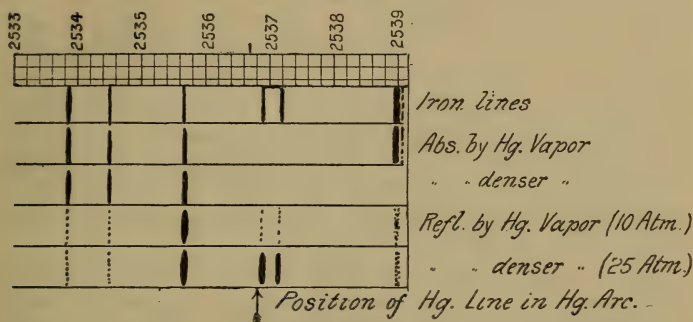
It was found that the spectrum of the iron arc showed a group of closely packed lines exactly in the region required, and it was accordingly put in place of the mercury arc. The slit was replaced on the quartz spectrograph, and photographs of the reflected image of the arc were taken with the bulb cold and heated to different temperatures. A very remarkable discovery was at once made, for it turned out that the iron line which was metallically reflected (2535.67) was about one Ångström unit on the short wave-length side of the absorption line. As the temperature and vapour density increased a second iron line was strongly reflected, this one coinciding almost exactly with the absorption line. It is in reality a double line, with wave-lengths 2536.90 and 2537.21.

To make absolutely sure that no error had occurred, I photographed the spectrum of the iron arc, passing the light through mercury vapour at different densities.

The iron line which first disappeared was the double one, which was not reflected until the mercury vapour was at its greatest density. The line which was metallically reflected by the vapour at a lesser density was not absorbed by the vapour even when its density was so great that four or five lines on the long wave-length side of the line first absorbed, were completely blotted out. This can be better understood by reference to fig. 2 (p. 191). In the upper line we have the group of iron lines in question. The next line shows the absorption by Hg vapour when its density is small, the fourth iron line (double, mean λ 2537) having disappeared. Just below is the absorption by dense vapour, the absorption having extended towards the visible region of the spectrum (much

farther than can be indicated in the figure), the third iron line being still transmitted, however. Below this is the reflexion from Hg vapour at about ten atmospheres, the third iron line being strongly reflected. In the lower line

Fig. 2.



we have the reflexion by vapour at 25 atmospheres, the third and fourth (double) iron lines appearing strongly reflected. This figure is a diagram, of course. Photographs are reproduced in figs. 3, 4, 5, & 6 (Pl. III.). Fig. 3 *a* is enlarged from fig. 5, and was made with a concave grating of two metres radius. The powerful reflexion of the third and fourth iron lines (indicated by arrows) is well shown. Fig. 3 *b* shows the absorption of the fourth iron line by the vapour at small density, while fig. 3 *c* shows that the reflexion of the third line is much stronger than that of the fourth by vapour at about ten atmospheres. In fig. 4 we have the same thing shown by the quartz spectrograph, the upper spectrum reflected from rare, the lower from dense mercury vapour; third and fourth lines indicated by arrows. Fig. 5 was made with the grating, and shows a wider range of the spectrum than the enlargement; the powerful selective reflexion of the two iron lines is beautifully shown. Fig. 4 was enlarged from fig. 6 (quartz spectrograph).

It is useless to speculate about the cause of this surprising apparent shift of the reflexion band until the absorption of a very thin layer of the vapour at very great density has been studied, for the absorption band may be shifted towards the short wave-length region as the density increases, though this is precisely the opposite to what we should expect. I have attempted to study this with small-bore tubes, but the thickness of the layer was too great.

It will be necessary to have a cell made of two thick plates of fused quartz separated by a distance of say 0.01 mm., and capable of standing a pressure of at least ten atmospheres. I have ordered the cell, made in prismatic form, with the plates in contact at one end, and separated by a distance of 0.3 mm. at the other. They will be fused together along the edges, a tube being provided for the introduction of the mercury and exhaustion. This cell will also be very useful in the further study of the reflecting power of the vapour in connexion with the state of polarization of the light. There are still many interesting points to investigate, and I am at the present time devising methods and apparatus for studying the gradual transition from diffuse scattering of the radiation, to metallic or specular reflexion. It seems probable that a good deal of new information regarding the mechanism of reflexion can be obtained in this way.

I have already made a preliminary investigation with a polarizing prism. The mercury arc was placed in such a position that the reflexion from the inner surface of the bulb took place at the polarizing angle. The image from the outer surface was hidden by a screen. The narrow line of light was then photographed with a quartz lens in combination with one half of a quartz Rochon prism. This prism gives two superposed spectra of an unpolarized source, one polarized vertically, the other horizontally. The light reflected from the inner surface of the bulb, being polarized by the reflexion, yields only a single spectrum. It was found, however, that when the bulb was heated red-hot the mercury line 2536 appeared *double*, showing that the vapour reflects unpolarized light. This is shown in Pl. III. fig. 7. The upper spectrum is of the light reflected from the hot bulb, the lower from the cold. In the latter the line to the extreme left is the 2536 line. In the upper spectrum we find this line not only relatively much stronger, but appearing in duplicate (indicated by the arrow).

It must be remembered that the two spectra given by half of a Rochon prism with an unpolarized source are superposed, the deviations being different, however. As will be seen, the 2536 line is the only one which appears twice, that is in the two spectra which the prism is capable of forming. The metallically reflected light being unpolarized is equally divided between the two spectra furnished by the prism. Both lines are therefore much brighter than the one in the lower spectrum.

One very important point in connexion with the specular

reflexion from an absorbing vapour is the very great density necessary before the phenomenon is exhibited. In a gas at atmospheric pressure the molecules are so close together that there will be about 80 to the wave-length, *i.e.* 6400 in a square the sides of which are equal to a light-wave in length. This would appear to be more than sufficient for the application of Huygens's principle: experiment shows, however, that the reflexion does not occur until we increase this density tenfold. I am inclined to think that it is a question of the suddenness with which the wave is stopped, rather than of packing the resonators close enough together to make the application of Huygens's principle possible. It appears to me probable that if the wave-train can penetrate to a depth of several wave-lengths into the medium, there will be no regular reflexion, regardless of the proximity of the resonators. An analogous case is that of two media of very different refractive indices, between which the transition is gradual instead of abrupt. Some authorities hold that reflexion will occur in this case, arguing, if I understand them correctly, that we can divide the transition layer into a large number of planes, each one of which will reflect a small amount. Even if this were the case it appears to me that we should have complete destructive interference, for the wave-trains reflected from the hypothetical planes would be gradually and progressively displaced with reference to each other and give us zero for a resultant. The same thing may be considered as taking place in the case of the resonators. A rigorous theoretical treatment of the resultant effect of the radiations from a system of closely packed resonators, excited by plane waves, considering several different depths of penetration, is much to be desired. At the present time apparatus is being constructed for the study of the anomalous dispersion of the vapour. As it is possible to work under known conditions of temperature, pressure, and density, the results will doubtless be of greater theoretical interest than those obtained with sodium vapour. The work on the fluorescence and absorption is completed.

In the study of the optical properties of mercury vapour I have been aided by a grant from the Rumford fund of the American Academy.

XXI. *The Damping of Mercury Waves.**By* Prof. R. W. WOOD*.

[Plate III. fig. 8.]

IN the course of some experiments with which I have been engaged during the past summer upon the construction of a reflecting telescope, by the uniform rotation of a large flat basin of mercury (see *Astrophysical Journal* for March 1909 : "The Mercury Paraboloid as a Reflecting Telescope"), I have found that the ripples can be almost completely abolished, even if they are of very large amplitude, by covering the surface of the metal with glycerine. It has recently occurred to me that castor-oil would be much better, for it is almost as viscous, and does not develope striæ by absorbing moisture from the air, which is the great objection to the use of glycerine. The oil can be obtained almost as transparent as water, and has been found to be almost as effective as glycerine in damping the ripples. It seems quite probable that the device will prove very useful in cases where a level reflecting surface is required.

To study the effect of different liquids in damping the waves I have recently employed a large electrically-driven tuning-fork, provided with a glass stylus which dipped in the mercury. The surface of the reflecting liquid was viewed through a pair of partially superposed narrow slits in thin cardboard, mounted on the prongs of a second electrically-driven fork of the same pitch. By throwing the forks slightly out of tune by loading with wax we can observe the waves (stroboscopically) slowly crawling out from the centre of disturbance, in the manner described by Lord Rayleigh, Boys, Vincent, and others. These waves cover the entire surface of the mercury. The light from a small source is rendered slightly convergent by a lens, before its incidence upon the mercury, the image of the source being thrown upon the vibrating slits. If now we cover the metal with a thin layer of glycerine or castor-oil, we find that the waves die out completely after travelling a distance of from one to three wave-lengths, depending upon the amplitude of the exciting fork. The experiment is the only one which I know of for showing damped waves, and gives us a most vivid idea of what happens to light when it enters an absorbing medium. A photograph of the damped waves taken when the exciting fork was very vigorously driven is reproduced on Plate III. fig. 8. The two forks were exactly in unison.

* Communicated by the Author.

when the photograph was taken, the waves appearing stationary. An exposure of ten seconds was given.

I have recently been speculating about the question of the flow of energy in cases where interference-fringes are formed, and have been making some experiments with interfering mercury-ripples. Some trouble was experienced from the reflexion of the waves from the sides of the dish as well as from waves which originated at the sides resulting from the jar communicated to the table by the tuning-fork. I found that both sets of disturbances could be prevented by pouring glycerine around the edge of the dish in the capillary depression formed by the mercury. The viscous fluid sticks to the edge of the dish, and shows little tendency to spread towards the centre, even if used in considerable quantities. This corresponds to painting the walls of the room black when making optical experiments.

In the case of interference-fringes formed by two sources vibrating with equal amplitudes, the flow of energy is along the hyperboloids it appears to me. With the mercury ripples and the stroboscopic fork, we can see the waves travelling along the curved hyperboloidal paths. So far as I know the question of energy-flow in the case of interference has not been discussed up to the present time, and there are some interesting points in connexion with it, such as the form of the wave-front which travels along a bright fringe, and whether, if passed through a slit and received by the eye, we should see both sources resolved or not. I am inclined to think that at least two bright fringes would have to be transmitted for resolution to take place, but have not tried the experiment yet. These matters will be postponed for a subsequent paper.

XXII. *On the Elliptic Polarization produced by the Direct Transmission of a Plane Polarized Stream through a Plate of Quartz, cut in a Direction oblique to the Optic Axis, with a Method of Determining the Error of a Plate supposed to be Perpendicular to the Axis.* By JAMES WALKER, M.A., Oxford*.

1. **L**ET the primitive stream with polarization-vector $\text{Exp}(i\omega t)$ be polarized in an azimuth α with respect to the principal section of the quartz plate. This may be replaced by the elliptically polarized stream represented by the vector

$$(\xi_1, \eta_1) = c_1(\cos \beta, -i \sin \beta) \text{Exp}\{i(\omega t + \epsilon_1)\},$$

* Communicated by the Physical Society: read November 27, 1908.

with its plane of maximum polarization (β being numerically less than $\pi/4$ and positive or negative according as the quartz is left- or right-hand) in the principal section, together with the oppositely polarized stream defined by the vector

$$(\xi_2, \eta_2) = c_2(-\sin \beta, -\iota \cos \beta) \text{Exp} \{ \iota(nt + \epsilon_2) \},$$

where

$$c_1 \text{Exp}(\iota \epsilon_1) = \cos \alpha \cos \beta + \iota \sin \alpha \sin \beta,$$

$$c_2 \text{Exp}(\iota \epsilon_2) = -\cos \alpha \sin \beta + \iota \sin \alpha \cos \beta.$$

These oppositely polarized streams traverse the plate with different speeds, and on emergence the second is retarded in phase relatively to the first by an amount δ , so that the emergent light will be in general elliptically polarized and may be denoted by

$$(u, v) = c(\cos \gamma, -\iota \sin \gamma) \text{Exp} \{ \iota(nt + \epsilon) \},$$

where

$$c(\cos \gamma \cos \theta + \iota \sin \gamma \sin \theta) \text{Exp} \{ \iota(\epsilon + \delta/2) \}$$

$$= c_1 \cos \beta \text{Exp} \{ \iota(\epsilon_1 + \delta/2) \} - c_2 \sin \beta \text{Exp} \{ \iota(\epsilon_2 - \delta/2) \},$$

$$c(\cos \gamma \sin \theta - \iota \sin \gamma \cos \theta) \text{Exp} \{ \iota(\epsilon + \delta/2) \}$$

$$= -\iota c_1 \sin \beta \text{Exp} \{ \iota(\epsilon_1 + \delta/2) \} - \iota c_2 \cos \beta \text{Exp} \{ \iota(\epsilon_2 - \delta/2) \},$$

θ being the angle between the principal section of the plate and the plane of maximum or minimum polarization of the resultant stream, according as γ is numerically less or greater than $\pi/4$.

Substituting for $c_1 \text{Exp}(\iota \epsilon_1)$ and $c_2 \text{Exp}(\iota \epsilon_2)$ and writing

$$\tan R = \sin 2\beta \tan \delta/2, \quad \tan \Delta/2 = \cot 2\beta \sin R, \quad . \quad (1)$$

we obtain

$$c(\cos \gamma \cos \theta + \iota \sin \gamma \sin \theta) \text{Exp} \{ \iota(\epsilon + \delta/2) \}$$

$$= \cos(\alpha + R) \cos \Delta/2 + \iota \cos \alpha \sin \Delta/2,$$

$$c(\cos \gamma \sin \theta - \iota \sin \gamma \cos \theta) \text{Exp} \{ \iota(\epsilon + \delta/2) \}$$

$$= \sin(\alpha + R) \cos \Delta/2 - \iota \sin \alpha \sin \Delta/2,$$

or

$$c \left\{ \cos \gamma \cos \left(\theta - \frac{R}{2} \right) + \iota \sin \gamma \sin \left(\theta - \frac{R}{2} \right) \right\} \text{Exp} \left\{ \iota \left(\epsilon + \frac{\delta}{2} \right) \right\}$$

$$= \cos \left(\alpha + \frac{R}{2} \right) \left(\cos \frac{\Delta}{2} + \iota \sin \frac{\Delta}{2} \right),$$

$$c \left\{ \cos \gamma \sin \left(\theta - \frac{R}{2} \right) - \iota \sin \gamma \cos \left(\theta - \frac{R}{2} \right) \right\} \text{Exp} \left\{ \iota \left(\epsilon + \frac{\delta}{2} \right) \right\}$$

$$= \sin \left(\alpha + \frac{R}{2} \right) \left(\cos \frac{\Delta}{2} - \iota \sin \frac{\Delta}{2} \right);$$

whence

$$\frac{\cos \gamma \cos \left(\theta - \frac{R}{2} \right) + \iota \sin \gamma \sin \left(\theta - \frac{R}{2} \right)}{\cos \gamma \sin \left(\theta - \frac{R}{2} \right) - \iota \sin \gamma \cos \left(\theta - \frac{R}{2} \right)} = \cot \left(\alpha + \frac{R}{2} \right) (\cos \Delta + \iota \sin \Delta),$$

which gives

$$\left. \begin{aligned} \frac{\cos 2\gamma \sin (2\theta - R)}{1 - \cos 2\gamma \cos (2\theta - R)} &= \cot \left(\alpha + \frac{R}{2} \right) \cos \Delta \\ \frac{\sin 2\gamma}{1 - \cos 2\gamma \cos (2\theta - R)} &= \cot \left(\alpha + \frac{R}{2} \right) \sin \Delta \end{aligned} \right\} \quad (2)$$

From these we obtain the equations

$$\left. \begin{aligned} \tan (2\theta - R) &= \tan (2\alpha + R) \cos \Delta, \\ \sin 2\gamma &= \sin (2\alpha + R) \sin \Delta, \\ \tan^2 \gamma &= \tan (\alpha + \theta) \tan (\alpha + R - \theta) \end{aligned} \right\} \quad (3)$$

that determine the character of the stream emerging from the quartz, β and δ and consequently R and Δ being supposed to be known.

2. For the discussion of the elliptic polarization it is more convenient to express $\tan 2(\theta - \alpha)$ and $\sin 2\gamma$ in terms of $\tan R$. Solving for $\tan 2(\theta - \alpha)$ we obtain

$$\begin{aligned} \tan 2(\theta - \alpha) &= \frac{\tan (2\alpha + R) \cos \Delta - \tan (2\alpha - R)}{1 + \tan (2\alpha + R) \tan (2\alpha - R) \cos \Delta} \\ &= \frac{\sin 2R \cos^2 \Delta/2 - \sin 4\alpha \sin^2 \Delta/2}{\cos 2R \cos^2 \Delta/2 + \cos 4\alpha \sin^2 \Delta/2} \\ &= \frac{\sin 2R - \sin 4\alpha \cot^2 2\beta \sin^2 R}{\cos 2R + \cos 4\alpha \cot^2 2\beta \sin^2 R} \\ &= \frac{2 \tan R - 2 \sin 2\alpha \cos 2\alpha \cot^2 2\beta \tan^2 R}{1 + (\cos 4\alpha \cot^2 2\beta - 1) \tan^2 R}. \end{aligned}$$

Also

$$\begin{aligned} \sin 2\gamma &= \sin 2\alpha \cos R \sin \Delta + \cos 2\alpha \sin R \sin \Delta \\ &= \frac{2 \sin 2\alpha \cot 2\beta \tan R + 2 \cos 2\alpha \cot 2\beta \tan^2 R}{1 + (\cot^2 2\beta + 1) \tan^2 R}, \end{aligned}$$

since

$$\sin \Delta = \frac{2 \tan \Delta/2}{1 + \tan^2 \Delta/2} = \frac{2 \cot 2\beta \sin R}{1 + \cot^2 2\beta \sin^2 R}.$$

These expressions have the form

$$y = 2(ax + bx^2)/(1 + cx^2),$$

and the maximum and minimum values of y are $y = ax_m$ occurring when

$$x = x_m = (b \pm \sqrt{b^2 + a^2 c}) / (ac).$$

3. First considering $\sin 2\gamma$, we see that $\sin 2\gamma = 0$, or the light is plane polarized, when $\tan R = 0$ and when $\tan R = -\tan 2\alpha$, the corresponding values of $\tan 2(\theta - \alpha)$ being 0 and $-\tan 4\alpha$. $\sin 2\gamma$ attains its maximum and minimum values $\sin 2\alpha \cot 2\beta \tan r_m$, when

$$\tan R = \tan r_m = \frac{1 \pm \sqrt{1 + \tan^2 2\alpha / \sin^2 2\beta}}{\tan 2\alpha / \sin^2 2\beta};$$

writing $\tan 2\alpha / \sin 2\beta = \tan \psi$, ψ being a positive angle less than π ,

$$\tan r_m = \sin 2\beta \cot \psi/2, \text{ or } -\sin 2\beta \tan \psi/2,$$

and since $\tan R = \sin 2\beta \tan \delta/2$, the maximum and minimum values of $\sin 2\gamma$ occur when $\delta = n\pi - \psi$.

The values of $\tan R$ corresponding to the maximum and minimum values of $\sin 2\gamma$ may be written

$$\tan R = \sin 2\beta \cdot x, \text{ where } x = \cot \psi/2 \text{ or } -\tan \psi/2,$$

$$\begin{aligned} \text{and } \tan 2\alpha &= -2 \sin 2\beta \cot \psi/2 / (1 - \cot^2 \psi/2) \\ &= 2 \sin \beta \tan \psi/2 / (1 - \tan^2 \psi/2) \\ &= -2 \sin 2\beta x / (1 - x^2); \end{aligned}$$

substituting these values in the expression for $\tan 2(\theta - \alpha)$, we find

$$\tan 2(\theta - \alpha) = 2 \sin 2\beta \frac{x(1 + x^2)(1 - \cos 4\beta \cdot x^2)}{(1 - x^2)(1 + x^2)(1 - \cos 4\beta \cdot x^2)};$$

and unless $x^2 = \sec 4\beta$

$$\tan 2(\theta - \alpha) = 2 \sin 2\beta \frac{x}{1 - x^2} = -\tan 2\alpha.$$

But when $x^2 = \sec 4\beta$

$$\tan 2\alpha = \sqrt{\cot^2 2\beta - 1}, \text{ or } \tan^2 2\beta = \cos^2 2\alpha;$$

hence corresponding to the maximum and minimum values of $\sin 2\gamma$, we have $\tan 2(\theta - \alpha) = -\tan 2\alpha$, unless $\cos 2\alpha = \pm \tan 2\beta$, a case that will be considered later.

Further, $\sin 2\gamma = \pm 1$, only if

$$\tan \psi/2 = \sin 2\alpha \cos 2\beta, \text{ or } \cot \psi/2 = \sin 2\alpha \cos 2\beta,$$

that is if $\cos 2\alpha = \pm \tan 2\beta$, and consequently, except in these cases, θ determines the plane of maximum polarization.

Hence, reserving for future consideration the cases in

which $\cos 2\alpha = \pm \tan 2\beta$, we see that as R increases from 0 to π , corresponding to a change of δ from $2n\pi$ to $2(n\pm 1)\pi$ according as β is positive or negative, the light is initially plane polarized in the primitive plane of polarization; it then becomes elliptically polarized of a sign, the same as or opposite to that of the plate according as $\sin 2\alpha$ is positive or negative: the ratio of the axes of the elliptic vibration attains its maximum value, the plane of maximum polarization being then either parallel or perpendicular to the principal section of the quartz, when $\tan R = \sin 2\beta \cot \psi/2$ or $= -\sin 2\beta \tan \psi/2$ according as β is positive or negative; when $\tan R = -\tan 2\alpha$ the light is again plane polarized in an azimuth symmetrical to the primitive plane of polarization with respect to the principal section of the plate; the sense of rotation then changes, and the ratio of the axes is again a maximum when $\tan R = -\sin 2\beta \tan \psi/2$ or $= \sin 2\beta \cot \psi/2$, the plane of maximum polarization being again either parallel or perpendicular to the principal section; and finally, when $R = \pi$, the light is plane polarized in the original plane*.

4. Turning now to $\tan 2(\theta - \alpha)$, we see that this attains its maximum and minimum values $\tan R_m$, when

$$\tan R = \tan R_m \\ = \frac{-\sin 2\alpha \cos 2\alpha \pm \sqrt{(\cos^2 2\alpha - \tan^2 2\beta)(\sin^2 2\alpha + \tan^2 2\beta)}}{\cos^2 2\alpha - \sin^2 2\alpha - \tan^2 2\beta}.$$

This, however, only gives a real value for $\tan R$, if $\cos^2 2\alpha > \tan^2 2\beta$, and when this is the case, writing

$$\cos^2 2\alpha - \tan^2 2\beta = \cos^2 \chi,$$

where χ is a positive angle less than $\pi/2$, we have

$$\tan R_m = \tan (\chi - 2\alpha) \text{ or } -\tan (\chi + 2\alpha),$$

the value of $\sin 2\gamma$ in these two cases being $\tan 2\beta / \cos 2\alpha$.

Thus if $\cos^2 2\alpha < \tan^2 2\beta$, the plane of maximum polarization rotates continuously as R increases; while if $\cos^2 2\alpha > \tan^2 2\beta$, it oscillates between two extreme positions, making an angle $\chi/2$ on either side of the principal section of the plate or the perpendicular plane according as $\cos 2\alpha$ is positive or negative†.

* The changes in the value of $\sin 2\gamma$ may also be conveniently traced from the formula

$$\sin 2\gamma = 2 \sin 2\alpha \cos 2\beta \sin \delta/2 \sin (\psi + \delta/2) / \sin \psi.$$

† This oscillation of the plane of maximum polarization was deduced by Croullebois (*Ann. de Ch. et de Phys.* [4] xxviii. p. 382 (1873)) for the special case of light initially polarized in the principal section of the quartz. He appears to have overlooked the fact that it requires in this case that $\tan 2\beta$ should be less than unity or $\beta < \pi/8$, a condition found by Monnory (*J. de Phys.* [2] ix. p. 277 (1890)). The condition for the general case does not appear to have been previously given.

The following tables give some of the principal cases of the changes of the plane of maximum polarization and of $\sin 2\gamma$ as R increases from 0 to π , β being taken as positive.

TABLE I.— $\cos^2 2\alpha > \tan^2 2\beta$.

$$0 < \alpha < \pi/4.$$

	$\tan R.$	$\tan 2(\theta - \alpha).$	$\theta.$	$\sin 2\gamma.$
1.	0	0	α	0
2.	$\tan(\chi - \alpha)$	$\tan(\chi - 2\alpha)$	$\frac{\chi}{2}$	$\frac{\tan 2\beta}{\cos 2\alpha}$
3.	$\frac{\tan^2 2\beta}{\sin 2\alpha \cos 2\alpha}$	0	α	$\sin 2\left(\tan^{-1} \frac{\tan 2\beta}{\cos 2\alpha}\right)$
4.	$\sin 2\beta \cot \frac{\psi}{2}$	$-\tan 2\alpha$	0	$\sin 2\alpha \cos 2\beta \cot \frac{\psi}{2}$
5.	$\cot 2\alpha$	$-\tan 4\alpha$	$-\alpha$	$\sin 2\left(\tan^{-1} \frac{\tan 2\beta}{\cos 2\alpha}\right)$
6.	$-\tan(\chi + 2\alpha)$	$-\tan(\chi + 2\alpha)$	$-\frac{\chi}{2}$	$\frac{\tan 2\beta}{\cos 2\alpha}$
7.	$-\tan 2\alpha$	$-\tan 4\alpha$	$-\alpha$	0
8.	$-\sin 2\beta \tan \frac{\psi}{2}$	$-\tan 2\alpha$	0	$-\sin 2\alpha \cos 2\beta \tan \frac{\psi}{2}$
9.	0	0	α	0

$$\pi/4 < \alpha < \pi/2.$$

	$\tan R.$	$\tan 2(\theta - \alpha).$	$\theta.$	$\sin 2\gamma.$
1.	0	0	α	0
2.	$\sin 2\beta \cot \frac{\psi}{2}$	$-\tan 2\alpha$	$\frac{\pi}{2}$	$\sin 2\alpha \cos 2\beta \cot \frac{\psi}{2}$
3.	$-\tan 2\alpha$	$-\tan 4\alpha$	$\pi - \alpha$	0
4.	$\tan(\chi - 2\alpha)$	$\tan(\chi - 2\alpha)$	$\frac{\pi}{2} + \frac{\chi}{2}$	$\frac{\tan 2\beta}{\cos 2\alpha}$
5.	$\cot 2\alpha$	$-\tan 4\alpha$	$\pi - \alpha$	$\sin 2\left(\tan^{-1} \frac{\tan 2\beta}{\cos 2\alpha}\right)$
6.	$-\sin 2\beta \tan \frac{\psi}{2}$	$-\tan 2\alpha$	$\frac{\pi}{2}$	$-\sin 2\alpha \cos 2\beta \tan \frac{\psi}{2}$
7.	$\frac{\tan^2 2\beta}{\sin 2\alpha \cos 2\alpha}$	0	α	$\sin 2\left(\tan^{-1} \frac{\tan 2\beta}{\cos 2\alpha}\right)$
8.	$-\tan(\chi + 2\alpha)$	$-\tan(\chi + 2\alpha)$	$\frac{\pi}{2} - \frac{\chi}{2}$	$\frac{\tan 2\beta}{\cos 2\alpha}$
9.	0	0	α	0

TABLE II.— $\cos^2 2\alpha < \tan^2 2\beta$.

$$0 < \alpha < \pi/4.$$

	$\tan R.$	$\tan 2(\theta - \alpha).$	$\theta.$	$\sin 2\gamma.$
1.	0	0	α	0
2.	$\cot 2\alpha$	$-\tan 4\alpha$	$\frac{\pi}{2} - \alpha$	$\sin 2\left(\tan^{-1} \frac{\tan 2\beta}{\cos 2\alpha}\right)$
3.	$\sin 2\beta \cot \frac{\psi}{2}$	$-\tan 2\alpha$	$\frac{\pi}{2}$	$\sin 2\alpha \cos 2\beta \cot \frac{\psi}{2}$
4.	$\frac{\tan^2 2\beta}{\sin 2\alpha \cos 2\alpha}$	0	$\frac{\pi}{2} + \alpha$	$\sin 2\left(\tan^{-1} \frac{\tan 2\beta}{\cos 2\alpha}\right)$
5.	$-\tan 2\alpha$	$-\tan 4\alpha$	$\pi - \alpha$	0
6.	$-\sin 2\beta \tan \frac{\psi}{2}$	$-\tan 2\alpha$	π	$-\sin 2\alpha \cos 2\beta \tan \frac{\psi}{2}$
7.	0	0	$\pi + \alpha$	0

$$\pi/4 < \alpha < \pi/2.$$

	$\tan R.$	$\tan 2(\theta - \alpha).$	$\theta.$	$\sin 2\gamma.$
1.	0	0	α	0
2.	$\sin 2\beta \cot \frac{\psi}{2}$	$-\tan 2\alpha$	$\frac{\pi}{2}$	$\sin 2\alpha \cos 2\beta \cot \frac{\psi}{2}$
3.	$-\tan 2\alpha$	$-\tan 4\alpha$	$\pi - \alpha$	0
4.	$\frac{\tan^2 2\beta}{\sin 2\alpha \cos 2\alpha}$	0	$\frac{\pi}{2} + \alpha$	$\sin 2\left(\tan^{-1} \frac{\tan 2\beta}{\cos 2\alpha}\right)$
5.	$-\sin 2\beta \tan \frac{\psi}{2}$	$-\tan 2\alpha$	π	$-\sin 2\alpha \cos 2\beta \tan \frac{\psi}{2}$
6.	$\cot 2\alpha$	$-\tan 4\alpha$	$\frac{3\pi}{2} - \alpha$	$\sin 2\left(\tan^{-1} \frac{\tan 2\beta}{\cos 2\alpha}\right)$
7.	0	0	$\pi + \alpha$	0

5. The transition from one of these cases to the other occurs when $\cos^2 2\alpha = \tan^2 2\beta$, and then

$$\tan 2(\theta - \alpha) = \frac{2 \tan R (1 - \tan 2\alpha \tan R)}{1 - \tan^2 2\alpha \tan^2 R} = \frac{2 \tan R}{1 + \tan 2\alpha \tan R}.$$

In this case θ increases from α to $\alpha + \pi/2$, as R increases from 0 to π : the light is circularly polarized when $\tan R = \cot 2\alpha$, and while for values of R less than that given by this equation θ gives the plane of maximum polarization, for greater values it determines the plane of minimum polarization*.

* When $\tan R = \cot 2\alpha$, $\tan 2(\theta - \alpha)$ of course becomes indeterminate, but it is easy to see that the change from determining the plane of maximum polarization to giving that of minimum polarization, or *vice versa*, occurs as θ passes through the values $(2n+1)\pi/4$.

The ratio of the axes of the elliptic vibrations may be easily determined from the formula

$$\tan^2 \gamma = \tan(\alpha + \theta) \tan(\alpha + R - \theta)$$

which in this case reduces to the simple form

$$\tan \gamma = \pm \tan(\theta + \alpha) \tan(\theta - \alpha),$$

the sign to be attributed to $\tan \gamma$ being determined from that of $\sin 2\gamma$.

The following table gives the principal changes relating to this case.

TABLE III.— $\cos^2 2\alpha = \tan^2 2\beta$.

$$0 < \alpha < \pi/4.$$

	$\tan R.$	$\tan 2(\theta - \alpha).$	$\theta.$	$\tan \gamma.$
1.	0	0	α	0
2.	$\cot 2\alpha$	$\cot 2\alpha$	$\frac{\pi}{4}$	1
3.	$-\cot 2\alpha$	∞	$\frac{\pi}{4} + \alpha$	$\tan\left(\frac{\pi}{4} + 2\alpha\right)$
4.	$-\tan 2\alpha$	$-\tan 4\alpha$	$\frac{\pi}{2} - \alpha$	∞
5.	$-\frac{\sin 2\alpha \cos 2\alpha}{1 + \cos^2 2\alpha}$	$-\tan 2\alpha$	$\frac{\pi}{2}$	$-\cot^2 \alpha$
6.	0	0	$\frac{\pi}{2} + \alpha$	∞

$$\pi/4 < \alpha < \pi/2.$$

	$\tan R.$	$\tan 2(\theta - \alpha)$	$\theta.$	$\tan \gamma.$
1.	0	0	α	4
2.	$-\frac{\sin 2\alpha \cos 2\alpha}{1 + \cos^2 2\alpha}$	$-\tan 2\alpha$	$\frac{\pi}{2}$	$\cot^2 \alpha$
3.	$-\tan 2\alpha$	$-\tan 4\alpha$	$\pi - \alpha$	0
4.	$-\cot 2\alpha$	∞	$\frac{\pi}{4} + \alpha$	$-\tan\left(\frac{\pi}{4} + 2\alpha\right)$
5.	$\cot 2\alpha$	$\cot 2\alpha$	$\frac{3\pi}{4}$	-1
6.	0	0	$\frac{\pi}{2} + \alpha$	$-\infty$

If α be between $\pi/8$ and $3\pi/8$, cases (3) and (4) in the above table are interchanged.

6. When the light is analysed in a plane making an angle ϕ with the principal section of the plate, the intensity of the radiation characterized by (θ, γ) is

$$I = c^2 \{ \cos^2 \gamma \cos^2 (\phi - \theta) + \sin^2 \gamma \sin^2 (\phi - \theta) \} \\ = \frac{1}{2} c^2 \{ 1 + \cos 2\gamma \cos 2(\phi - \theta) \}.$$

Now from (2) and (3)

$$\begin{aligned} \cos 2\gamma \sin 2\theta \cos R - \cos 2\gamma \cos 2\theta \sin R \\ = \sin 2\gamma \cot \Delta = \sin (2\alpha + R) \cos \Delta \\ \cos 2\gamma \sin 2\theta \sin R + \cos 2\gamma \cos 2\theta \cos R \\ = 1 - \tan \left(\alpha + \frac{R}{2} \right) \sin 2\gamma / \sin \Delta \\ = 1 - \tan \left(\alpha + \frac{R}{2} \right) \sin (2\alpha + R) = \cos (2\alpha + R); \end{aligned}$$

whence

$$\begin{aligned} \cos 2\gamma \sin 2\theta &= \cos 2\alpha \sin R \cos R (1 + \cos \Delta) \\ &\quad + \sin 2\alpha (\cos^2 R \cos \Delta - \sin^2 R) \\ \cos 2\gamma \cos 2\theta &= \cos 2\alpha (\cos^2 R - \sin^2 R \cos \Delta) \\ &\quad - \sin 2\alpha \sin R \cos R (1 + \cos \Delta). \end{aligned}$$

Now

$$\begin{aligned} 1 + \cos \Delta &= \frac{2}{1 + \tan^2 \Delta / 2} = \frac{2}{1 + \cot^2 2\beta \sin^2 R} \\ &= \frac{2}{\cos^2 R + \sin^2 R / \sin^2 2\beta}, \end{aligned}$$

$\therefore$

$$\begin{aligned} \sin R \cos R (1 + \cos \Delta) &= \frac{2 \tan R}{1 + \tan^2 R / \sin^2 2\beta} = \sin 2\beta \sin \delta \\ \cos^2 R \cos \Delta - \sin^2 R &= \frac{2}{1 + \tan^2 R / \sin^2 2\beta} - 1 \\ &= 2 \cos^2 \frac{\delta}{2} - 1 = \cos \delta \\ \cos^2 R - \sin^2 R \cos \Delta &= 1 - \frac{2 \tan^2 R}{1 + \tan^2 R / \sin^2 2\beta} \\ &= 1 - 2 \sin^2 2\beta \sin^2 \frac{\delta}{2} \\ &= \cos^2 2\beta + \sin^2 2\beta \cos \delta. \end{aligned}$$

Hence

$$\begin{aligned}\cos 2\gamma \cos 2\theta &= \cos 2\alpha (\cos^2 2\beta + \sin^2 2\beta \cos \delta) - \sin 2\alpha \sin 2\beta \sin \delta \\ &= \cos 2\alpha \cos^2 2\beta + \sin 2\beta \sqrt{\sin^2 2\alpha + \cos^2 2\alpha \sin^2 2\beta} \\ &\quad \cos(\delta + \psi)\end{aligned}$$

$$\begin{aligned}\cos 2\gamma \sin 2\theta &= \cos 2\alpha \sin 2\beta \sin \delta + \sin 2\alpha \cos \delta \\ &= \sqrt{\sin^2 2\alpha + \cos^2 2\alpha \sin^2 2\beta} \sin(\delta + \psi),\end{aligned}$$

where $\tan \psi = \tan 2\alpha / \sin 2\beta$,

and

$$\begin{aligned}I &= \frac{1}{2}c^2 [1 + \cos 2\alpha \cos^2 2\beta \cos 2\phi \\ &\quad + \sqrt{\sin^2 2\alpha + \cos^2 2\alpha \sin^2 2\beta} \{ \sin 2\beta \cos 2\phi \cos(\delta + \psi) \\ &\quad + \sin 2\phi \sin(\delta + \psi) \}] \\ &= \frac{1}{2}c^2 [1 + \cos 2\alpha \cos^2 2\beta \cos 2\phi \\ &\quad + \sqrt{\sin^2 2\alpha + \cos^2 2\alpha \sin^2 2\beta} \sqrt{\sin^2 2\phi + \cos^2 2\phi \sin^2 2\beta} \\ &\quad \cos(\delta + \psi - \psi')],\end{aligned}$$

where $\tan \psi' = \tan 2\phi / \sin 2\beta$.

This formula contains the theory of the interference phenomena that are obtained when a conical pencil of polarized light traverses the plate and is subsequently analysed.

It may also be employed for the case in which the stream that passes through the polarizer, quartz plate and analyser is a parallel pencil of white light that is investigated with a spectroscope. With a sufficiently thick plate, β may be regarded as constant over a region of the spectrum corresponding to a variation of δ by several multiples of π , and we see that the minima of intensity occur for the radiations for which $\delta = (2n+1)\pi + \Psi$, where

$$\tan \Psi = \frac{\sin 2\beta (\tan 2\phi - \tan 2\alpha)}{\sin^2 2\beta + \tan 2\phi \tan 2\alpha},$$

the intensity then being

$$I = \frac{1}{2}c^2 \{ 1 + \cos 2\alpha \cos^2 2\beta \cos 2\phi - \sqrt{1 - \cos^2 2\alpha \cos^2 2\beta} \sqrt{1 - \cos^2 2\beta \cos^2 2\phi} \}.$$

Thus as the analyser is turned, the dark bands travel along the spectrum, changing their intensity as they progress: they are perfectly black when $\phi = \pi/2 \pm \alpha$, and are least marked when $\phi = 0$ and $\pi/2$.

With a plate perpendicular to the optic axis, $\beta = \pi/4$, and

$$I = c^2 \cos^2 \frac{\delta - \Psi}{2} = c^2 \cos^2 \frac{\delta - 2(\phi - \alpha)}{2};$$

hence as the analyser is turned, the bands traverse the spectrum, retaining the same intensity.

When the plate is parallel to the optic axis, $\beta=0$ or is at any rate very small, and

$$I = \frac{1}{2}c^2\{1 + \cos 2\alpha \cos 2\phi + \sin 2\alpha \sin 2\phi \cos \delta\};$$

when $\phi=0$ or $\pi/2$, there are no bands, and in other cases the bands occur where $\delta=(2n+1)\pi$ or $2n\pi$, according to the sign of $\sin 2\alpha \sin 2\phi$. Thus when $0<\alpha<\pi/2$, the bands occur at the points for which $\delta=(2n+1)\pi$ so long as $0<\phi<\pi/2$, their intensity being zero, when $\phi=\pi/2-\alpha$; and as ϕ increases through $\pi/2$, they change their position to that of the former maxima of intensity, becoming perfectly black when $\phi=\pi/2+\alpha^*$.

7. Let us now suppose that the light emergent from the plate is examined with a Savart's analyser.

On leaving the quartz, the stream may be represented by the components

$$(u, v) = c(\cos \gamma, -\iota \sin \gamma) \text{Exp} \{\iota(nt + \epsilon)\}$$

polarized in planes making angles θ and $\theta + \pi/2$ with the principal section of the plate. Hence if μ be the angle that the principal section of the quartz makes with that of the first plate of the analyser, the stream emerging from the Savart's plate will have components

$$\xi = \{\cos \gamma \cos (\theta + \mu) + \iota \sin \gamma \sin (\theta + \mu)\} \text{Exp} \{\iota(nt + \epsilon' + D/2)\}$$

$$\eta = \{\cos \gamma \sin (\theta + \mu) - \iota \sin \gamma \cos (\theta + \mu)\} \text{Exp} \{\iota(nt + \epsilon' - D/2)\}$$

polarized in the principal sections of the first and second plate, where $D=D_2-D_1$, D_1, D_2 being the relative retardations of phase introduced by the two plates.

On analysis in an azimuth ϕ , these give a stream

$$E = \xi \cos \phi + \eta \sin \phi$$

$$\begin{aligned} &= [\{\cos \gamma \cos (\theta + \mu) + \iota \sin \gamma \sin (\theta + \mu)\} \cos \phi \text{Exp} (\iota D/2) \\ &+ \{\cos \gamma \sin (\theta + \mu) - \iota \sin \gamma \cos (\theta + \mu)\} \sin \phi \text{Exp} (-\iota D/2)] \\ &\quad \times \text{Exp} \{\iota(nt + \epsilon')\}, \end{aligned}$$

* The above results agree with Beaulard's description of the phenomena (*J. de Phys.* (3) ii. p. 399 (1893)). Croullebois, however, gives a different account of the phenomena in the first and last cases (*loc. cit.* p. 391).

and the intensity, obtained by multiplying this by the conjugate expression, is

$$\begin{aligned} I &= \{ \cos^2 \gamma \cos^2 (\theta + \mu) + \sin^2 \gamma \sin^2 (\theta + \mu) \} \cos^2 \phi \\ &\quad + \{ \cos^2 \gamma \sin^2 (\theta + \mu) + \sin^2 \gamma \cos^2 (\theta + \mu) \} \sin^2 \phi \\ &\quad + \{ \cos 2\gamma \sin 2(\theta + \mu) \cos D - \sin 2\gamma \sin D \} \sin \phi \cos \phi \\ &= \frac{1}{2} [1 + \cos 2\gamma \cos 2(\theta + \mu) \cos 2\phi \\ &\quad + \sin 2\phi \{ \cos 2\gamma \sin 2(\theta + \mu) \cos D - \sin 2\gamma \sin D \}], \end{aligned}$$

and when, as is ordinarily the case, $\phi = \pi/4$

$$I = \frac{1}{2} \{ 1 + \cos 2\gamma \sin 2(\theta + \mu) \cos D - \sin 2\gamma \sin D \}.$$

In order, then, that the bands may disappear, we must have

$$\sin 2\gamma = 0 \quad \text{and} \quad \sin 2(\theta + \mu) = 0;$$

that is, the light emergent from the quartz must be plane polarized in the principal section of one of the plates of the Savart's analyser—a result that is obvious from elementary considerations.

Now the light on leaving the quartz is plane polarized (1) when $\delta = 2n\pi$, the plane of polarization being then the same as it was initially; (2) when $\tan \delta/2 = -\tan 2(\alpha - \mu)/\sin 2\beta$, α being now the azimuth of the initial plane of polarization with respect to the principal section of the first plate of the analyser, and in this case $\tan 2(\theta + \mu - \alpha) = -\tan 4(\alpha - \mu)$. Hence in the first case, in order that the bands may disappear, the light must be initially polarized in one of the principal sections of the Savart's plate; and in the second case the principal section of the quartz must be in the azimuth $\mu = k\pi/4 + \alpha/2$.

It follows, then, that if the initial polarization be such that there are no bands before the introduction of the quartz plate, there will be a disappearance of the bands whenever the relative retardation of phase due to the quartz plate is $\delta = 2n\pi$, and if the principal section of the quartz bisect the angle between the principal sections of the Savart's plate*, there will be a further disappearance whenever $\delta = (2n+1)\pi$. These results afford a method of setting the principal section of a quartz plate perpendicular to the axis of a spectrometer and of determining the error in the cutting of a plate that is supposed to be normal to the optic axis.

The collimator is furnished with a web, placed by the

* The second of the above cases becomes identical with first when $\alpha = 0$, if k be even, and when $\alpha = \pi/2$, if k be odd, since the value of δ then becomes $2n\pi$.

ordinary method perpendicular to the axis of the instrument. The telescope is turned to one side, and a telescope with a Savart's analyser is inserted in its place with its axis in the direction of that of the collimator, and the analyser is adjusted so that, when plane polarized light passes, the fringes are parallel to the web of the collimator. This sets the plane bisecting the angle between the principal sections of the plates of the analyser perpendicular to the turning axis of the spectrometer. This adjustment being made, the polarizer is turned until the fringes disappear exactly in the centre of the field.

The quartz plate is then placed at the centre of the table of the instrument, and by means of the telescope of the spectrometer its faces are made parallel to the turning axis, and the reading determined for which its normal is along the axis of the collimator.

With the polarizer adjusted as above, the fringes will in general be restored when the light traverses the quartz, but a series of positions can be found for which they vanish over a narrow streak extending across the field, and if the quartz plate be turned about its normal so as to bring its optic axis perpendicular to the turning axis of the spectrometer, there will be, intermediate to these, further positions at which the fringes are absent over a small area at the centre of the field.

In this way the principal section of the quartz is set perpendicular to the turning axis of the instrument, and a determination of the angles of incidence, for which the centres of the zones of disappearance of the fringes are at the centre of the field, will give by calculation or by the graphic method employed by Beaulard* the direction of the optic axis with respect to the faces of the quartz.

XXIII. *The Effect of Temperature on the Rate of Production of Uranium X.* By R. W. FORSYTH, A.R.C.Sc., Imperial College of Science and Technology †.

IN the following paper an account is given of experiments carried out to determine whether the rate of production of Uranium X from a salt of uranium depends upon the temperature of the compound in which the transformation is taking place. Calculations have shown that the quantity of radium present in the crust of the Earth, if evenly distributed

* *Thèse de Doctorat*, Marseille, 1893.

† Communicated by Prof. the Hon. R. J. Strutt, F.R.S.

throughout the globe, is more than sufficient to account for the Earth's internal heat. If the rate of production of Uranium X from uranium is retarded at high temperatures it is possible that the subsequent transformation products, including radium, may exist in relatively smaller quantities in the hotter strata beneath the cooler crust. Experiments have been performed on the rate of production of Uranium X in specimens kept respectively at atmospheric temperatures and at 1000° C.

To prepare a salt of uranium as free as possible from Uranium X, the method described by Sir William Crookes (Proc. Roy. Soc. lxvi. 1900) was employed. Uranium nitrate was dissolved in ether in a separating funnel, and a little freshly precipitated ferric hydrate was added. The ethereal portion of the solution was separated from the aqueous part, containing the bulk of the Uranium X, and allowed to evaporate to dryness. The residue was treated in a similar manner, and after three successive separations a salt was obtained containing an amount of Uranium X which would be produced in a day from the inactive uranium salt. The nitrate was then ignited and the resulting oxide thoroughly well mixed and divided into two equal portions. These were enclosed in quartz tubes, one being kept at the temperature of the laboratory and the other heated in a gas muffle-furnace to a temperature of 1000° C. The rate of formation of Uranium X was studied by measuring the β -ray activity as evidenced by the rate at which the specimens discharged a gold-leaf electroscope. The normal leak of the electroscope was determined several times throughout the investigation, and varied from 4.3 to 5.0 scale-divisions per hour over the range used in the experiments.

In making an experiment the cold specimen was placed in a definite position above the electroscope, and the time taken for the gold-leaf to move over a fixed part of the eyepiece scale of the observing microscope was recorded. The cold specimen was then replaced by the one which had been heated, and a similar observation was made. In no case was the hot specimen out of the furnace for more than two hours for any observation. Making suitable allowance for the normal leak of the instrument, the rate of leak in scale-divisions per hour for each of the specimens was calculated and the ratio between them determined. The variation of this ratio with time served as an index of the rates of growth in the specimens.

The following details of one of the experiments may be taken as typical of others obtained.

Experiment commenced March 19th, 1909.

Normal leak = 4.3 divisions per hour.

Cold specimen.		Hot specimen (1000° C.).			
Date.	Corrected leak.	Date.	Corrected leak.	Interval.	Ratio (cold/hot).
March 19th ...	10.3 divs. per hr.	Mar. 19th ...	6.1 divs. per hr.	...	1.69
„ 26th ...	58.6 „ „	„ 26th ...	26.2 „ „	7 days.	2.24
„ 29th ...	73.5 „ „	„ 29th ...	34.2 „ „	10 „	2.15
April 1st ...	84.7 „ „	April 1st ...	45.4 „ „	13 „	1.87
„ 15th ...	140.0 „ „	„ 15th ...	62.3 „ „	27 „	2.25

It was found in all cases that the change in the ratio *cold/hot* was not sufficiently decided to show that the rate of production of Uranium X at 1000° C. was sensibly different from that at ordinary atmospheric temperatures.

In conclusion, I wish to thank Professor Strutt for suggesting these experiments and for his help and interest throughout.

XXIV. *The Motion of Electrons in Solids. Part II.—Radiation of all Wave-lengths in a perfectly Reflecting Enclosure. Natural Radiation. Dependence of Natural Radiation on the Law of Force. By J. H. JEANS, M.A., F.R.S.**

A general Formula for the Emission of Radiation of all Wave-lengths.

25. **I**N the first part of this paper we discussed radiation of great wave-length only: our first task in discussing radiation of all wave-lengths must be to find general analytical expressions for the radiation emitted by moving electrons, which shall be valid throughout the whole spectrum.

Consider an element of volume dv , so small that its linear dimensions may be neglected in comparison with the wave-length of any radiation under discussion. We regard the motion of an electron in this element as the creation of a series of imaginary electric doublets. Let M (components

* Communicated by the Author.

M_x, M_y, M_z) be the moment of the doublets in dv at any instant, then in the notation already used,

$$\frac{dM_x}{dt} = \Sigma eu = i_x dv. \quad . \quad . \quad . \quad (46)$$

Here Σ , which in the previous paper denoted summation over a great number of electrons, will now, on account of the smallness of dv , represent (except in special cases) a sum over zero or one electron only.

Taking the element dv as origin, the components of magnetic force at x', y', z' arising from the motion parallel to Ox of any electron or electrons which may happen to be in dv , will be *

$$\left(0, -\frac{z'}{r}, \frac{y'}{r}\right) \frac{d}{dr} \left(\frac{1}{r} \frac{dM_x}{dt}\right)$$

or, by equation (46),

$$\left(0, -\frac{z'}{r}, \frac{y'}{r}\right) \frac{d}{dr} \left(\frac{1}{r} i_x dv\right), \quad . \quad . \quad . \quad (47)$$

where, in calculating the field at time t , M_x and i_x are to be evaluated at time $t-r/V$.

26. Consider now a small prism of infinitesimal cross section $dy dz$ extending from $x=-l$ to $x=l$. The small piece $dx dy dz$ of this prism is to replace the former dv .

The total current parallel to Ox which flows through this prism at any point will be $i_x dx dy$. Let us call this j_x .

The value of j_x at any point of the prism can be expanded in the form

$$j_x = \frac{1}{\pi} \int_{q=0}^{q=\infty} (A_q \cos qx + B_q \sin qx) dq, \quad . \quad . \quad . \quad (48)$$

where

$$A_q = \int_{-l}^l j_x \cos qx dx, \quad B_q = \int_{-l}^l j_x \sin qx dx.$$

Ultimately, when the elements of current are identified with individual electrons, these equations become

$$A_q = \Sigma eu \cos qx, \quad B_q = \Sigma eu \sin qx, \quad . \quad . \quad (49)$$

the summation being through the prism of length $2l$.

The values of A_q, B_q of course vary with the time. Again using Fourier's theorem, we put

$$A_q = \frac{1}{\pi} \int_{p=0}^{p=\infty} (\alpha_{qp} \cos pt + \alpha'_{qp} \sin pt) dp, \quad . \quad . \quad . \quad (50)$$

* Larmor, 'Æther and Matter,' p. 223.

where

$$\alpha_{qp} = \int_0^t A_q \cos pt \, dt, \quad . \quad . \quad . \quad . \quad . \quad (51)$$

$$\alpha'_{qp} = \int_0^t A_q \sin pt \, dt. \quad . \quad . \quad . \quad . \quad . \quad (52)$$

The value of B_q can be similarly resolved into harmonic terms, with typical coefficients β_{qp}, β'_{qp} . The whole motion is now resolved into regular waves. There are waves of all possible frequencies, and the waves of any single frequency are of all possible wave-lengths. Consequently these waves travel with all possible velocities.

27. The magnetic force at time t produced at a sufficiently distant point x', y', z' in direction l', m', n' by the x components of velocity of the electrons inside the prism, is, by expression (47),

$$(0, -n', m') \frac{d}{dr} \left(\frac{1}{r} \int_{x=-l}^{x=l} j_x dx \right), \quad . \quad . \quad . \quad (53)$$

in which j_x is evaluated at time $t - r/V$.

From equation (50) the value of A_q at time $t - r/V$ is

$$A_q = \frac{1}{\pi} \int_{p=0}^{p=\infty} \left\{ \alpha_{qp} \cos p \left(t - t_0 - \frac{l'x}{V} \right) + \alpha'_{qp} \sin p \left(t - t_0 - \frac{l'x}{V} \right) \right\} dp,$$

where t_0 is the time required for radiation to travel from the middle point of the prism ($x=0$) to x', y', z' .

This gives, as the value of the integral in expression (53),

$$\begin{aligned} \int_{x=-l}^{x=l} j_x dx &= \frac{1}{\pi^2} \int_{x=-l}^{x=l} dx \int_{p=0}^{p=\infty} \int_{q=0}^{q=\infty} \alpha_{qp} \cos qx \cos p \left(t - t_0 - \frac{l'x}{V} \right) dp dq \\ &\quad + \text{similar terms in } \alpha'_{qp}, \beta_{qp}, \beta'_{qp}, \\ &= \frac{1}{2\pi^2} \int_{p=0}^{p=\infty} \int_{q=0}^{q=\infty} \alpha_{qp} \left[\frac{\sin \left\{ p(t-t_0) + \left(q - \frac{l'p}{V} \right) l \right\}}{q - l'p/V} \right. \\ &\quad \left. + \frac{\sin \left\{ p(t-t_0) - \left(q + \frac{l'p}{V} \right) l \right\}}{q + l'p/V} \right] dp dq, \\ &\quad + \dots - \text{a similar function of } -l, \\ &= \frac{1}{\pi} \int_{p=0}^{p=\infty} (\alpha_{qp})_{q=l'p/V} \cos p(t-t_0) dp + \dots \\ &= \frac{1}{\pi} \int_0^\infty [(\alpha_{qp} + \beta'_{qp}) \cos p(t-t_0) \\ &\quad + (\alpha'_{qp} - \beta_{qp}) \sin p(t-t_0)] dp, \quad . \quad (54) \end{aligned}$$

in which q may now be regarded as simply an abbreviation for $l'p/V$.

We have in effect imagined the motion of the electrons parallel to Ox to be resolved into a doubly infinite series of regular waves corresponding to all values of p and q . Analysis has now shown that the only waves which contribute at all to the radiation are those for which p and q are connected by the relation $q = l'p/V$. Thus the only waves in the prism parallel to Ox from which the radiation is appreciable are those with an exponential factor $e^{ipt - il'px/V}$. It follows that when the whole three-dimensional motion of electrons is resolved into waves of electrons travelling in all directions with all possible frequencies and wave-lengths, the only waves which contribute to the radiation in the direction l', m', n' are those having the exponential factor

$$e^{ip\{t - (l'x + m'y + n'z)/V\}}.$$

These are waves of electrons travelling in the direction l', m', n' with the velocity of light. In fact it can easily be shown, by direct physical reasoning, that the disturbances sent out in the direction l', m', n' by the different elements of a wave of electrons which is not of this kind, must annihilate one another by interference.

28. Putting

$$\gamma_p^2 = (\alpha_{qp} + \beta'_{qp})^2 + (\alpha'_{qp} - \beta_{qp})^2, \quad . \quad . \quad (55)$$

we see that equation (54) can be expressed in the form

$$\int_{x=-l}^{x=l} j_x dx = \frac{1}{\pi} \int_0^\infty \gamma_p \cos p(t - \epsilon) dp.$$

On replacing q by pl'/V , we have (*cf.* equations (51) and (49))

$$\alpha_{qp} = \int_0^t \sum eu \cos pt \cos \frac{l'px}{V} dt,$$

so that, from this and similar equations,

$$\begin{aligned} \alpha_{qp} + \beta'_{qp} &= \int_0^t \sum eu \cos p\left(t - \frac{l'x}{V}\right) dt \\ \alpha'_{qp} - \beta_{qp} &= \int_0^t \sum eu \sin p\left(t - \frac{l'x}{V}\right) dt. \end{aligned}$$

Squaring and adding, we obtain

$$\gamma_p^2 = \int_0^t \int_0^t \sum \sum e^2 u_1 u'_2 \cos p\left(t_1 - t_2 - \frac{l'(x_1 - x'_2)}{V}\right) dt_1 dt_2, \quad (56)$$

the summation extending over all possible pairs of electrons, one being taken at instant t_1 and the other at instant t_2 .

From the law of distribution of velocities there is known to be no correlation between the velocity-components of two different electrons, either at the same or at different instants. Hence in equation (56), the whole value of γ_p^2 must arise from terms in which the two electrons concerned are identical: we must have

$$\gamma_p^2 = \int_0^t \int_0^t \Sigma e^2 u_1 u_2 \cos p \left(t_1 - t_2 - \frac{l'(x_1 - x_2)}{V} \right) dt_1 dt_2. \quad (57)$$

29. In dealing with radiation of great wave-length we neglect p/V , and therefore replace $p \left(t_1 - t_2 - \frac{l'(x_1 - x_2)}{V} \right)$ by $p(t_1 - t_2)$.

The value of γ_p^2 now becomes

$$\gamma_p^2 = \int_0^t \int_0^t \Sigma e^2 u_1 u_2 \cos p(t_1 - t_2) dt_1 dt_2, \quad (58)$$

and this is readily seen to be identical with the value obtained in our previous analysis for the corresponding quantity *, as of course it ought to be.

Formula (58), in which p/V is neglected, may also be interpreted in another way: it is a formula for radiation of all wave-lengths, in which the finiteness of the velocity of propagation is neglected. It is of interest to notice that from formula (58) which had already been obtained, we could have deduced the more general formula (57) by an appeal to the principle of relativity.

Still another meaning can be given to formula (58): it is a general formula for radiation of all wave-lengths, in which the Doppler-effect is neglected. In all observed radiation the influence of the Doppler-effect on the partition of energy in the spectrum is insignificant †, so that we may take (58) to be a formula for radiation of all wave-lengths under natural conditions.

Formula (58) is independent of the direction l' , m' , n' . Adding to it the corresponding quantities which originate in the velocities of the electrons parallel to Oy and Oz , we obtain, as in Part I., for the total emission per unit volume in time t ,

$$\frac{2\mu}{3\pi V} \int_{p=0}^{p=\infty} p^2 \left(\int_0^t \int_0^t \Sigma e^2 (u_1 u_2 + v_1 v_2 + w_1 w_2) \cos p(t_1 - t_2) dt_1 dt_2 \right) dp. \quad (59)$$

* The present $p^2 \gamma_p^2$ is identical with the previous $A_p^2 + B_p^2$.

† At 0°C . $u/V = \frac{1}{30000}$; in the sun, at 6000°C ., $u/V = \frac{1}{6000}$. It is easily shown that the error introduced by neglecting the Doppler-effect is of the order of u^2/V^2 .

This formula can be used, as in Part I., for the calculation of radiation. It is of interest to notice that it can be readily transformed to another known formula.

If we replace $\cos p(t_1 - t_2)$ by $\cos pt_1 \cos pt_2 + \sin pt_1 \sin pt_2$, and integrate by parts with respect to t_1 and t_2 simultaneously, we obtain for the emission

$$\begin{aligned} & \frac{2\mu}{3\pi V} \int_{p=0}^{p=\infty} \left\{ \int_0^t \int_0^t \Sigma e^2 \left(\frac{du_1}{dt} \frac{du_2}{dt} + \dots \right) (\cos pt_1 \cos pt_2 \right. \\ & \quad \left. + \sin pt_1 \sin pt_2) dt_1 dt_2 \right\} dp \\ &= \frac{2\mu}{3\pi V} \int_{p=0}^{p=\infty} \left\{ \left(\int_0^t \Sigma e \frac{du}{dt} \cos pt \right)^2 + \left(\int_0^t \Sigma e \frac{du}{dt} \sin pt \right)^2 \right\} dp, \end{aligned}$$

which is the formula used by Sir J. J. Thomson*. On integration with respect to p , the total radiation per unit volume per unit time is verified to be

$$\Sigma \frac{2\mu}{3V} e^2 f^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

Radiation in a Perfectly Reflecting Enclosure.

30. Every material substance ought, on the electromagnetic theory of light, to be perfectly reflecting for waves of infinite wave-length, while all actual substances are transparent for waves of very short wave-length.

It follows that in any actual enclosure, the radiation of great wave-length will be retained indefinitely, so that equilibrium (*i. e.* equipartition) of energy must be established between this radiation and the matter in the enclosure (a consequence of general dynamical theory which we have already verified), but the radiation of short wave-length will escape without ever attaining to energy-equilibrium.

On the other hand in an ideal (but in practice unattainable) enclosure, from which no radiant energy can escape, no matter how short its wave-length, there ought to be equilibrium of energy between matter and radiant energy of all wave-lengths. The law of partition of radiant energy in such an enclosure ought, therefore, to be that given by the theorem of equipartition of energy, namely,

$$8\pi RT \lambda^{-4} d\lambda. \quad . \quad . \quad . \quad . \quad . \quad . \quad (61)$$

* Phil. Mag. (6) vol. xiv. p. 217.

The formulæ which have now been obtained not only enable us to verify this last conclusion, but also give us an insight into the mechanism by which equipartition is arrived at and maintained.

31. In §§ 2-7 of Part I. an expression was obtained for c , the conductivity, on the supposition that the motion of the electrons was checked or modified only by the forces acting upon the electrons from the matter. We must now consider also the possibility of the motion of the electron being modified by the pulses and waves of radiation through which it passes on its path. If, as will be found to be the case, the partition of radiation is that given by formula (61), the expectation of electric force arising from the radiation at any point is infinite, or, more strictly, is limited only by the imperfections in the reflecting powers of the walls or by the imperfect homogeneity (coarse-grainedness) of the æther, both of which disturbing factors we have agreed to neglect*. Thus the forces produced by the radiation preponderate over those originating in the matter. We must still picture the electrons as checked in their motion by the molecules of matter, but must also regard them as being continually buffeted by waves and pulses of free radiation.

32. Whatever forces act, it is clear that the general argument of §§ 2-7 may still be used, provided we take account of encounters with pulses and waves of radiation as well as with molecules. The value of κ will then depend on the partition of energy in the æther, but, whatever its value, the relation expressed by equation (10) will still be true, namely,

$$c = \frac{\kappa}{1 + \frac{\kappa^2 \rho^2 m^2}{N^2 e^4}} \quad \dots \dots \dots (62)$$

Also, provided this same value of κ is still used, equation (38) is also true, namely

$$i_2 = i_1 e^{-\epsilon(t_1 - t_2)}, \quad \dots \dots \dots (63)$$

where $\epsilon = Ne^2/m\kappa$.

These equations are still subject to the limitation that they are true only if the time intervals $2\pi/p$ and $t_1 - t_2$ are great

* Let λ_0 be, roughly speaking, the shortest wave-length which can exist in, and be retained by, the enclosure. The radiant energy per unit volume obtained by integration of (61) is $8\pi RT/3\lambda_0^3$. Half of this must be the mean value of $E^2/8\pi$, where E is the electric force resulting from the presence of the radiation. Thus the values of E are arranged (and it can be shown that they are arranged according to Maxwell's law) about the mean square value $E^2 = 32\pi^2 RT/3\lambda_0^3$.

compared with the time of a complete rearrangement of the velocities of the electrons. But when the æther is filled with pulses and short waves of sufficient intensity, this latter interval may be treated as infinitesimal, and equations (62) and (63) may be supposed true for all values of p and $t_1 - t_2$.

33. Proceeding as before (§ 19), we integrate expression (59) with the help of relation (63) and obtain for the emission per unit volume per unit time,

$$\int \frac{4p^2 RT \kappa \mu}{\pi V \left(1 + \frac{\kappa^2 p^2 m^2}{N^2 e^4}\right)} dp. \quad . \quad . \quad . \quad . \quad (64)$$

For simplicity suppose the enclosure (of volume A) to be completely filled with homogeneous matter (K, μ). Let the radiant energy per unit volume, analysed into waves of different frequencies, be $\int E_p dp$.

The total radiant energy of frequency between p and $p + dp$ in the whole enclosure is $A E_p dp$. The rate at which this disappears owing to absorption by matter is as in Part I. (§ 18),

$$\frac{\kappa}{1 + \frac{\kappa^2 p^2 m^2}{N^2 e^4}} 4\pi V^2 \mu A E_p dp, \quad . \quad . \quad . \quad . \quad (65)$$

in which $V^2 \mu$ replaces C^2/K . The rate at which the energy in question is added to by new emissions from the matter is at once obtainable from formula (64). Combining loss and gain, we find for the complete rate of change in E_p ,

$$\frac{dE_p}{dt} = \frac{\kappa \mu}{1 + \frac{\kappa^2 p^2 m^2}{N^2 e^4}} \left\{ \frac{4p^2 RT}{\pi V} - 4\pi V^2 E_p \right\}. \quad . \quad (66)$$

34. The steady state is given by $dE_p/dt = 0$, and therefore by

$$\begin{aligned} E_p dp &= \frac{1}{\pi^2 V^3} RT p^2 dp, \\ &= 8\pi RT \lambda^{-4} d\lambda, \quad . \quad . \quad . \quad . \quad (67) \end{aligned}$$

which is exactly the value required for equipartition of energy. Moreover, it will readily be found that formula (67) exactly expresses the steady state even after allowing for the corrections required by the Doppler-effect, which in our present analysis have been neglected. Thus the steady state is one of absolute equipartition of energy.

35. Equation (66) shows further that if E_p has not the value required for equipartition of energy, it will always move towards that value. The total energy, if not equally distributed between the different vibrational degrees of freedom, must pass towards this state. The "time of relaxation" for radiation of frequency p is

$$\frac{1}{4\pi V^2 \kappa \mu} \left(1 + \frac{\kappa^2 p^2 m^2}{N^2 e^4} \right).$$

For long waves this becomes $K/4\pi C^2 \kappa$, which is equal to the "time of relaxation" for the dissipation of irregularities of electric charge inside the same conductor*.

In other words, the energy of great wave-length adjusts itself to equipartition exactly as rapidly as electric charges adjust themselves to the equalization of potential: the radiation of shorter wave-length adjusts itself at a slower, but still comparable, rate.

There is one reservation, a very important one, to be made to all this. Equation (66) assumes the æther to be already filled with an adequate number of "pulses" and waves of short wave-length. If this condition is not at first satisfied, progress towards a state of equipartition of energy of short wave-length will begin slowly, for the creation of pulses and short waves is known to be a very slow process (§ 21). The theorem of equipartition shows, however, that the final state must be one of equipartition.

Natural Radiation.

36. If the reflecting powers of the walls of the enclosure fall short of perfection in any degree, the system may no longer be treated as conservative, and there is no justification for expecting equipartition of energy (*cf.* § 21). To find the partition of radiant energy we fall back on our detailed analysis of § 28.

Natural radiation, whether examined when streaming freely from a hot body or when taken from an enclosure with the most perfectly reflecting walls obtainable, is found, as a matter of observation, to be almost totally devoid of waves of the shortest wave-lengths. This is what we should in any case expect from our knowledge of the slowness with which energy of short wave-length is generated, and of the imperfect retaining power for such waves of all actual substances. The analysis of §§ 30-33 assumed the presence in abundant quantity of waves of short wave-length: this analysis must now be modified to suit natural conditions.

* Jeans, 'Electricity and Magnetism,' § 396.

The symbol κ has already been regarded as dependent on the partition of radiant energy. In passing to the new conditions now under consideration, κ acquires a new value but not a new meaning; it stands in the analysis just as before, and equation (62) remains true for the new value of κ as well as for the old, except that, as in Part I., its validity is confined to small values of p .

Equation (63) also is no longer true except for certain sufficiently great values of the time-interval $t_2 - t_1$. As a consequence, equation (64) no longer expresses the emission of radiation except for waves of great wave-length.

Let us replace this formula for the emission by the perfectly general formula

$$\int \frac{4p^2 RT \kappa \mu}{\pi V \left(1 + \frac{\kappa^2 p^2 m^2}{N^2 e^4}\right)} \Theta_p dp, \quad \dots \quad (68)$$

which will be supposed true from $p=0$ to $p=\infty$. Here Θ_p is a function of p and of T , and the constants of matter. The formula (68) is perfectly general because Θ_p may be a perfectly general function of p and of these other quantities.

Our analysis has already proved that Θ_p must become equal to unity when p is small, and must fall off exponentially with p when p is great.

Formula (65) will give the absorption provided p is small. Thus at sufficiently low temperatures the formula will give the absorption accurately through those parts of the spectrum in which the energy is appreciable. To avoid too great complication, we shall assume formula (65) to be accurate for all values of p with which we are concerned. Our results will therefore be strictly true for low temperatures, but will be only approximations for higher temperatures.

37. Using the assumed value (68) for the emission, it is found that equation (66) must be replaced by

$$\frac{dE_p}{dt} = \frac{\kappa \mu}{1 + \frac{\kappa^2 p^2 m^2}{N^2 e^4}} \left\{ \frac{4p^2 RT}{\pi V} \Theta_p - 4\pi V^2 E_p \right\}, \quad \dots \quad (69)$$

so that instead of equation (67) for the steady state, we have

$$E_p dp = \frac{1}{\pi^2 V^3} RT p^2 \Theta_p dp. \quad \dots \quad (70)$$

$$= 8\pi RT \lambda^{-4} \Theta_p d\lambda. \quad \dots \quad (71)$$

From what has already gone before (§ 16), it will be clear that the known qualitative properties of radiation will be accounted for if, on actual calculation, Θ_p is found to be a function only of T/p and of constants which are the

same for all kinds of matter. Any other form for Θ_p would be contrary to experience: a form in which T and p entered otherwise than through the single variable T/p would be in opposition to Stefan's law and to Wien's displacement-law, while a form of Θ_p which varied for different kinds of matter would contradict the fact that the radiation in a cavity is independent of the substance of the walls. It should be noticed that we are at present concerned only with facts about Θ_p which are deducible from experiment; these facts are undoubtedly true, quite apart from any theories we may hold as to why they are true.

38. We shall first consider what are the conditions, in terms of the electron-theory, that Θ_p shall have the required functional form. If these conditions are found to be such as we can readily believe to be satisfied in actual matter, then we shall be able to apply a further test to the electron theory by actually evaluating the function Θ_p and comparing the value so found with experiment.

The functional form required for Θ_p can be expressed by the equation

$$\Theta_p = \phi\left(\frac{ap}{T}\right), \quad . \quad . \quad . \quad . \quad . \quad (72)$$

in which a is a constant, the same for all matter, of which the physical dimensions are those of T/p , and ϕ is a function which may involve other constants, but must be the same for all kinds of matter. Let us assume Θ_p to be of this form and examine what inferences can be drawn from equation (72).

Let us put

$$ap = uT,$$

so that u is a pure number, and Θ_p is a function of u only. Expression (68), which represents the total emission per unit volume per unit time, now becomes

$$\begin{aligned} & \frac{4RT^3 \mu N e^2}{\pi V m a^2} \int_{u=0}^{u=\infty} \frac{\kappa u m T}{N e^2 a} \cdot \frac{\phi(u) u du}{1 + \left(\frac{\kappa u m T}{N e^2 a}\right)^2} \\ &= \frac{4RT^3 \mu N e^2}{\pi V m a^2} \Phi\left(\frac{\kappa m T}{N e^2 a}\right), \quad . \quad . \quad . \quad (73) \end{aligned}$$

where Φ is a new function which is the same for all matter.

The total emission per unit volume per unit time is, however, known to be (*cf.* equation (60))

$$\frac{2 N e^2 \mu}{3 V} \bar{f^2},$$

where $\bar{f^2}$ is the mean value, at any instant, of the squares of

the accelerations of the electrons. Hence, as a condition that equation (72) may be true, we must have

$$\overline{f^2} = \frac{6RT^3}{\pi m a^2} \Phi \left(\frac{\kappa m T}{N e^2 a} \right). \quad . \quad . \quad . \quad . \quad (74)$$

39. We wish to evaluate both sides of this equation in terms of electron-motion. Let u, v, w be the components of velocity of an electron at any point x, y, z at which the potential is V . The equations of motion of the electron are

$$\frac{du}{dt} = -\frac{e}{m} \frac{\partial V}{\partial x}, \text{ \&c.}, \quad . \quad . \quad . \quad . \quad (75)$$

so that

$$f^2 = \frac{e^2}{m^2} \left\{ \left(\frac{\partial V}{\partial x} \right)^2 + \left(\frac{\partial V}{\partial y} \right)^2 + \left(\frac{\partial V}{\partial z} \right)^2 \right\}.$$

Hence, if E^2 denote at any instant the mean of the squares of the electric intensities at the points occupied by electrons we have

$$\overline{f^2} = \frac{e^2 E^2}{m^2}. \quad . \quad . \quad . \quad . \quad . \quad (76)$$

The evaluation of the right-hand member of equation (74) is more difficult, as it involves the evaluation of κ .

If we denote by $\mathfrak{S}$ the operator

$$\frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z},$$

we obtain from (75)

$$\frac{d^n u}{dt^n} = -\frac{e}{m} \mathfrak{S}^{n-1} \frac{\partial V}{\partial x}, \quad . \quad . \quad . \quad . \quad (77)$$

where, on the right, $\frac{\partial}{\partial t}$ operates only on u, v, w , and $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ operate only on V .

Let the electrons in unit volume move so that the small departures from Maxwell's law produce an infinitesimal current i_x parallel to Ox , this being given (*cf.* equation (2)) by

$$i_x = \Sigma eu.$$

The summation of equation (77) over all electrons in unit volume leads to

$$\left(\frac{d}{dt} \right)^n i_x = -\frac{e^2}{m} \Sigma \mathfrak{S}^{n-1} \frac{\partial V}{\partial x}. \quad . \quad . \quad . \quad . \quad (78)$$

If we take the expectation of value of each term on the

right, we of course obtain the expectation of the value of the term on the left. In taking these expectations of value, we put

$$\begin{aligned}\Sigma u &= i_x/e, \quad \Sigma v = \Sigma w = 0. \\ \Sigma u^2 &= \Sigma v^2 = \Sigma w^2 = RT/m, \quad \Sigma uv = \dots = 0. \\ \Sigma u^3 &= \Sigma v^3 = \dots = 0, \text{ \&c.}\end{aligned}$$

Also there is no correlation between velocities and position, and we may put

$$\Sigma \frac{\partial V}{\partial x} = \Sigma \frac{\partial V}{\partial y} = \Sigma \frac{\partial V}{\partial z} = 0,$$

and the same for all differential coefficients of odd order. Equation (78) now assumes the form

$$\text{expectation of } \left(\frac{d}{dt}\right)^n i_x = p_n i_x, \quad . \quad . \quad (79)$$

where p_n depends only on RT/m , e/m , and on averages of differential coefficients, and of products of differential coefficients of V^* .

Corresponding to a definite value i_1 of i_x at time t_1 , the expectation of i_x at time t_2 , say i_2 , is given by

$$i_2 = i_1 + (t_2 - t_1) \left(\frac{di_x}{dt}\right)_{t_1} + \frac{1}{2} (t_2 - t_1)^2 \left(\frac{d^2 i_x}{dt^2}\right)_{t_1} + \dots,$$

in which the expectation of every term has to be taken. Thus from equation (79)

$$i_2 = i_1 + (t_2 - t_1) p_1 i_1 + \frac{1}{2} (t_2 - t_1)^2 p_2 i_1 + \dots, \quad . \quad (80)$$

also, for great values of $t_2 - t_1$, the relation is known to be

$$\begin{aligned}i_2 &= e^{-\epsilon(t_2 - t_1)} i_1 \\ &= i_1 - (t_2 - t_1) \epsilon i_1 + \frac{1}{2} (t_2 - t_1)^2 \epsilon^2 i_1 +, \quad . \quad . \quad (81)\end{aligned}$$

where, as before, $\epsilon = Ne^2/m\kappa$.

For great values of $(t_2 - t_1)$, equations (80) and (81)

* Relation (79) can be derived more simply, and by a method which some may think more rigorous, by an application of statistical mechanics.

Let all possible states of the electrons in a unit of volume be represented in the appropriate generalized space. Let S denote the region or regions of this space for which $\Sigma u = i_x/e$, where i_x has a definite given value. Equation (78) is true for the configuration represented by every point in the generalized space: it is therefore true throughout S . Multiply it by the element of volume in the generalized space, integrate throughout S , divide by the volume of S , and we obtain equation (79) at once.

become identical. It follows that, as regards the quantities involved, and the algebraic dimensions of these quantities, ϵ^n must be similar to p_n —we can evaluate ϵ^n , to within numerical constants, by evaluating p_n . Or, again, ϵ^2 must be given, except for numerical constants, by the value of p_{n+2}/p_2 . But the difference between p_{n+2} and p_n lies in operating, before averaging, with $\mathfrak{S}^2$. It follows that ϵ^2 is similar, as regards the quantities involved and their algebraic dimensions, to $\mathfrak{S}^2$, regarded as a multiplier.

The operator $\mathfrak{S}^2$ consists of sixteen terms of which typical terms are

$$u^2 \frac{\partial^2}{\partial x^2}, \quad \frac{\partial}{\partial t} u \frac{\partial}{\partial x}, \quad \frac{\partial^2}{\partial t^2}.$$

Regarded as multipliers, the dimensions of these terms are those of

$$u^2 \left(\frac{\partial V}{\partial x} \right)^2 / V^2, \quad \frac{du}{dt} \frac{\partial V}{\partial x} / V, \quad \left(\frac{du}{dt} \right)^2 / u^2. \quad \dots \quad (82)$$

The potentials and velocities of the different electrons are arranged according to the law of distribution

$$A e^{-(2eV + mu^2 + \dots)/2RT} dx dy dz du dv dw,$$

so that the average values of eV and mu^2 are each of dimensions RT. We know that the average value of mu^2 is RT, and can take the average value of eV to be αRT , where α will, in general, depend on the structure of the particular substance involved. The value of $\frac{du}{dt}$ is given by equation (75). Substituting in the terms (82), we find that all the terms are of the same dimensions, namely, $\frac{e^2}{RTm} \left(\frac{\partial V}{\partial x} \right)^2$, which may be expressed as the dimensions of $e^2 E^2 / RTm$.

We must accordingly have an equation of the form

$$\epsilon = \frac{Ne^2}{m\kappa} = \frac{\beta eE}{\sqrt{RTm}}, \quad \dots \quad (83)$$

where β is a numerical constant which may depend on the structure of the particular substance,

40. The values of f^2 and κ are given by equations (76) and (83). Substituting these values in equation (74) we obtain

$$\frac{e^2 E^2}{m^2} = \frac{6RT^3}{\pi m a^2} \Phi \left(\frac{\sqrt{RT^3 m}}{\beta a e E} \right),$$

which is satisfied if

$$\Phi(x) = \frac{\pi}{6\beta^2} \frac{1}{x^2}. \quad \dots \quad (84)$$

Since Φ is to be the same for all substances, β must be the same for all substances.

41. Going back to equation (73), and substituting this form for Φ , we find that we must have (writing v for $\kappa mT/Ne^2a$)

$$\int_{u=0}^{u=\infty} \frac{v}{1+v^2} \phi(u) u du = \Phi\left(\frac{v}{u}\right) = \frac{\pi}{6\beta^2} \frac{u^2}{v^2} \dots (85)$$

The fraction v/u does not depend on p , and is therefore independent of u . Calling it γ , the equation becomes

$$\int_{v=0}^{v=\infty} \frac{v^2}{1+v^2} \phi\left(\frac{v}{\gamma}\right) dv = \frac{\pi}{6\beta^2}, \dots (86)$$

and since ϕ is the same for all substances, this can only be satisfied if γ is the same for all substances. It must clearly be a pure numerical constant.

We now have

$$\gamma = \frac{v}{u} = \frac{\kappa mT}{Ne^2a}$$

so that

$$\kappa = \frac{\gamma Ne^2a}{mT}, \dots (87)$$

or, in simpler form,

$$\kappa = \frac{Nb}{T}, \dots (88)$$

where b is a universal constant.

42. For pure metals κ is known from experiment to be approximately proportional to the inverse temperature, and it is natural to expect κ to be proportional to N . There is, therefore, nothing inherently improbable in equation (88). We see that the observed differences between the temperature coefficients of κ and the values which they would have if κ varied exactly as T^{-1} must be attributed either to variations in N as the temperature changes, or to differences between the value of κ measured and the value with which we are here concerned. Such a difference would arise, for instance, if the resistance of a solid was not entirely made up of resistances such as we have been concerned with—as, for example, if the solid were not homogeneous, but had a crystalline structure. In any case equation (88) ought only to be true for very low values of T .

The Law of Force.

43. To reconcile experiment with electron-theory, we have found that at low temperatures the value of κ must be of the form given by equation (87). But actual calculation has shown that κ must have the form given by equation (83). Comparing these two values for κ , we find

$$E^2 = RT^3 m / e^2 \alpha^2 \beta^2 \gamma^2. \quad \dots \quad (89)$$

Thus the average square of the force ($e^2 E^2$) acting on an electron must be of the form gT^3 , where g is the absolute constant $Rm/\alpha^2 \beta^2 \gamma^2$. We can easily see why $e^2 E^2$ increases with T ; at higher temperatures the kinetic energies of the electrons are greater, and the electrons consequently penetrate to regions at which the force is greater. The knowledge of the exact relation between E^2 and T enables us to find the law of force under which the electrons move.

44. Consider first a temperature $T=0$. The electrons have no kinetic energy, and so fall into positions of equilibrium. Thus $E^2=0$, in accordance with equation (89).

Considering next small values of T , we can find the nature of these positions of equilibrium. Suppose, first, that these positions were represented by ordinary minima of potential (if such could exist), then at a small temperature T the electrons would oscillate about these positions. The average squared force ($E^2 e^2$) would be proportional to the average square of the displacement, and therefore to T , and equation (89) would not be satisfied. Thus the zero value of E^2 which indicates rest in a position of minimum potential is not a limiting solution of equation (89).

There is only one alternative— E^2 must vanish when $T=0$, on account of the electrons being at infinity, or being so far removed from the centre of force that the forces acting on them are negligible.

Suppose, first, that each electron is acted on by only one centre of force of law μ/r^n . The potential is $(n-1)\mu/r^{n-1}$, so that at temperature T the law of distribution of distances is $e^{-(n-1)\mu/RT} r^{n-1} dr$, and the value of $e^2 E^2$ (the average value of μ^2/r^{2n}) is found to be proportional to $T^{2n/n-1}$. This gives relation (89) if $n=3$, and if μ is the same for all kinds of matter.

If the electrons are acted on by more than one centre of force, it is readily found that E^2 will not vary exactly as any power of T , and must moreover involve the distances between the different centres of force; these would be different for different kinds of matter.

45. Thus we must suppose that, in all kinds of matter

alike, the radiation proceeds from electrons describing orbits about centres repelling as the inverse cube of the distance, these centres being of equal strength in all kinds of matter. The influence of forces varying as other powers of the distance, and of the presence of centres of force other than that primarily involved, must be looked for in departures from the laws of Stefan and Wien.

It will be remembered that Sir J. J. Thomson, in his paper already referred to, comes to the conclusion that Stefan's and Wien's laws point to collisions with molecules which have to be similar for all matter, and repel according to the law μ/r^3 . The present writer ventures to think that the similar centres of force may be found to be the positive electrons. The ultimate test of any such conjecture must, however, be the calculation of the radiation function and comparison with experiment.

Conclusion.

46. It may be of value to collect and summarize those of the results obtained which have reference to the radiation problem, arranging them in their logical order.

I. It has been verified, by analysing the motion of the electrons, that radiation in a perfectly reflecting enclosure, when in a steady state, must be distributed between the different wave-lengths according to the law of equipartition (§ 34). We have found formulæ for the rate of progress towards this state, and for the "time of relaxation" (§ 35).

II. It has been shown that when the walls of the enclosure are incapable of retaining radiation of short wave-length (as all actual substances are) an entirely different partition of energy is to be looked for. We have supposed this new partition to be such that the energy of frequency p has Θ_p times its equipartition value.

III. It has been verified, by analysing the motion of the electrons, that $\Theta_p = 1$ when p is small, and that Θ_p must fall off exponentially with p when p is great. Both these results are in agreement with experiment.

[The first result $\Theta_p = 1$ had been obtained by Lorentz* on the suppositions of undisturbed free-paths and instantaneous collisions, but it seemed desirable to have a further calculation free from these suppositions†.]

IV. The various results obtained have given some insight as to why Θ_p starts with unit value and gradually falls off as p increases. We have found ‡ (§ 26) that the irregular

* Amsterdam Proceedings, 1902-3, p. 666.

† Phil. Mag. xvii. p. 253.

‡ Cf. also Phil. Mag. xvii. p. 254.

motion of the electrons can be resolved into regular trains of waves. Only those waves contribute to the radiation which travel with the velocity of light (§ 27). These waves emit and absorb radiation with a frequency equal to their own, and there will clearly be rapid interchange of energy between the waves of electrons and the radiation of the same frequency in the æther. So long as we deal only with waves of wavelength great compared with the distances of adjacent electrons, the collection of electrons can be treated as a continuous electric medium. The degrees of freedom of the electrons may be supposed to be the waves in this medium, and these are in energy-equilibrium both with the matter and with the waves in the æther. We see at once the dynamical necessity for the result that $\Theta_p = 1$ when p is small.

As we advance up the spectrum we cannot suppose that there are waves of the electric medium for all values of p , for if this were so the finite number of electrons would possess an infinite number of degrees of freedom. If p_1 and p_2 are two adjacent values of p , there may be only one degree of freedom of the electric medium for the values p_1 and p_2 of p , and for all the values between. The vibrations of this degree of freedom may at one instant exchange energy with the vibrations in the æther of frequency p_1 , at another instant with those of frequency p_2 , and so on. If the various vibrations of the æther are retained indefinitely there must ultimately be partition of energy between all (result I.), but if not there must be a falling off from equipartition values.

V. The dependence of Θ_p on p will vary with different structures of matter. All the known phenomena of radiation are accounted for if Θ_p is a function only of T/p and of universal constants.

According to the thermodynamical theory, Θ_p must necessarily have this form. The thermodynamical theory, however, applies (if at all) only to the radiation inside a perfectly radiation-tight enclosure. Then Θ_p has the required form, for it is equal to unity for all values of T and p (result I.), but this does not give any assistance towards finding a formula for natural radiation.

VI. The required form of Θ_p is obtained only if the law of force acting on the electrons is that of the inverse cube of the distance.

VII. Since it is known that there are terms in the law of force which fall off as the inverse square, and since the electrons must always be acted on by more than one centre of force, it follows, if our analysis is sound, that Stefan's and Wien's laws must be only approximations.

April 28th, 1909.

XXV. *Notices respecting New Books.*

Cours de Physique. By H. BOUASSE. Vols. II., IV., & V.
Paris : Librairie Ch. Delagrave.

TO compile a new textbook of physics at the present day is no light task, since the subject is being enriched in so many ways in consequence of modern discoveries on both the experimental and the theoretical sides. M. Bouasse himself has felt this especially in connexion with the development of Thermodynamics (Vol. II.). The happy time is past when the lot of a professor was merely to explain certain general principles and apply them to some stereotyped examples; or rather, as we should prefer to say, the happy time has come when that is no longer necessary.

The present textbook is one which deals mainly with the theoretical side, though the results of theory are usually illustrated by numerical calculations applied to particular experimental cases. M. Bouasse possesses the quality of most French writers of being a very lucid exponent of his subjects. The general impression conveyed by these volumes is that they are very tightly packed; there is nothing prolix, at any rate, in their style; and yet we do not know of any clearer exposition than we are here presented with. Our own preference is that the experimental side should be considered side by side with the theoretical, and the absence of experimental details may be thought by others as well as ourselves to be a drawback. On the understanding, however, that the book is not intended to be used alone, we can confidently recommend it. The parts of the subject which are dealt with in the volumes at present before us are: Thermodynamics, Theory of Ions, Optics, Electro-optics, Hertzian Waves.

The Theory of Valency. By J. NEWTON FRIEND.
Longmans, Green & Co. 1909. Price 5s.

THIS forms one of the Textbooks of Physical Chemistry edited by Sir Wm. Ramsay. The fact that a special volume should be needed dealing with a peculiar part of a subject like this, is sufficient evidence of the very rapid progress of knowledge. No volume on this subject exists prior to this one in English, and only one in German; consequently it practically opens new ground in the way of separate publication. The subject is a very fascinating one, and our author, who can speak with weight, has put it forward in an interesting and thorough fashion. In spite of the many difficulties which the subject presents, there can be no doubt that the idea of valency is a valuable one, especially in the light of modern electrical developments. At the same time the present reviewer wonders whether it is not being pressed too far. The possibility, at any rate, of the existence of fractional valencies is certainly suggested by the phenomena of loose aggregates such as in the formation of hydrates. It also is suggested by such results as those of Ternent Cooke (p. 57), according to which the vapour

density of zinc is found to be some 12 per cent. higher when in an atmosphere of argon than in one of nitrogen. Such facts as these make one pause contemplatively, and sympathise more completely with the ideas of Lodge expressed on p. 137 rather than with the main ideas underlying this volume.

Transactions of the International Union for Co-operation in Solar Research. Vol. II. Manchester University Press.

THIS volume, which is edited by Prof. A. Schuster, contains the verbatim report of the Third Conference of the International Union held at Meudon in 1907. Amongst the papers communicated and printed here are one on the determination of the wave-length of the red ray of cadmium as a fundamental standard of wave-lengths by MM. Benoît, Fabry and Perot; the work of MM. Fabry and Buisson on measurements of wave-lengths made to establish a system of spectroscopic standards; the measurement of Solar Photographs made with the spectroheliograph by George E. Hale; and the report of the Committee on Sun-spot spectra drawn up by the Secretary (A. Fowler), including a list of publications referring to Sun-spot spectra. The volume is crowded with interesting and valuable matter.

XXVI. *Intelligence and Miscellaneous Articles.*

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

IN a paper of April 1909 in this Magazine, the Earl of Berkeley and C. V. Burton treat the problem of finding the influence of gravity upon a binary solution. I wish to call the attention of the authors to the fact, which they seem to have overlooked, that a complete solution of the problem is to be found in a paper* by the writer published Oct. 1906. The cases of a rotating solution and a solution subject to the earth's field were specially treated, and in the latter case numerical calculations were carried out for solutions of cane-sugar and potassium hydrate. Further, the following equation was found:—

$$\rho - \rho_0 = \left(\frac{\partial \pi}{\partial c} \right)_p \frac{dc}{dn} \frac{1}{K} + \rho \frac{\partial \pi}{\partial p}.$$

This important relation connecting the concentration gradient with the variation of osmotic pressure with hydrostatic pressure is identical with the relation expressed in equation (7) in their paper.

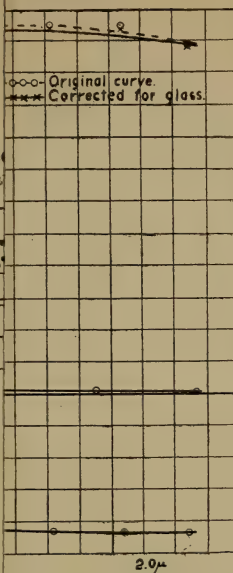
I am, Gentlemen,

Yours faithfully,

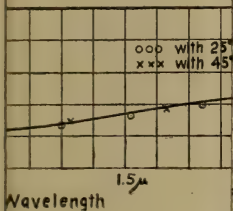
L. VEGARD.

* Christiania Vid. Selsk. Skr. No. 8, 1906; Phil. Mag. May 1907.

μ thickness.



n on reflexion from steel
and low dispersion array



Wavelength 1.5μ

2.0μ

0.—Curves showing the variation of transmission with wave-length for the various films.

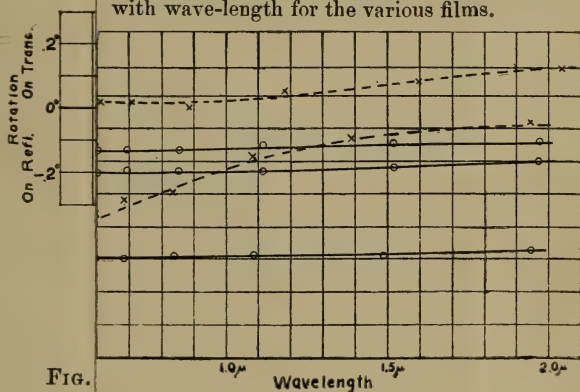


FIG.

sion

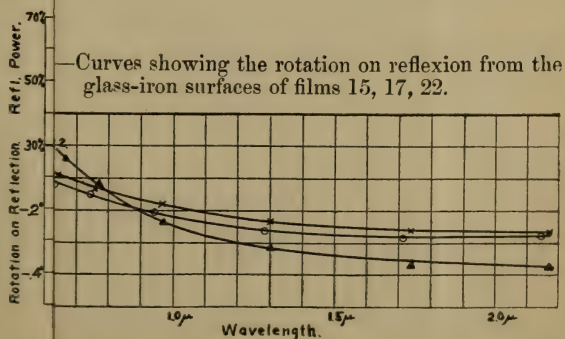


FIG. 3.—Curves for rotation on transmission through Fluorite, showing effect of using high and low dispersion.

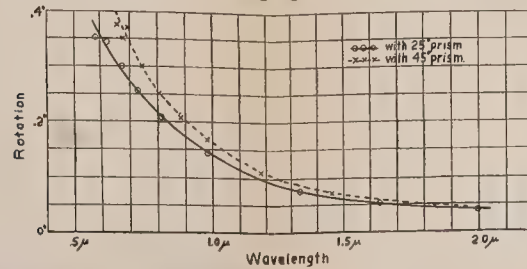


FIG. 4.—Curves for rotation on reflexion from steel, showing measurements with high and low dispersion arrangements.

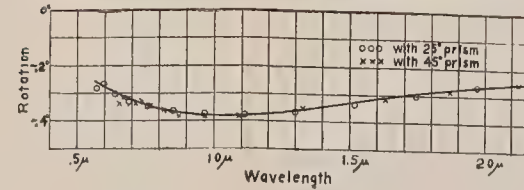


FIG. 6.—Curves for film 15, of 58 μ thickness.

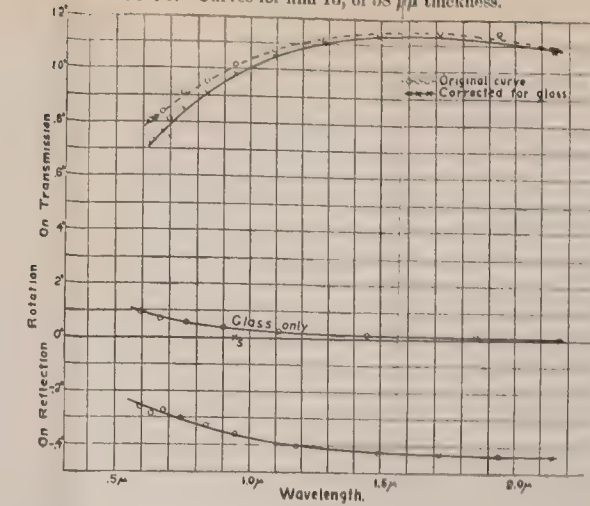


FIG. 7.—Curves for film 17, of 30 μ thickness.

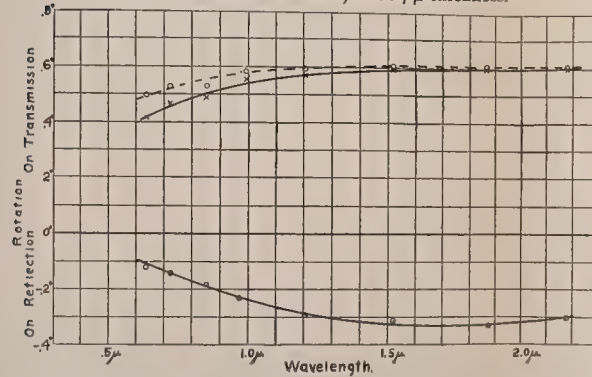


FIG. 8.—Curves for film 22, of 13 μ thickness.

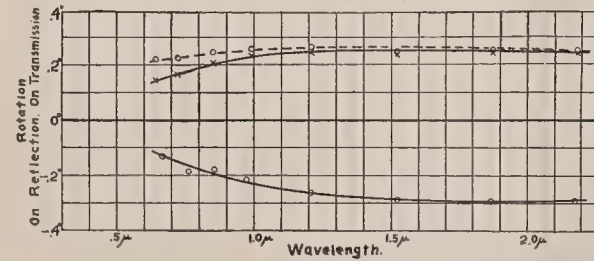


FIG. 9.—Curves for film 24, of 8 μ thickness.

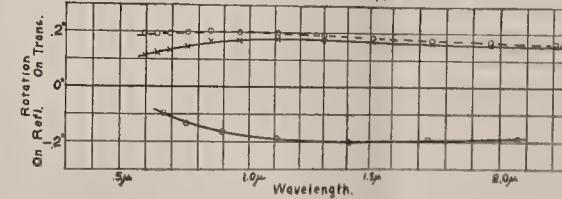


FIG. 10.—Curves showing the variation of transmission with wave-length for the various films.

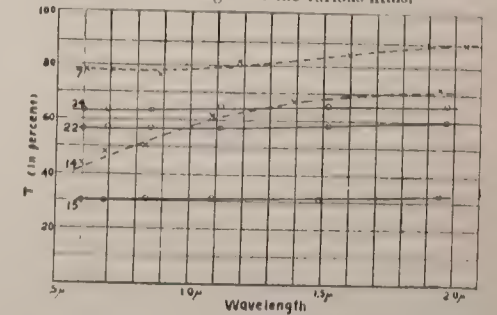


FIG. 11.—Curves for partially oxidized film 7. Transmission curve has been corrected for the rotation of the glass. On the reflexion curve, points plotted as triangles were determined with the rear glass surface covered with Canada balsam and lamp-black.

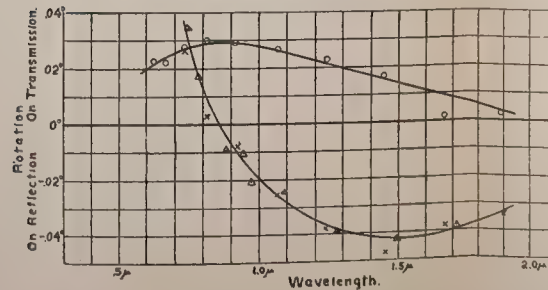


FIG. 12.—Curves for partially oxidized film 14. Transmission curve has been corrected for rotation due to glass.

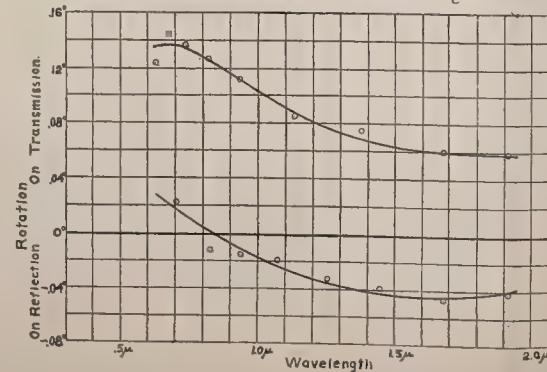


FIG. 13.—Curves for rotation on reflexion and for reflecting power of opaque, partially oxidized, film 29. The reflecting power curve of pure iron film 15 is shown for comparison.

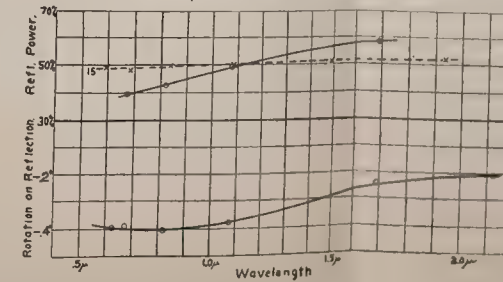


FIG. 14.—Curves showing the rotation on reflexion from the glass-iron surfaces of films 15, 17, 22.

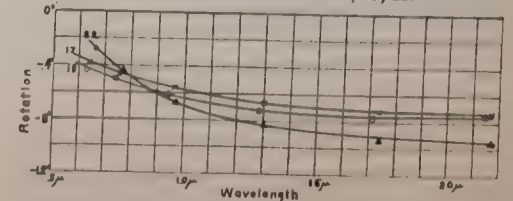


FIG. 4.

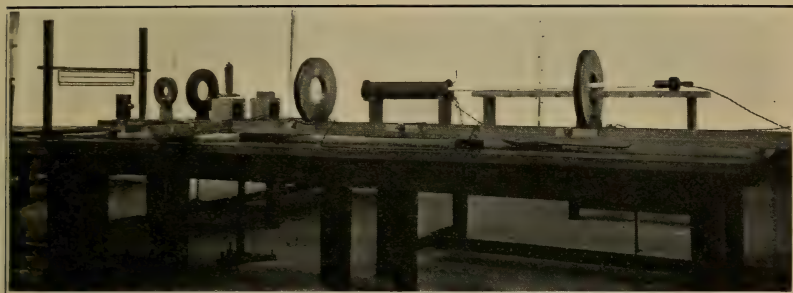


FIG. 4 A.



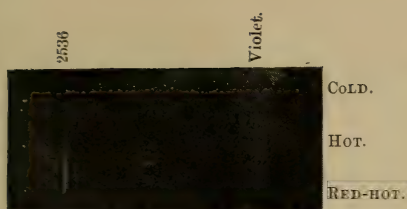


FIG. 1.

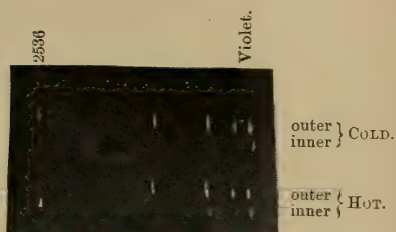


FIG. 2.

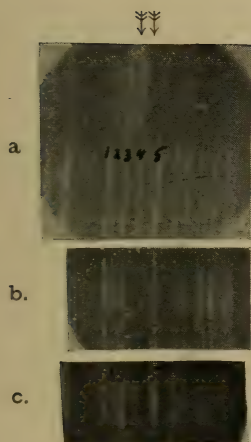


FIG. 3.



FIG. 4.



FIG. 5.



FIG. 6.



FIG. 7.

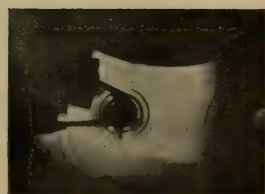


FIG. 8.—Mercury Waves.

THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[SIXTH SERIES.]

AUGUST 1909.

XXVII. *The Discontinuity of Potential at the Surface of Glowing Carbon.* By J. A. POLLOCK, A. B. B. RANCLAUD, and E. P. NORMAN*.

OWING to the projection of ions by hot substances, a discontinuity of potential will occur at the surfaces of the electrodes in any circuit in which these latter are formed of heated materials. The value of potential discontinuity has been calculated by Professor Richardson † from considerations connected with the gas theory of metallic conduction, and has been found in the case of carbon by Mr. Duddell ‡, who observed at the surface of the cathode of the ordinary carbon arc a forward electromotive force of 6.1 volts, and a back electromotive force of 16.7 volts at the surface of the anode.

In a circuit with one heated electrode in air at ordinary pressure, the projection of ions from the hot surface necessitates the establishment of a potential-difference between the electrodes if the current in the circuit is to be zero. If electrons alone were projected, a layer of gas close to the hot surface would become negative to the solid and the potential-difference between the electrodes for zero current might be taken as a measure of the surface discontinuity of potential. When positive ions are emitted as well as electrons, owing to the difference in the distances from the electrode at which the two classes of ions are stopped by

* Communicated by the Authors. Read before the Royal Society of New South Wales.

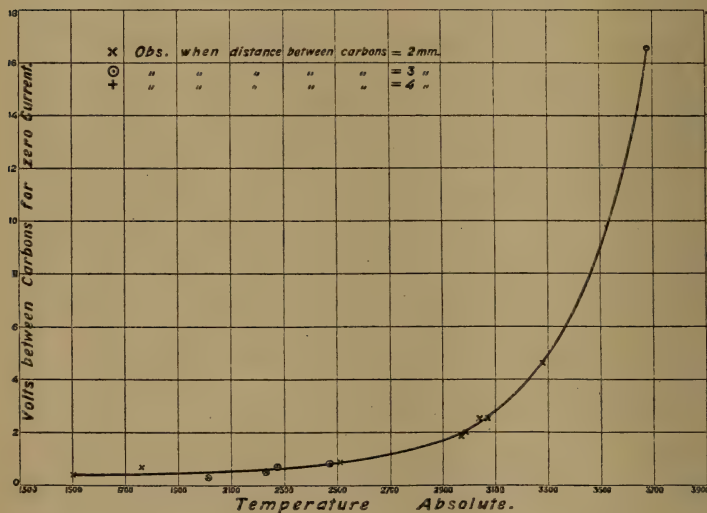
† Richardson, Phil. Trans. A. cci. p. 497 (1903).

‡ Duddell, Phil. Trans. A. cciii. p. 305 (1904).

collision with the gas molecules, the electric force may change sign in the near neighbourhood of the heated surface. In this case the potential-difference between the electrodes for zero current, with appropriate sign, must be less than the value of the surface discontinuity which would be due to the projection, alone, of that class of ions which conditions the sign of the potential-difference.

With the apparatus described in a previous paper* the potential-difference for zero current has been found in the case of glowing carbon at various temperatures. The experiments were a continuation of the work already described, and in the paper just mentioned will be found full details of the method of investigation. The observations, which fulfil the condition that the values should be independent of the distance separating the electrodes, are shown in the figure. For zero current the hot carbon was positive to the cooler electrode in all cases.

Potential-differences for zero current.



The heated electrodes were cored carbon rods, 0.5 centimetre in diameter, supplied by Messrs. Siemens Brothers for use with their Lilliput arc lamps, with the exception of the one employed for the observation at 3040° absolute which was a squared rod, 0.5 centimetre on the side, of solid Conrady carbon Marke C. As the measure with this material agrees well with the other results, it may be concluded that

* Pollock & Ranclaud, *Phil. Mag.* March 1909, p. 366.

values of the potential-difference for zero current with hot electrodes of different makes of carbon do not seriously differ.

At 3120° absolute, with 50 volts between the electrodes, the ratio of the flow of negative electricity from the hot carbon to the flow of positive, when the sign of the potential-difference was reversed, was as 20 to 1.

On the assumption that at high temperatures the potential-difference for zero current measures the surface discontinuity of potential, as the number of electrons then projected per second far exceeds that of positive ions, the curve in the figure has been continued to the value, 16·7 volts, found by Mr. Duddell (*loc. cit.*), for the back electromotive force at the anode of an arc between solid Conradty horis carbons. This measure has been plotted for the temperature of the crater, 3690° absolute, determined by Messrs. Waidner and Burgess* from observations with a Holborn-Kurlbaum optical pyrometer, a similar instrument to that with which we have estimated the other temperatures. Mr. Duddell gives 6·1 volts as the measure of the forward electromotive force at the arc cathode; this value corresponds, on the curve of potential-differences for zero current, to a temperature of 3375° absolute, which, if the assumption already stated is legitimate, may be taken as an estimate of the temperature of the cathode of the carbon arc.

With a heated iron wire in the place of the hot carbon of the previous experiments, the potential-difference for zero current was 0·85 volt at 1410° absolute and 0·25 volt at 1570°, the hot wire being, in both cases, negative to the cooler electrode. The potential-difference with a platinum wire, not specially treated, at 1580° absolute, was 0·40 volt, the hot wire being again negative. With a Nernst filament, designed for 0·25 ampere at 90 volts and used under these conditions, the potential-difference for zero current was 0·25 volt. In this case the filament was positive to the cooler electrode, and with a potential-difference between the electrodes of 45 volts, the ratio of the flow of negative electricity from the hot filament to the flow of positive, when the sign of the potential-difference was reversed, was as 33 to 1.

Professor Richardson (*loc. cit.*) gives, for the number of corpuscles shot off from unit area of a hot conductor per second, the expression $A\theta^{\frac{1}{2}}\epsilon^{-\frac{b}{\theta}}$, in which A depends on the number of corpuscles per unit volume of the conductor, and b on the work done by a corpuscle in passing through the

* Waidner & Burgess, *Phys. Rev.* xix. p. 255 (1904).

surface layer, θ being the absolute temperature. The formula, considering A and b as constants, very well represents the observations of its author and others on the saturation currents from hot bodies through the range of temperature for which it has been employed; from such observations the values of b for certain substances have been determined. In the theory by which the expression is deduced, b is equal to Φ/R , where Φ is the work done by a corpuscle in passing through the surface layer, and R a gas constant, equal to $p/N\theta$, p being the pressure and N the number of molecules per cubic centimetre. The discontinuity of potential is thus represented by b/Re , where e is the ionic charge.

The following are the values of the discontinuity of potential as calculated by Professor Richardson:—

	Temperature absolute.	Discontinuity of potential.
Sodium	490°–700°	2.45 volts.
Platinum	1378–1571	4.1 „
Carbon	1520–1770	6.1 „

These results, which are not appreciably altered by recalculation with the new value of the ionic charge determined by Professor Rutherford and Dr. Geiger*, seem singularly high. If bR/e really represents the discontinuity of potential, the fact that the discontinuity varies somewhat rapidly with temperature for higher values shows that b cannot be considered altogether constant.

In a recent paper Professor Richardson and Mr. Brown† originate the theory of a method for finding the kinetic energy of the electrons projected from heated materials, and they have experimentally determined the component, perpendicular to the hot surface, of the velocity with which electrons are projected from glowing platinum, the result for a temperature of 1650° absolute being 1.5×10^7 centimetres per second. This velocity of projection, in the case of glowing carbon, may, on certain assumptions, be estimated from the results already given in the present paper. As suggested by one of us‡, a point of view is possible from which the work done in connexion with the passage of the electrons through the surface-layer may be considered as represented by their translational energy when they emerge into the gas. In such a case the surface discontinuity of potential, V , may be

* Rutherford & Geiger, Proc. Roy. Soc. A. xxxi. p. 162 (1908).

† Richardson & Brown, Phil. Mag. xvi. p. 353, Sept. 1908.

‡ Pollock, Phil. Mag. March 1909, p. 361.

expressed by the equation

$$V = \frac{1}{2} \frac{m}{e} v^2,$$

where m is the mass of the electron, e the ionic charge, and v the component, in the direction of the field, of the velocity with which the electrons are projected from the hot surface. If this equation holds, on the assumption previously stated, we may deduce from the curve in the figure that the velocity with which electrons are projected from glowing carbon varies from 1.5×10^8 centimetres per second at 3375° absolute to 2.5×10^8 centimetres per second at 3690° absolute. In a similar way, from the result previously given, a lower limit to the velocity with which electrons are projected from the Nernst filament may be calculated as 3×10^7 centimetres per second.

The Physical Laboratory,
The University of Sydney,
November 28th, 1908.

XXVIII. *Vibration Curves of Violin G String and Belly.*
By E. H. BARTON, D.Sc., F.R.S.E., Prof. of Experimental
Physics, and T. J. RICHMOND, B.Sc., Research Scholar,
University College, Nottingham*.

[Plates IV.-VI.]

AT the end of 1904 it occurred to one of us that it would be of interest to trace the various changes in vibration form experienced as the motion passes from a vibrating string to the ear by means of the bridge, belly, and air in the sound-box. For although the string is the source of the vibrations concerned, and the pitch of all those vibrations depends on the dimensions, nature, and tension of the string, yet it is only after the occurrence of the modifications referred to that the ear receives the impression and affords the hearer any evidence of the *quality* of the tone produced by the instrument as a whole. Hence, although both by theory and experiment the vibrations of a string have been investigated and their character known, whether excited by plucking, striking, or bowing, there still remains much to be done before we have full information as to the qualities of sound actually received from any musical instrument. Or, in brief, though we have theoretically a sufficient knowledge of

* Communicated by the Authors.

the quality of tones emitted by vibrating *strings*, we have not the like knowledge of the quality emitted by stringed *instruments*.

The preliminary work was done with the monochord and dealt with the belly *, the air † in the sound-box, and the bridge ‡ respectively.

The present paper is the first instalment of work on the violin and is in the main confined to the *g* string.

Speaking generally, perhaps the chief distinction between the curves here obtained for the violin belly and those for the monochord lies in the fact that to the *eye* the violin-curves appear simpler and smoother. Often that means, however, that they have a fuller retinue of upper partials, but without undue prominence of any one of them, as was sometimes noticeable with the monochord. But perhaps it is still too soon to draw such conclusions safely. Incidentally the pitch of best resonance was found by use of the French-horn, and a wandering of the vibrations of the belly over its surface was detected.

Optical Arrangements.—The general arrangement of the apparatus for the experiments may be understood from fig. 1 (Pl. IV.). In this T_1 and T_2 indicate the two tables, the first carrying the lantern *L* and the lenses, the second holding the violin *V*. These tables were each mounted on pieces of rubber and lead to cut off the vibrations of the one from communication *viâ* the floor to the other.

Since together with the vibrations of the belly, those of the string were to be recorded also, the light coming from the electric arc-lantern needed dividing into two parts. One part passed through the vertical slit *S*, then through a lens L_1 (with diaphragm), which focussed a real image of the slit on the string. The light then passed to the plane mirror *M*, held just behind the string. This reflected a diverging beam of light to the lens L_2 , which was thus focussed upon the photographic plate *P*, the light passing through the opening of the door *D* to the dark room.

Since the first real image of the slit was focussed upon the string the second image on the plate was crossed by the shadow of the string. Thus, as the string rises and falls in its vibration, the shadow of it falls and rises on the plate. This of course would only produce a blur if the plate were at rest. But the plate is shot in horizontal rails by a catapult of indiarubber cords, so arranged that the velocity

* Barton & Garrett, Phil. Mag. July 1905, pp. 149–157.

† Barton & Penzer, Phil. Mag. Dec. 1906, pp. 576–578.

‡ Barton & Penzer, Phil. Mag. April 1907, pp. 446–452.

is uniform as the plate passes the doorway and receives the light. Quarter-plates were used and generally those called "Warwick Rainbow Isochromatic Fast, Backed," though sometimes "Imperial Process."

Consider now the second portion of the light from the lantern. This was deflected by the plane mirrors M_1 and M_2 , and so directed to the small hole H in a copper plate. It passed thence to the concave mirror by Hilger, 1.2 cms. in diameter (fixed on a lever), and marked m in the figure. This mirror focussed the real image of the hole on the plate just below the image of the slit crossed by the shadow of the string. Hence, as the string and belly vibrate, and the plate shoots past, we obtain a photographic negative, the positive print of which gives the *string's* vibration in *black* upon a white ground in the upper half, and the *belly's* vibration in *white* on a dark ground on the lower half. The precise arrangement of the concave mirror will be dealt with in connexion with fig. 4. The distances M to L_2 and L_2 to P in fig. 1 were respectively of the order 52 and 132 cms. Hence, on the negatives, the string's motion is magnified $132/52 = 2.54$ times nearly.

Mounting the Violin.—The method of holding the violin for the experiments (and in a way intended to imitate the actual grip of a player's shoulder, chin, and left hand) is shown in fig. 2 (Pl. IV.).

Two blocks screwed on the table supported a stout brass rod, carrying a board which was thus capable of turning about a horizontal axis. The board was turned to a suitable inclination and then clamped in position on a block as shown. This board carried two other blocks, that to the right having a groove, lined with wash-leather, to admit the neck of the violin, which was then held down by the indiarubber cord passing over the peg-box, as seen in the photograph. These arrangements imitate the player's left hand. Near the tail-piece of the violin, *i. e.* at the left end of the figure, wash-leather lies under the back of the violin, which was held down by a swinging block over which an indiarubber cord passed. These pieces imitate the player's shoulder and chin; the ordinary chin-rest was left in position. In order to be able to photograph the motions of the single string, although all four were present, the finger-board was removed from the violin, the mirror M (fig. 1) placed just behind the string under examination, and the other strings kept out of the path of the beam. Thus, when the *g* string was being dealt with (which is the one farthest removed from the lantern and plate) the *d'* and *a'* strings

(occupying the middle positions) were raised on a specially prominent part of the bridge, and the e'' string (nearest to the lantern and plate) was held down by a cord passing round the neck of the violin. Thus, although three of the strings were kept out of the way the conditions were but slightly altered from those of actually playing the violin. All the strings were there and in tune, two were on a rather higher part of the bridge, which would slightly increase the pressure on the belly, and one was tied down, which is only the same as its being pressed down by the performer's finger when "stopping" it for a particular note.

Optical Lever for Belly Vibrations.—The violin used was an old English instrument without any name or date, but full size and of ordinary model, as seen by fig. 3 (Pl. IV.) .

It is easily found by scattering sand or only touching lightly with the finger-tips, that the belly of the violin is very dead near the edges where supported by the ribs which attach it to the back. But spots just inside the f holes are specially lively and were accordingly thought preferable as places whose motion should be recorded. A spot on the right hand of the belly was chosen and a little wedge of soft wood fastened upon it, so that the upper surface of this wedge was level when the violin was mounted. On this upper surface a piece of microscope cover-glass was then fastened by shellac so as to form a support for one leg of the three-legged optical lever used to record the vibrations. The other two legs rested upon a conical hole and V-groove on a small table or disk of brass carried by an adjustable bracket. Thus the optical lever was carried on the geometrical arrangement of the "hole, slot, and plane," and turned about the hinge formed by the hole and slot as the plane rose and fell. The lever was of aluminium provided with steel legs and with the mirror had a mass of 1.124 gram. It was held firmly down by an indiarubber ring obtained by cutting a slice off a tube. The lever, mirror, and rubber ring complete weighed 1.254 gram. The adjustable bracket which carried the optical lever can be seen better in the detached view of fig. 4.

In both views the hole and slot are shown in white.

Magnification of the Belly's Motion.—The distance of the moving foot of the optical lever from the axis through the hole and slot was 6 mm., and that from the mirror to the photographic plate was 176 cms. (see $m. P$ in fig. 1). Hence the magnification of the belly's motion on the original negatives was $2 \times 176 \div 0.6 = 587$ nearly. Thus, on the original plates, or any prints reduced from them, the

magnification of the belly's motion is $(587 \div 2.54)$ or say 231 times that of the string's motion.

The General View of Experimental Arrangements is shown in fig. 5, which gives at a glance the scale and disposition of the various parts, optical and acoustical.

Results.—The chief results obtained are shown in the curves 1–40 on Plates V. & VI. These are partially explained by the marginal letterpress; other necessary details may be noted here.

PLATE V.—The whole of the curves 1–20 on this first plate deal with the *g* string at concert-pitch (about 203.4 per second). This is the fourth or so-called silver string, and always elicited a sufficient motion of the point of the belly under examination.

Curves 1–3 show the vibrations of the string and belly when the string was bowed at one-sixth, one-seventh, and one-eleventh respectively of its length from the bridge. The string had a vibrating length of 32.5 cms. from bridge to nut and was usually illuminated at a point 14 cms. from the bridge, *i. e.* at 0.43 of its length. The speed at which the plates were shot can be inferred from the number of periods or complete vibrations occurring in the length of the plate ($4\frac{1}{4}$ inches). Thus, three vibrations occupying $3 \div 203.4$ of a second were photographed on the length of $4\frac{1}{4}$ inches, which gives about 24 feet per second for the speed of the plate, which was the normal usage in all the curves except Nos. 4 and 5.

Curves 4 and 5.—It was noticed that in plucking the *g* string at $\frac{2}{3}$ of its length from the bridge (or at other places like $\frac{1}{4}$ or $\frac{1}{5}$ &c.) the vibrations of the point of the belly under examination speedily rose to a maximum, died away again, attained a second maximum, and so on, for about five successive maxima, occurring at intervals of about one second or rather less.

By placing one ear *close* to either of the *f* holes the same effect of successive maxima or “beats” could be heard. When the observer's ears were about a yard away from the violin this effect of beats was practically gone, thus showing that the maxima previously observed were merely maxima of the vibrations occurring at *that spot*, and that when the vibrations there diminished, those of some other part of the belly increased; so that at a moderate distance the total effect from the various parts of the belly was devoid of these successive waxings and wanings. To record these fluctuations of amplitude on a single plate necessitated its exposure for several seconds, and thus the motion would be far too

slow to show the thousand vibrations, or thereabouts, then occurring. It was accordingly moved quite slowly by hand, all trace of the individual vibrations being therefore lost, but the successive swellings and subsidences of the vibrations being well shown. Curve 4 shows about half these effects in its entire length. Curve 5, in which the plate was moved still slower, comprises the whole series of waxings and wanings in its length. The sudden, almost instantaneous, reponse of the belly at the start is specially noticeable.

Curves 6–10 show the effect of plucking the string with the fleshy finger-tip as is usual when the violin is played *pizzicato*; the points of attack being respectively at $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{7}$ of the string's length from the bridge, the plates being shot quickly with the catapult as at first.

Curves 11–13 show the vibrations obtained when the string was struck with a hammer consisting of an ordinary lead-pencil wrapped at the end with several thicknesses of wash-leather, the places of striking being respectively $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{7}$ the string's length from the bridge.

Curves 14–17 give the effects of plucking the string with the point of a lead-pencil, which may be termed a plectrum, the points of excitation being respectively at $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{7}$ of the string's length from the bridge.

Curves 18–20, concluding the first set, show the vibrations elicited by striking the string with an unpadded lead-pencil at the points $\frac{2}{3}$, $\frac{1}{4}$, and $\frac{1}{7}$ from the bridge.

It is very noteworthy that all these cases of plucking and striking give vibrations of the belly and of the string itself of a very smooth flowing character, being quite different from those with the metal string and the monochord belly*, &c.

PLATE VI.—The curves 21–40 on this plate show vibrations at various pitches from the *g* string and the other strings. As before, the chief points are indicated by marginal notes; but some further details are given here.

Curves 21–22 show the vibrations elicited by the *g* string sounding its octave harmonic or second partial. It was touched at the middle with a finger of the left hand, and bowed with the right at the points $\frac{1}{12}$ and $\frac{1}{16}$ of the *whole* length of the string from the bridge. The large amplitudes of the belly's vibrations are noticeable in contrast with the very small motions of the string.

Curves 23–24 show the vibrations from the *g* string “stopped” or held down by a clamp at three-fourths of its

* See Plate I. fig. 10, Phil. Mag. July 1905; also others in Phil. Mag. Dec. 1906 and April 1907.

length from the bridge, so as to sound middle c' (say 271.1 per second).

Curves 25-28.—As the strings of a violin are occasionally tuned to other pitches, either to agree with accompanying instruments or for special effects, the g string was tuned a semitone down and up, also a tone and a tone and a half up. The effects obtained are shown in curves 25-28 respectively, the bowing throughout being at one-seventh of the string's length from the bridge. It can be seen from the prints that, when the string was sharpened more and more, the belly responded so powerfully that, to avoid an undue amplitude, the string needed bowing more and more gently. In fact, for the $b\flat$ the string's motion is barely visible in the original plate, though the belly's motion is so large. (See also notes on curves 38-40.)

Curves 29-30 show the vibrations from the g string when flattened and sharpened a semitone and plucked with the finger at one-seventh.

Curves 31-33 show the vibrations from the combined effect of the g string at concert-pitch and the d' string (normally at 305 per second) tuned to b , c' and at concert-pitch respectively. The excitation is by bowing both strings at one-seventh. The curious character of these curves, amounting in one place to an undercutting (curve 31) is due to the fact that the vibrations of the d' string caused the mirror to rotate slightly about a vertical axis, thus moving the spot of light horizontally on the photographic plate. Hence these curves have not the true character of displacement-time curves. Accordingly, no curve is given, with the present arrangement, from the vibrations of the d' string *alone*. But it is easily seen that a small motion, whether vertical or horizontal, *superposed* upon a displacement-time graph of much larger amplitude gives very similar results; these combined effects are therefore included now.

Curves 34-35 show the combined effects of d' string at concert pitch, (i.) with the g string, also at concert pitch, and both plucked at $\frac{1}{7}$, and (ii.) with the g string sounding its octave harmonic, and both bowed at $\frac{1}{16}$.

Curves 36-37.—Since the d' string vibrating alone failed to act satisfactorily with the present arrangement, it was at first thought that the same would apply still more to the a' and e'' strings, which are thinner and higher in pitch. It was indeed true of the e'' string. But the a' string, on the contrary, evoked, by bowing, such powerful vibrations of this particular spot of the belly that it was very difficult to bow so lightly as to keep these vibrations within reasonable

limits on the plate. The bowing at $\frac{1}{10}$ was out of the question, and we were driven to adopt the positions $\frac{1}{5}$ and $\frac{1}{7}$ from the bridge, which produce softer passages.

Curves 38-40.—It was noticed in connexion with curves 26-28 that, as the sound of the string approached $b\flat$, the response of the belly was very powerful. This led to the idea that $b\flat$, or some note near it, was the pitch of best resonance of the instrument. A French-horn was accordingly tried with its bell just over the violin belly and the notes $a\sharp$, $b\flat$, and $b\sharp$ sounded in succession, a plate being shot each time. The string, of course, did not respond, being tuned to g ; it therefore shows as a straight black line on the prints. The belly's motions are, however, shown by curves of moderate amplitude in prints 38 and 40 for the extreme pitches and by one of greater amplitude in print 39 for the intermediate pitch of $b\flat$ (say 244 per second). This is accordingly the pitch of the instrument as a resonator, when we judge it by the vibrations of the spot of the belly under examination.

University College, Nottingham.
May 11th, 1909.

XXIX. *The Absorption, Fluorescence, Magnetic Rotation and Anomalous Dispersion of Mercury Vapour.* By R. W. WOOD, *Professor of Experimental Physics, Johns Hopkins University* *.

[Plates VII. & VIII.]

SHORTLY after Hartly's observation that the vapour of mercury exhibited fluorescence under the stimulus of the light from the electric spark, I commenced a careful study of the optical properties of the vapour, in the hope that phenomena analogous to those which have been observed in the case of sodium vapour might be found. Though the vapour has proved disappointing in some respects, a number of very interesting observations have been made, some of which appear to be quite new.

I have already described the remarkable absorption-band at wave-length 2536, which is probably the most unsymmetrical band ever observed, and the modifications produced in its appearance caused by the admixture of any chemically inert gas ('*Astrophysical Journal*,' 1907).

In the present paper I shall briefly review the results set forth in the above note, as they have some bearing upon the

* Communicated by the Author.

other observations. The study of the vapour has been attended with great difficulty, since all of the interesting things are in the ultra-violet, and can be picked up only by the aid of photography.

The work has been suspended from time to time, and then taken up again as new methods of attack have suggested themselves.

I have at last succeeded in devising a satisfactory piece of apparatus for the study of the anomalous dispersion of the vapour. This is much to be desired, for the vapour, unlike that of sodium, does not attack the quartz vessel in which it is contained. It is thus possible to work with prisms of known angle, and vapour of known and uniform density. If we work with prisms we shall need a cement capable of resisting the vapour and of standing a temperature of say 300° . I have some hopes of a mixture of the fused haloids of silver and am now making experiments in this direction. A quantitative determination of the dispersion would be of very great interest, on account of the asymmetry of the absorption-band.

The absorption spectrum of the vapour is shown on Pl. VII. fig. 1. A small globule of mercury was placed in a quartz bulb 3 cms. in diameter, which was thoroughly exhausted and sealed. The spectrograms were taken in succession, the temperature of the bulb being gradually raised. There are three distinct absorption-bands, all in the ultra-violet region. The one at wave-length 2536 appears first as a pair of fine lines, not unlike the D lines in appearance, one of them ($\lambda=2539\cdot4$) relatively much weaker than the other ($\lambda=2536\cdot7$). As the density of the vapour increases they fuse together, forming a single band which then widens in a remarkable manner towards the region of longer wave-length, its boundary on the other remaining almost fixed in position.

In addition to the absorption-band at wave-length 2536, there is a group of narrow bands just above the three bright cadmium lines further down in the ultra-violet. These bands do not appear until the vapour has acquired a considerable density: they form a series, the head of which is turned towards the visible spectrum. The individual bands which make up the group decrease in intensity with decreasing wave-length and lie closer together, resembling a Balmer series on a very small scale. The wave-lengths of the four bands which make up the group are as follows:—2346·1, 2339·4, 2334·4, 2331·2. They can be seen in figs. 1 and 7 of Pl. VII. They are so nearly fused together that it is

doubtful if they will appear separated in the reproduction. There is also a general absorption further down in the ultra-violet, which wipes out the last cadmium line first, and then the others in succession. The apparent absorption-band at the extreme right of fig. 1, Pl. VII., is merely the region of minimum sensibility in the blue-green region of the orthochromatic plate.

If the bulb contains air or any other chemically inert gas at atmospheric pressure, the band (2536) widens symmetrically at first, attaining a width of about 8 Ångström units. Beyond this point a further increase in the density of the mercury vapour causes a widening in one direction only, as is the case when the vapour is *in vacuo*.

If, instead of sealing the mercury up in a bulb filled with air, we place it in a quartz flask provided with a long neck and gradually raise the flame below the flask, we get a remarkable series of spectrograms (Pl. VII. fig. 4). The band widens and then appears to drift towards the longer wave-lengths, without further increase in width. This apparent drift is due to the expulsion of the air by the boiling mercury, the band contracting on one side more rapidly than it widens on the other.

This action of the air in modifying the appearance, and in some cases the apparent position of the absorption-band, cannot be attributed to chemical action, for the same effect was found with hydrogen, nitrogen, and helium. As we shall see presently, the vapour is deprived of its power of fluorescing when it is mixed with another gas.

It was found, by making a more careful study, that the effect of air upon the band at 2536 was a little more complicated than was at first supposed. *In vacuo* the broadening is almost entirely in the direction of longer wave-lengths. If air is present a hazy band appears on the short wave-length side of the line, and if the time of exposure and vapour density are just right the band and line are separated by a narrow strip slightly lighter than the band. The form of the absorption curve in the two cases is indicated in fig. 1.

A photograph is reproduced on Pl. VII. fig. 3, short wave-lengths being to the left. It is difficult to obtain an enlargement which shows the lighter region between the line and the band, though it is not difficult to make it out in the negative. The appearance of the band suggests that mercury + air gives us some sort of a molecular aggregate

Fig. 1.



which broadens the 2536 line and shifts it towards the shorter wave-length region. The shifted and broadened band and the narrower 2536 band appeared to be disconnected, and it occurred to me that it might be possible to obtain the broadened band alone. It seemed as if we might be dealing with a molecular aggregate mixed with free mercury molecules, in which case it might be possible to get the effect of the aggregate alone by employing a very long column of air containing but very little mercury vapour. I accordingly fitted up a steel tube 3 metres long, containing 20 small porcelain boats each containing a few drops of mercury, and closed at the ends with quartz windows. The appearance of the two bands was the same as before however. The tube was surrounded by a second tube heated by a line of very small flames obtained by drilling pin-holes at intervals along a long piece of gas-pipe. The very interesting observation was made that the 2536 line was quite distinct in the photograph of the absorption spectrum, *even when the tube was at room temperature*. This gives us a means of studying mercury vapour at low temperatures, and determining whether it can be prevented from entering an exhausted receiver by any of the absorbing devices (*e.g.* gold-leaf) frequently suggested. It will also be possible to determine its vapour density at lower temperatures than has hitherto been possible. It was found that if the tube at room temperature was exhausted, the line became much fainter, almost disappearing. On readmitting the air it immediately resumed its full intensity. No trace of the broadened and displaced band appeared at room temperature, owing to the insufficient density of the vapour. The effect of admitting the air is precisely analogous to the observations made by Ångström on the CO band in the infra-red. It is a pressure effect, the effect of admitting the air being equivalent to compressing the actual amount of mercury vapour present until it is at atmospheric pressure. Ångström's paper should be referred to in this connexion. I have observed the same thing with sodium vapour for many years, but never felt quite sure as to the interpretation of the phenomenon. The D lines widen and fuse together the moment nitrogen or hydrogen is admitted to one of the steel tubes. In the case of sodium vapour, however, I could not feel sure that the actual number of sodium molecules per cubic cm. had not been increased by the admission of the gas, for the gas would hinder diffusion to the cooler parts of the tube, and it might easily be possible for us to have the metallic vapour at the centre of the tube at a pressure as

great as that of the atmosphere in the tube. I have long since abandoned the viscosity theory which I first proposed to explain the apparent peculiar behaviour of the vapour, and now believe that even in the highly exhausted tubes we can never have the vapour at a greater pressure than that of the residual gas. The error resulted from an over-estimate of the density of the metallic vapour, based upon its optical properties.

This pressure effect is not to be confused with the phenomenon studied by Humphreys and Mohler, for it occurs at pressures below one atmosphere, and there is no shift, the band merely becoming wider and blacker, precisely as if more mercury vapour were present.

Fluorescence.

The fluorescence of the vapour is best shown by enclosing a drop of the metal in an exhausted quartz bulb, heating the bulb over a Bunsen burner turned down very low, and passing a powerful spark between zinc or cadmium electrodes placed as close as possible to the bulb. The colour of the fluorescent light is a bluish green mixed with a good deal of white, *i. e.* it embraces a region of the spectrum extending from the red-yellow well down into the ultra-violet. The colour of the fluorescent light varies somewhat with the nature of the electrodes used. The spectrum appears continuous even with a concave grating of 2 metres radius.

Photographed with a small quartz spectrograph, the spectrum is found to be continuous, extending roughly from the yellow down to wave-length 3000, with a very pronounced minimum at wave-length 3600. It was at first suspected that the visible band and the ultra-violet band might be excited by different radiations, and a week or more was spent in photographing the spectrum emitted by the vapour when stimulated by lines isolated from the spark-spectrum by an auxiliary quartz spectrograph. Very little, if any, difference could be detected however. It was found that if air was in the bulb it was impossible to excite any fluorescence. If, however, we boil the mercury in a flask it fluoresces brightly as soon as it is in brisk ebullition, and if the absorption spectrum is photographed at the moment at which fluorescence appears, we find that the absorption-band has contracted on its short wave-length side to its position when the vapour is *in vacuo*. The fluorescence spectrum is shown on Pl. VII. figs. 2 & 5, excited in this case by the light from the cadmium spark. The cadmium lines appear as well owing to diffused light. As will be seen, there is in addition

to the continuous spectrum, a bright line (indicated by an arrow on fig. 2) which is not present in the spark spectrum, and which coincides in position with the sharp absorption-line 2536·7, shown by vapour of small density. It was at first thought that this line was excited by the bright cadmium line which falls within the region of the expanded part of the absorption-band. The zinc spark had a bright line which lies even nearer the mercury line, and it was expected that the emission line would be stronger in the case of zinc excitation. To my surprise it was impossible to obtain any trace of the line using the zinc spark as a stimulus. The fluorescence spectrum was then photographed when excited by the cadmium lines in succession *but in no case was any trace of the line found in the spectrum.* This seemed quite baffling, and much time was spent in repeating the work using longer exposures. The results were all negative however. It was then found that when the image of the spark was thrown upon the bulb with a quartz lens and the bright spot of fluorescent light photographed with the spectrograph the line was absent in the spectrum. The aluminium spark was found to be even more efficient than the cadmium in bringing out the line. (See fig. 6. The aluminium spark spectrum is recorded below for comparison. Fluorescent line 2536 marked with arrow.)

This at once made me suspicious that the bright line might be excited by the very short waves, which penetrate quartz with difficulty, and I arranged my auxiliary quartz spectrograph to deliver the light of the last aluminium line which can be observed with ordinary apparatus. This light excited a feeble fluorescence, and by giving a very long exposure I was able to secure its spectrum. The line appeared in his photograph, faint but unmistakable. I then focussed the spark upon the bulb with the quartz lens, placing the lens much nearer the bulb than before, after having made a rough calculation of its focal length for these very short waves. Again the line appeared. As a final test I placed the spark close to the bulb and took a series of spectra, interposing in succession quartz plates of increasing thickness. It was found that a plate 8 mm. in thickness reduced the intensity of the fluorescent line fully one half, while a plate 18 mm. caused its disappearance from the spectrum entirely.

The mystery was thus completely solved. The cadmium spectrum has lines of much shorter wave-length than any shown by zinc. The cadmium lines which excite the 2536

line in the fluorescent spectrum are more refrangible than any which appear in the photographs.

The difficulties which accompanied the solution of this problem were still further complicated by the fact that even with the aluminium spark placed close to the bulb, the line was often found to be absent in the spectrum. The cause of this was eventually found. The line only appears when the vapour is quite rare, *i. e.* when the temperature of the bulb is comparatively low. The best density is that at which visible fluorescence first appears and is quite faint. The line then comes out strong. If the temperature is raised a trifle and the density of the vapour increased, the visible fluorescence becomes very bright, but no trace of the line 2536 appears in the spectrum. This circumstance gave a great deal of trouble in the labour of securing the line with the monochromatic aluminium radiation (shortest wavelength) as the vapour density was at first arranged so as to give the brightest fluorescence. In fig. 2 are reproduced four spectra taken under identical conditions as to time of exposure, position of the cadmium spark, &c. The upper was taken with highly attenuated vapour, and while showing the bright line distinctly (see arrow) exhibits only a very faint trace of the continuous spectrum fluorescence. In the third spectrum the line is barely visible, while the fluorescence is much brighter. In this case the vapour was denser. In the fourth the bright line has disappeared, and the continuous spectrum is at its maximum brilliancy. The fluorescent line is double like the absorption-line, 2536.7 strong, 2539.3 faint. Both appear in fig. 2.

Effect of Temperature upon the Fluorescence.

If a very small drop of mercury is sealed up in an exhausted bulb it can be completely vaporized. The temperature can now be raised to a bright red heat without danger of bursting the bulb. It was found that as the temperature was raised the fluorescence diminished gradually in intensity and finally disappeared entirely. On removing the flame the fluorescence presently reappeared again. No effect of temperature on the appearance of the absorption-bands could be detected.

If a quartz flask with a long neck is filled to a depth of several millimetres with mercury, and the flame of a Bunsen burner is made to play around the bulb heating it red hot, fluorescence can be detected only close to the surface of the

boiling metal. The appearance is quite striking, a layer of glowing vapour clinging close to the metal, reminding one of the discharge at the cathode in a vacuum-tube. The temperature of the vapour when it leaves the metal is about 360, consequently it fluoresces brightly: as it rises its temperature is speedily raised to the point at which fluorescence disappears.

A still better way of showing the effect of temperature upon the fluorescence is illustrated in fig. 1, Pl. VIII. The exhausted bulb previously described is used. It is heated from below by a small flame from a Bunsen burner until the fluorescence excited by a cadmium spark placed as close as possible is at its brightest. The small pointed flame from a blowpipe is then directed against the side for a few seconds. The fluorescence promptly disappears at the super-heated spot, reappearing again as soon as the local heating is stopped.

If the ultra-violet light of the spark is focussed at the centre of the bulb, a narrow cone of brilliant green light extends from the point at which the light enters the bulb nearly to the opposite wall, as shown on Pl. VIII. fig. 2. The flame of a blast-lamp directed against the wall of the bulb at the point where the fluorescence begins causes the luminous cone to retreat from the wall to a distance of several millimetres, much as the positive column separates from the cathode in a vacuum-tube discharge. A photograph of the effect is shown on Pl. VIII. (right-hand figure).

The failure of the 2536 line to appear in the fluorescence spectrum except when the vapour is at comparatively small density is very remarkable. It occurred to me that it might be due to disappearance of diffuse radiation by the combination of the secondary waves from the molecular resonators into a regularly reflected wave. I have shown in a previous paper ("Selective Reflexion of Monochromatic Light by Mercury Vapour" in this Journal, *suprà*, p. 187) that this takes place when the vapour is illuminated by the light of the 2536 line from the mercury arc, and its density increased to ten or more atmospheres. It is possible that something of the same nature might explain the failure of the line to appear when the vapour was excited by the very shortest ultra-violet waves. It must be remembered that in this case there is a change of wave-length involved, and it is hard to see how the necessary phase relations for selective reflexion could hold. Experiment showed that the hypothesis was

untenable. The spectroscope was directed so as to view the image of the aluminium spark reflected at the inner surface of the bulb, everything else being screened off by closing the iris diaphragm of the instrument. The vapour was given the density which showed the brightest visible fluorescence. If the 2536 radiations were present, going off as a regular (*i. e.* not diffuse) wave, the line ought to appear under these conditions, but no trace of it appeared on the plate.

I see no way of explaining its failure to appear by absorption, for the whole surface of the bulb on one side is illuminated by the light of the spark, and the spectrograph receives the light from the outermost surface of the vapour mass.

I have not yet determined whether an elevation of temperature causes the disappearance of the line, as it does of the continuous spectrum, for the bulbs which I now have are not well adapted to the complete investigation of the temperature effect. I am having two connecting bulbs made, one of which can be raised to a very high temperature, the other being held at a lower temperature, the density of the vapour being the same in both. This bulb should furnish more precise information.

The failure of the 2536 line to appear in the fluorescent spectrum when the vapour has a considerable density, and the complete destruction of the visible fluorescence by the elevation of temperature, I regard as the two most important points brought out by the work.

The 2536 line is connected in some way with an absorption-band in the vicinity of the wave-length 1860, for it appears only when the vapour is stimulated by the last aluminium line, and the cadmium lines which are completely stopped by 2 cms. of quartz. It is not brought out by stimulation with the other brilliant cadmium and zinc lines, which excite a very powerful visible fluorescence. Stimulation by the 2536 line from the mercury arc probably brings out the 2536 fluorescent line, though I have not proved this as yet, for it is difficult to be sure that we have removed all possibility of diffused light. The fact that we get selective reflexion of this light when the vapour is dense, indicates that in all probability we have a diffuse radiation at lesser densities. If the ultra-violet spectrum of the cadmium spark is focussed upon the bulb, it is found that the brightest fluorescence is excited by the lines which are most strongly absorbed, which is what we should expect.

Anomalous Dispersion and Magnetic Rotation.

Strong anomalous dispersion has been found at the 2536 line, as shown in fig. 8, Pl. VII. These photographs were made by the method of crossed prisms in the manner which I have employed in the study of sodium vapour. The mercury was placed in a long steel tube, closed by quartz windows and heated along the under side by a row of small flames. As will be seen from the photographs, the curvature of the spectrum is much more marked on the short-wave-length side of the band, which is the steep side. A quantitative study of the dispersion of the vapour will be made next year, by an interferometer method, which will doubtless be of considerable value in connexion with the theory of dispersion, for it will be possible to determine the exact density of the vapour, the length of the column, and its temperature, which was never possible in the case of sodium vapour. The determination of the dispersion curve will also be extremely interesting on account of the asymmetrical nature of the absorption-band.

The magnetic rotation of the plane of polarization was also investigated in the vicinity of the 2536 line. The effect of the absorption-band upon the rotation was very marked on the short wave-length side, but there was little or no increase on the other side, doubtless due to the broadening of the band in this direction. The light from the cadmium spark was passed through a polarizing prism (Foucault), then through the quartz bulb between the poles of the electromagnet, a Fresnel biquartz parallelepiped, and a second polarizing prism, the image of the fringes formed by the Fresnel parallelepiped being focussed on the slit of the quartz spectrograph. Nicol prisms cannot be used for the ultra-violet on account of the absorption by the film of Canada balsam. The direction of the rotation resulting from the 2536 line was the same as for the D lines of sodium, from which we may infer that the line is due to negative electrons.

A further study of the dispersion and magneto-optics of the vapour will be commenced as soon as the necessary apparatus of fused quartz has been constructed. A wide tube with plane-parallel end-plates also of fused quartz welded directly to the tube is being made by Heraeus.

XXX. *On the Flow of Energy in a System of Interference Fringes.* By R. W. WOOD, *Professor of Experimental Physics, Johns Hopkins University* *.

[Plate VIII. fig. 3.]

THE interference minima formed by two similar sources of light form a system of confocal hyperboloids, and the question of the flow of energy in this case, or any similar case, does not appear to have been discussed. Energy is obviously flowing out from both sources at its normal rate, but the direction of flow is perhaps not quite obvious. Suppose the minima equal to zero, which is nearly correct at the centre of the system. Energy evidently cannot cross a plane along which there is no disturbance.

In stationary waves, if the nodes are absolutely at rest, which is the case if the two wave-trains are of equal amplitude, we cannot speak of a flow of energy across them. A node may be considered as having the properties of a perfect reflector, that is to say the point acquires the power of reflecting as a result of the arrival of a wave travelling in the opposite direction. We are thus forced to the conclusion that the flow of energy in the case of the interference-fringes must be along the hyperboloids, that is along curved paths. We can show this experimentally by means of ripples in mercury excited by two needles mounted on the prong of a tuning-fork. If we view the mercury surface through a narrow slit opened and closed by the vibrations of another fork slightly out of tune with the first, we see the waves (stroboscopically) creeping slowly along the surface, and following the lines of the hyperboloids. Two questions now naturally occur to us. How does the energy get into the bright fringes, if the dark fringes are supposed to act as barriers? and what is the nature of the wave that is travelling along a bright fringe? In regard to the first question: the dark fringes are never absolutely black, as no one of them is equidistant from both sources. The amplitudes are therefore slightly different, and there will be a flow of energy in the direction of the disturbance having the larger amplitude. Though it may be very slight at any given point, it is ample to account for the flow along the hyperboloid. We can take as an analogous case two parallel sheets of cloth tightly stretched, and very close together. Consider water forcing its way into the space between the two sheets from both sides. A very small flow across unit cross-section will give us a

* Communicated by the Author.

large flow across unit section taken perpendicular to the sheets.

We may, however, have a fringe which is absolutely black, for there is nothing to prevent us from considering the sources as vibrating with a difference of phase of 180° . This makes the centre of the system dark, and equal to zero, and it must act as a barrier to the flow of energy from both sources. In other words, the central fringe can be considered as acting as a perfect mirror, and we can regard the fringes as formed by the interference of these reflected waves with the direct. If the flow of energy is along the hyperboloids, it is evident that in the region between the sources the flow is in a direction nearly perpendicular to the rays. We can watch this flow with the mercury ripples and tuning-forks. (I find that a ring of castor-oil or glycerine poured around the edge of the mercury surface prevents disturbing reflexion from the walls, and to a large extent waves due to jars from the table.) The bars of light perpendicular to the line joining the vibrating sources slide out sideways, each one of them forming one of the waves which travel along the hyperboloid. We can perhaps get a better idea of what happens if we consider what is going on in a bright fringe outside of the region between the sources. What is the type of wave, and is it capable of showing us both of the sources if it alone is allowed to enter the eye? If we regard the dark fringes as absolutely dark, that is as perfect reflectors, we must regard the waves as travelling between them as between two silver walls. The incidence is very oblique, *i.e.* the wave is nearly perpendicular to the reflecting plane; and if we consider the wave as a portion of a sphere with its centre at one of the sources, the wave after reflexion from the interference plane will be a portion of a sphere with its centre of curvature at the other source. This process will repeat itself over and over again, a given portion of the wave-front appearing to come first from one source and then from the other. The bright fringe will then contain two groups of wave-fronts inclined to each other at a small angle. These can be seen with the tuning-fork waves, and on looking over some of Mr. Vincent's photographs, published in this Journal some time ago, I found one which showed the phenomenon very clearly. An enlargement of a portion of this photograph is reproduced on Plate VIII. fig. 3, the inclined wave-fronts showing especially well above the point marked X.

Though each bright fringe contains two wave-fronts, we cannot "resolve" the sources with them, for calculation shows that their width will be insufficient. In other words,

if we screen off the other bright fringes, passing the waves in the one through a slit, the slit width necessary turns out to be just what is needed to prevent resolution.

In the region between the sources we must regard the same thing as going on, the only difference being that here the incidence is more nearly normal. The waves are stationary on the line joining the sources, but as soon as we get off this line we must regard the stationary waves as oozing out in all directions, the velocity of the oozing increasing with the distance from the line.

I realize that the language which I have used here is not very exact, but it is not easy to visualize exactly what is going on, and still harder to put it into words. If this note serves to direct attention to a new view-point of a well-known phenomenon, it will have accomplished all that I had intended it to do.

XXXI. *The Conductivity of Dielectrics under the Action of Radium Rays.* By B. HODGSON, B.Sc., Prize Demonstrator in Physics, Armstrong College, Newcastle-on-Tyne*.

A STUDY of the conductivity of dielectrics under the action of radium rays is extremely important in view of the fact that the work already done on this problem is throwing light on the nature of dielectric conductivity itself, and perhaps also on the nature of polarization in dielectrics.

The early investigators† were inclined to look upon dielectric conductivity as electrolytic in action, but recent work seems to point to a conduction similar to that occurring in a dense gas.

The conductivity acquired under the action of radium rays was demonstrated by P. Curie, Bécquerel, and Becker, and recently the phenomena have been studied in great thoroughness by Jaffé‡.

Curie §, Bécquerel ||, and Becker ¶ showed that dielectrics under the action of radium rays increase in conductivity, and noticed some departure from Ohm's law.

Jaffé divided the current into two parts :

- (1) obeying Ohm's law,
- (2) an ionization current.

* Communicated by Prof. H. Stroud.

† Von Schweidler, *Ann. d. Phys.* xxiv. p. 711 (1907).

‡ Jaffé, *Ann. d. Phys.* xxv. p. 257 (1908).

§ Curie, *Comp. Rend.* cxxxiv. p. 420 (1902).

|| Bécquerel, *Comp. Rend.* cxxxvi. p. 1173 (1904).

¶ Becker, *Ann. d. Phys.* xiii. p. 394 (1904).

The Ohm's law current was exceedingly small for pure substances, and it was subtracted from the observed effect and the remaining ionization current studied. This latter shows evidence of consisting of two parts:

- (1) due to mobile ions similar to those existing in an ionized gas ;
 - (2) due to slowly moving ions which appear to be of the nature of electrolytic ions.
- (1) is saturated by a field of 1000 volts per cm.
 - (2) shows no signs of saturation in fields of 7000 volts per cm.

Jaffé took extraordinary care to obtain pure dielectrics ; but even with the most elaborate precautions it appears impossible to get an absolutely pure specimen. In the work described below substances were obtained as pure as possible, and only experimented with if the ordinary conduction current was of the order of 10^{-13} ampere with an applied field of about $100 \frac{\text{volt}}{\text{cm.}}$

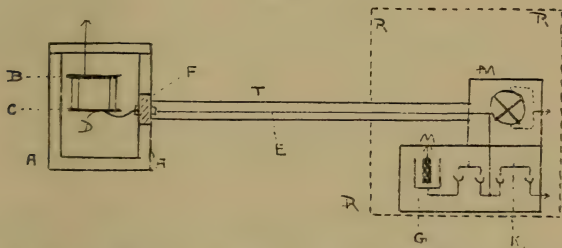
The method used was to measure first the current through the dielectric without the radium near and then with the radium, the difference giving the ionization current.

Apparatus.

One of the many difficulties was to eliminate all possibility of leak through air ionized by the radium, which would give either a spurious effect or cause such leak as to mask any effect which might exist. After many trials the following simple form of apparatus had the desired effect.

The substance under examination was placed in a thick lead box A (fig. 1), 11 cm. square and of sides 2 cm. thick.

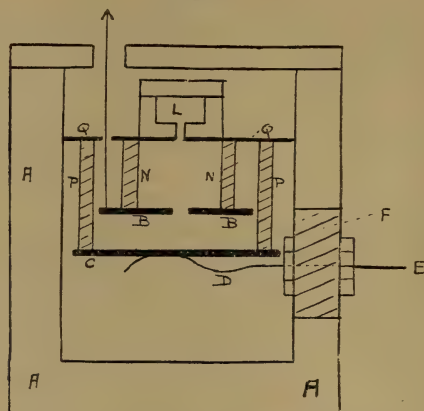
Fig. 1.



Two electrodes B and C were supported inside by means of ebonite stems, B being connected during an experiment to one terminal of a set of storage-cells, and C to one set of quadrants of a Dolezalek electrometer, by means of the spring D and the wire E, both held in a cylinder of ebonite F.

To prevent leak across the ebonite supports the following arrangement was employed. The electrode B (fig. 2) was

Fig. 2.



supported by ebonite stems *NN*, and *C* by similar stems *PP*, from an earthed plate *Q*. This avoided effectively any spurious effect due to defective insulation.

The radium used contained about 5 mg. of pure radium bromide and was sealed in a glass vessel with a thin mica cover. When acting on the dielectric it was placed in the lead box *L* and the rays passed through 2 cm. of liquid before reaching that between the electrodes.

*K*₁ (fig. 1) was a key which enabled the electrode *C* and electrometer to be "earthed" or insulated at will.

G was a capacity added when necessary to lessen the rate of movement of needle. To prevent any leak due to ionized air the following arrangement was used. The wire *E* was passed through a glass tube *T* covered with tinfoil and earthed, and filled with heavy paraffin oil of high insulation. By this means air was removed from all connexions right up to the electrometer, and so chances of leak due to air-ionization were eliminated. This oil proved a splendid insulator, and when the system was charged up to 1 volt no leak could be detected in 6 min.

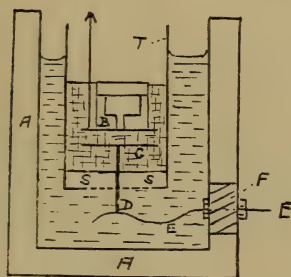
The electrometer, keys and condenser were enclosed in earthed tin boxes *M*, *M*. *R*, *R* was a large earthed wire cage. In performing an experiment the electrode *B* was connected to a set of storage-cells and the key *K* was lifted, insulating the electrode *C*. The needle then began to move, and scale readings were taken every $\frac{1}{4}$ minute. A knowledge of the capacity of the system and the sensitiveness

of the electrometer enabled the current to be found. When no E.M.F. was applied no current could be detected, so that the radium rays themselves did not charge up the system appreciably.

In all cases given the results were got by applying a + E.M.F., as Jaffé showed that the ionization current was the same on applying either a + or - E.M.F.

The following substances were experimented with:—heavy paraffin oil (density .89), solid paraffin, vaseline, glass, and ebonite. The oil and vaseline were poured into the lead box, and in the case of the latter it was allowed to solidify. For the solid paraffin an additional piece of apparatus was used (fig. 3). Molten paraffin was poured round the electrodes

Fig. 3.



C, B, held in a tin vessel T. C was supported by a stem D passing through a hole in the vessel, and insulated from it by a layer of sealing-wax S.

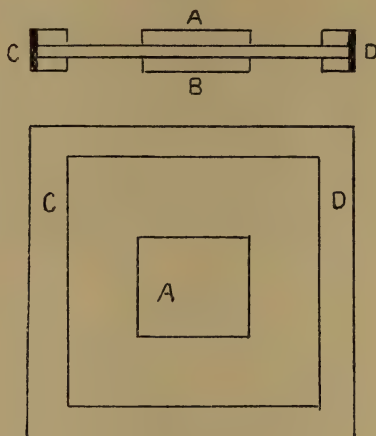
This vessel was placed in the lead box A, the stem D touching the spring E, and so making contact with the electrometer. Oil was poured into the lead box to exclude air.

In the case of glass and ebonite thin sheets were used. On opposite sides of these were pasted pieces of tinfoil, B making a sort of condenser. To prevent surface leak a guard-ring of tinfoil CD was pasted round the edges and earthed during an experiment (fig. 4, p. 256).

In performing an experiment it was necessary to allow the conduction current to pass for some time. The current at first was large and gradually reached a minimum value. At this stage the radium was put in position and allowed to ionize the dielectric and the current again measured. The

following results show the increase of conductivity immediately after insertion of radium. In the cases marked thus *

Fig. 4.



in Table I. no deflexion was got after some minutes. The sensitiveness of the electrometer throughout the experiments was about 200 scale-divisions per volt, and the capacity of the electrometer and connected electrode 0.000096 microfarad.

The capacity G (fig. 1) was 0.002454 microfarad.

TABLE I.

Substance.	Current without radium.	Current with radium.	Field strength.
Heavy paraffin oil...	*0.0 $\times 10^{-13}$ amp.	5.1 $\times 10^{-13}$ amp.	190 $\frac{\text{volt}}{\text{cm.}}$
" " "...	0.13 " "	1.9 " "	90 "
Solid paraffin	*0.0 " "	2.2 " "	260 "
Ebonite	*0.0 " "	1.5 " "	520 "
Vaseline.....	1.7 $\times 10^{-12}$ "	2.2 $\times 10^{-12}$ "	190 "

If the radium was removed and the current measured from time to time, the E.M.F. being continually applied, it was found that the current diminished, and finally reached its initial value (Table II.).

TABLE II.

<i>Solid Paraffin.</i>						
Time	10.40	11.0	11.2-11.9	11.13	11.24	11.40
Current (10^{-13} amp.)	0.0	0.0	2.2	.8	.5	.3
Radium	out	out	in	out	out	out
<i>Heavy Paraffin Oil.</i>						
Time	12.5	12.10-12.33	12.42	2.15	3.52	
Current (10^{-13} amp.)	0.0	5.1	3.3	2.5	1.8	
Radium	out	in	out	out	out	

Fig. 5.

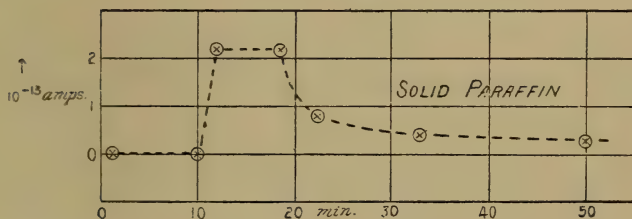
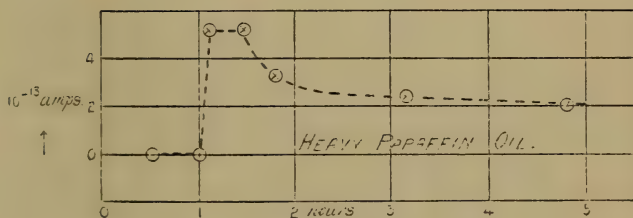


Fig. 6.



The complete recovery took a considerable time, one specimen of paraffin having a conductivity equal to that before the action of radium only after the lapse of 13 hours.

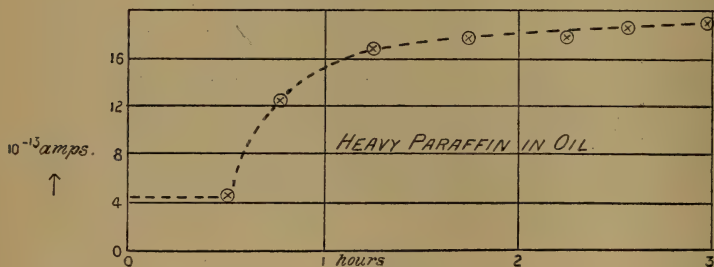
If the radium was allowed to act continuously and the current measured from time to time, it was found that the

conductivity increased. This is shown in the case of paraffin and vaseline in Table III.

TABLE III.

<i>Vaseline.</i>							
Time	10.36	10.52	11.18	12.46	3.58		
Current (10—12 amp.)	1.8	1.8	2.2	3.5	4.9		
Radium	out	out	in	in	in		
<i>Heavy Paraffin Oil.</i>							
Time	10.30	10.47	11.18	11.48	12.18	12.36	12.55
Current (10—13 amp.)	4.8	12.4	17.0	17.4	17.8	18.2	19.1
Radium	out	in	in	in	in	in	in

Fig. 7.



The curves obtained, showing the behaviour of dielectrics under the action of radium rays, are similar in character to those obtained by Jaffé with very pure substances. The immediate increase due to radiation was observed in benzene, toluene, aniline, and carbon disulphide, in addition to those mentioned above. The vaseline used was the unpurified commercial sort, and it was rather irregular in its action. Its conductivity proper decreased very rapidly, so much so as to mask completely the increase due to radiation, if sufficient time was not allowed to elapse to produce its own minimum conductivity.

These experiments have been conducted in the Physical Laboratory of Armstrong College, Newcastle-on-Tyne, under Prof. H. Stroud and Mr. H. Morris-Airey, to whom my best thanks are due.

XXXII. *Note on the Psychological Accommodation of the Lenses of the Eye.* By A. L. HODGES*.

[Plate IX.]

IF one chances to look at a picture or drawing showing in perspective a road leading off into the distance he can feel the lens of his eye changing focus, as if the object were really getting a greater distance away. Now of course all parts of the picture are practically the same distance from the eye and no such change in shape of the lens should occur. I append a simple perspective drawing to illustrate (Pl. IX.).

Run the eye rapidly from the seemingly nearest part of the track to the farthest. The eye consciously and by will accommodates itself to the seemingly increased distances. This is readily perceived by trying it. This should throw it out of focus. I have found that it does with some people and does not with others. Dr. Horace Richards, to whose attention I called the phenomenon, seems to think that the eye is fooled into changing its focus, and immediately on coming to rest again at the end of the line finds out its mistake and re-focusses. I have tried this myself and feel a perceptible jerk on letting the eye come to rest. But on several students whom I persuaded to try it, the final re-focussing effect was not noticed, they claiming that they could see equally well along the whole line but still perceiving the change in the accommodation. I think it a rather interesting little effect and one not devoid of interest to physicists. The question is: If the brain can see an upright image where the eye sees an inverted one, why cannot the brain fool itself into seeing a clearly defined object where the eye sees a blurred one? Because it certainly pursues the method that always gives a clearly defined image in real life, and the sub-conscious reasoning—if such there be—possibly proves to the brain that the blurred image falling on the retina is really a sharply defined one.

* Communicated by the Author.

XXXIII. *The Viscosity of Water.* By RICHARD HOSKING, B.A. (Camb.), *Physics Master, Sydney Grammar School**.

IN a paper recently published† in the *Philosophical Magazine* I explained how I had obtained three absolute values for the viscosity of water at different temperatures. I have since examined my reductions more carefully, and have found that the values are, in c.g.s. units, **·005500** at 50° C., **·008926** at 25° C., and **·017897** at 0°·05 C. The last value reduces to **·017928** at 0° C.

Knibbs‡ has deduced the absolute value **·013107** at 10° C. from Poiseuille's observations, and has shown that the data of every other experimenter whose work was published earlier than 1895, including Thorpe and Rodger, were not satisfactory for absolute values. So far as I have been able to examine the work published since 1895, I have reached a similar conclusion with regard to it.

In order, therefore, to obtain, if possible, a complete table of absolute values between the temperatures 0° C. and 100° C. I have revised all my earlier work§, and reduced my observations again by applying in the reduction formula, as given in my last paper, the value $m=1\cdot158$ ||, instead of $m=1\cdot000$, which I had previously used, and the value $n=1\cdot64$ ¶. My results obtained in this way are contained in Table A. Each value is obtained by the reduction of double sets of observations. The table contains also the kinetic-energy correction in each case. The values are reduced to even temperatures by means of the formulæ to be given in Table B (p. 262), which reproduce the observed values with remarkable closeness. It is this fact which justifies the method.

These values agree with the absolute values at the temperatures 0° C., 10° C., 50° C., and also (by interpolation) at 25° C., and it is reasonable to suppose that at the other temperatures also they are the correct absolute values. From the graph, values at the odd 5 degrees, for which there were no experiments, were obtained very carefully.

* Communicated by the Author. Read before the Royal Society of N. S. Wales.

† *Phil. Mag.* April 1909.

‡ *Journal and Proceedings of the Royal Society of N. S. Wales*, vol. xxxi.

§ *Phil. Mag.* March 1900, May 1902, May 1904.

|| Boussinesq's theoretical correction is 1·12, but experimentally I obtained the average value **1·158** for four tubes, which had been treated in exactly the same way as those used in these earlier experiments.

¶ For these tubes n seems to have the value 1·64, for with that value inserted, the reduced values for viscosity at 0° C., 10° C., and 50° C. agree with the absolute values at those temperatures as given at the beginning of this paper.

TABLE A.

Temp.	K.E. term $\times 10^5$.	Viscosity $\times 10^5$.	Reduction $\times 10^5$.	Viscosity at even temperature.
0° C.	6.5	1785	+6	.01791
0.25	9.5	1780	+15	.01795
0.21	11.0	1780.5	+13	.01793
0.21	11.0	1779.5	+13	.01793
5.71	7.6	1489	+33	.01522
6.53	10.4	1452	+70	.01522
9.89	11.0	1314	-4	.01310
10.05	12.5	1310	+2	.01312
12.44	11.9	1221	+87	.01308
12.51	9.2	1223	+89	.01312
17.07	9.9	1083	+59	.01142
17.10	9.9	1082	+60	.01142
20.37	11.2	997	+9	.01006
20.44	11.2	998	+10	.01008
20.72	14.4	989	+17	.01006
20.72	14.4	988	+17	.01005
30.25	13.7	798	+4	.00802
30.40	13.7	793	+6	.00799
30.78	17.1	787	+13	.00800
30.78	16.0	786	+13	.00799
40.07	16.2	658	+1	.00659
40.00	16.0	657	± 0	.00657
41.74	19.3	636	+21	.00657
41.74	19.3	635	+21	.00656
50.10	18.4	550	+1	.00551
50.30	18.9	548	+3	.00551
51.82	21.2	533	+16	.00549
51.82	21.0	532	+16	.00548
59.90	18.4	468	-1	.00467
60.13	18.3	470	+1	.00471
60.67	23.0	463	+5	.00468
60.67	22.8	464	+5	.00469
65.30	21.0	434	+2	.00436
65.38	21.0	434	+2	.00436
69.91	19.7	410	-0.5	.004095
70.10	20.8	405	+0.5	.004055
71.58	25.3	396	+9	.00405
71.58	25.3	396	+9	.00405
80.27	22.5	357	+1	.00358
79.90	20.8	356	± 0	.00356
81.19	26.6	350	+5	.00355
81.19	26.6	350	+5	.00355
90.00	22.5	316	± 0	.00316
90.10	29.0	314	+1	.00315
90.10	29.4	316	+1	.00317
90.10	29.3	316	+1	.00316
95.8	23.9	298	+3	.00301
97.7	23.6	291	+8	.00299
97.7	23.6	291	-7	.00284

The formula which best represents the results is of the type $\eta_t = \eta_0 / 1 + \kappa_1 t + \kappa_2 t^2$, but it is found that the range of temperature for each set of constants must not be greater than 25 degrees.

In Table B are given the constants and the temperatures to which they apply, in connexion with the above formula.

TABLE B.—Constants in the formula $\eta_t = \eta_0 / 1 + \kappa_1 t + \kappa_2 t^2$.

Formula.	Temperature limits.	κ_1 .	κ_2 .	η_0 .
(1)	0° C.— 25° C.	·03445	·000235	·017928
(2)	25° C.— 50° C.	·03544	·000195	·017928
(3)	50° C.— 75° C.	·0364	·000175	·017928
(4)	75° C.—100° C.	·0390	·000142	·017928

Knibbs* has tabulated the relative fluidities (reciprocal of viscosity) of Poiseuille, Graham, Rosencranz, Slotte, Noack, and Thorpe and Rodger, as redetermined by himself; and if my results be treated in the same way and compared with the mean values, it will be seen that the agreement is very close as far as 50° C. These relative values are given in Table C.

TABLE C.—Relative Fluidities of Water 0° C.—100° C.

Temp.	Poiseuille. 1846.	Graham. 1861.	Rosencranz. 1877.	Slotte. 1883.	Noack. 1886.	Thorpe & Rodger. 1894.	Average Values.	Hosking.
0° C.	1000	1000	1000	1000	1000	1000	1000	1000
5	1177	1183	...	1181	1178	1176	1179	1178
10	1363	1369	...	1372	1375	1365	1369	1369
15	1557	1580	...	1574	1575	1562	1570	1569
20	1771	1796	...	1785	1796	1772	1784	1782
25	1992	2021	...	2005	2020	1992	2006	2008
30	2226	2240	...	2233	2233	2225	2231	2241
35	2479	2493	...	2473	2484	2466	2479	2477
40	2738	2758	2730	2717	2726	2716	2731	2730
45	3010	3018	3020	2961	2974	2976	2993	2988
50	...	3308	3270	3214	3214	3246	3260	3260
55	...	3549	3540	3477	3479	3517	3512	3530
60	...	3834	3720	3744	3763	3797	3772	3823
65	...	4165	3980	4017	...	4086	4062	4112
70	...	4461	4210	4294	...	4385	4338	4414
75	...	...	4430	4571	...	4693	4565	4719
80	...	...	4620	4849	...	5003	4824	5037
85	...	...	...	5128	...	5316	5222	5352
90	...	...	5460	5409	...	5633	5501	5674
95	...	...	...	5695	...	5955	5825	5977
100	...	...	...	5983	...	6282	6132	6314

* Journal and Proceedings of Royal Society of N. S. Wales, vol. xxx.

Table D contains the absolute values for the viscosity of water at 5 degree intervals from 0° C. to 100° C. as determined from the whole of my experiments on water. The

TABLE D.—Absolute Viscosity of Water.

Temperature.	Absolute Viscosity. (Hosking)	Viscosity (Computed). (by formula)	Thorpe & Rodger's Values.
0° C.	·017928	·017928 (1)	·01778
5 	·01522	·015218	·01512
10 	·013105	·013107	·01303
15 	·01142	·011422	·01138
20 	·01006	·010055	·01004
25 	·008926	·008926 (2)	·00893
30 	·00800	·00801	·00799
35 	·00724	·00723	·00721
40 	·00657	·00657	·00655
45 	·00600	·00600	·00597
50 	·005500	·00550 (3)	·00548
55 	·00508	·00508	·00506
60 	·00469	·00470	·00468
65 	·00436	·00436	·00436
70 	·00406	·00406	·00405
75 	·00380	·00380 (4)	·00379
80 	·00356	·00356	·00355
85 	·00335	·00336	·00334
90 	·00316	·00317	·00316
95 	·00300	·00300	·00299
100 	·00284	·00284	·00283

values computed by formulæ are also given, as well as Thorpe and Rodger's values, which are in close agreement with mine, except at the lowest temperatures. I consider that my values are correct as far as the last figure given in each case.

XXXIV. *The Proposed International Unit of Candle Power.*
By CLIFFORD C. PATERSON*.

(From the National Physical Laboratory.)

THE intercomparison of light units between the National or other standardizing Laboratories of America, France, Germany, and Great Britain has been proceeding at intervals since 1905.

The values which have been obtained for the ratios between the different units are now found to be in sufficiently close accord to warrant the establishment of a working basis of

* Communicated by the Physical Society: read June 11, 1909.

agreement between this country, America, and France in the matter of a common unit of Candle Power.

The writer has been conducting the photometric measurements connected with the work, in this country, and it is the intention in this paper to give the results of the comparisons which have been made, and to explain briefly those facts connected with the different standards concerned which have a bearing on the agreement which has been reached.

The possibility of agreement between the British and French units was demonstrated by Dr. Glazebrook in a paper on light standards read before the B.A. at Dublin in 1908*. The chief factor in the present movement has been the desire of the authorities in the United States to establish one unit for both Gas and Electrical industries in that country, and the possibility of their adopting a unit which should be identical with those existing in Great Britain and France led them to take the initiative in an attempt to obtain international cooperation. The agreement which has resulted has the approval of the Metropolitan Gas Referees and now forms the subject of an announcement which is reproduced on page 273.

It is not necessary for the purpose of this paper to enter into a detailed description of the various standard lamps and units referred to in the memorandum. Dr. J. A. Fleming discussed the question of light standards very fully in his paper before the Inst. of Elect. Engineers, vol. 32, to which reference should be made†. Some facts, however, connected with the units in question have a more especial bearing on the experimental results, and should be borne in mind in connexion with the table of ratios given on pages 270, 271.

The British Unit.—As defined in the above-mentioned recommendation, the unit of Candle Power in this country is the Harcourt 10 C.P. Pentane Lamp burning in an atmosphere at normal barometric pressure and containing 8 litres of water-vapour per cubic metre as measured by a ventilated hygrometer.

A difficulty arises in the use of all flame standards in connexion with the method to be employed for measuring the humidity. When flame lamps are burning in a closed room it is well known that their candle power diminishes, due probably to the vitiation of the air in the immediate neighbourhood

* B.A. Report, 1908, Dublin, "The Photometric Standard of the National Physical Laboratory"; also 'Journal of Gas Lighting,' vol. 103, 1908, p. 713.

† See also paper by the author, Journ. Inst. Elect. Eng. vol. xxxviii. p. 271.

of the flame*. Two standards will not necessarily diminish in Candle Power at the same rate, and it is therefore necessary to take readings after the air of the room has been changed and before the C.P. of the lamps has had time to be affected. The method of measuring humidity must therefore be a rapid one, and it is now generally agreed that from considerations of accuracy and quickness of reading the ventilated hygrometer is the best instrument to use†. In the German and French comparisons this has been used, but in the English comparisons (as reported to the Photometric Commission meeting in Zurich in 1907) the unventilated hygrometer was employed‡, and in the tables published in the proceedings of the Commission the author's results are given in terms of humidity measured by this instrument. The ratios tabulated in the present paper are corrected so as to be in terms of the ventilated hygrometer. In each case values are taken for the humidity at which each lamp, in the country where it forms the standard, is considered to give its nominal Candle Power.

The Unit of the United States of America.—In the initial adoption of a unit of Candle Power the United States of America endeavoured to make its value as nearly as possible the same as that accepted at the time in this country§. This was before Prof. Vernon Harcourt and the Metropolitan Gas Reterees (London) had established the 10 Candle Pentane Lamp on the present definite basis.

The American Inst. of Electrical Engineers recommended the derivation of their unit from the Hefner Lamp by increasing its value in the ratio of 0.88 to 1. This seemed at the time, from different observers' work, to be the most probable ratio between the Hefner and British units. The gas industry in America, however, did not follow this course but developed their unit along the lines of the 10 Candle Pentane Lamp||. The result is that there has been, up to now, an appreciable difference between the units adopted in the two industries in that country. The Illuminating

* Report Amer. Gas Inst., "Methods of taking C.P. of Gas," Illum. Eng. 1909, p. 203.

† Proc. Roy. Soc. Edinburgh, vol. xliii., 1905; also "Zur kenntniss des Ventilierten Psychrometers," Akademische Abhandlung der Fakultat der Universitat zu Upsala, by Aron Svensson, 1898.

‡ See B.A. Report, Dublin, 1908.

§ Bulletin of the Bureau of Standards, vol. iii., no. 1, p. 65; Report to the American Gas Institute on "A Unit of Light," Journal of Gas Lighting, vol. 104, 1908, p. 426.

|| "The Working Standards of Light and their Use in the Photometry of Gas," Ch. O. Bond, Franklin Inst. 1908.

Engineering Society and other bodies took the matter up energetically, and the Bureau of Standards, Washington, now has the support of the leading institutions in America, in defining the value of a common standard, to be accepted throughout the States. This Institution has ascertained by means of electric intercomparisons the ratio of their present unit to those of Germany, France, and Great Britain respectively *, and has arranged to adjust the value of the American unit as already indicated.

German Unit.—The unit accepted in Germany is the light given by the Hefner Lamp burning in an atmosphere at normal barometric pressure and containing 8·8 litres of water vapour per cubic metre. The researches of Liebenthal † at the Reichsanstalt on the Hefner Lamp and the variation of its C.P. with atmospheric change were the earliest systematic experiments undertaken of this nature and are too well known to require more than passing mention.

French Unit.—The Candle Power Unit adopted by the Electrical Industry in France is the Bougie Decimale. This is the 20th part of the light given out by a sq. cm. of platinum at the temperature of solidification. The unit was suggested by M. Violle and adopted by the Congrès International des Electriciens in 1881.

This standard has been found very difficult to reproduce and the French authorities still use the Carcel lamp, burning colza oil, as the standard for all photometric work.

A determination of the value of the Carcel lamp in terms of the Violle platinum standard has only been made once. This was by M. Violle himself in 1884 ‡. Measurements were made by him using two or three photometric methods, and all his values except one showed the bougie decimal to be a little less than 4 per cent. greater than the Carcel unit.

A multiplying factor of 1·04 for the Carcel unit was therefore given by him, and has been adopted ever since for reducing the values in terms of one standard to those of the other. As no account was taken by M. Violle of the pressure and humidity of the atmosphere in which the Carcel lamp was burning, the accepted figure of 1·04 must be regarded as liable to a certain inaccuracy due to this cause. It should be remarked, also §, that no correcting factor has as yet been determined for the variations in the C.P. of the Carcel lamp

* Ref. cit.

† "Zeitschrift für Instrumentenkunde," vol. xv. 1895, p. 157.

‡ Séances of the French Physical Soc., May to July, 1884.

§ "Rapport de Trois Lamps," Laporte & Jouaust, Bull. Soc. Inst. des Elect. 2nd série, tome vi. no. 58.

due to atmospheric changes. Hence, in the table given later on in the paper a correcting factor has had to be assumed in cases where the Carcel lamp is corrected for a difference of humidity.

Accuracy of Comparisons.—It is well to explain in giving the results of experiments that different limits of accuracy must be attributed to photometric measurements of different types of standards.

It is usual in giving photometric results to write down the fourth figure, but even in the *most favourable* circumstances this must be written small and the value considered liable to an error of + or - 0.1 per cent. In the case of the best comparisons of electric sub-standards this inaccuracy should not be occasioned by imperfection in the bench or photometer head nor yet to the electrical measurements, but must be attributed, in the author's opinion, chiefly to want of constancy in the individual who is making the photometric observations.

In some of the comparisons which are tabulated the electrical measurements are probably not so accurate as in others. The fuller appreciation, however, of the exact values of the national and international electrical units which has recently resulted from the labours of the International Conference, makes it possible now to attain an accuracy which leaves nothing to be desired from this point of view.

As matters stand now, undoubtedly the photometric comparisons in which the highest precision is attainable are those between properly seasoned electric glow-lamps of the same coloured light. With a potentiometer which is accurate to one part in 10,000 and a substitution method of photometric comparison* on a bench which can be read to 0.5 mm., an accuracy is attainable with a set of good sub-standards in which the fourth figure is almost definite.

When, on the other hand, comparisons are made against or between flame standards, the probable inaccuracy is greater.

How much the inaccuracy is must depend largely on the flame adjustments and the consistent behaviour of the standard in question. It also depends upon the accuracy of measurement of atmospheric conditions and the precise knowledge which we have of their effect on the light of the standard lamps.

It follows from this that a relatively large number of observations must be made, when using a flame standard,

* See "Photometry of Electric Lamps," by Dr. J. A. Fleming, M.A., F.R.S., Journ. Inst. Elect. Eng. vol. xxxii. p. 144.

if the same order of accuracy is to be attained that is possible with a much smaller number of electric comparisons. It must further be remembered when considering the question of photometric measurements to two or three parts in a thousand that the estimation of the height of the flame in some lamps, or the exact reproduction of the standard conditions, may not be identical when carried out by different observers. Hence it is conceivable that owing to this cause the observations in one laboratory on some flame standard may differ consistently by a small amount from those in another on the same standard. This, however, is not the case when electric sub-standard comparisons are made if the electrical measuring apparatus is accurate. To a certain extent, therefore (in some cases more than others), a flame standard needs to be "interpreted" when its absolute value is desired to a high accuracy.

The value of electric sub-standards comparisons thus becomes apparent. If (as is generally the case) a Laboratory has sets of electric sub-standards which have been compared at intervals for years with the primary flame standard whose value they represent, an opportunity is given for realising the absolute value of this unit to an accuracy which could hardly be attained with certainty by others who might endeavour to reproduce it in a single set of observations, however carefully made. When these electric sub-standards are intercompared through the medium of a travelling set of lamps, there is no reason why we should not obtain accurate knowledge of the relative values of the different units as each is interpreted in the country where it is the recognized standard.

The ratios between the four units of light given in the Table are the results of measurements which have been made at the specified laboratories in the four countries concerned. Other determinations were made previous to these*, but the standards used for obtaining the British Unit were of several different forms and the atmospheric conditions have not always been taken into consideration. I have deemed it desirable, therefore, to insert only the more recent determinations, in all of which the 10 Candle Harcourt Pentane Lamp has been used and atmospheric changes have been allowed for.

The Table is divided into two portions. Columns 1 to 9 give the various ratios obtained through the medium of

* For a discussion of these, see J. A. Fleming, "The Photometry of Electric Lamps," *ref. cit.*

electric lamps which have been measured at some or all of the laboratories. These have been chiefly initiated by the Americans, who have from time to time sent sets of lamps to Europe to have values assigned in London, Paris, and Berlin. It is not suggested that all the results given in the Table should receive equal weight. In some of the electric comparisons the conditions allowed of a greater accuracy than in others, when fewer lamps were employed and time only allowed a single set of measurements to be made.

Columns 10, 11, and 12 contain the values which resulted from the intercomparison of flame-standards undertaken at each of the laboratories. These were initiated by the International Commission on Photometry and gave a set of ratios which brought the knowledge of the relative values of the candle power units to within an accuracy of about $\pm$ or -1 per cent. As in the case of the electric comparisons, the conditions in some cases probably allowed of a higher accuracy than in others—but the results of all the measurements have been tabulated in order that the bearing may be seen of each on the agreement which has been established.

The first series of ratios (columns 1 to 9) may therefore be regarded as representing the ratios of the standards as they are interpreted in the countries to which they belong, whilst in the second series we have the interpretation of the values of the standard lamps by experimenters who are not so accustomed to their manipulation.

Lines marked A, B, C, give the values of the standards in terms of the Pentane unit. D, E, F, give them in terms of the Hefner and similarly other sets are in terms of the Bougie Decimale and the Bureau of Standards Candle.

Without going into detailed comments upon the experiments from which each ratio in the table is derived it will suffice to say that as far as the electric lamp comparisons are concerned, greatest stress should be laid on the results in columns 4 and 7. This is partly on account of the large number of lamps employed, and also because of the more prolonged measurements made. Line J illustrates the high accuracy it is possible to secure in comparisons of this nature.

In the certification of glow-lamps in terms of the Hefner Unit the Reichsanstalt only give candle-power values to the nearest one per cent. If the average of 10 or 12 lamps is taken the error thus introduced is probably not great, but when the number is small, appreciable inaccuracies may be introduced into the mean, and the rather low value obtained

TABLE giving determinations of the Ratios of
 Pentane = British. Bougie Decimale = French.

		ELECTRIC					
COLUMN		1.	2.	3.	4.	5.	6.
Tests conducted by {		Sharp, 1903.	Paterson, 1905.	Fleming, 1905.	Hyde, 1906.	Laporte & Jouaust, 1907.	Laporte, 1907.
Number of Lamps.....		...	6	3	12	6	2
A	$\frac{\text{Hefner}}{\text{Pentane Unit}}$		0.89 ₀	0.88 ₅	0.89 ₃	0.90 ₈	
B	$\frac{\text{Bougie Decimale}}{\text{Pentane Unit}}$				0.99 ₈	1.01 ₃	1.00 ₃
C	$\frac{\text{Bur. of Standards}}{\text{Pentane Unit}}$				1.01 ₆		
D	$\frac{\text{Pentane Unit}}{\text{Hefner}}$		1.12 ₃	1.13 ₀	1.12 ₀	1.10 ₁	
E	$\frac{\text{Bougie Decimale}}{\text{Hefner}}$	1.11 ₈			1.11 ₆	1.11 ₃	
F	$\frac{\text{Bur. of Standards}}{\text{Hefner}}$				1.13 ₈		
G	$\frac{\text{Pentane Unit}}{\text{Bougie Decimale}}$				1.00 ₂	0.98 ₇	0.99 ₇
H	$\frac{\text{Hefner}}{\text{Bougie Decimale}}$	0.89 ₄			0.89 ₆	0.89 ₈	
I	$\frac{\text{Bur. of Standards}}{\text{Bougie Decimale}}$				1.01 ₈		
J	$\frac{\text{Pentane Unit}}{\text{Bur. of Standards}}$				0.98 ₄		
K	$\frac{\text{Hefner}}{\text{Bur. of Standards}}$				0.87 ₉		
L	$\frac{\text{Bougie Decimale}}{\text{Bur. of Standards}}$				0.98 ₂		

* Pentane values corrected to a humidity of

National Candle Power Units from 1903 to 1908.

Hefner = German.

Bureau of Standards = U.S.A.

COMPARISONS.			DIRECT COMPARISONS.			
7.	8.	9.	10.	11.	12.	13.
Rosa, 1908. Spring.	Rosa, 1908. Autumn.	Laporte, 1908.	Paterson.	Liebenthal.	Perot & Laporte.	Photometric Commission, Zurich,* 1907.
12	6	11				
0.89 ₆			0.90 ₂	0.90 ₄	0.92 ₁	0.90 ₂
1.00 ₈		1.00 ₈	1.00 ₉	1.00 ₉	1.01 ₉	1.00 ₈
1.01 ₅	1.01 ₆					
1.11 ₆			1.10 ₉	1.10 ₆	1.08 ₆	1.10 ₉
1.12 ₄			1.11 ₈	1.11 ₆	1.11 ₇	1.11 ₈
1.13 ₁						
0.99 ₂		0.99 ₂	0.99 ₁	0.99 ₁	0.98 ₁	0.99 ₂
0.89 ₀			0.89 ₄	0.89 ₅	0.89 ₅	0.89 ₄
1.00 ₆						
0.98 ₅	0.98 ₄					
0.88 ₄						
0.99 ₄						

8 litres of water vapour per cubic metre of air.

by Fleming in 1905 may be attributed to the fact that only 3 lamps were tested*.

As regards the flame-lamp comparisons, it will be noticed that Perot and Laporte (Column 12) found a value for their Pentane lamp which was appreciably lower than that obtained by other observers. Except for this difference the agreement between the ratios is fairly close. The exceptionally close agreement shown in columns 10, 11, and 12 for the ratio Bougie Dec./Hefner can only be attributed to a coincidence, since in the experiments from which two of the three ratios were determined the observations varied between 3 and 4 per cent. from the mean.

The chief point of interest in these comparisons is the near coincidence of the value of the Bougie Decimale with that of the Pentane unit as indicated by lines B and G. An inspection of these values indicates that the Bougie Decimale, as interpreted by the Laboratoire Central, may be slightly larger than the Pentane unit—but the amount is less than 1 per cent. When we remember that, at present, the value of the Bougie Decimale depends on the interpretation of the Carcel lamp and the ratio between it and the platinum unit, determined by Violle in 1884, it must be admitted that this small apparent difference is well within the limits of the errors of experiment.

The second point to notice is the difference of 1·6 per cent. between the units of the Bureau of Standards and the National Physical Laboratory. It is generally recognized† that the unit at present adopted by the Gas interests in the States is about 4 per cent. smaller than the Bureau of Standards unit. It will then be seen that by lowering the value of their unit by 1·6 per cent. the Bureau comes into exact agreement with this country and approximately halves the difference between the units employed by the Gas and Electrical industries in the States.

A further point of interest and importance which results from the comparisons (see line A in the table) is that the Hefner unit is in the ratio 9/10 to the new candle. The French authorities have for some time taken the ratio Hefner/Bougie Dec. as 0·895. It is of interest, therefore, to see from lines A and H how nearly the value for the Hefner unit in terms of both Pentane and Bougie Dec. units

* See Journ. Inst. Elect. Eng. vol. xxxviii. p. 311: Discussion by Dr. Fleming of the Author's paper on "Investigation of Light Standards etc."

† Report of Committee on Nomenclature and Standards, Annual Conv. Illum. Eng. Soc. Oct. 5, 1908, Dr. A. C. Humphreys.

approaches the figure 0.90. The mean of all the ratios Hefner/Pentane comes to 0.90₀ and those of Hefner/Bougie Dec. to 0.89₅, so that although the comparisons between the Pentane and Bougie Decimale units indicate a difference of 0.8 per cent., the same units compared through the Hefner Standard only appear to differ by 0.5 per cent.

The author's acknowledgments are due to Dr. R. T. Glazebrook, F.R.S., Director of the National Physical Laboratory.

APPENDIX.

Copy of Announcement made in France, America, and Great Britain, relative to the proposed International Unit of Light.

In order to determine as accurately as possible the relations between the photometric units of America, France, Germany, and Great Britain, comparisons have been made at different times during the past few years at the Bureau of Standards, Washington; at the Laboratoire Central d'Electricite, Paris; at the Physikalisch-Technische Reichsanstalt, Berlin, and at the National Physical Laboratory, London.

The unit of light at the Bureau of Standards has been maintained through the medium of a series of incandescent electric lamps, the values of which were originally intended to be in agreement with the British unit, being made 100/88 times the Hefner unit.

The unit of light at the Laboratoire Central is the bougie decimale, which is the twentieth part of the standard defined by the International Conference on Units of 1884, and which is taken, in accordance with the experiments of Violle, as 0.104 of the Carcel lamp.

The unit of light at the Physikalisch-Technische Reichsanstalt is that given by the Hefner lamp burning at normal barometric pressure (76 cm.) in an atmosphere containing 8.8 litres of water-vapour per cubic metre.

The unit of light at the National Physical Laboratory is that given by the 10 candle power Harcourt Pentane lamp burning at normal barometric pressure (76 cm.) in an atmosphere containing 8 litres of water-vapour per cubic metre.

In addition to the direct intercomparison of flame standards carried out recently by the national laboratories in Europe, one comparison was made in 1906 and two in 1908 between the American and European units by means of carefully seasoned carbon filament electric standards, and as a result

of all the comparisons, the following relationships are established between the above units.

The Pentane unit has the same value within the errors of experiment as the bougie decimale. It is 1.6 per cent. less than the standard candle of the United States of America, and 11 per cent. greater than the Hefner unit.

In order to come into agreement with Great Britain and France, the Bureau of Standards of America proposed to reduce its standard candle by 1.6 per cent. provided that France and Great Britain would unite with America in maintaining the common value constant, and with the approval of other countries would call it the International Candle. The National Physical Laboratory, London, and the Laboratoire Central d'Electricité, Paris, have agreed to adopt this proposal in respect to the photometric standardization which they undertake, and the date agreed upon for the adoption of the common unit and the change of unit in America is April 1, 1909.

The following simple relations will therefore hold after that date :

$$\begin{aligned}\text{Proposed New Unit} &= 1 \text{ Pentane Candle.} \\ &= 1 \text{ Bougie Decimale.} \\ &= 1 \text{ American Candle.} \\ &= 1.11 \text{ Hefner Unit.} \\ &= 0.104 \text{ Carcel Unit.}\end{aligned}$$

Therefore 1 Hefner Unit = 0.90 of the proposed unit.

The Pentane and other photometric standards in use in America will hereafter be standardized by the Bureau of Standards in terms of the new unit. This, within the limits of experimental error, will bring the photometric units for both gas and electrical industries in America and Great Britain and for the electrical industry in France to a single value, and the Hefner unit will be in the simple ratio of 9/10 to this international unit.

The proposal to call the common unit of light to be maintained jointly by the national standardizing laboratories of America, France, and Great Britain, the "International Candle" has been submitted to the International Electrotechnical Commission, and through it to all the countries of the world which are represented on that Commission.

It is to be hoped that such general approval will be secured, and that in the near future the term "International Candle" for the new unit will have official international sanction.

XXXV. *Primary and Secondary Gamma Rays.*

By A. S. EVE, M.A., D.Sc., McGill University, Montreal *.

WHATEVER may be the views held about Bragg's theory of the nature of the Röntgen and gamma rays, it is certain that his able advocacy had led to a notable increase of our knowledge of the properties of both types of radiation. The gamma rays in particular are so little affected by physical conditions that new methods of investigation are essentially valuable. Therefore, the plan adopted by Bragg of examining the incident and emergent secondary radiations, consisting of electrons or corpuscles, from a reversible pair of dissimilar metals traversed by the gamma rays, appears to be a promising line of attack. He has himself published † a few results showing the changes produced in the ionization current of an electroscope, when four plates, two above and two below, of dissimilar substances, are traversed by a pencil of γ rays, and are interchanged in the four possible arrangements. In this manner he has proved the asymmetry of the incident and emergent secondary radiations.

The writer has for some months made an experimental study of effects produced in a similar way with primary and secondary γ rays from radium C, and with the primary γ rays from uranium X. The main results are capable of considerable simplification, although the underlying ultimate processes remain obscure.

The first paper on Secondary Radiation due to the β and γ rays of radium was published ‡ in December 1904, and the results then obtained have been on the whole well substantiated by later observers. The work of McClelland §, however, placed the theory of absorption of the β rays and the laws of reflected or *incident* secondary radiation on a sound basis, and a host of other workers in the field have contributed to steady advance in various directions. In particular McClelland found that the amount of incident secondary radiations from various elements followed the order of atomic weights. Such radiations have been proved to come from a very slight depth, and to consist of high velocity negatively charged particles. Using relatively large masses, Kleeman ¶ and the writer ¶¶

* Communicated by the Author.

† Phil. Mag. May 1908.

‡ Phil. Mag. Dec. 1904.

§ Trans. Roy. Dublin Soc. xxvii. p. 9 (1906).

¶ Phil. Mag. May 1908.

¶¶ Phil. Mag. Aug. 1908.

have proved the existence of secondary radiations of the γ -ray type, less penetrating than the primary γ radiations to which such secondary rays are due.

The transmitted or *emergent* corpuscular secondary radiations at first received less attention, although it was shown* in 1904 that for varying thickness of the secondary radiator the intensity of the emergent radiation followed a curve of the well-known type whose equation involves the difference of two simple exponentials. A. S. Mackenzie† has called attention to the remarkable fact that there travels in the direction of the γ rays, when absorbed, a stronger stream of negative secondary rays than in the reverse direction. This fact, substantiated by Bragg, is one of the arguments in the chain of reasoning which led him, in spite of able opposition and many difficulties of experiment and theory, to controvert the æther-pulse theory.

Bragg‡ has proved that the emergent secondary corpuscular radiation from various elements does not follow the order of atomic weights, but that the curve, plotted with atomic weights as abscissæ and intensities of secondary radiation as ordinates, roughly resembles a parabola with vertex downwards, such as may be seen in fig. 3 of this paper. Again, there is some divergence in the views and results of Bragg and Kleeman, for the latter found evidence of selective absorption, while Bragg and Madsen attributed the observed effects to the hardening of the rays. Also Kleeman, using both primary and secondary γ rays, divided the metals examined into three well-marked groups; but Bragg's work, and Madsen's, indicates no sharp line of demarcation into groups. The present work was undertaken to investigate this point also; and it will be seen that the results favour the conclusions of Bragg and Madsen.

Apparatus.

A rectangular brass electroscope (fig. 1), $18 \times 16 \times 12$ cm., was used, but both ends were removed and covered with thin aluminium foil. It requires about 12 layers of this foil to stop the α rays of radium; so that it may be expected, and it was found, that in the experiments described the foil produced a negligible effect on the secondary rays investigated, when proceeding from the surfaces of plates close to the aluminium. The radium, 14 mg. pure RaBr_2 , was placed

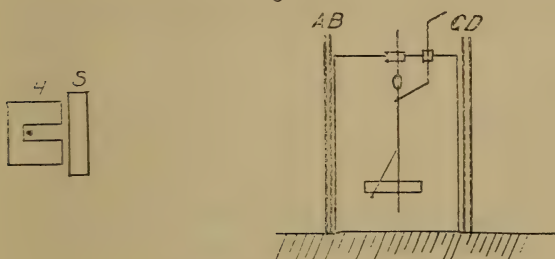
* Phil. Mag. Dec. 1904.

† Phil. Mag. July 1907.

‡ Phil. Mag. Dec. 1908.

sometimes entirely inside a closed hollow cylinder of iron or lead of the desired thickness, sometimes in a cylinder such as H,

Fig. 1.



with an open end so that a cone of rays was produced passing through the electroscope, 12 cm. long. The results obtained by these two methods were not different when suitable metal screens were placed at S. Plates of various substances were placed at A, B, C, D; these will always be referred to as *plates*, and in the order A, B, C, D, and the electroscope will always be understood as between B and C. It must further be noted that the γ rays from the radium C traversed the electroscope and produced ionization, and that the secondary rays from the top, bottom, and two walls of the electroscope further contributed to the total ionization; and no attempt has been made to make deductions for these. It was merely desired to ascertain the effects of reversing the plates at A and B, and those at C and D.

An example will make this clear.

Plates at A.	B.	C.	D.	Divisions a minute.
Al	Pb	Pb	Al	 36·4
Pb	Al	Pb	Al	 36·0
Al	Pb	Al	Pb	 30·1
Pb	Al	Al	Pb	 29·7

The last column gives the four readings obtained for the stated four arrangements of the plates. The order given was always followed; and in future the results may be given as percentages, thus,

(Al, Pb) 100, 99, 83·5, 81.

Substituting carbon for aluminium the readings gave

(C, Pb) 100, 105, 80, 84.

Bragg has published about a dozen readings of this character. I find that, for reasons subsequently stated, the product of the extreme is nearly equal to the product of the mean readings.

The plates used in the experiments measured about 17×21 cm., and their respective thickness is stated—

Lead	1.7 mm.	Iron, steel.....	1.83 mm.
Silver	4.6	„ wrought ..	6.00
Tin	4.0	Aluminium	1.72
Zinc	0.45	Carbon	15.0
Copper	3.05		

The zinc and aluminium plates were rather too thin, so a few were clamped together as required.

Effect of Reversing Plates C and D.

When γ rays pass through the electroscope and when the back plates C and D alone are varied, the resulting changes in the rate of discharge of the electroscope depend simply on the atomic weight of the substance at C, provided that plate is one or two millimetres thick. For the plate D, unless very thick, does not send back through C sufficient secondary radiation to affect the ionization in the electroscope to a noticeable extent. If the plate D were several centimetres thick it would, of course, send back through C secondary γ rays, the additive effect of which could be detected.

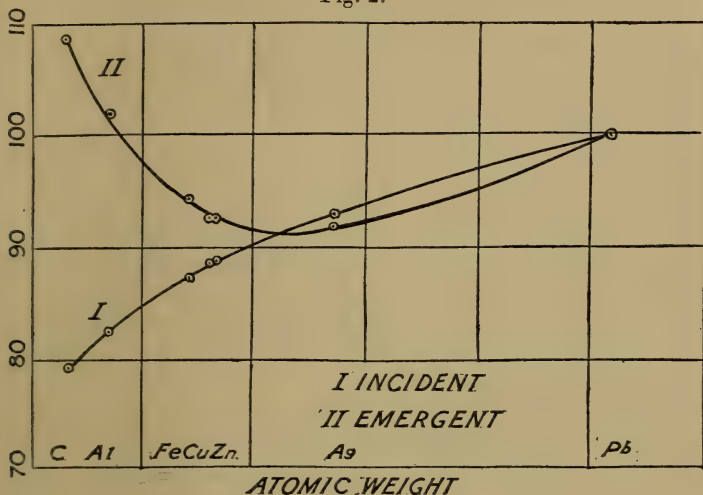
In the actual experiments, keeping at A and B any plates of a definite material, and using at S screens of iron 2 cm., or lead 0.6 cm., or lead 2 cm., the effects of varying C are shown in the following table and in curve I. (fig. 2).

TABLE I.

Plate C.	Atomic Weight.	Ionization Current.			
		Iron 2.1.	Pb 0.6.	Pb 2.0.	Mean.
Pb	206.9	100	100	100	100
Ag	107.9	93	92.8	...	92.9
Zn	65.4	88.3	89.5	89	88.9
Cu	63.6	87.6	89.0	87	88.3
Fe	55.9	86.9	87.8	87	87.3
Al	27.1	81.8	83.0	81	82.4
C	12.0	78.0	80.7	78	79.3

There is some evidence of a slight change due to the hardening of the rays by screening. The results given, the

Fig. 2.



mean of many experiments, might have been expected and predicted from the known character of the incident secondary radiation due to γ rays, which, like that due to β rays, follows the order of atomic weights.

Effect of Reversing Plates A and B.

If a plate of any given substance is kept constantly at C, and if readings of the electroscope are taken when two different metal plates are at A and B, and also for those same plates in reverse order, then the two readings are not the same. The change observed is not due to an alteration of the intensity of the γ rays traversing the electroscope, but to the negative electrons, or corpuscles, which constitute the emergent secondary radiation from plate B. Hence the advantage of the reversal method, using two plates; for with single plates there is an uncertain decrease in the γ rays depending on thickness and density. Nor is it necessary to use a magnetic field to remove negative rays approaching the face of A from the screens, for these are absorbed by the plates.

It has been shown by many observers that when very thin plates are placed in the path of the γ rays near an electroscope, the effects due to such thin plates are small, but on increasing

their thickness the ionization current in the electroscope rises quickly to a maximum, owing to the increase of the emergent corpuscular secondary radiation from the plate. Further increase of thickness causes a very gradual decrease in ionization owing to the slow absorption of the γ rays. In fact the curve has an equation involving approximately the difference between two simple exponentials. Provided, therefore, that the plates at A and B are more than one or two millimetres thick, it makes but a minute difference, in the reversal method, if they are several millimetres thick, so gradual is the absorption of the γ rays. The changes produced in the electroscope on reversing A and B are, then, mainly dependent not on the thickness, but on the atomic weight of the plate in position B. This conclusion was confirmed experimentally. An exception must be made in the case of lead; for an increase of thickness of the lead plates hardens the γ rays rapidly unless they have previously passed through screens of lead.

The method of procedure adopted was as follows. Tin and lead plates were placed at A and B and a measurement taken; the plates were reversed, and again a reading made. The readings in the electroscopes were in the ratio of 100 to 92. Lead was similarly compared with all the other substances, as shown in the first column of Table II. Then tin, assumed 92, was compared with the remaining substances, as given in the second column, and so forth.

TABLE II.

RaBr₂ 14 mg. in lead 0.6 cm. thick. Distance 15 cm.

							Mean. 100
Pb	100						100
Sn	92	92					92.0
Zn	92.4	93	92.4				92.6
Cu	92.4	92	94.0	92.4			92.6
Fe	94.1	94.3	95.5	93.5	94.1		94.3
Al.....	102.3	102.2	101.8	102.0	102.8	102.3	102.2
C	109.2	107.5	110.8	108.2	108.5	109.1	108.9

The corresponding curve II. (fig. 2) shows the effect of the atomic weight of plate B on the electroscope reading. Similar curves were obtained by Bragg*, who used 11 elements. It appears difficult or impossible to estimate the proper deduction to be made from the mean values in the right-hand column so as to arrive at the absolute values of the emergent secondary

* Trans. Roy. Soc. S. Aust. 7. xxxii. (1908).

radiations. The readings given in Table II. were obtained with a vertical cylindrical zinc electroscope 21 cm. high, 16 cm. diameter, having a thin aluminium base with the radium placed below it at 15 cm. distance. The plates A and B were clamped close to the aluminium.

Using the electroscope previously described (fig. 1) the mean results obtained, for different screens as stated, and in the manner just described, are given in Table III.

TABLE III.

Screens:— Plates.	Lead Cylinder, 0·6 cm.	Lead, 2·2 cm.	Iron Cylinder, 2·1 cm.
Pb	100	100	100
Ag	90		88
Zn	90	94·4	87·6
Cu	90	96·5	87·0
Fe	91	99·0	87·0
Al	98	107·6	90·0
C.....	104	119·4	95·2

The corresponding three curves are shown in fig. 3 (p. 282). The changes in the relative positions of carbon and lead according to the nature and thickness of the screens employed are very striking, and at first sight suggest selective absorption; and a number of experiments were made to examine this peculiarity.

Lead, aluminium, and carbon plates were compared with screens of lead and iron.

Screens:—	LEAD.				IRON.	
	0·6 cm.	0·72.	1·5.	2·6.	2·1.	5·7.
Pb	100	100	100	100	100	100
Al	98	99	102·5	106	91	90·5
C.....	104			117	98	99·0

In this case the radium was in vessel H (fig. 1), and screens were interposed near H. Zinc, copper, carbon, and paraffin

Fig. 3.

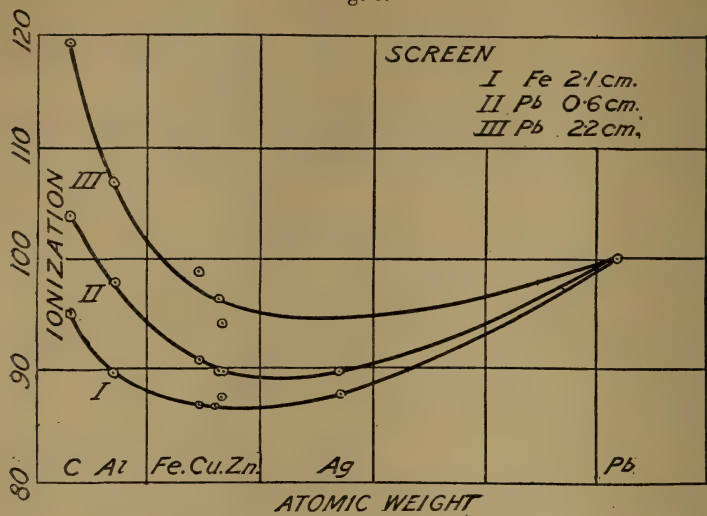
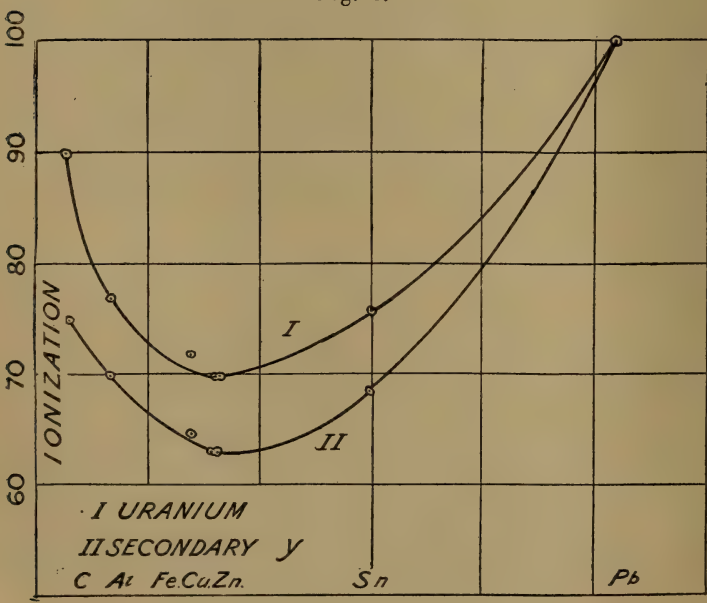


Fig. 5.



screens were also used, and these resembled iron, not lead, in the effects produced. It is then easy to produce a change with less than a centimetre of lead which cannot be brought about by many centimetres of iron. This peculiarity was first noticed by Bragg and Madsen *, and it cannot be attributed to selective absorption because a variation in the character of the screens causes corresponding modifications in the radiations from other pairs of plates, such as copper-carbon or copper-aluminium.

After the γ rays have passed through iron screens similar changes can be produced by a moderate thickness of lead. Thus,—

Screens	Fe 2·1.	{ Fe 2·1 Pb ·16	Fe 2·1. Pb ·48.	Fe 2·1 cm. Pb ·64 cm.
Pb	100	100	100	100
Al	91	93	99	100

Iron, however thick, appeared incapable of producing equivalent results.

It is possible that the true explanation is a little complex. When primary γ rays pass through iron they give rise to secondary γ rays, softer than the primary which caused them, but nevertheless capable of penetrating a considerable thickness of iron. The same is true of zinc, copper, and other substances of a like order of atomic weight. When, however, primary γ rays pass through lead, secondary γ rays are produced to a less extent than with iron, or they are so rapidly absorbed that they are more difficult to detect. Hence thick iron screens do not harden the rays, but lead screens do so rapidly.

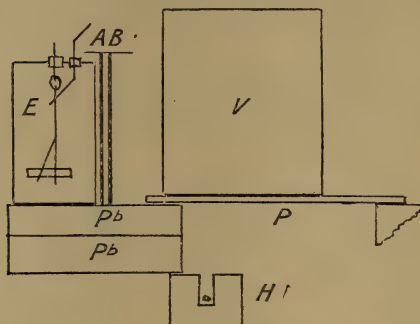
When the U-shaped curves shown in the preceding figures have the left branch high compared with the right, it seems to indicate hard rays; when low, soft rays. In order to put this to the test, I examined the γ rays from uranium X, and also the secondary γ rays from iron produced by radium C. Both types of radiation are known to be softer than the primary γ rays of radium.

* Phil. Mag. Dec. 1908.

Secondary γ Rays.

A strong and satisfactory source of secondary γ rays was obtained by the arrangement shown in fig. 4. 14 milligrams

Fig. 4.



of radium bromide, placed in a hollow cylinder H, sent a cone of rays through the platform P to the secondary radiator V. The secondary γ rays were measured by the electroscope E, after passing through the plates A and B. Both electroscope and plates were well screened from the radium by much lead. A correction was made for natural leak and for all rays entering the electroscope otherwise than through A and B, and the amount of this deduction could be well estimated by placing many thick plates between V and E. Three different platforms were tried, of lead, iron, and wood respectively, each about 6 mm. thick. The results obtained differed somewhat; but as the differences were too small to measure correctly the mean values were taken. When a large lead cylinder served as radiator at V, the secondary γ rays were feeble, because they were to a large extent absorbed in the lead itself. Four bricks placed on end, close together, proved a good radiator. A tinned iron pail, 17 cm. high, 15 cm. in diameter, was also used, and could be filled with iron filings, water, sand, or coal. Such different radiators did not give well-marked differences in their secondary γ radiations; and I worked, therefore, mainly with bricks, or with iron filings as radiator, and with a wooden platform.

The absorption of the secondary γ rays by lead plates is shown by the following :—

Thickness of Absorbing Lead.	Divisions a minute.	λ .
0.0 mm.	7.13	3.4
1.7	4.01	2.5
3.4	2.61	2.3
5.1	1.77	1.9
6.8	1.34	1.6
8.5	.95	1.4
10.2	.75	

The last three values of λ are not exact.

The values of λ and of $\lambda/\text{density}$ for thicknesses between 2 to 3 mm. and 5 to 6 mm. were also found for various substances.

Absorber.	SECONDARY.		PRIMARY.	
	λ .	λ/d .	λ .	λ/d .
Lead	2.4	.21	.64	.056
Zinc	.86	.121	.28	.039
Copper	1.09	.122		
Iron	.84	.118		
Aluminium ...	.21	.08	.10	.039

In the right columns are values for the more easily absorbed primary γ rays as found by McClelland; these serve to illustrate the relative softness of the secondary.

The secondary γ rays were next investigated by the method of reversal of pairs of plates at A and B, precisely as described earlier in this paper when using primary rays. Taking the mean values for brick or iron radiators, on wood, iron, and lead platforms, the following results were obtained.

Emergent secondary radiation using secondary γ rays.

Pb.....	100	Fe	64.5
Sn.....	68.5	Al	70.0
Zn.....	63.0	C	75.0
Cu	63.5		

The curve obtained, shown in fig. 5, is typical of easily absorbed γ rays.

Gamma Rays from Uranium X.

About $1\frac{1}{2}$ kilograms of uranyl nitrate were placed in a cylindrical glass vessel so as to form a layer about 6 cm. thick. This was placed beneath the vertical cylindrical electroscope described earlier in the paper, plates were fixed close to the electroscope and the emergent secondary radiation was observed by the reversal method. The γ rays from uranium X are feeble, and the measurements were difficult and uncertain; but the results obtained were:—

Uranium, γ rays. Emergent secondary radiation.

Pb	100	Fe	72
Sn	76	Al	77
Cu	70	C.....	90
Zn	70		

The curve is given in fig. 5, and it shows that the γ rays from uranium X resemble those of Ra C more nearly than they resemble the secondary γ rays due to Ra C. It would be interesting to obtain the curve more correctly with uranium X concentrated in the manner recently described by Soddy*.

The five curves seen in figs. 3 and 5 show the gradual changes in the emergent secondary radiation from very soft to very hard γ rays.

Reversals with Four Plates.

It is now possible to explain the results obtained by reversing the order of plates at the front and back of the electroscope. Let the plates A, B, C, D consist two of aluminium, two of lead; then the ionization currents in the electroscopes for the arrangements stated have the following relative values, when the γ rays have passed through a screen of lead 0.6 cm. thick.

	A.	B.	C.	D.	Eve.	Bragg.
I.....	Al	Pb	Pb	Al	100	100
II.....	Pb	Al	Pb	Al	98.5	99.5
III.....	Al	Pb	Al	Pb	83.5	81.0
IV.....	Pb	Al	Al	Pb	81.1	82.5

* 'Nature,' 28 Jan. 1909.

The results obtained by Bragg* were found with an arrangement different from mine, for he placed the radium close to the electroscope.

To calculate the effect of reversing plates A and B we have to look at Table III. or at curve II., fig. 3, and thence obtain the fact that the reversal from Al, Pb to Pb, Al decreases the ionization from 100 to 98. This explains the change from I. to II., and also from III. to IV. just above.

To calculate the effect of reversing C and D we see in Table I., or from curve I. (fig. 2), that lead at C gives 100 whereas aluminium at C gives 84.

Thus II. and IV. are 98 per cent. of I. and III. respectively, but III. and IV. are 84 per cent. of I. and II. respectively. Hence, of course, the product of the extremes I. and IV. must equal the product of the means II. and III. The calculated and observed values for Al, Pb are respectively—

I.	II.	III.	IV.
100	98	84	82
100	98·5	83·5	81·1

Or, again, taking aluminium and copper plates,—

	I.	II.	III.	IV.
Observed	100	103·5	94·3	96·8
Calculated	100	103·4	93·4	96·6

In the following Table IV. (p. 288) are given the results for many sets of four plates; and successive columns show,—reference number, nature of screen, thickness, and distance from the radium to the near face of the electroscope. In the fifth column the substances used as plates are set forth, and then follow,—the results for the arrangements I., II., III., IV., as explained above; the products of the extremes, and of the means, also their ratio, which has an average value 0·993.

From No. 14 we see that the reversals of plates of nearly equal atomic weights, such as Fe, Cu, produce very slight changes within the electroscope.

The hardening effect of lead is well seen by contrasting No. 1 and 2; for on increasing the thickness from 0·6 to 2·0 cm., II. changes from 98·5 to 106·5, and III. from 83·5 to 78·0. No such alterations take place when the thickness of iron is increased as in Nos. 3 and 4. The same general results are found in Nos. 5 to 9 when carbon and lead are used.

* Phil. Mag. May 1908.

TABLE IV.

	Screen.	en. thick.	Distance.	Plates.	I.	II.	III.	IV.	E.	M.	E/M.
1.	Pb	0.6	41	Al, Pb	100	98.5	83.5	81.4	8140	8230	0.99
2.	Pb	2.6	28	"	100	106.5	78.0	84.0	8400	8320	1.01
3.	Fe	2.1	41	"	100	93.3	80.7	73.2	7320	7540	0.97
4.	Fe	5.7	28	"	100	90.0	80.7	71.4	7140	7260	0.98
5.	Pb	0.6	41	C, Pb	100	104.6	79.1	84.0	8400	8270	1.01
6.	Pb	2.6	28	"	100	117	76.0	90.0	9000	8920	1.01
7.	Fe	2.1	41	"	100	97.0	82.0	78.2	7820	7950	0.98
8.	Fe	2.7	25	"	100	98	79.5	76.0	7600	7770	0.98
9.	Fe	5.7	28	"	100	98.5	79.5	75.5	7550	7820	0.97
10.	Fe	2.4	25	Cu, Pb	100	86.0	84.0	72.2	7220	7240	1.00
11.	Pb	0.6	41	Al, Cu	100	107.5	93.4	99.5	9950	10000	0.99
12.	Fe	2.1	41	Al, Cu	100	103.5	94.3	96.8	9680	9780	0.99
13.	Fe	2.1	41	C, Cu	100	108	94	101	10100	10100	1.00
14.	Pb	0.6	25	Fe, Cu	100	100	99	100	10000	9900	1.01

Mean 0.993

Hence we may conclude that the γ rays themselves which pass through the electroscope are unaffected by the reversal of plates A and B ; also that each face contributes its own corpuscular secondary radiation to an extent depending only on the atomic weight of the substance of which the interior surface layer of that face is composed. This secondary radiation is almost independent of the thickness, provided the latter exceeds a millimetre or two. Moreover, the secondary radiations are so absorbed that there is little or no evidence of any continuous rebound from face to face. Each surface produces its own secondary radiation to an extent which is mainly independent of the character of the other faces.

Summary.

1. Change of hardness of the γ rays makes little difference in the relative intensities of the incident corpuscular secondary radiation from various elements, and the intensities follow the order of atomic weights of the radiators. (Table I. and curve I. fig. 2.)

2. Change in hardness of the γ rays makes a marked difference in the relative intensities of the emergent corpuscular secondary radiation from various elements. Such radiations do not follow the order of atomic weight. (Table III., fig. 3.)

3. Hardening of the γ rays is readily produced by lead screens, but not by iron screens of great thickness.

4. On hardening the γ rays the emergent radiation from carbon and aluminium is increased relatively to heavier substances ; on softening, decreased. (Figs. 3, 5.)

5. With various radiators, and with the secondary rays due to the primary γ rays from radium C, the values of λ and λ/d , and the emergent corpuscular secondary radiations were observed (fig. 5). These all indicate the soft character of the secondary γ rays.

6. The γ rays from uranium X caused emergent radiation of a character showing that these rays are softer than those from radium C, and harder than the secondary γ .

7. The gradual changes in the character of the emergent corpuscular secondary radiation were observed when the exciting rays were varied from a very hard to a very soft type. (Figs. 3, 5.)

8. When reversals of plates are made, as described, the four readings obtained are nearly in proportion (Table IV.)

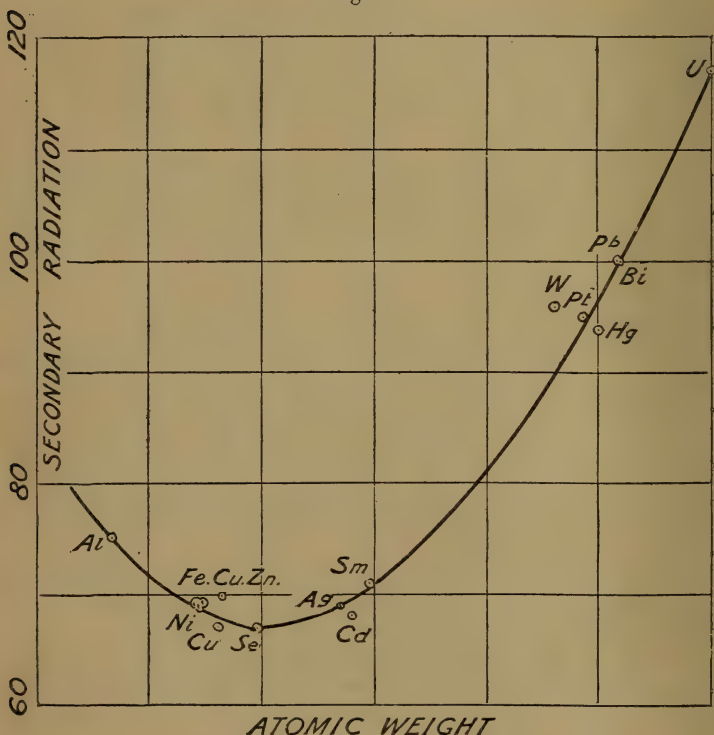
Addendum.

Since completing this work I have obtained a copy of F. E. Hackett's paper in the Transactions of the Royal Dublin Society (Jan. 1909) entitled "The Secondary Radiations excited by γ Rays."

His method differed from mine, for his apparatus was so arranged that the emergent secondary rays passed obliquely to a testing vessel which was not itself traversed by the exciting γ rays.

His results are shown graphically by me in fig. 6. As the primary γ rays were screened by only 2 mm. of lead, it is evident that these rays will partly consist of a soft type; and indeed the shape of the curve (fig. 6) is such that it lies

Fig. 6.



between curves I. and II. fig. 5, that is, between the curves for uranium and secondary γ . It will be remembered that the uranium γ rays in my experiment were hardened by passage

through the uranyl nitrate itself which contained the radiating uranium X. If more lead had been used by Hackett his curves would, I think, resemble those in fig. 3. It is still an open question how near the points lie to a definite curve. So far as I can judge, repeated observations and increased care in attention to details tends to bring the points nearer and nearer the curves shown in this paper, which for soft rays are nearly parabolic.

On the other hand, the points for incident rays do not seem to be on a curve; and there is therefore good reason to expect that the points for emergent rays will show a corresponding irregularity.

XXXVI. *The Arthur Wright Electrical Device for evaluating Formulæ and solving Equations.* By ALEXANDER RUSSELL, M.A., D.Sc., and ARTHUR WRIGHT, M.I.E.E.*

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- VII. The Use of the Arms of the Bridge.
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- IX. Equations of Degrees higher than the Third.
- X. Imaginary Roots.
- XI. Solutions of Equations containing Miscellaneous Functions.
- XII. Solution of Transcendental Equations.
- XIII. Tracing Curves Electrically.
- XIV. Conclusion.

I. *Introduction.*

IF an ordinary algebraical expression consisting of several terms be computed by the slide-rule or by logarithmic tables, each term must be computed separately and the sum of the negative terms subtracted from the sum of the positive terms. If all these operations were done mechanically it would often save time and lessen the risk of error. In a few cases the form of the expression can be altered with advantage so as to adapt it to logarithmic computation, but there are many cases where the terms of the expression have to be evaluated separately, and in these cases it saves mental labour to do the operations mechanically.

The electrical device described below, which was invented

* Communicated by the Physical Society: read June 11, 1909.

by Mr. Arthur Wright, enables us to evaluate almost all mathematical expressions by simple mechanical and electrical operations. In the present model the accuracy is not high, being of the order of one per cent.; but this accuracy suffices in most engineering calculations. The apparatus, however, can be easily elaborated to give a far higher accuracy. The main object of the authors in this paper is to explain the principles on which it works, and to point out a few of its many applications in practical calculations.

Although some of the operations described below—as, for example, the finding of the roots of a numerical algebraical equation of the n th degree—can easily be done by the expert mathematician, yet even in his case the apparatus can be a help, as it gives him at once the approximate values of the roots. In general, he can then find them to any required degree of accuracy by the methods given in the Theory of Equations.

II. *Historical.*

The history of calculating machines, and an excellent description of many of them, is given by M. d'Ocagne in his book *Le Calcul Simplifié* (1905). Valuable information is also given in Professor Henrici's article on "Mathematical Instruments" in the *Encyclopædia Britannica* (10th edition). We need only mention, therefore, the statical method* of solving equations due to Professor Vernon Boys, the Torrès logarithmic arithmophore†, which is a very elaborate piece of mechanism, and the extremely ingenious electrical method‡ due to M. F. Lucas, of finding the real and imaginary roots of an equation of the n th degree. The last method, however, is not rigorously accurate and would be very laborious to apply in practice.

III. *The Slide Resistances.*

In the electrical device described below the principle of the ordinary logarithmic slide-rule is combined with addition and subtraction, by utilising the laws according to which resistances combine in series or parallel. The products found by the slide-rule method are represented either by the resistances or by the reciprocals of the resistances of certain wires. In the latter case the currents through them can easily be added or subtracted by a Wheatstone Bridge Method. Hence the sums or differences of the products can be found.

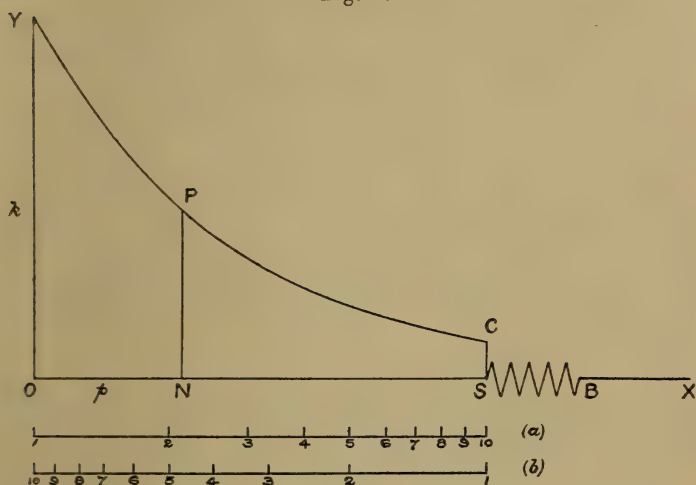
* Phil. Mag. vol. xxi. p. 241 (1886).

† M. d'Ocagne, *l. c. ante*, p. 123.

‡ *Comptes Rendus*, t. cvi. p. 1072 (1888).

The method of constructing the slide resistances (fig. 1) is as follows:—A template of thin insulating material is made

Fig. 1.



in the shape OSCY, shown in fig. 1. The equation to the curve YPC is

$$y = k 10^{-x/h},$$

where $OY=k$, $OS=h$, and $SC=k/10$.

This plate is wound uniformly with a hundred turns of No. 36 insulated manganin wire, the temperature coefficient of which is negligible, and the wires are practically parallel to OY. At points along OS the insulation of this wire is removed, and contact with the wires is made by the contact-finger described below. The scale OS is logarithmic. If p , for instance, be the reading at the point N, we have

$$ON = h \log p,$$

and thus

$$y = PN = k/p.$$

The resistance of the wire between O and N will be approximately proportional to the area ONPY. This area is given by

$$\begin{aligned} \int_0^x y \, dx &= k \int_0^x 10^{-x/h} \, dx \\ &= \frac{hk}{\log_e 10} \left(1 - \frac{y}{k}\right). \end{aligned}$$

If, then, the wire on the frame have a resistance $9R/10$,
Phil. Mag. S. 6. Vol. 18. No. 104. Aug. 1909. X

and a coil SB of resistance $R/10$ be put in series with it, the resistance from N to B will equal $(y/k)R$, that is, R/p .

When the resistances are fixed on the device, the resistance between N and B is in circuit, and this always equals R/p , where p is the reading on the top scale [(a), fig. 1]. If we have n of these slide resistances in parallel, and their readings are $p_1, p_2, \dots p_n$, the sum of the currents will be

$$(E/R) (p_1 + p_2 + \dots + p_n),$$

where E is the applied potential-difference.

Instead of having the logarithmic scale placed as in (a), fig. 1, we may reverse it and place it as in (b). If p' be the value of ON read on this scale, we obviously have $p' = 10/p$; and hence the resistance between N and B is $(R/10)p'$. If the n resistances be now connected in series, their combined resistance will be

$$(R/10) (p'_1 + p'_2 + \dots + p'_n);$$

and if E be the applied potential-difference, the current flowing through them will be

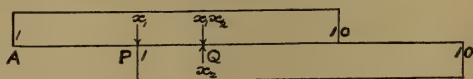
$$(10 E/R) / (p'_1 + p'_2 + \dots + p'_n).$$

In some problems it is more convenient to use the scale placed as in (a), and in others it is more convenient to use it as in (b). In what follows, unless otherwise stated, we shall suppose that it is placed as in (a).

IV. *Multiplication. The Index Line.*

The method of performing multiplication is practically identical with that utilised in the ordinary slide-rule, which was originally designed by Seth Partridge*. In Partridge's instrument two logarithmic scales (fig. 2) slide with their edges

Fig. 2.



in contact. If the reading on the top scale at P equals x_1 , and on the lower scale at Q equals x_2 , the reading on the top

* 'The Description and Use of an Instrument called the Double Scale of Proportion.' London, 1671. Near the beginning of the book we read: "Here might have been expected a print of the rule, but in regard to its sliding it could not well be demonstrated: wherefore I thought good to advertise that this Scale and all other Mathematical Instruments are accurately made by Mr. Walter Hayes at the Cross-Daggers in More-Fields, next door to the Popes-Head-Tavern, London." Slide rules, therefore, were for sale in London 238 years ago.

scale at Q equals x_1x_2 . This follows because by the construction of the scales,

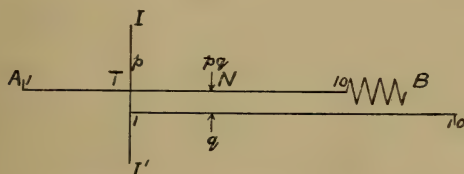
$$AP = h \log x_1, \quad PQ = h \log x_2,$$

and thus

$$\begin{aligned} AQ &= AP + PQ \\ &= h \log x_1 + h \log x_2 = h \log x_1x_2. \end{aligned}$$

In the Arthur Wright device the slide resistances are movable. If we wish to form the product pq , we move the slide so as to make the index-line II' (fig. 3) lie over

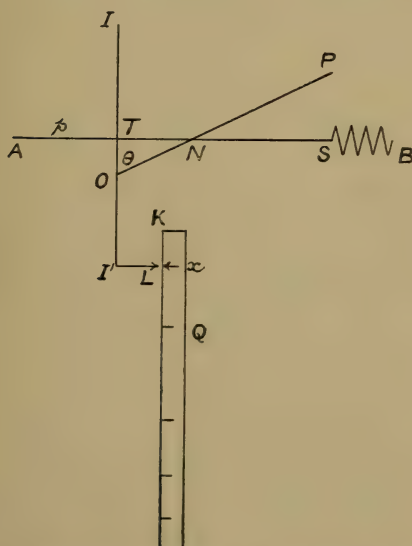
Fig. 3.



the reading p , and we then move the contact-finger until it makes contact at N, where TN, on the logarithmic scale, is q . The resistance NB will then equal R/pq .

V. The Contact Fingers.

Fig. 4.



The ordinary contact finger is a straight piece of metal wire ONP (fig. 4), which is capable of rotation about a

point O fixed on a sliding bar parallel to and vertically over the index-line II'. This bar can move in the direction of its length. A pointer L attached to it moves along a fixed logarithmic scale KQ. The length of this scale is made equal to h , and so

$$KL = h \log x = OT.$$

The contact finger OP can be adjusted so as to make any desired angle θ with the index-line II'. If the contact be made at N, we have

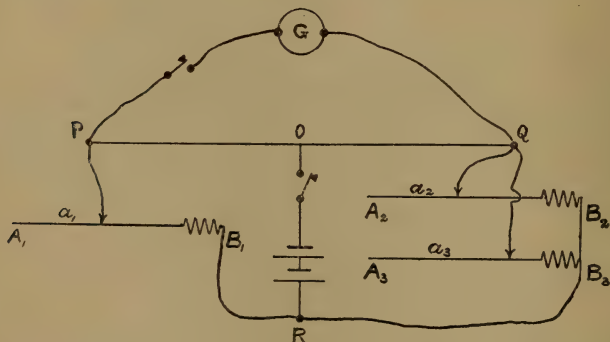
$$TN = OT \tan \theta = h \tan \theta \log x = h \log x^{\tan \theta};$$

and thus, if $AT=p$, the resistance from N to B will be $R/(px^{\tan \theta})$. By giving various values to θ we can, with the help of a table of natural tangents, obtain all positive integral or fractional powers of x . If N lie between A and T, θ is negative, and so we obtain all the negative integral or fractional powers of x .

VI. Addition and Subtraction.

In performing addition and subtraction, the action of the apparatus can be understood from the following diagram. P and Q are the terminals of a wire bridge, of which O is the middle point (fig. 5). One pole of a battery of dry cells is

Fig. 5.



connected with O and the other pole is connected with the B ends of the slide resistances, A_1B_1 , A_2B_2 , and A_3B_3 . A galvanometer connects P with Q. Let us suppose that we have to find the value of $a_2 + a_3$. We connect Q with the contact-fingers of A_2B_2 and A_3B_3 respectively, and set the fingers at the marks a_2 and a_3 on their respective scales. We then join P with the contact-finger of A_1B_1 , and closing the keys we adjust the position of this finger until there is no

deflexion on the galvanometer. In this case let the reading on the slide A_1B_1 be a_1 . If E be the potential at R , and V be the potential at P or Q , then, since the currents in PO and QO are equal, we have

$$(E - V)/(R/a_1) = (E - V)/(R/a_2) + (E - V)/(R/a_3),$$

and thus $a_1 = a_2 + a_3$.

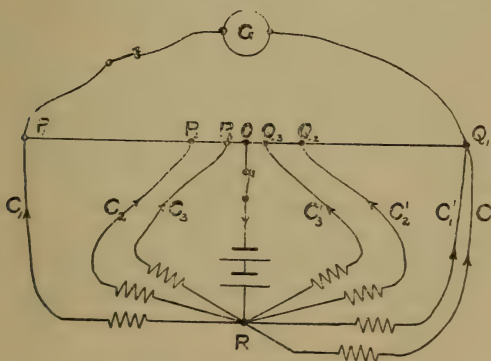
To find $a_2 - a_3$, all we have to do is to change the connexion of the finger of A_3B_3 from Q to P . Proceeding as before, a_1 now gives us the value of $a_2 - a_3$.

When subtracting approximate values, it has to be remembered that the percentage error in the result may be large, especially when $a_2 - a_3$ is small compared with a_3 . This difficulty, which arises when subtracting the approximate values of nearly equal quantities, affects all approximate methods. In our case it can only be overcome by increasing the accuracy of the device.

VII. The Use of the Arms of the Bridge.

Terminals are connected at points P_1, P_2, P_3, Q_3, Q_2 , and Q_1 of the bridge wire (fig. 6). These points are chosen so that if the resistance P_1Q_1 be $2r$, the resistances of OP_1, OP_2 , and OP_3 are equal to $r, r/10$, and $r/100$, and also to the resistances of OQ_1, OQ_2 , and OQ_3 respectively.

Fig. 6.



Let the magnitudes of the currents in RP_1, RP_2, RP_3, RQ_3 , and RQ_2 be C_1, C_2, C_3, C_3' , and C_2' respectively.

Let us also suppose that the currents in the two branches joining R and Q_1 are C and C_1' respectively.

The resistances in these circuits are the slide resistances described above. When there is no deflexion on the galvanometer joining P_1 and Q_1 , the potentials of these two

points are equal, and so the differences of their potentials from that at O are equal, and thus we find that

$$C_1r + C_2(r/10) + C_3(r/100) \\ = Cr + C_1'r + C_2'(r/10) + C_3'(r/100).$$

Hence

$$C = C_1 + C_2/10 + C_3/100 - (C_1' + C_2'/10 + C_3'/100).$$

Let E be the potential of R, and V_1 the potential of P_1 and Q_1 (fig. 6). Then, if V_2, V_3, V_2', V_3' be the potentials of P_2, P_3, Q_2 , and Q_3 , and $p_1, p_2, p_3, q_1, q_2, q_3$, and x be the readings on the scales of the slide resistances in the branches RP_1, RP_2 , &c., we have

$$(E - V)x = (E - V_1)p_1 + (E - V_2)(p_2/10) + (E - V_3)(p_3/100) \\ - \{ (E - V_1)q_1 + (E - V_2')(q_2/10) \\ + (E - V_3')(q_3/100) \};$$

and therefore

$$x = p_1 + p_2/10 + p_3/100 + \Delta_p - (q_1 + q_2/10 + q_3/100 + \Delta_q),$$

where

$$\Delta_p = \frac{V_1 - V_2}{E - V_1} \cdot \frac{p_2}{10} + \frac{V_1 - V_3}{E - V_1} \cdot \frac{p_3}{100},$$

and

$$\Delta_q = \frac{V_1 - V_2'}{E - V_1} \cdot \frac{q_2}{10} + \frac{V_1 - V_3'}{E - V_1} \cdot \frac{q_3}{100}.$$

The readings p_1 and q_1 on the scales cannot be less than 1, and the readings p_2, p_3, q_2 , and q_3 cannot be greater than 10. Hence, remembering that $(V_1 - V_2)/(E - V_1)$ cannot be greater than $(V - V_3)/(E - V_1)$, we see that if $(V_1 - V_3)/(E - V_1)$ be equal to or less than $1/53$, Δ_p is not greater than the hundredth part of $p_1 + p_2/10 + p_3/100$. Similarly, if $(V_1 - V_3')/(E - V_1)$ be equal to or less than $1/53$, Δ_q is not greater than the hundredth part of $q_1 + q_2/10 + q_3/100$. We have therefore to arrange the relative values of the resistances of the slides and the bridge wire so that this may be true. As we have pointed out above, however, the inaccuracy in the value of x depends on the relative values of x and $p_1 + p_2/10 + p_3/100 + \Delta_p$. When these quantities are nearly equal, approximate methods of computation fail.

By Ohm's law, we see from fig. 6 that

$$\frac{V_1 - V_2}{\frac{9}{10}r} = \frac{E - V_1}{R_1}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and

$$\frac{V_2 - V_3}{\frac{9}{100}r} = \frac{E - V_1}{R_1} + \frac{E - V_2}{R_2}, \quad . \quad . \quad . \quad (2)$$

where R_1 and R_2 are the resistances of the circuits RP_1 and RP_2 . Hence it readily follows that

$$\frac{V_1 - V_3}{E - V_1} = \frac{9}{10} \cdot \frac{r}{R_1} + \frac{9}{100} \left(\frac{r}{R_1} + \frac{r}{R_2} \right) + \frac{81}{1000} \cdot \frac{r^2}{R_1 R_2}.$$

If R be the resistance of a slide, the minimum possible value of both R_1 and R_2 is $R/10$. Hence the maximum possible value of $(V_1 - V_3)/(E - V_1)$ is

$$9 \frac{r}{R} + 1.8 \frac{r}{R} + 8.1 \left(\frac{r}{R} \right)^2,$$

and this is less than $1/53$ when $2r/R$ is not greater than $1/287$.

Similarly, if ρ , R_1' , and R_2' be the resistances of the slides, the readings on which are x , q_1 , and q_2 respectively, we see that

$$\begin{aligned} \frac{V_1 - V_3'}{E - V_1} &= \frac{9}{10} \left(\frac{r}{\rho} + \frac{r}{R_1'} \right) + \frac{9}{100} \left(\frac{r}{\rho} + \frac{r}{R_1'} + \frac{r}{R_2'} \right) \\ &\quad + \frac{81}{1000} \left(\frac{r}{\rho} + \frac{r}{R_1'} \right) \frac{r}{R_2'}. \end{aligned}$$

In the very unlikely case when $\rho = R_1' = \frac{R}{10} = R_2'$, $(V_1 - V_3')/(E - V_1)$ has its maximum possible value, and in this case it will be less than $1/53$ if r/R be not greater than $1/1100$. Hence the desired accuracy can be attained by making the resistance R of each slide 550 times greater than the resistance of the bridge wire.

It is to be noticed that the effect of connecting a slide resistance with P_2 instead of P_1 is to divide its value by 10. Hence, if we were adding 8.8 to 0.75, we would put the contact-finger of one slide on 8.8 and connect it with P_1 , and the contact-finger of another slide on 7.5 and connect it with P_2 . Suppose also that the value of x was 123. In this case we could only obtain a balance when it was connected with Q_3 .

VIII. *Cubic Equation.*

Let us suppose that the cubic equation is

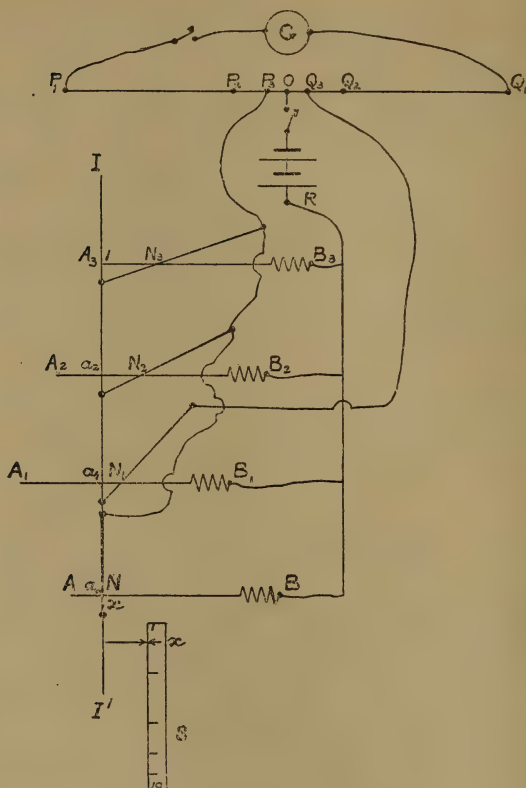
$$x^3 + a_2 x^2 - a_1 x + a_0 = 0,$$

where a_2 , a_1 , and a_0 are positive numbers.

The arrangement of the apparatus for solving this equation

300 Dr. Russell and Mr. Wright : *The Wright Electrical*
 is shown in fig. 7. The bridge-wire is the same as in fig. 6.
 We move the slides AB, A_1B_1 , A_2B_2 , and A_3B_3 so that the

Fig. 7.



readings 1, a_2 , a_1 , and a_0 on their logarithmic scales are on the index-line II' . The ends of the contact-fingers touching AB, A_2B_2 , and A_3B_3 are connected with P_3 and the finger touching A_1B_1 with Q_3 . The fingers through N, N_1 , N_2 , and N_3 make angles $\tan^{-1}0$, $\tan^{-1}1$, $\tan^{-1}2$, and $\tan^{-1}3$ with the index-line. The battery and galvanometer-keys being closed, the bar to which the fingers are connected is gradually lowered. When the galvanometer deflexion is zero let us suppose that x is the reading on the logarithmic scale S. In this case we have

$$\frac{E-V_3}{R} + \frac{E-V_3}{R} + \frac{E-V_3}{R} = \frac{E-V_3}{R},$$

$$\frac{1}{x^3} + \frac{1}{a_2x^2} + \frac{1}{a_0} = \frac{1}{a_1x},$$

and thus $x^3 + a_2x^2 - a_1x + a_0 = 0$. Hence x is one of the roots of this equation.

When a contact-finger, N_1 for instance, reaches the end of its slide N_1B_1 will be $R/10$. If we now move the slide A_1B_1 so so that A_1 is on the index-line, unclamp the contact-finger and move it parallel to itself until N_1 is over A_1 and then reclamp it, the resistance of N_1B_1 will be R . If, then, we disconnect the finger N_1 from Q_3 and connect it with Q_2 the deflexion on the galvanometer will not be altered, and so we can continue to increase x . It will be seen that as x increases from 1 to 10 the contact-finger N_3 traverses A_3B_3 three times. The first time N_3 gets to the end of its scale it is connected with P_2 and moved back to A_3 . The second time it reaches the end it is connected with P_1 and again moved back.

The device enables us to find any real root of the equation, greater than 10^n and less than 10^{n+1} . All we have to do is to find the root of the equation

$$x^3 + (a_2/10^n)x^2 - (a_1/10^{2n})x + a_0/10^{3n} = 0,$$

lying between 1 and 10, and multiply the result by 10^n . By solving a similar subsidiary equation we can find the approximate values of the roots of the equation which are less than unity.

We may use Newton's rule to find more accurate values of the roots. If a , for instance, be an approximate value of a real root of $f(x) = 0$, $a - f(a)/f'(a)$ gives usually a very much closer approximation x_1 . The failing case is when we have two roots very nearly equal to one another. The device always indicates when this occurs. If two roots are each equal to a , or if they are approximately equal to a , the galvanometer deflexion instead of passing to the other side of zero when x becomes greater than a returns to the same side.

In practice if $f'(a)$ is large when $f(a)$ is zero the device is very sensitive, but when $f'(a)$ is small and in general, therefore, when the roots are equal the device is unsensitive.

To find approximate values of the negative roots of the original equation we find by the device the roots of

$$x^3 - a_2x^2 - a_1x - a_0 = 0.$$

The imaginary roots* are most readily found by first finding as accurate a value x_1 as possible of the real root. We then divide the cubic expression by $x - x_1$ and equate the dividend to zero. The roots of this quadratic equation give the approximate values required.

* Cf. C. P. Steinmetz, 'Transient Electric Phenomena,' p. 136.

IX. *Equations of Degrees higher than the Third.*

The device can be usefully employed to find the values of the roots of equations higher than the third with sufficient accuracy for practical work. When calculating, for example, the requisite resistances for the motor controller of an electric car, the following equation * has to be solved

$$x^{n/(n-1)} + a_1x - a_0 = 0,$$

where n is the number of steps in the rheostat.

To solve this equation three only of the slides shown in fig. 7 are required. AB is connected with Q_3 , and A_1B_1 and A_2B_2 are connected with P_3 . The finger making contact with A_2B_2 is inclined at an angle $\tan^{-1}n/(n-1)$ with II' , and the fingers on A_1B_1 and AB are inclined at angles of 45° and 0° respectively. The value of x is then increased until a balance is obtained.

When solving equations containing large numbers of terms it is sometimes convenient to divide the equation by a suitable power of x . In this case the fingers making contact with slide resistances representing terms containing negative powers of x are inclined to the left of II' (fig. 8).

Let us consider, for example, a sextic equation. As shown above, we alter it so that the required root or roots are multiples or submultiples of the equation

$$x^6 - a_5x^5 + a_4x^4 + a_3x^3 - a_2x^2 - a_1x + a_0 = 0,$$

which lie between 1 and 10.

Dividing this equation by x^3 we get

$$x^3 - a_5x^2 + a_4x + a_3 - a_2x^{-1} - a_1x^{-2} + a_0x^{-3} = 0.$$

In general, seven slide resistances will be required, but if when x is put equal to unity any term is less than the hundredth part of the sum of the terms of the same sign preceding it, that term can be neglected. The slide resistances are first moved until the readings on the index-line are $a_0, a_1, a_2, \dots$ &c., as shown in fig. 8. The contact-fingers are next turned round to the requisite angles $\tan^{-1}3, \tan^{-1}2, \dots \tan^{-1}-3$, respectively. The ends of the contact-fingers are then connected with suitable points on the arms of the bridge.

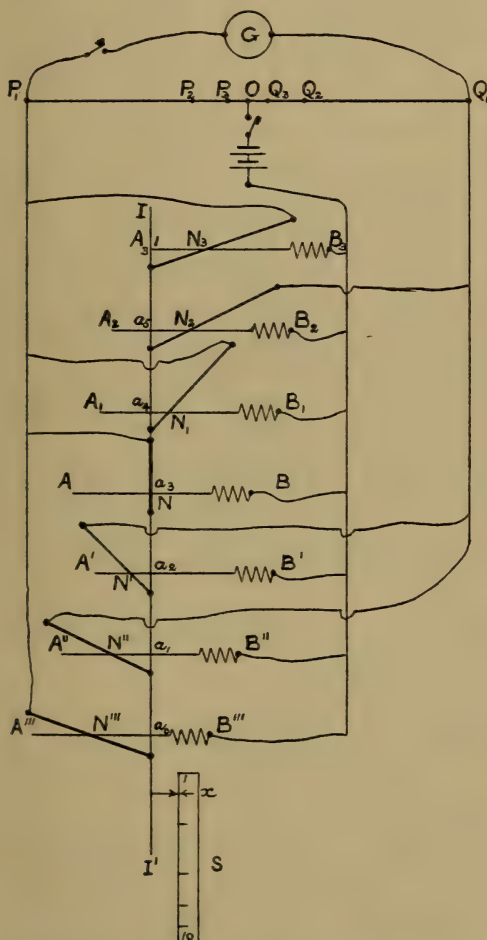
At first sight it might be thought that two extra pairs of contact-points P_4, Q_4 and P_5, Q_5 would be required in the bridge arms. If a one per cent. inaccuracy, however, is

* E. Wilson, 'Electrical Traction,' vol. i. p. 42.

permissible this is not necessary. To illustrate this point, and also the method of using the device in this case, let us consider the numerical equation

$$x^3 - 5.7x^2 + 0.12x + 8.6 - 0.04x^{-2} + 1.3x^{-3} = 0.$$

Fig. 8.



Putting x equal to unity we see that the term 0.04 can be neglected compared with 5.7 , only five slide resistances therefore are required. We connect A_3B_3 with P_2 , A_2B_2 with Q_2 , A_1B_1 with P_3 and make the reading on it 1.2 , AB with P_2 and $A'''B'''$ with P_2 . The finger on the slide $A'''B'''$ gets to the end A''' of its scale first. We then disconnect it from P_2 and connect it with P_3 . We also alter the reading

on the slide resistance $A'''B'''$ to 10. When the fingers on the resistances A_2B_2 and A_3B_3 get to the ends of their scales we connect them with P_1 and Q_1 respectively. When, however, the finger on the resistance A_3B_3 gets for the second time to the end of its scale we disconnect altogether the wires connected with P_3 , and move the wire connected with P_2 to P_3 and the wire connected with Q_1 to Q_2 . In working the device these operations seem quite natural and little thinking is required. It is also easy to see that if a one per cent. inaccuracy is permissible it is unnecessary to have more than three pairs of terminals on the bridge arms.

X. *Imaginary Roots.*

In solving certain engineering problems in connexion with finding the amplitudes, the damping factors, and the periods of certain mechanical and electrical oscillations, a necessary step is finding the imaginary roots of certain algebraic equations. Quadratic equations present no difficulty, and we have already shown how approximate values of the imaginary roots of cubic equations can be found.

With the biquadratic equations, however, which occur when discussing the theory of the parallel running of alternators*, the oscillations set up in coupled electric circuits in wireless telegraphy†, &c., both pairs of roots are sometimes imaginary. In this case we proceed as follows:—Let $x + yi$ be a root of the equation $f(z) = 0$. Then $f(x + yi) = 0$, and hence, expanding by Taylor's theorem, we have

$$f(x) - \frac{y^2}{2} f''(x) + \dots + iy \left\{ f'(x) - \frac{y^2}{3} f'''(x) + \dots \right\} = 0,$$

and thus we must have

$$\left. \begin{aligned} f(x) - \frac{y^2}{2} f''(x) + \frac{y^4}{24} f^{iv}(x) &= 0 \dots (a) \\ \text{and} \quad f'(x) - \frac{y^2}{6} f'''(x) &= 0 \dots (b) \end{aligned} \right\}.$$

From (b) we get y^2 in terms of x , and substituting this value of y^2 in (a) we get an equation of the sixth degree to find x . The two real roots of this equation can be found by the machine arranged in the manner shown in fig. 8, and the pairs of corresponding values of y are given at once by (b). Approximate values of the four imaginary roots can thus be rapidly found. If a higher degree of accuracy be required we can either use Newton's method of approximation, or better apply Horner's method to the auxiliary sextic.

* A. Russell, 'Alternating Currents,' vol. ii. p. 184.

† J. A. Fleming, 'Electric Wave Telegraphy,' p. 209.

Theoretically we can always obtain approximate values of all the roots real or imaginary of an equation of the n th degree by the device, as y^2 can always be easily eliminated from the equations corresponding to (a) and (b) by Sylvester's method*.

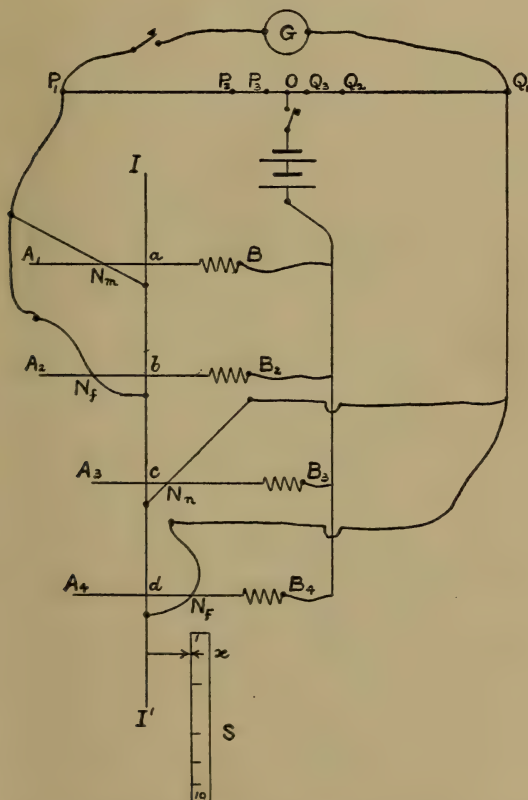
XI. Solution of Equations containing Miscellaneous Functions.

For this purpose some of the contact-fingers are made from wires bent into the shape of certain curves. Suppose, for example, we desire to find the roots of the equation

$$\frac{a}{x^m} + \frac{b}{f(x)} = cx^n + dF(x),$$

where the values of $f(x)$ and $F(x)$ have been computed, or found experimentally, for values of x .

Fig. 9.



The wire passing through the point N_f (fig. 9) is bent so

* Burnside and Panton, 'Theory of Equations,' p. 296.

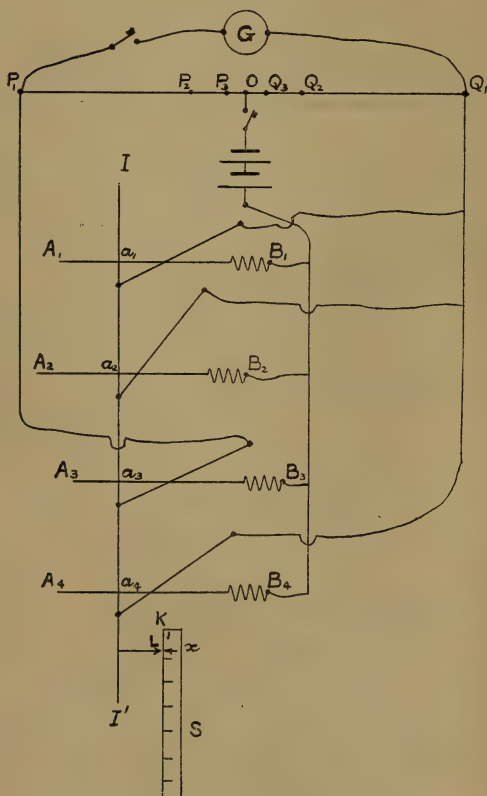
that if y be a horizontal ordinate and x the vertical abscissa on the logarithmic scale, $y = h \log f(x)$, where h is the length of the scale on the slide resistances. Similarly the equation to the wire through N_F is $y = h \log F(x)$.

The contact-fingers through N_m and N_n are set so that their inclinations to the vertical are fixed at $-\tan^{-1} m$ and $\tan^{-1} n$ respectively. The rod II' is then gradually lowered and the readings on the logarithmic scale S , when the deflexion on the galvanometer is zero, give the roots of the equation lying between 1 and 10.

The use of a bent contact-finger to take into account the "hysteresis loop" of iron would be of use in certain electrical problems.

XII. Solution of Transcendental Equations.

Fig. 10.



Let us suppose that the equation we have to solve is

$$a_1 l_1^{b_1 x} + a_2 l_2^{b_2 x} - a_3 l_3^{b_3 x} + a_4 10^x = 0.$$

In this case (fig. 10) the scale S is an ordinary scale, the length KL being equal to $h\alpha$. The various contact-fingers are adjusted so that the angles they make with the index-line II' are $\tan^{-1}(b_1 \log l_1)$, $\tan^{-1}(b_2 \log l_2)$, $\tan^{-1}(b_3 \log l_3)$ and 45° respectively. We connect the contact-fingers as in fig. 10 and vary x . The values of x for which there is no deflexion on the galvanometer are the roots of the given equation lying between 1 and 10. By altering the given equation to one whose roots are ten times smaller we find the roots lying between 10 and 100, and proceeding in this way we can get approximate values of all the roots.

By similar settings of the contact-fingers, and by using both logarithmic and ordinary scales, approximate values of the roots of very complicated equations can sometimes be easily found.

XIII. *Tracing Curves Electrically.*

Suppose we desire to find the graph of the curve

$$y = a_p x^p + a_q x^q - a_r x^r + \dots = f(x),$$

where $p, q, r, \dots$ may be positive, fractional, or negative indices. We set the contact-fingers at angles $\tan^{-1} p$, $\tan^{-1} q, \dots$ with the index-line, and move the slide resistances until the readings on the scales are $a_p, a_q, \dots$. The contact-fingers on the slide resistances representing the positive terms are then connected with P, and the other contact-fingers with Q. In addition we have a slide resistance Y with a vertical finger, which is also connected with Q. The fingers are moved down through a given distance x on the logarithmic scale, and the value of the reading y on the slide resistance Y when there is no deflexion of the galvanometer is then found. In this way simultaneous values of x and y can be rapidly obtained, and hence we can readily plot the curve.

The curve could also be traced automatically by making the spot of light from a mirror galvanometer, connected between P and Q, fall on a strip of sensitive paper which is constrained to move so that its velocity is always proportional to the rate at which x is increasing. The points where the trace cuts the line of zero deflexion would give the roots of the equation $f(x)=0$, and the turning points of the trace would give the roots of $f'(x)=0$. In the same way the integral curve

$$y = \int_0^x f(x) \partial x,$$

could be drawn automatically.

If the slide resistances were made of large size so that they could carry appreciable currents, recording ammeters and voltmeters could be employed to trace the curves.

XIV. Conclusion.*

We think that if engineers and physicists recognize that approximate values of roots of very complicated equations can be easily obtained, it may considerably extend the usefulness of theory. Authors are often diffident to publish results which can only be utilized when the roots of equations of degrees higher than the third can be found, or when equations involving miscellaneous functions can be solved. In these cases we hope that a knowledge of the methods of using logarithmic slide resistances described above may be of practical value by showing how the theoretical results can be immediately utilized.

We hope also that this preliminary sketch will induce others to improve the method, and to apply it to other and possibly more practical uses. It seems, for example, particularly suited to harmonic analysis as the integrals representing the coefficients of $\sin nx$ and $\cos nx$ in the expansion of $f(x)$ can be readily found.

In conclusion, we shall quote from the quaint but spirited preface to Seth Partridge's book on the slide rule (1671).

"I am sure here is a good Subject, a good piece of Cloath, if the Garment be not marred in the making; if it be, the fault is in the botching Taylor, not in the stuffe."

XXXVII. On the Localization of the Direction of Sounds.

By LOUIS T. MORE, Ph.D., Professor of Physics, The University of Cincinnati*.

IN a paper, written with the co-operation of Dr. H. S. Fry †, I published some experiments which indicated that the phase relations of sound-waves played a part in our determination of the direction from which sounds were heard. The experiments had been made some years previously and laid aside until further work could be done. They certainly showed that, by altering the phase difference of the waves entering the two ears and without appreciably changing the intensity, the direction of the sound could apparently be made to change. The weak point in our results was that the sound did not return to the medial

* Communicated by the Author.

† More and Fry, Phil. Mag. [6] xiii. p. 452 (1907).

plane of the head, shift to the other side and back again as the cycle of phase difference was completed. In the meanwhile, this was shown to be attainable by the experiment of Lord Rayleigh *. Later Professors Myers and Wilson †, using an improved form of our apparatus, made the sound shift from right to left and back again several times, as the phase difference was steadily increased, and they plotted curves from their results showing agreement between the observed and theoretical directions based on this idea. They also advanced an ingenious theory to account for the effect of the phase. Our knowledge of this subject has recently been increased by other investigators. Mr. T. J. Bowlker ‡ records observations, in which he found this influence of phase difference, and that the direction of the sound shifted over to the opposite side, but the value of his results was diminished by the form of his apparatus. By the use of short metal tubes, attached to the ears and with the open ends a very considerable distance from the source of the sound, too much importance was given to disturbances due to the direct transmission of the sound to the ears and to the effects of resonance. Apparently these, at times, were more evident than the effect sought for. Dr. Knight Dunlap § has brought out the curious fact that each person has a preference in locating the direction of a sound, which is independent of the position of the source and which varies from time to time in the same individual. The cause of this preference has not been discovered. In the same journal Mr. Joseph Peterson || has a monograph on combination tones and allied problems, in which he has collected a large amount of information on this obscure subject, and he has added a critical review of the various theories extant.

The experiments in this paper were made with an apparatus similar in principle to that in my former work, but with the improvement introduced by Myers and Wilson, which permits of a continuous variation of the difference of phase at the two ears. The apparatus (fig. 1, p. 310) consists of a brass tube, 260 cm. long and 1.5 cm. in diameter, with

* Lord Rayleigh, *Phil. Mag.* (6) xiii. p. 214 (1907).

† Myers and Wilson, *Proc. Roy. Soc.* vol. lxxx. p. 260 (1908). See also the *Brit. Journ. Psychol.*, vol. xi. p. 363, where a more complete account is given.

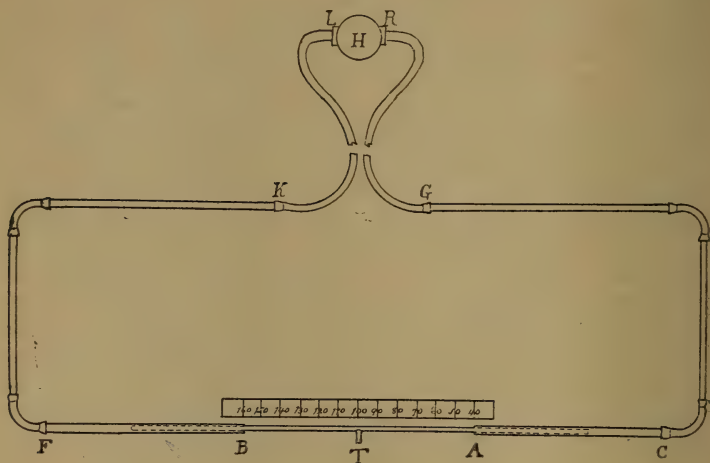
‡ Bowlker, *Phil. Mag.* (6) vol. xv. p. 318 (1908).

§ Dunlap, *Psychological Rev.* vol. x. no. 1 (1909).

|| Peterson, *Psychological Rev.* vol. ix. no. 3 (1908).

a short brass T-piece soldered to its middle point. This tube slides freely in two other brass tubes, AC and BF, each 140 cm. long. The ends of these are extended by

Fig. 1.



glass tubes, C to G and F to K, of the same diameter, and joined together by rubber tubing. In the early part of the work, two ear-caps were attached at G and K. The ear-caps were made of wooden disks, 9 centimetres in diameter, stuffed with soft annular pads of cotton-wool covered with silk. But in all the observations recorded two long rubber tubes, each 800 cm. in length and of a diameter equal to that of the other tubes, were attached at G and K and the ear-caps added to their free ends. The tubes A to G and B to K were clamped to the top of a table, and the long tubes GR and KL were laid side by side on the floor to an adjacent room, where the listener sat with the ear-caps attached to his head, H, as shown. Much care was given to making the whole system symmetrical and the two branches of exactly the same length. An examination of the results shows that this was obtained. The position of the T-piece of the slider was read from a scale, graduated in centimetres, fastened near it, the middle or symmetrical point fixed at 100.

The tuning-forks used in the experiments were clamped vertically in a box covered with wadded cloth to deaden the

sound on the outside and were struck with a constant force by a soft hammer operated by a lever outside the box. A glass funnel, with its shank protruding through the side of the box, collected the sound which then entered the slider tube by a rubber tube connected to the shank of the funnel and to the T-piece.

If rubber stoppers were inserted in the orifices of the tubes at the ear-caps, and the latter placed over the ears, no sound from the fork could be heard, but with the tubes free it could be heard clearly and the intensity was sufficient to locate its direction accurately. I found this arrangement to be much better than to have both listener and the fork in the same room, for two reasons. In the latter case the sound passing through the outside air directly to the listener caused confusion with the sound passing through the apparatus, when decisions were made. Also the listener involuntarily tried to guess the position of the slider independently of the sound heard, even though a large screen concealed it entirely from sight. The desire of the listener to guess the position of the slider was also lessened by seating him so that the line through his ears was at right angles to the sliding tube.

Unmounted Koenig tuning-forks were used as sources, and by sliding the T-piece back and forth, any difference of phase was obtainable at the two ears. In the series of observations which follow, I noted the apparent direction of the sound and, after recording it, my assistant, Mr. P. B. Evens, who managed the forks and slider, told me the position of the latter by the scale. The sequence of the positions was quite irregular and not known to me previously. The apparent direction of the sound was recorded as: middle, when the sound seemed to lie in the medial plane of the head; right or left when directly from either side; and a fourth, half, or three-fourths right or left as the angle increased by $22\frac{1}{2}^{\circ}$ from the middle to the sides.

Figs. 2 to 11 (p. 312) show the results of such a set of observations. They have been plotted in the convenient manner adopted by Myers and Wilson. The abscissæ represent the position of the T-piece in centimetres and the ordinates the apparent direction of the sound, above the axis to indicate from the right, and below it, the left. The points plotted are not means of several readings taken for the same position of the slider, but separate judgements. The dotted lines show the theoretical positions of the points.

Fig. 2. ($n = 64$).

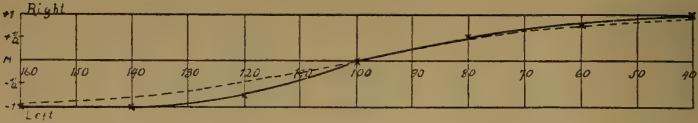


Fig. 3. ($n = 128$).

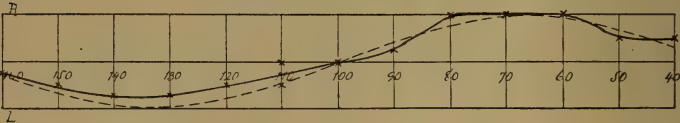


Fig. 4. ($n = 192$).

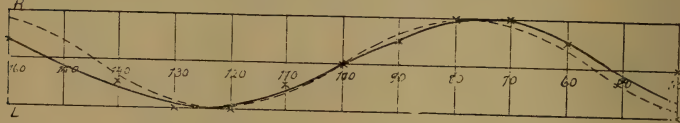


Fig. 5. ($n = 256$).

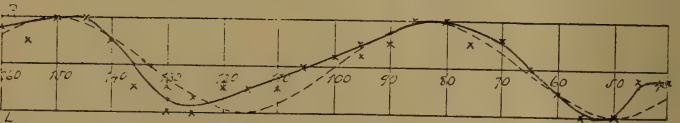


Fig. 6. ($n = 320$).

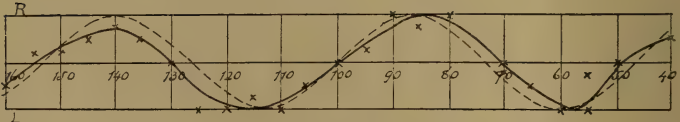


Fig. 7. ($n = 384$).

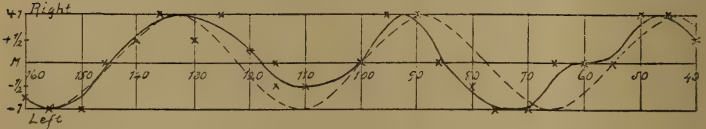


Fig. 8. ($n = 512$).

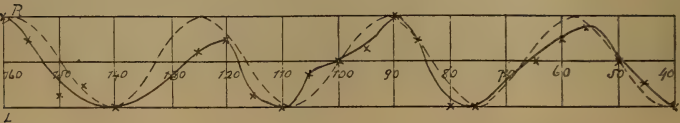


Fig. 9. ($n = 640$).

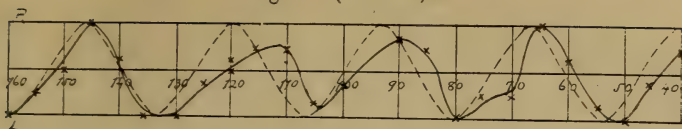


Fig. 10. ($n = 768$).

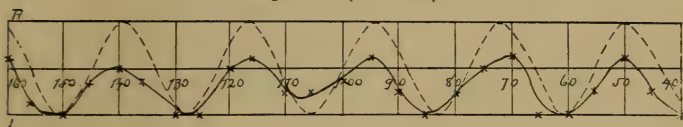
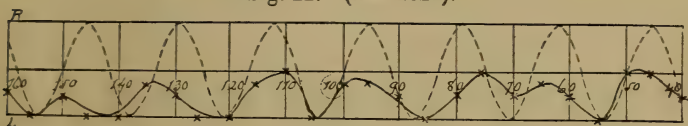


Fig. 11. ($n = 1024$).



If the phase does influence the location of the sound then no difference of phase or a difference of any whole number of half wave-lengths should cause the sound to come from the front or back and as the difference of phase increases to one fourth a wave-length, the sound should swing around to the side, and then go back to the middle as the difference increases further to a half wave-length. From a half to a whole wave-length the cycle is the same, but from the opposite side, and so on to any number of wave-lengths. The curves show this clearly, the sound apparently coming from the side whose phase is in advance. Since the shifting of the slider doubles the phase-difference, the wave-lengths have been divided by two, so that they would agree when the scale-readings were plotted instead of phase-differences.

In the series of observations given, it is unnecessary to record them in tables as they can be read easily from the curves, which are plotted from individual readings. The observations on the direction were made by myself in all of them and the close agreement to the theoretical curves which are shown dotted, I attribute, not to any keenness of hearing on my part, but to the precaution of having the sounds made in a different room and muffled by striking the forks in a padded box, thus entirely eliminating the distraction caused by hearing the sound otherwise than through the tubes.

We may conveniently divide the curves into two groups. Figures 2 to 7 inclusive, showing records of forks C, ($n = 64$ dble. vib.) to g' ($n = 384$ d. v.), give almost perfect

agreement between the experimental and theoretical curves. In figures 8 to 11 inclusive there is a progressively increasing discrepancy between the curves. In all cases I noted changes of direction which followed a sinusoidal curve, but there was besides an increasing difficulty of locating sounds which theoretically should be from the right. When the frequency 1024 d.v. was reached in the series, all the sounds apparently came from the left in varying degrees. The judgement for these high frequencies was also somewhat further confused, as shown by an inclination to locate sounds near the middle or slightly to the left. With practice the regularity of the curves was improved but they invariably showed this decided preference in location. On trying the judgement of Mr. Evens in locating directions, I found that he had an even more decided preference, but his was for the right side, and also that 1024 d.v. was practically the limit of his power of determining direction in this manner.

The cause for this preference, which agrees with Dr. Dunlap's conclusions, is unknown, but there are some facts which apparently have a bearing on it. My own hearing is about normal, although my right ear is a little more acute. Mr. Evens is not normal, as he heard with considerable difficulty with his right ear. In both cases then the region of preference lies on the weaker side. In the next place the half wave-length for frequency 1024 is about 16 cm., the average distance between the ears. For waves of such length the head must cast a very appreciable sound shadow and so make possible the determination of direction by an immediate comparison of the intensities at the two ears. The mind would then have no training for phase-differences with these waves. But these facts are possibly only coincidences.

We may judge then from these experiments that c'' ($n = 512$) is near to my limit for accuracy of judgement by phase-differences and that accuracy decreases for higher pitches, becoming untrustworthy at a pitch of about 1024 d.v.

Although no detailed investigation was made with sounds of a higher pitch, yet a qualitative trial with a fork of pitch 3000 d.v. approximate, showed that absolutely no sensation of direction existed. The fork was struck and then held close to the orifice of the T-piece, while the latter was slowly moved to the right or left. The only result was the sensation of a very shrill uniform sound a long way off, whose direction was impossible to locate.

It has not yet been shown whether the perception of

direction requires the use of two ears, and it is hard to determine because total deafness in one ear alone is rather rare. Experiments made on Mr. Evens and others who are hard of hearing in one ear, prove that they locate directions about as readily and as accurately as those whose hearing is quite normal. But these do not answer the question as the auditory nerve in each ear is usually unaffected, the trouble being with the drum or some other mechanical defect, and sufficient sound reaches the inner ear to produce some sensation. I was fortunate in finding a person whose physical ears were perfect but who was totally deaf in one ear, and normal in the other, in fact the good ear was abnormally acute.

Miss S., after an attack of spinal meningitis at the age of four years, entirely lost the sense of hearing in the *left* ear. Examination by aurists showed that the nerve of this ear was atrophied, and the power of hearing gone. She tells me that she experiences no difficulty except that she has no faculty of telling the direction from which sounds come. On hearing a sound she turns her head indifferently in either direction until the object making the sound is seen.

To test her statement the following experiments were made.

1. The ear-caps were applied to the head and a fork sounded, as in the previous experiments. The right (good) ear was stuffed with cotton and the orifice of the tube into the ear-cap closed with a rubber stopper. The tube to the left (deaf) ear remained open. No sound was heard.

2. Same arrangement as in 1, except that the fork was mounted on a resonator and held directly to the T-piece. In this case also no sound was heard.

The intensity of the sounds used in these and in the previous experiments was evidently too feeble to produce sensation by conduction through the head.

3. A test of locating sounds was made in the same manner as described earlier in this paper. In all positions of the slider, Miss S. merely heard a uniform tone issuing from the right tube. She apparently could not understand what I meant by asking if she noted any shift or change in the direction of the sound, as it seemed to her to be without direction.

4. Lastly, a test was made to see if she could determine direction without the use of the apparatus. She was seated blindfolded in the centre of a room. As a source, a fork with frequency 256 d. v. was used, mounted on its resonator. She apparently could determine direction to an extremely

limited degree when the sound was made on her right, but she was quite in error for sounds from the medial plane or from the left. This inaccurate power of determining direction I attribute entirely to changes in loudness. If the fork was held well to the right side and the mouth of the resonator gradually and slowly closed by the palm of the hand, she stated that the fork had been moved to her left side.

The use of two ears for comparison is probably necessary for any but a most rudimentary determination of directions of sounds. What power there is, seems to be due to the increase or decrease in the intensity of the sound produced by a decided lateral position of the sounding body. There is no faculty of locating by phase relation in such a case.

Combination of Two Tones.

The tone given by a musical instrument usually consists of a fundamental combined with several overtones of relatively feeble intensity, and we undoubtedly locate their position by the fundamental wave, ignoring the higher tones. This, for example, was found to be the case when an organ-pipe was used, as no confusion occurred from the presence of its harmonics. But the conditions are altered when two forks of different pitch are struck with equal force, or still better the shriller one a little harder, and held to the T-piece simultaneously.

1. Forks c' ($n=256$) and c'' ($n=512$) were tried in this manner. The two tones combined so that the separate notes were heard with great difficulty. At 100 the direction was middle, at 40, $1/2$ left, and at 160, $1/2$ right. Comparison with figs. 5 and 8 shows that the lower tone exerted the greater influence.

The results obtained with chords of less perfect harmony were not so simple.

2. Forks c' ($n=256$) and e' ($n=320$).—At 100, the tones combined in a chord, whose direction was middle, and the component tones were not heard.

At 40, e' was heard distinctly and by the right ear only with a location of full right; c' was heard by the left ear only with a direction of half left. Besides these, the combination chord was noted as being rather indistinct and near the middle.

At 160, e' full left, and c' one-half right. The chord, indistinct and one-fourth right.

Figs. 5 and 6 show that the direction of the separate notes followed the curve as if each were alone present, and that the location of the chord was rather more influenced by the lower tone.

3. Forks c' ($n=256$) and g' ($n=384$).—In general, the lower tone was completely blotted out, the chord was heard either to the right or left, and the higher tone was usually quite distinct and its direction clearly indicated. For the most part the direction of the chord was most strongly influenced by the theoretical direction of the higher component. But at certain positions of the slider, the sound g' became quite feeble and even inaudible, leaving only the chord. The positions where this occurred were when the theoretical directions of the two component waves, taken separately, were both in coincidence, either right, left, or middle.

Professors Myers and Wilson have offered an ingenious theory to explain the effect of phase-difference on hearing. According to this theory the difference of phase is a primary cause of the changes in locating sounds, and acts by producing an inequality between the intensities of the sound inside the ears. For example, a sound-wave enters the right ear with a certain phase, and at the inner ear encounters and combines with a portion of the sound which entered the left ear and passed through the head to the inner right ear. Similarly, the sound-wave enters the left ear but at a different phase, and there meets sound passing through the head from the right side. They assume "That the displacements in the internal ear due to the two sets of waves are in opposite directions. It should be said that the principal reason for making this assumption is that it enables an explanation of the lateral effects to be given." Now the phases of the waves entering the ears from without are different, therefore the diminution of the intensities of these waves by the combination with the waves conducted through the head is unequal in the two ears. And we are supposed to judge the direction of the sound by comparing the two unequal intensities of the combined waves in the internal ears. In further explanation, they say, "The distance between the ears through the head is small, and the velocity of sound through the bones probably very high, so that we should not expect a reversal of the effect due to this cause, unless the frequency were very great. But with high frequencies the lateral effects cannot be obtained. The amount of sound which must get through the head to produce an appreciable

difference between the intensities at the two internal ears is not large, because since the two amplitudes are added, an imperceptible amount getting through might produce an appreciable difference of intensity."

The theory is very ingenious at first sight, but it does not seem adequate when considered closely, as there are a number of facts which contradict it, and others which are not explained by it.

In the first place, the head always casts something of a sound shadow, even with deep tones, and the intensity at the averted ear is always a little less, no matter what the length of the sound-wave may be. This direct difference of intensity is probably as great, if not considerably greater, than the difference of intensity caused by the interference of the sound in the internal ears. If so, why is it not simpler to refer localization of sound always to differences of intensity of this kind at the outer ears?

This opinion is supported in several ways. In my experiments the sounds were purposely made feeble, so feeble that sufficient sound did not travel through the head to produce any sensation. It is possible that even if they did not produce a direct sensation, yet they might cause an appreciable effect by interference. But this is hardly likely when we remember that these feeble tones were as readily affected in direction as loud ones. Certainly such minute interferences should at least render the effect less obvious. Also if one tube were suddenly pinched, so as largely to reduce the direct effect of intensity at one ear, no confusion was caused to the judgement, yet the relative intensities of the compound tones in the two internal ears must have been as suddenly and as greatly altered, and this should by their theory shift the direction of the sound. This is not the case. An experiment was devised which makes this objection very forcible. A thick felt pad was inserted between one ear-cap and the head. In the pad a hole was cut the size of the opening of the passage into the ear and in line with it. The direct sound entering the ear was thus decreased but slightly while the amount of sound conducted through the head from this side was much diminished, since the sound which would have struck the skull in the neighbourhood of the orifice of the ear was stopped. Now if the pad was suddenly removed the resultant intensity of the sound in the opposite internal ear was decreased, because the sound conducted through the head was increased and relatively much more than the intensity at the inner ear on the pad side, consequently the direction

of the sound should shift unmistakably. No such effect could be detected.

So far we have discussed only the simplest case, when the sounds approach through tubes and impinge only on the ears. But if the theory is to hold for the practical locating of sounds in the open air, the idea of interference in the inner ears becomes still more inadequate to account for the facts. Let a musical note be struck behind a person and at an angle of 20° with the medial plane, then besides the sounds travelling through the head from one ear directly to the other, we have the entire back part of the head struck by the waves of sound. These waves travel through the head to the inner ears with different lengths of paths, at varying angles, and together make an intensity certainly much greater than the waves going from one ear to the other. It would be extremely difficult to predict what the resultant amplitude would be when all these motions with their differing phases compounded with the sound entering the external ear. Locating sounds would be to say the least a complicated matter.

It is also readily understood that this theory would lead us to expect that simple musical tones would be easier to locate than noises which have not so simple a wave form, while the contrary is proved by experiment.

None of these objections applies to a simple and direct influence of phase relations on our hearing, as they do not affect the phase of the waves.

The only objection to the idea that the ear is capable of detecting the phase of a sound or at least the difference between the phases of two sounds, is that it is difficult to reconcile with our theories of audition. But is it not a fact that we know extremely little about the mechanism of audition, and still less about the resultant nervous stimulation? The experimental facts to me, at least, seem irreconcilable to the theory of locating sounds in all cases by intensity variations, and none of the objections made in this paper are pertinent to the appreciation of phase-differences. It is of course possible that neither of these is the direct cause, which may be something in the sound-wave affecting us by some unknown psychological influence.

University of Cincinnati,
April 1909.

XXXVIII. *A Method of producing an intense Cadmium Spectrum, with a proposal for the use of Mercury and Cadmium as Standards in Refractometry.* By T. MARTIN LOWRY, D.Sc., F.C.S.*

OF the different line spectra that are available for spectroscopic standards—hydrogen, mercury, cadmium, &c.—the simplest and purest is undoubtedly the cadmium spectrum. The visible spectrum is made up of four strong lines (red, green, blue, and dark blue), which are so narrow and of such a high degree of purity in respect of the absence of satellites that they have been used by Michelson to produce interference-bands of an order of retardation that has apparently never been reached in the case of any other lines. Michelson's measurements of the wave-lengths of the three chief Cadmium lines :—

Cd red.....	6438·4722	10 ⁻¹⁰ metre.
Cd green.....	5085·8240	„ „
Cd blue	4799·9107	„ „

have indeed formed the standards from which all other wave-lengths have been deduced. It is therefore evident that the cadmium spectrum is destined to play an extremely important part in optical determinations of all kinds. Unfortunately, the difficulty of producing a cadmium lamp which shall burn steadily and give out light of high intensity has been so great that the four cadmium lines have been used only very occasionally in optical experiments.

Sodium.

The standard monochromatic light employed almost universally for refractometric and polarimetric measurements has been the yellow flame-spectrum of sodium, which has the advantage of being produced with very great readiness, but with all the drawbacks inseparable from the use of a doublet, instead of a single line, as a standard. Thus in determining the refractive index n_D of a liquid, the Pulfrich refractometer gives readings for the less refrangible constituent, whilst a hollow prism mounted on a spectroscope gives an average value for the two constituents, unless indeed the resolution be sufficient to read them separately. In polarimetric work the double character of the sodium line renders it impossible to secure a proper extinction for large values of α_D , since one

* Communicated by the Physical Society : read June 25, 1909.

wave-length is transmitted with considerable intensity when the other is extinguished, and in addition there is always some uncertainty as to the "optical mass-centre" of the doublet, which may indeed vary in different types of sodium-lamp, on account of changes in the relative intensity of the two constituents *. It should also be noted that the sodium flame emits a considerable amount of light of other colours, which in accurate work, or in reading large rotations, must be removed by filtering through a coloured screen, or, better, by means of a spectroscopic eyepiece (Perkin).

Hydrogen.

The hydrogen lines,

$$H_{\alpha} \text{ (red), w.-l.} = 6560.04,$$

$$H_{\beta} \text{ (blue), w.-l.} = 4861.49,$$

$$H_{\gamma} \text{ (violet), w.-l.} = 4340.66,$$

have been universally employed with the sodium doublet in refractometric work when dispersion-values were required. The choice has been wholly one of convenience and has no other merit to recommend it. The vacuum-tube, though easily fitted up, can hardly be seriously considered as a source of light. The red line is by far the strongest, and has been used with advantage to produce interference-fringes in measurements of length †, but would be utterly useless for polarimetric work in which the source of light must be viewed through a Nicol's prism set within 2° or 3° of the extinction position. The violet line is unpleasantly weak even for refractometric measurements, and demands the use of the full power of a six-inch coil, with an efficient optical condenser, before readings can be made with any degree of comfort. The hydrogen spectrum has the further disadvantage of showing, at least in an ordinary vacuum-tube, an almost continuous back-ground of weak lines. Although, therefore, refractometers are regularly sent out with tables for the sodium and the three hydrogen lines—and no data whatever for light of any other wave-length—it is evident that this position is radically unsound and cannot be permanently maintained.

* Compare Landolt, *Optische Drehungsvermögen*, 1898, pp. 362 *et seq.*

† See, for instance, Tutton's measurements of the coefficients of expansion and of elasticity of crystal-plates (Phil. Trans. 1903, A. 202, p. 143). Compare also Tutton, Proc. R. S., June 10, 1909.

Choice of Standards.

The essential properties for a standard source of light are, (1) that it should be of sufficient intensity to be used for *all* the various types of optical measurements, so that, for instance, refractive indices and optical and magnetic rotatory powers may be determined for the same wave-lengths, (2) that it should be strictly monochromatic and as far as possible free from satellites, and (3) that it should be produced with sufficient readiness to render it generally available. These requirements, as has been shown, are only partially fulfilled by sodium light and fail completely in the case of the hydrogen spectrum. The purpose of the present communication is to suggest that the spectra of mercury and of cadmium fulfil most of the essential conditions outlined above, and to describe a method by which the cadmium spectrum may be rendered more generally available for spectroscopic work.

The suggestion—which is made on the basis of practical experience in the actual measurement of optical and magnetic rotations and of refractive indices for a large range of wave-lengths (see for instance Proc. Roy. Soc. 1908, 81. p. 472)—that sodium should give place to mercury and cadmium as a chief standard source of light, is fully supported by the theoretical considerations recently advanced by Bates (“Spectrum Lines as Light Sources in Polariscopic Measurements,” Bureau of Standards, Bulletin, 1906, ii. p. 239) and by Nutting (“Polarimetric Sensibility and Accuracy,” *ibid.* p. 249, “Purity and intensity of Monochromatic Light Sources,” *ibid.* p. 439). The former author has worked out a formula showing the errors due to the use of a doublet in polarimetry, and has redetermined the ratio of the sodium-yellow and mercury-green rotations for quartz; the latter has developed formulæ in reference to polarimetric sensibility, and spectral purity. The two points in these papers that bear directly on the practical problem now under consideration are, (1) the confirmation by Bates of the purity of the mercury green line, which gave very sharp readings in the case of a quartz plate of about $5\frac{1}{2}$ mm. thickness; this point is, however, seriously discounted by the fact that he professes to read the sodium doublet to $0\cdot0001^\circ$, and gives the ratio of sodium to mercury to six significant figures ($0\cdot850944 : 1$), (2) the statement by Nutting that on one basis of reckoning the “spectral purities” of cadmium green, mercury green, and sodium yellow, are represented by the ratios $\frac{1}{100,000}$, $\frac{1}{10,000}$, and $\frac{1}{700}$, whilst on another basis the “specific

impurities" of the mercury green and sodium yellow lines are given by the ratios $\frac{1}{1,000,000}$, $\frac{1}{2000}$; these figures serve to show that the change of principal standard now proposed on the basis of practical polarimetric work is fully justified by minute spectroscopic tests on the lines themselves.

Mercury.

The use of the enclosed mercury arc as a source of light in spectroscopy dates back to 1860 (J. H. Gladstone, "On the Electric Light of Mercury," *Phil. Mag.* [4] xx. pp. 249-253), but its use in polarimetric measurements was apparently introduced by Disch (*Ann. Phys.* (4) xii. p. 1155) in 1903, who made use of the Arons lamp. The Bastian mercury lamp, which has been in use in my own laboratory since 1906, and at the Central Technical College since 1907, has the advantage of being a commercial article of much lower cost; it is constructed with a suitable resistance in the holder, so that it can be plugged into the ordinary lighting circuit without using a resistance-frame or any special leads. This lamp is unfortunately no longer on the market, though it is still constructed to order by the Brush Electrical Engineering Company; but silica lamps of moderate price are promised which may prove to be as economical in working as, and even more efficient in illumination than, the earlier glass lamps.

Of the six chief mercury lines,

5790.49	} a yellow doublet,
5769.45	
5460.97	a splendid green line,
4358.58	a strong violet line,
4078.03	} at the extreme limit of the visible spectrum,
4046.78	

two, the green and the violet, have already proved to be of the utmost value in polarimetry, and are likely in the future to prove of equal value in the measurement of refraction and dispersion.

Their use in polarimetry has been due to the following considerations. For accurate measurements of the specific rotatory power of a substance, and to any even larger extent for tracing the course of chemical changes (isomeric change, sugar-hydrolysis, &c.) by polarimetric observations, it is essential to use an intense source of light in association with a very small half-shadow angle, since only thus can a maximum of sensitiveness be secured: it is also desirable to use a light

of relatively short wave-length, in order that the actual readings may be large *, without incurring the loss of optical intensity and the fatigue which result from the use of blue light. The intense green mercury line, which can be read with a considerably smaller half-shadow angle, and gives readings about 15 per cent. larger than the sodium doublet, is therefore much superior to the traditional standard, apart altogether from the question of spectral purity. In the latter respect the contrast is extreme ; in the case of quartz I have been able, without any noticeable loss of accuracy, to secure readings showing a total rotation of 50 right angles for the mercury green line, two independent series of determinations giving average values 4487.78° and 4487.79° ; sodium under similar conditions gave no extinction at all. The green mercury line promises, indeed, wholly to replace the sodium doublet as a chief standard in polarimetric work, and it is highly desirable that it should acquire as quickly as possible a like predominance in the measurement of refractive indices, and in all other optical determinations.

The violet mercury line has proved indispensable in the measurement of rotatory dispersion on account of its extraordinary brilliancy. In spite of the low sensitiveness of the eye for light of such small wave-length it has been found possible to read this line with a half-shadow angle of only 6° , and to secure series of readings (each an average of 10 settings) which only differed from one another by a hundredth of a degree. The violet line is less pure than the green, as it is accompanied by two satellites of smaller wave-length, but these are so weak that they cannot be seen at all in the polarimeter, and cannot, therefore, produce any appreciable disturbance in the readings. The yellow doublet is made up of two lines separated by about three times as great an interval as in the case of sodium ; for small rotations they may be read as one line, but I have also been able, by using a narrow slit, to read them separately ; they are, however, altogether unsuited for general use.

For refractometer work the mercury lines are at least as easily available as those of hydrogen ; a warmed vacuum-tube containing a drop of mercury gives the lines with greater brilliance than those of hydrogen, and it is therefore not unreasonable to suggest that—as a minimum concession to the correlation of optical measurements of various kinds—the use of $H\beta$ 4861 and H_γ 4341 shall be abandoned in favour of Hg 5461 and Hg 4359 in future refractometric work, and

* As a rule the specific rotation is doubled on passing from yellow to violet.

that tables for these wave-lengths shall be supplied as a matter of course with instruments of the Pulfrich pattern. It may be noted that the mercury and hydrogen violet lines differ by only 18 Ångström units, the mercury line having the longer wave-length: in a Pulfrich instrument the two lines are indistinguishable, but the edge that is read with a hydrogen-mercury vacuum-tube (such as is sometimes sent out with the instrument) is due to mercury and not hydrogen. The adaptation of a polarimeter for use with mercury light costs about £2, with a further £3 for the lamp.

Cadmium.

The cadmium spectrum is much less easy to produce than that of mercury. Michelson made use of a strongly heated vacuum-tube with aluminium electrodes connected to platinum wires passing through the glass. This was improved upon by Hamy (*Comptes Rendus*, 1897, cxxiv. p. 749) who used a copper heating-jacket and external electrodes, thus avoiding the risk of cracking the hot glass by wires passing through it. I have had no personal experience of such lamps, but am doubtful whether they would give a sufficiently intense light for use in polarimetry. The amalgam lamp with an arc enclosed in silica can be made to give a splendid series of lines for use in spectroscopy, but I have found that it is useless for polarimetric work, since even the green cadmium-line can only be read with a half-shadow angle of nearly 20° . Apparently the current is carried mainly by the mercury, and the other metals show only weakly in the spectrum. I have not been able as yet to find any description of an enclosed cadmium arc, though I believe they have been tried—evidently not with complete success, or the results would be more widely known.

The method set out below is not put forward as the ideal way of producing an intense cadmium spectrum, but rather as an intermediate stage in the development of the perfect cadmium lamp of the future. It was found that brilliant spectral lines could be sent into the polarimeter by using an arc burning between metallic poles rotating in opposite directions. Copper, for instance, gave a valuable series of lines, and brass electrodes were found to be very efficient for developing a zinc spectrum, the red line Zn 6364, and the three blue lines Zn 4811, Zn 4722, and Zn 4680, standing out very distinctly from the copper lines. A brilliant cadmium spectrum could be produced by melting the metal onto copper electrodes, but it soon burned off, and in any case it

was difficult to avoid a displacement of the readings by the appearance of the copper line Cu 5106, as a ghostly partner of the green cadmium line Cd 5086. After many unsuccessful attempts a workable method of producing the cadmium spectrum was found in the use of an alloy of silver and cadmium. It is perhaps not very widely known that these metals are isomorphous, and form an excellent series of alloys. These have the advantage that no eutectic is formed, the melting-points throughout the series lying above that of cadmium and over a considerable range approximating somewhat closely to the melting-point of silver. Thus whilst the addition of 28 per cent. of copper (mp. 1082°) lowers the melting-point of silver from 960° to 780° , the addition of 28 per cent. of cadmium (mp. 322°) only lowers the melting-point to 860° . Nearly 50 per cent. of cadmium must be added to lower the melting-point to 780° , and even a 60 per cent. alloy melts as high as 700° . These alloys, which can be turned up like pure silver, were supplied by Messrs. Johnson and Matthey in the form of rods $\frac{1}{4}$ inch in diameter and $1\frac{1}{4}$ inch long.

For spectroscopic work a tiny arc can be burnt quite steadily between the points of the rods, in great contrast to the behaviour of pure cadmium, which splutters very badly and gets choked up with oxide, even when the current is kept so small as not to melt the metal.

For polarimetric work a greater intensity of light is desirable, and this is obtained by using a heavier current, and rotating the electrodes in opposite directions (compare Baly, *Spectroscopy*, p. 370) in order to maintain the arc in a central position. The rods of alloy were screwed for half their length into copper cylinders $\frac{5}{8}$ inch in diameter, which served the double purpose of cooling the electrodes—a point of some importance—and connecting them with the iron spindles by means of which the rotation was produced. When run at the highest intensity both rods become red hot, and one of the copper cylinders is usually luminous, but it is not desirable to over-run the arc since even if the electrodes do not melt the cadmium distils out irregularly and causes a certain amount of spluttering.

The electrodes are filed up before the arc is started, and are carefully adjusted so as to run true to centre; alternatively they may be allowed to burn until the ends are flat and then used without further attention except to adjust the length of the arc from time to time. The cadmium spectrum thus produced is of great brilliance—the green line is even brighter than that of mercury, and can be read with a half-shadow angle of 3° or less. From some points of view

it would be a better chief standard than mercury green, as it is considerably brighter and gives readings about 15 per cent. higher, but in view of the greater trouble involved in producing the light, and persuading it to burn steadily, it is better to use it as a secondary line for the study of rotatory dispersion. The red and blue lines are also very bright and can be read quite easily. The dark blue line is of much less intensity, and is not likely to be widely used, as it is difficult to read, and does not differ sufficiently in wave-length from the light blue line to justify the extra trouble involved; this observation applies, however, only to the existing arrangements, as it is quite possible that when a more powerful source of steady light is available the dark blue line may prove to be of considerable value in shortening the gap between Cd 4800 and Hg 4359.

Unlike copper the silver spectrum does not clash at all with that of cadmium; the brilliant silver green lines

5471.72 }
5465.66 } doublet (compare sodium)
5209.25 }

are separated from the cadmium green, Cd 5086, by an interval nearly as great as that which separates the two cadmium blues, and the only other line that shows at all strongly in the spectrum is a line in the far-violet, perhaps Ag 4055.

In conclusion: It is suggested that the mercury line Hg 5461 should be used as chief standard in optical work of all kinds, and that dispersion should be measured from this line to Hg 4359 instead of from H_{α} 6561 or H_{β} 4861 to H_{γ} 4341. As secondary standards are suggested the flame spectra Li 6708 and Na 5893, purified spectroscopically, together with the three cadmium lines Cd 6438, Cd 5086, Cd 4800, giving a well-distributed series of seven wave-lengths.

6708,	6438,	5893,	5461,	5086,	4800,	4359.
Red	red	yellow	green	green	blue	violet.

130 Horseferry Road, Westminster, S.W.

Note added July 1909.—I am glad to find that the desirability of a change in the standard wave-lengths for use in refractometry is apparently recognized by other workers: in particular, Dorn & Lohmann in their measurements of the refractive indices of liquid crystals (*Ann. Physik*, June 10th, 1909, [4] xxix. pp. 535-565) have used the series Li 6708, Na 5893, Hg 5461, Hg 4359, which is identical with a series that I am using for the measurement of the refractive dispersions of the alcohols and acids of the aliphatic series, and differs from the series of seven lines used in the measurement of rotatory dispersion only in the omission of the three cadmium lines.

XXXIX. *The Recombination of the Ions in Air at Different Temperatures.* By HENRY A. ERIKSON, *Assistant-Professor of Physics, University of Minnesota* *.

IN this paper are given the results of an investigation, as far as it has progressed, of the recombination of the ions produced by the γ and β rays of radium in air at different temperatures, the density of the air remaining constant and equal to that of the normal atmosphere.

It is now known that the previous experimental results obtained in this connexion were rendered uncertain by diffusion, owing to the fact that the electrodes were too close together in the case of some of the densities involved †.

It was shown experimentally by L. L. Hendren ‡, in connexion with his work on recombination at different pressures, that the effect of diffusion upon the coefficient of recombination (α) in the case of air is very small when the electrodes are 2 cm. apart and the density and temperature of the air are those of the atmosphere.

It was concluded from this that, with the electrodes 4 cm. apart and a similar density, the effect of diffusion would be so small, even at different temperatures, that whatever variation was found in α would be mainly due to causes other than diffusion.

If the effect of diffusion may be neglected at room-temperature, it may be neglected with more safety still at lower temperatures. At higher temperatures one cannot be so certain.

The Ionization-Chamber.

The final form of ionization-chamber decided upon was a copper sphere 11 cm. in diameter, having at its centre a sphere 3 cm. in diameter. This inner sphere, with its attached tube, was of glass covered with copper, electrically deposited. These two spheres formed the electrodes and were therefore 4 cm. apart. Any irregularity due to the connexion with the inner sphere was avoided by means of a cone, as shown in fig. 1.

The radium was contained in a small glass sphere and placed at the centre of the two spheres forming the chamber. This makes a very regular arrangement, for which the

* Communicated by Sir J. J. Thomson.

† Phil. Mag. vol. vi. p. 655 (1903).

‡ Physical Review, vol. xxi. p. 314 (1905).

constant, on the assumption that the ionizing effect of the rays varies inversely as the square of the distance from the source, may be readily obtained as follows :

$$Q = \int_{r_1}^{r_2} \frac{g_0}{r^2} dv = \int_{r_1}^{r_2} \frac{g_0}{r^2} (4\pi - \omega) r^2 dr = g_0 (r_2 - r_1) (4\pi - \omega) \quad . \quad (A)$$

$$N = \int_{r_1}^{r_2} n dv = \int_{r_1}^{r_2} \sqrt{\frac{g_0}{\alpha r^2}} (4\pi - \omega) r^2 dr = \frac{1}{2} \sqrt{\frac{g_0}{\alpha}} (r_2^2 - r_1^2) (4\pi - \omega).$$

$$\therefore \quad \alpha = \frac{g_0}{4N^2} (r_2^2 - r_1^2)^2 (4\pi - \omega)^2.$$

But from equation (A)

$$g_0 = \frac{Q}{(r_2 - r_1) (4\pi - \omega)},$$

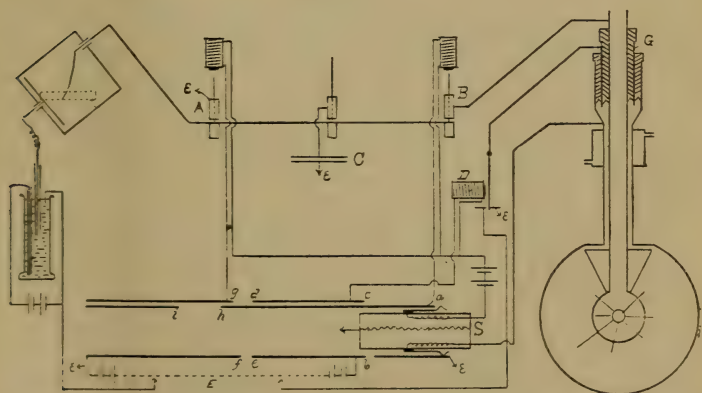
$$\therefore \quad \alpha = \frac{(r_2 + r_1) (r_2^2 - r_1^2) (4\pi - \omega)}{4} \frac{Q}{N^2} = K \frac{Q}{N^2},$$

where Q is the total number of ions produced by the rays per second, N the number existing in the gas when equilibrium is established, and ω the solid angle of the cone. It is seen that K is a constant which depends only upon the dimensions of the chamber.

Experimental Method and Arrangement of Apparatus.

The method used in this investigation was essentially the same as that employed by Hendren (*loc. cit.*).

Fig. 1.



The arrangement of the apparatus was as shown in fig. 1.

Q was measured in the ordinary way by obtaining the time (T) required for the leaf of the electroscope to move a certain number of divisions when the key A was open, 1000 volts being applied to the outside of the chamber. During this measurement the capacity (C) was added to the capacity (C_1) of the electrode system.

N was obtained by measuring the total charge in the gas when equilibrium had been established. This was accomplished by means of the contacts made by the horizontal slide S , which could be released and made to move by means of a suspended weight. When the slide was in the position shown in fig. 1 everything was connected to earth. When the slide is released contact is made at a opening the key B , thus insulating the inner electrode of the chamber. A little later the slide makes contact at b , which connects the ionization-chamber to a high potential E . Considerable difficulty was here experienced in preventing a partial loss of the free induced charge through leakage to the guard-ring G . It was found, however, that this could be overcome in a very satisfactory manner by applying a voltage of about $\frac{3}{5} E$ to the guard-ring G at the same instant that E was applied to the chamber, and then connecting the guard-ring to earth again at the same time that the chamber was earthed. This was accomplished by the relay D when the slide made contact at c and broke contact at d . At e the slide disconnects the chamber from the high potential E and an instant later earths the chamber through contact at f .

When the slide makes contact at g the key A is opened, thereby insulating the leaf of the electroscope. At h the inner electrode is connected to the leaf by closing key B , the charge $C_1 V_1$ collected on the electrode causing the leaf to deflect. Contact at i opens the key B again and insulates the leaf. From the deflexion the potential V_1 to which the charge has raised the capacity C_1 of the electrode system is determined.

The collected charge $C_1 V_1$ requires two corrections: During the time (t) that E was applied to the chamber, the rays produced a quantity Q_{et} which must be subtracted. Also during the time (t') from break of contact at e to make of contact at i a back-leak took place which must be added. The leak per second (L) was readily determined by charging the electrode system to a potential a few divisions higher and noting the time (T_1) required for the leaf to move to a point the same number of divisions lower than the

deflexion caused by the charge C_1V_1 . The amount of loss due to the back-leak then is :

$$Lt' = \frac{C_1(V_3 - V_4)t'}{T_1}.$$

From these determinations we have :

$$Q = \frac{(C + C_1)V}{T300e},$$

$$N = \left[C_1V_1 - \frac{(C + C_1)V}{T}t + \frac{(V_3 - V_4)C_1t'}{T_1} \right] \frac{1}{300e}.$$

$$\therefore \alpha = K \frac{(C + C_1)V300e}{T \left[C_1V_1 - \frac{(C + C_1)V}{T}t + \frac{(V_3 - V_4)C_1t'}{T_1} \right]^2}.$$

The times t and t' were readily obtained from the tracing of a tuning-fork on the surface of a smoked glass placed upon the slide. Care is necessary at this point; t must be sufficiently large to allow all the ions to pass to the electrodes, and E must be so high that the time required for an ion to pass the distance between the electrodes permits only a negligible amount of recombination. These points are readily tested experimentally by using different values for E . It was found that with $E=600$ volts and $E=1000$ volts, the same value for α was obtained. This test was applied at the temperature of liquid air as well as at room-temperature. The value of E used throughout the entire work was, however, 1000 volts.

In order to be certain that the deflexions obtained are due to the ions produced by the rays it is necessary to have the apparatus so constructed that the radium may be removed and the manipulations described above repeated without the rays. The inner spherical electrode was therefore constructed so as to open to the outside through the tube serving as its support. It was found to be possible to get all parts so perfect that no motion of the leaf was obtained without the rays, and care was taken to have this as the working condition throughout the entire work.

The capacity C consisted of two circular plates placed 2 mm. apart. The capacity of this condenser was computed according to Kirchhoff's formula (Abhandl. p. 113) giving 700 cm. The capacity C_1 was obtained from comparison with C by method of mixtures.

The value of each deflexion in volts was obtained from a potentiometer, a cadmium cell being used as the standard.

The lowest temperature was obtained by immersing the chamber in liquid-air. The next higher temperature was obtained by packing CO₂ snow about the chamber, the liquid-air tube being used as the containing vessel.

Temperatures higher than that of the room were obtained by inverting the chamber and heating it by means of a concentric electric coil, the whole being imbedded in dry sand.

Results.

In order to show the magnitude of the different quantities involved, a set of observations with their substitution in the expression for α is here given.

Time in seconds required for the leaf to move 10 divisions (=·611 volt) :

$$10, 10, 9\cdot9, 10\cdot2, 10\cdot1, 10, 10, 10, 10, A_v = 10.$$

The deflexion in scale-divisions due to charge C_1V_1 :

$$23\cdot6, 24\cdot4, 24\cdot5, 24\cdot4, 24\cdot3, 23\cdot8, 23\cdot8, 24\cdot2, 24\cdot4, 24\cdot6, 24\cdot5, \\ A_v = 24\cdot23 (=1\cdot56 \text{ volts}).$$

$$K = \frac{1}{4}(r_2 + r_1)(r_2^2 - r_1^2)(4\pi - \omega) \\ = \frac{1}{4}(5\cdot84 + 1\cdot55)(5\cdot84^2 - 1\cdot55^2)(4\pi - \cdot561) = 710.$$

$$Q = \frac{(C + C_1)V}{T300e} = \frac{(700 + 47)\cdot771}{10} \frac{1}{300e} = 5\cdot64 \times 10^8, \\ (e = 3\cdot4 \times 10^{-10}).$$

$$N = \left[C_1V_1 - \frac{(C + C_1)V}{T} t + \frac{(V_3 - V_4)(C + C_1)}{T} t' \right] \frac{1}{300e} \\ = \left[47 \times 1\cdot56 - \frac{747 \times \cdot771}{10} \frac{30\cdot2}{120} \right. \\ \left. + \frac{(1\cdot60 - 1\cdot37)747}{13\cdot56} \frac{12\cdot4}{120} \right] \frac{1}{300e} = 5\cdot87 \times 10^8.$$

$$\alpha = K \frac{Q}{N^2} = 710 \frac{5\cdot64 \times 10^8}{(5\cdot87 \times 10^8)^2} = 710 \times 4\cdot82e = 3420e = 1\cdot16 \times 10^{-6}.$$

In the following Tables are given all the results obtained up to the present time.

TABLE II.

	Q_e .	+12° C.		+64° C.		100° C.		155° C.	
		N_e .	α .	N_e .	α .	N_e .	α .	N_e .	α .
1...	19.82×10^{-2}	18.17×10^{-2}	6.00 Ke						
	19.66 "					25.94×10^{-2}	2.95 Ke		
	19.46 "							29.42×10^{-2}	2.24 Ke
2...	19.16 "	19.96 "	4.82 "						
	20.16 "					31.10 "	2.08 "		
	22.13 "	21.83 "	4.64 "						
3...	22.13 "			26.21×10^{-2}	3.22 Ke				
	22.13 "					31.69 "	2.20 "		
	21.53 "							35.12 "	1.81 "
4...	22.20 "	23.15 "	4.14 "						
	22.20 "			25.97 "	3.29 "				
	22.20 "					31.00 "	2.31 "	34.74 "	
	22.20 "								1.84 "
Av.	20.51×10^{-2}	20.78×10^{-2}	4.90 Ke	26.09×10^{-2}	3.25 Ke	29.93×10^{-2}	2.38 Ke	33.10×10^{-2}	1.96 Ke

The value of K (namely 710) strictly applies to the results given in Table II. only. The results in Table I. were obtained with the radium contained in a small glass tube which it only partly filled. This distribution, it is reasonable to suppose, leads to a constant of different value. In order to avoid convection-currents at the higher temperatures due to the cooler air in the neck, it was necessary to invert the chamber. It was then thought best to transfer the radium to a small glass sphere, which it completely filled. Owing to the thinness of the glass in this case a greater ionization was obtained, giving rise to a greater value of Q_e .

Observations with the air at room-temperature were always taken before the temperature was changed. The results that belong to the same set are connected by brackets in each of the Tables.

The value of N and the absolute value of α , at the different temperatures, are given in Table III. The values of N obtained from Table I. were reduced to the same order as those of Table II., the ratio used being $\frac{20.78}{15.0}$, the ratio of the values of N at 12° in the two cases. The absolute values of α for the two lowest temperatures were obtained by using for K_1 the value 0.692, obtained by placing $5.03K_1 = 4.90K$, both being for 12°C . Q was found, within the experimental error, to be independent of the temperature.

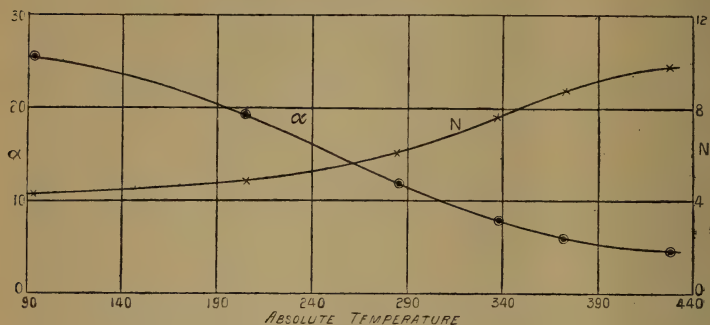
TABLE III.

Absolute Temperature, C.	N.	α .
91°	4.28×10^8	2.55×10^{-6}
205	4.81 "	1.92 "
285	6.10 "	1.18 "
337	7.67 "	0.785 "
373	8.80 "	0.59 "
428	9.74 "	0.47 "

These results are shown graphically in fig. 2 (p. 336).

It is seen from this that as the temperature of the air increases, the less readily do the ions recombine, and that the change, with temperature, of the coefficient of recombination is represented by a compound curve, the point of inflexion of which is at about 250° absolute temperature.

Fig. 2.



The writer intends to continue this investigation, using higher temperatures and also different gases.

In conclusion I wish to express my sincere thanks to Prof. Thomson for suggesting this problem and for his kind interest during this research.

June 2, 1909.

XL. A Device to prevent Backlash in the Toothed Wheels and Rack-and-Pinion Gears of Scientific Instruments. By JAMES ROBERT MILNE, D.Sc., Assistant to the Professor of Natural Philosophy in the University of Edinburgh*.

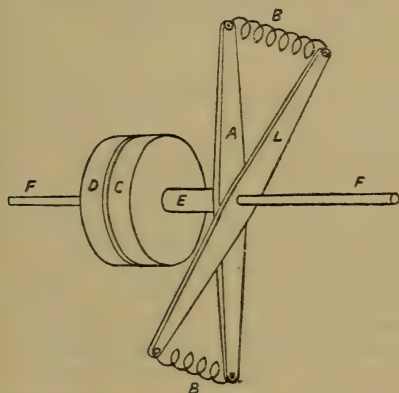
MANY scientific instruments have either toothed wheels or a rack-and-pinion as a part of their mechanism. When a measured motion has to be transmitted by these parts, it is essential that there should be no backlash; and even if these parts are only used for making an adjustment, they should have as little backlash as possible.

The usual way of overcoming backlash is to fit the teeth of the wheels very closely to one another. This can only be accomplished by very skilled workmanship; and however accurately the teeth fit at first, in course of time they begin to wear, and develop backlash. Moreover, closely fitted teeth work with a certain amount of stiffness, which is frequently a drawback.

* Communicated by Dr. C. G. Knott.

The author's device, which is free from the above objections is as follows:—

The smaller of the two wheels that gear together, or the pinion-wheel in the case of a rack-and-pinion, is divided in half, so as to make two wheels similar to the original one, although only half as thick. They are represented at C and D in the figure—the teeth not being shown. Both



C and D continue to gear with the other wheel—or with the rack—as before. * D is fixed to the axle F; the wheel C, the sleeve E, and the arm A are fixed together as one piece, loose on the axle. The arm L is fixed on the axle. The effect of this arrangement is to force the two wheels C and D in opposite directions, because of the tension of the springs B, B. Therefore the corresponding teeth on C and D turn past each other until they completely fill up the spaces between the teeth of the wheel—or the rack—with which they gear; and hence with this arrangement there can be no backlash.

In the above device the movement in one direction is positive, in the other it takes place through the intervention of the springs; which therefore must be of sufficient strength to transmit the force without yielding. The author finds that in the case of scientific instruments this condition is attained without difficulty, a comparatively slight strength of spring being sufficient.

The author has had the device for some time in use in connexion with the rotation of the Nicol's prism of a spectrophotometer, and it has proved very successful.

* The following is the way in which the author made use of this device, but other ways might be used to suit other circumstances.

XLI. *Notices respecting New Books.*

An Introduction to the Theory of Infinite Series. By T. J. I'A. BROMWICH. Macmillan & Co. 1908.

A Course of Pure Mathematics. By G. H. HARDY. Cambridge University Press. 1908.

Plane Geometry for Advanced Students.—Part I. By CLEMENT V. DURELL. Macmillan & Co. London: 1909.

THE authors of these three books deserve the thanks of teachers and of serious students for placing in their hands well-planned and clearly written treatises on certain important parts of pure mathematics. Of the three the Geometry is the most elementary, and may be studied with great profit by any intelligent student familiar with the equivalent of Euclid's first six books. With the exception of a few obvious sphere and plane problems in inversion and a short paragraph on solid geometry, the geometry is throughout plane and deals with the properties of triangles, plane quadrilaterals, and circles. The demonstrated properties are arranged under 93 Theorems, while the numerous examples to each chapter contain a large number of other interesting theorems which apparently do not attain to the status of the ninety-three. The diagrams are excellent and add much to the beauty of the page. Occasionally the author indicates instructive statical interpretations of the geometrical theorems. One definition only have we noted as being incomplete. On page 68 we read that "Any quantity which can be completely represented by a straight line of appropriate length, drawn in an appropriate direction, is called a vector." A finite rotation can be so represented, and yet it is not a vector; for two finite rotations cannot be added vectorially. Part II. is promised shortly. It will deal with the higher applications to conics and other more recondite aspects of modern geometry. An interesting feature of Mr. Durell's book is the series of historic notes prefixed to most of the chapters.

Mr. Hardy's Course of Pure Mathematics will almost certainly become very useful in University classes. It lays important stress on points which are apt to be slurred over when a student passes from the ordinary algebra to the study of the calculus. The author himself regards the fourth chapter on limits of functions of an integral variable as one of the leading features of the book. This is followed by a discussion of the limits of functions of a continuous variable, and then Chapter VI. on Derivatives and Integrals constitutes a "first chapter in the differential and integral calculus." Subsequent chapters deal with Taylor's Theorem, with definite integrals, with questions of convergency, with infinite integrals, and with the theory of logarithmic, exponential and circular functions. In introducing the complex variable in

Chapter III., Mr. Hardy makes use of the conception of displacement in a plane; and ostensibly makes the properties of the complex variable depend upon a definition of Multiplication of Displacements (§ 28). The vector addition of displacements is not necessarily in a plane; why then should their multiplication be confined to a plane? There is a looseness of argument in the statement that "if any definition of such product (*i. e.* of two displacements) is to be of any use, the product of two displacements *must itself be a displacement.*" Finally, led ostensibly by a geometrical relation which is said to suggest the "right definition," he finds that

$$[x, y] [x', y'] = [xx' - yy', xy' + yx'].$$

But if there had been no complex variable to guide the assumptions, would it not have been as reasonable to have assumed

$$[x, y] [x', y'] = [xx' + yy', xy' - yx']?$$

This indeed is very nearly Hamilton's definition of the product of two vectors, and is one which leads to spacial symmetry. However, having got what he calls the "right definition," Mr. Hardy finds it "convenient to represent the pair of real numbers xy by the symbol $x+iy$." This gives the assumed multiplication law when $i^2 = -1$, and lo! the thing is done. In spite of the footnote on page 67, very few students will accept this demonstration without at the same time taking for granted that a displacement, a vector, and the complex variable are names for one and the same thing. By calling the usual geometrical representation of the complex variable a displacement, writers on function theory do a disservice to the student who nine cases out of ten will continue to believe that displacements in their *kinematical* significance can be multiplied together to produce a displacement—a statement which is devoid of any real physical meaning. Chapter III. could have been made as useful without any reference to displacements or vectors. Fortunately the doubtful logic does not in the least affect the subsequent use of the complex variable in its true analytical significance.

The first book on our list is much more advanced than the others and appeals to a more limited circle of students. The question of infinite series is one of far-reaching importance, and it has been discussed in more or less detail by several of our recent writers, such as Whittaker, Pierpont, and Carslaw. And now from the able pen of Professor Bromwich we have a systematic up-to-date account of infinite series and much that hinges on them. To make the argument as complete as possible the author has found it necessary to add three appendices which occupy nearly one quarter of the whole book. Appendix I. gives an arithmetic treatment of convergence and limits based on Dedekind's definition of irrational numbers, and might have been incorporated in the main argument

of the early part of the book. But this certainly would have hindered the progress of the average student, whose mathematical intuitions predispose him to take a good deal for granted, and whose appreciation of the rigorous arithmetical foundation cannot be expected to be lively until the conception of limits and the tests of convergency are familiar. Just as the child learns the rules of arithmetical operations long before he understands them, so it is throughout the whole of mathematical study. It is the historic method. Appendix II. is a brief discussion of the logarithmic and exponential functions, the former being defined in the familiar integral form, and the fundamental properties being deduced therefrom. A very similar section will be found in Mr. Hardy's book. It is valuable as giving the theory of the logarithm independent of the series. The third and longest Appendix Mr. Bromwich introduced so that he might be able to use the theory of infinite integrals in his investigations in Chapter IX. on Non-convergent and Asymptotic Series. This additional matter has the single disadvantage of making the book more bulky; but it gives practically everything that is needed for the complete study of the subject. Besides, it is always good to have Mr. Bromwich's presentation of a connected series of theorems.

Both Mr. Bromwich and Mr. Hardy prefer the continental notation $\arcsin x$ for the arc whose sine is x instead of the more usual English form $\sin^{-1} x$. Notations are largely a question of custom. Yet the latter has its advantages and fits well in with functional notation generally. If $y = \sin x$, then $\sin^{-1} y = x$. On the other hand, $\sin x = y$ and $x = \arcsin y$ fall in with no other reciprocal notational scheme in recognized use.

XLII. *Intelligence and Miscellaneous Articles.*

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

WE wish to express our regret at having overlooked the fact that one of Mr. Vegard's equations is, in effect, equivalent to one of ours. It may not be out of place to mention that our main theoretical results are obtained independently of this equation; but it is satisfactory to find that the wholly different line of argument used by Mr. Vegard leads him to a conclusion, which, as far as it is applicable, agrees with our investigation.

Yours faithfully,

BERKELEY,
C. V. BURTON.

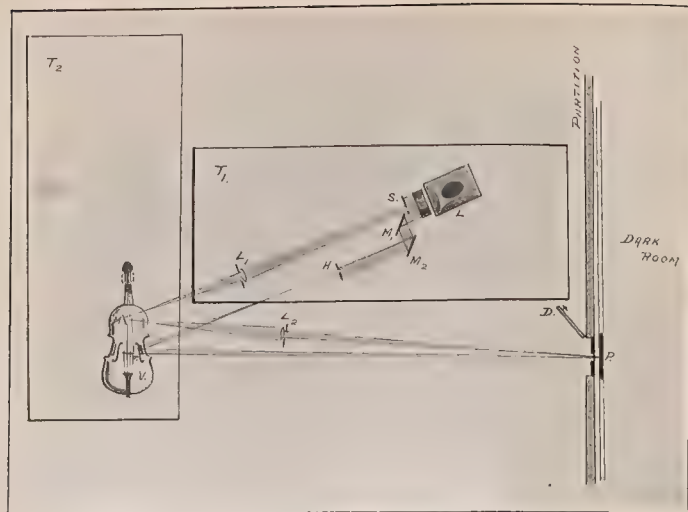


FIG. 1.—Diagram of Optical Arrangements.



FIG. 2.—Violin as mounted for the Experiments.



FIG. 3.—Violin with Bracket for Optical Lever.

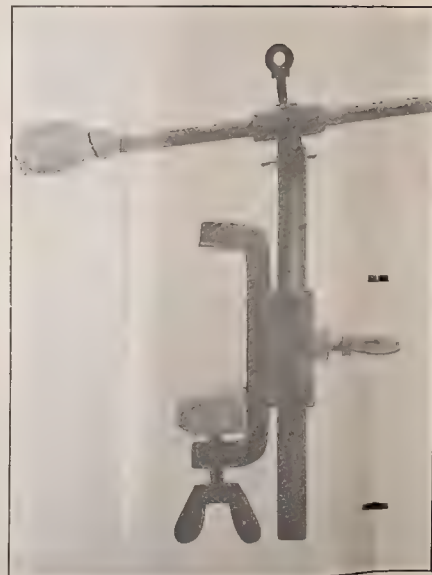


FIG. 4.—Bracket for Optical Lever.

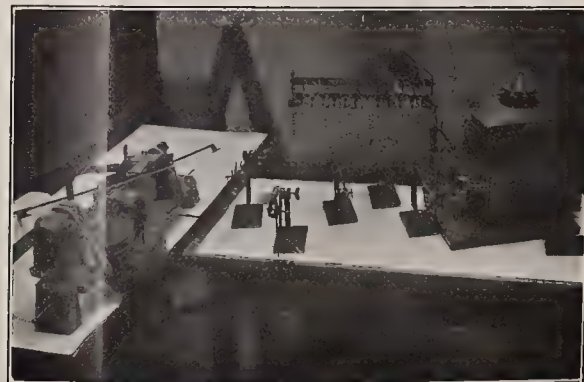


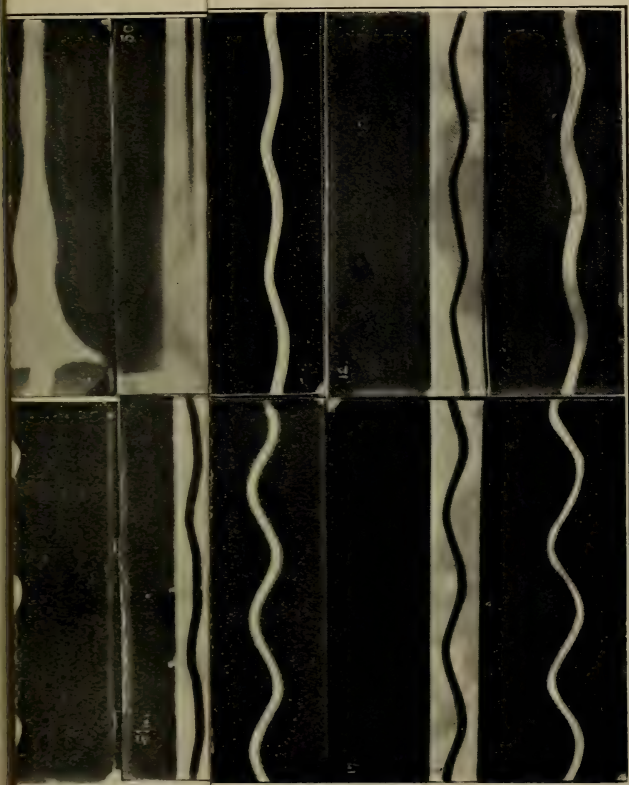
FIG. 5.—Experimental Arrangements.

N G=ST

but plate moved s
bration-curves but
riations of amplitut $\frac{1}{6}$.

Plucked at $\frac{1}{7}$.

10



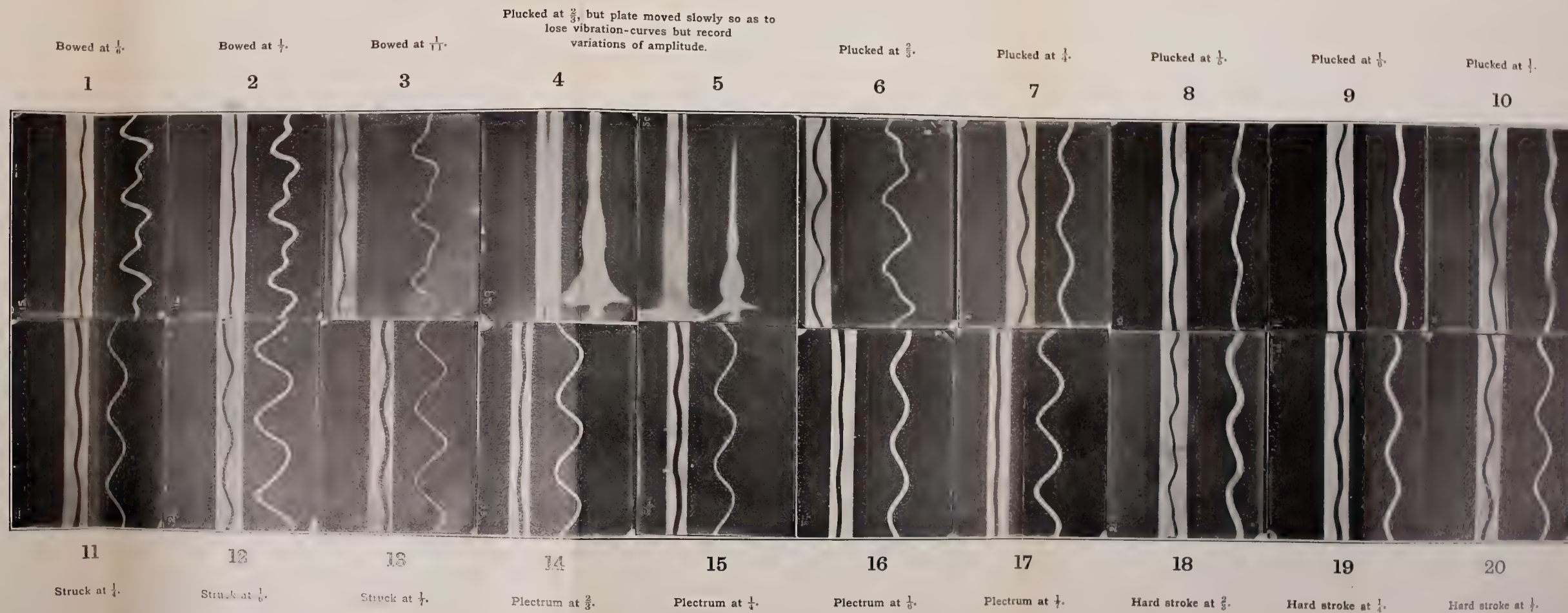
20

$\frac{1}{3}$.

Pat $\frac{1}{4}$.

Hard stroke at $\frac{1}{7}$.

OPEN G=STRING AT CONCERT PITCH.



IBRA

and middle string plucked at $\frac{1}{7}$ and tuned to

owed at $\frac{1}{16}$.

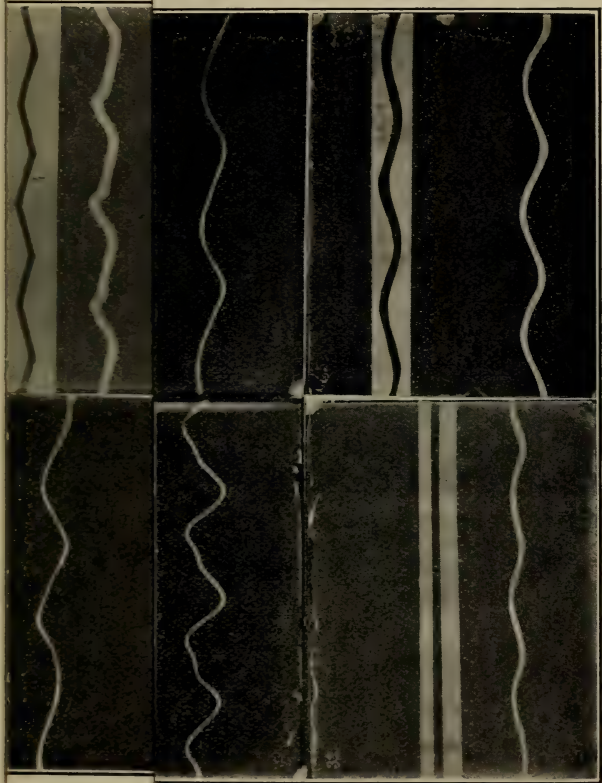
g $\flat$

a $\flat$

24

29

30



34

39

40

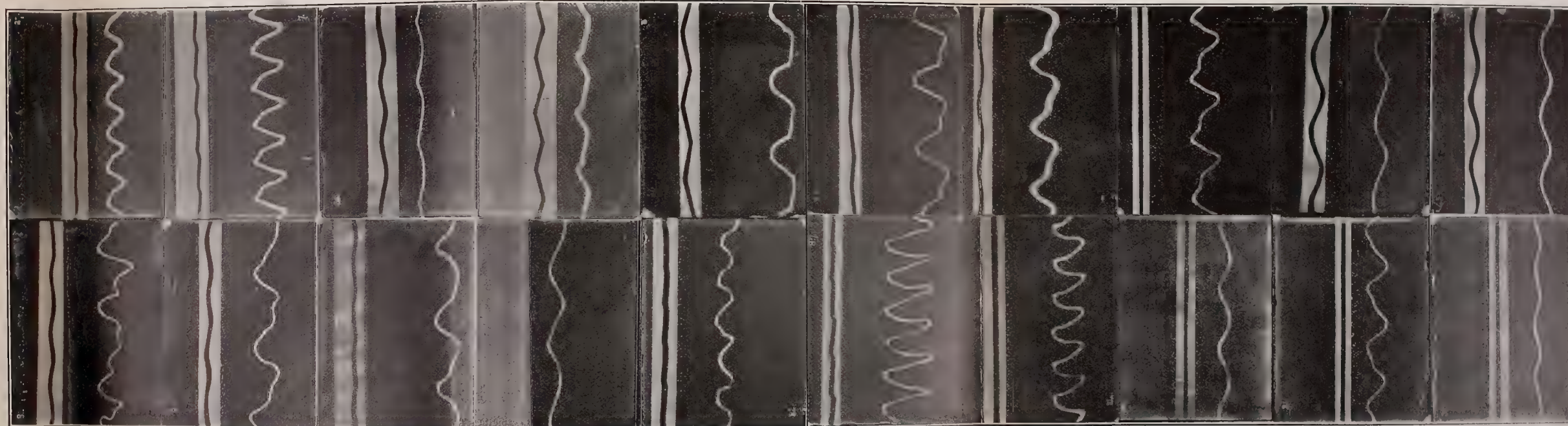
g and g string
concert pitch

b $\natural$

plucked at $\frac{1}{7}$.

Horn giving above notes.

<i>g</i> string sounding octave harmonic		<i>g</i> string stopped at $\frac{3}{4}$ to sound middle <i>c</i>		Open <i>g</i> string bowed at $\frac{1}{7}$ and tuned to sound				Open <i>g</i> string plucked at $\frac{1}{7}$ and tuned to	
Bowed at $\frac{1}{12}$.	Bowed at $\frac{1}{16}$.	Bowed at $\frac{1}{7}$.	Bowed at $\frac{1}{16}$.	$f^{\#}$	a^b	<i>a</i>	$b^?$	g^b	a^b
21	22	23	24	25	26	27	28	29	30



31	32	33	34	35	36	37	38	39	40
<i>d'</i> string tuned to <i>b</i> with <i>g</i> string at concert pitch.	<i>d'</i> string tuned to <i>c'</i> with <i>g</i> string at concert pitch.	<i>d'</i> string and <i>g</i> string both at concert pitch.	<i>d'</i> string and <i>g</i> string both at concert pitch.	<i>d'</i> string at concert pitch with <i>g</i> string sounding octave harmonic.	Bowed at $\frac{1}{3}$.	Bowed at $\frac{1}{7}$.	$a^{\sharp}$	b^b	$b^{\sharp}$
Both bowed at $\frac{1}{7}$.			Both plucked at $\frac{1}{7}$.	Bowed at $\frac{1}{10}$.	Open <i>a'</i> string.		Belly resounding to French Horn giving above notes.		

2536
2537

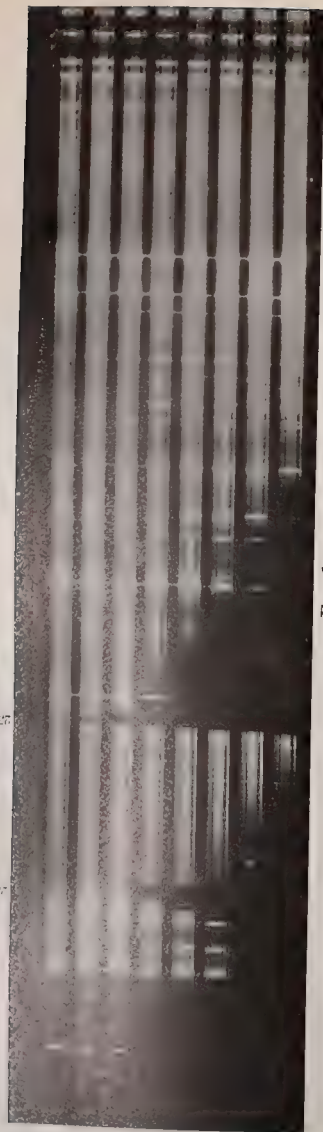
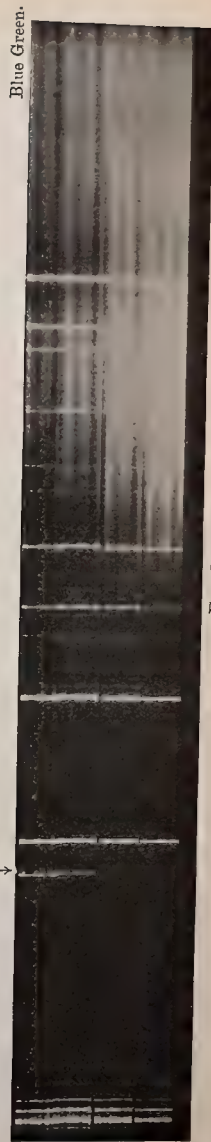


FIG. 1.

2538
↓



Blue Green.

FIG. 2.



In vacuo.

FIG. 3.

FIG. 5.

2536

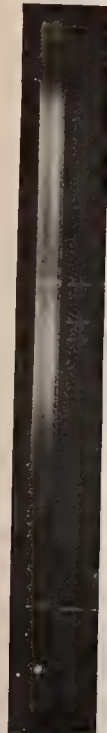


FIG. 8.

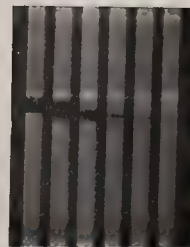


FIG. 4.

FIG. 6.

2546



FIG. 7.



FIG. 9.



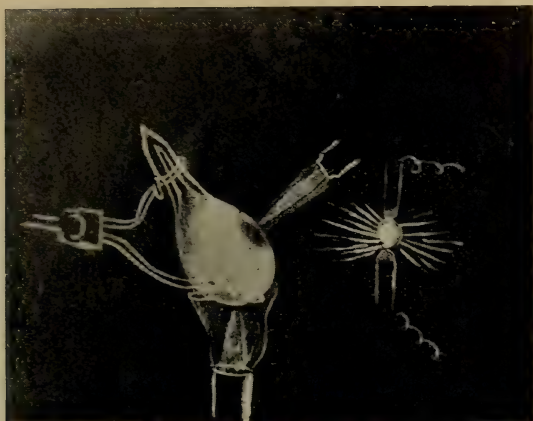


FIG. 1.



FIG. 2.

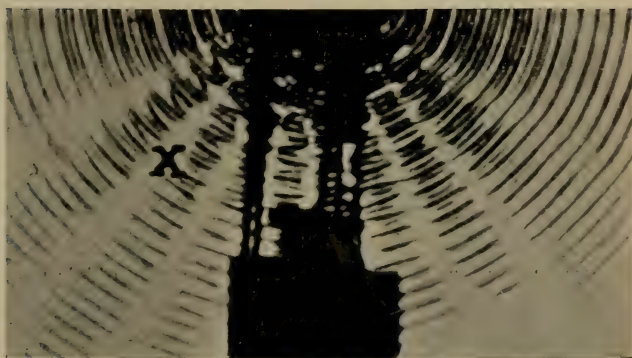
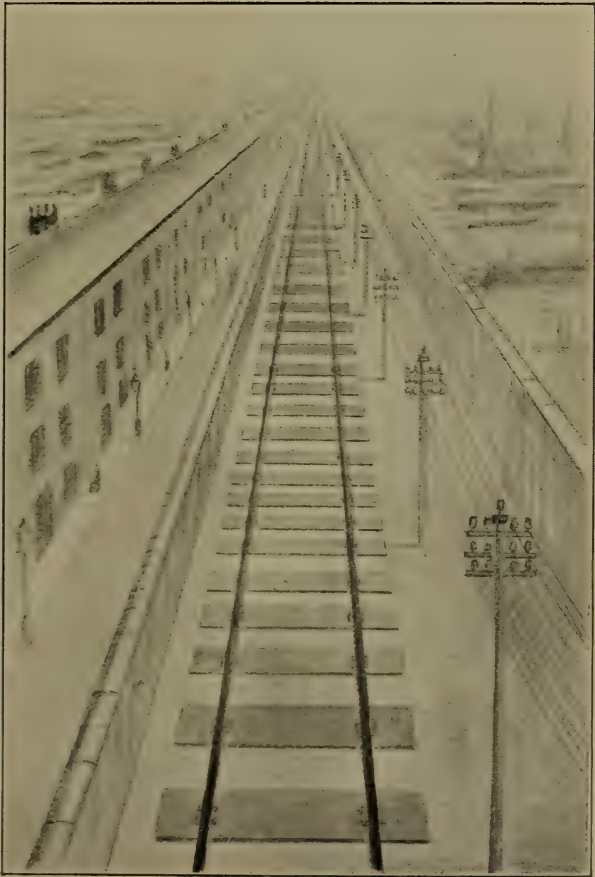


FIG. 3.



INDEXED

THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[SIXTH SERIES.]

SEPTEMBER 1909.

XLIII. *The Ions of Gases.* By WILLIAM SUTHERLAND*.

RECENT accumulations of experimental work on this subject show the need of a more comprehensive theory than has yet been advanced. The following is an attempt to state the fundamentals of a theory wide enough to contain the chief phenomena now known. The simplest theoretical ion in a gas is an atom charged with an electron, just as in electrolytic solutions. But the calculations of J. J. Thomson, Langevin, and others, seem to show that the smallest ions in gases consist of a few molecules, say two or three. P. Phillips in his experiments on Ionic Velocities in Air (Proc. Roy. Soc. lxxviii. 1906, p. 167), with a wide range of temperature finds the number of molecules in a gaseous ion to vary apparently with the temperature. It will be shown immediately that this variation of the size of the small ion in gases is only apparent, that the experimental ion is the simple ion of theory and electrolysis, and that the seeming change of size is due to the same cause as the seeming change of size of molecules with temperature in the viscosity of gases and allied phenomena before the effects of cohesive force were allowed for in the kinetic theory of gases. When the electric force of the simple ion in a gas is introduced into the calculation of the ionic mobilities to be expected at the different temperatures in the experiments of

* Communicated by the Author.

Phillips, it removes all necessity for ascribing varying complexity to that ion. Wellisch ('Nature,' lxxix. 1908, p. 148) has already shown by reasoning which is not supported by the present investigation, that by considering the electric force of an ion in the same way as cohesional force in the theory of gaseous viscosity, it becomes no longer necessary to hold that the linear dimensions of the smallest ions in gases betoken a complex structure for them. But in the larger ions formed by spraying electrolytic solutions into flames and hot gases we find on a more elaborate scale just such a variable gaseous ion as the simplest has hitherto been supposed to be. As an extreme case of this type we have the large ion discovered by Langevin in the atmosphere. Of this ion we know little more than its small mobility except the demonstration by Pollock (Austr. Assoc. for the Adv. of Sci. 1909) that this mobility varies considerably with the humidity of the atmosphere. Premising that the mobility of an ion is its velocity in cm. per second under an electric force of a volt per cm. we shall for convenience divide gaseous ions into three classes, those having at about 15° C. mobilities of the order of magnitude 2, 0·02, and 0·002. No doubt these types merge continuously each into the next. They can be specified more accurately according to the following definitions. Let the ion formed of atom and electron be called a nucleolus, such an ion surrounded by molecules forming a solid or liquid mass be called a nucleus, and let a collection of molecules in the form of a vapour round a nucleus or nucleolus be called an envelope; then the three types of gaseous ion are (1) nucleolus alone, (2) nucleolus and envelope, and (3) nucleus and envelope. The third type merges into the visible drop of fog and rain, though it is probably distinguished from these by the fact that the majority of the molecules in the nucleus and envelope are more directed by central electric force from the electron than by the mutual cohesional forces. The present theory will be divided into three sections, each devoted to one of these types of ion with a reference to theories of condensation and a summary.

1. *The Ion formed by Atom or Radical with Electron.*

Let the ion of charge e move with velocity u in the direction x under uniform fall of potential whose rate is dE/dx , then, if the viscous resistance of the gas to its motion with unit velocity is F

$$Fu = e dE/dx \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If cohesive and electric force were neglected, the molecules of gas being treated as forceless spheres of radius a_3 , mass m_3 , and number per cm.³ N_3 , suffix 1 allotting the corresponding quantities to the ion, the expression found for Fu in the kinetic theory of "perfect" gases is

$$Fu = \frac{4}{3} N_3 (a_1 + a_3)^2 \left\{ 2\pi (v_1^2 + v_3^2) / 3 \right\}^{\frac{1}{2}} \frac{m_1 m_3}{m_1 + m_3} u \quad . \quad . \quad (2)$$

The immediate problem is to find how we must modify this expression to provide for the greater frequency of collisions caused by electric and cohesive force. The treatment can proceed almost exactly on the lines of my paper on the Viscosity of Gases and Molecular Force (Phil. Mag. [5] xxxvi. 1893, p. 507). Select an ion and a molecule which are initially far enough apart to exert negligible force on one another and are each destined to have their next collision with one another. Let r_1 and r_3 be their distances from the centre of mass of the two, so that $m_1 r_1 = m_3 r_3$ and let $r_1 + r_3$ their distance apart be denoted by r . Let the cohesive force between them be $-d\phi(r)/dr$ and the electric force between the electron of the ion and the molecule be $-d\psi(r)/dr$, then using z for $1/r$ we have for the motion of the molecule relative to the centre of mass the usual equation

$$\frac{d^2 z_3}{d\theta^2} + z_3 + \frac{1}{m_3 h_3^2 z_3^2} \left(\frac{d\phi(r)}{dr} + \frac{d\psi(r)}{dr} \right) = 0 \quad . \quad . \quad (3)$$

in which h_3 is twice the area swept out by r_3 as it describes the angle θ . From this for the equation of relative motion we get

$$\frac{d^2 z}{d\theta^2} + z + \frac{m_1 + m_3}{m_1 m_3 h^2 z^2} \left(\frac{d\phi(r)}{dr} + \frac{d\psi(r)}{dr} \right) = 0, \quad . \quad . \quad (4)$$

and then the first integral of this gives the equation of energy

$$\frac{1}{2} h^2 \left\{ \left(\frac{dz}{d\theta} \right)^2 + z^2 \right\} = \frac{1}{2} v^2 = \frac{m_1 + m_3}{m_1 m_3} \left\{ \phi(r) + \psi(r) \right\} + \frac{1}{2} V^2 \quad (5)$$

in which v is the relative velocity when ion and molecule are at distance r apart, and V is the relative velocity when r is so great that the relative orbit nearly coincides with its asymptote. Let b be the length of the perpendicular from the ion on the asymptote so that $h = bV$. When the relative orbit is such that there is no collision, we can determine the

minimum value of r by the² condition $dz/d\theta=0$. Denote the reciprocal of this distance by w which is given by

$$\frac{1}{2}h^2w^2 = \frac{m_1+m_3}{m_1m_3} \left\{ \phi(r) + \psi(r) \right\} + \frac{1}{2}V^2 \quad \dots \quad (6)$$

or

$$\frac{m_1+m_3}{m_1m_3} \left\{ \phi(r) + \psi(r) \right\} - \frac{1}{2}b^2V^2w^2 + \frac{1}{2}V^2 = 0.$$

There must be a collision if $1/w$ is less than a_1+a_3 . Hence the greatest value of b for which a collision is possible is given by

$$\frac{m_1+m_3}{m_1m_3} \left\{ \phi(a_1+a_3) + \psi(a_1+a_3) \right\} - \frac{b^2V^2}{2(a_1+a_3)^2} + \frac{1}{2}V^2 = 0,$$

or

$$b^2 = (a_1+a_3)^2 \left(1 + \frac{2(m_1+m_3) \{ \phi(a_1+a_3) + \psi(a_1+a_3) \}}{m_1m_3V^2} \right) \quad \dots \quad (7)$$

Thus it is proved that to adapt (2) to the case where electric and cohesional forces are operative $(a_1+a_3)^2$ is to be replaced by the right hand of (7). Just as the corresponding result in the theory of the viscosity of a gas applies strictly only to gases above their critical temperature and at moderate densities, that is to say, under conditions which allow the relative orbit of a pair of colliding molecules seldom to consist of a curve of finite range, so the result just obtained is strictly applicable only when the relative orbit of ion and molecule is seldom a curve of finite range. For the resistance to an ion we now get from (2) with $m_1v_1^2 = m_3v_3^2$, $V^2 = v_1^2 + v_3^2$.

$$Fu = \frac{4}{3}N_3(a_1+a_3)^2 \left(\frac{2\pi}{3} \cdot \frac{m_1m_3}{m_1+m_3} m_3v_3^2 \right)^{\frac{1}{2}} \left\{ 1 + \frac{\phi(a_1+a_3) + \psi(a_1+a_3)}{m_3v_3^2/2} \right\} u. \quad \dots \quad (8)$$

To adapt this for comparison with experimental results we put $m_3v_3^2$ proportional to T the absolute temperature, and remembering that N_3 varies as $1/T$, we get F proportional to $T^{-\frac{1}{2}}(1+C'/T)$ where C' is proportional to

$$\phi(a_1+a_3) + \psi(a_1+a_3).$$

If then in (1) we make dE/dx a volt per cm., we get for the relation between ionic mobility u and temperature T the equation

$$u = \frac{A'T^{\frac{1}{2}}}{1+C'/T} \quad \dots \quad (9)$$

in which A' and C' are parameters having values characteristic of each gas, the dashes being used to distinguish them

from the corresponding parameters in the similar equation for the viscosity of a gas. We can test the validity of (9) by means of the experiments of Phillips (*loc. cit.*). In air for the positive ion $A'=0.222$ and $C'=509.6$, and for the negative ion $A'=0.222$ and $C'=333.3$. These give the following calculated mobilities for comparison with the experimental.

TABLE I.

	T	411	399	383	373	348	333	285	209	94
Pos. {	u exp. ...	2.00	1.95	1.85	1.81	1.67	1.60	1.39	0.945	0.235
ion. {	u cal. ...	2.01	1.95	1.86	1.81	1.68	1.60	1.34	0.934	0.336
Neg. {	u exp. ...	2.495	2.40	2.30	2.21	2.125	2.00	1.785	1.23	0.235
ion. {	u cal. ...	2.487	2.42	2.33	2.27	2.12	2.03	1.73	1.24	0.474

Down to $T=209^\circ$ the mobilities of both ions are given with satisfactory accuracy by the theoretical formula (9), but at 94° the formula fails, as theoretical considerations led us to expect that it would, because 94 is about 30 below the critical temperature of air. It is notable that at this lowest temperature the experimental mobility is the same for both ions. This shows that at this temperature some rather profound change has occurred in the conditions of movement of the two ions. From the values of A' and C' it appears that at high temperatures the mobilities of the two ions will tend to become equal again. It appears also that the difference in the mobilities is caused by the difference in the values of C' which are proportional to the potential energy of ion and molecule in contact. If we compare these values, namely 509.6 and 333.3, with 113 the value of C the similar parameter in the viscosity of air, which is proportional to the potential energy of two molecules of air in contact, we see that in both cases the potential energy of ion and molecule is larger than that of molecule and molecule. This is what we should expect from our knowledge that the potential energy of molecule and molecule is like that of two electric doublets of charges e and moment es in which s is less than the diameter of a molecule. It seems then we must trace the remarkable difference in the mobilities of the two ions to differences of their potential energies when in contact with a molecule of air. Unfortunately we do not know experimentally what the ions are. In pure gases we may infer that the ions are atoms charged with a positive or a negative electron, but we do not yet know whether in the

presence of impurity there may be a tendency for one or both sorts of electron to associate specially with the impurity. But in spite of these remaining uncertainties the interpretation of the experiments by (9) proves that the small ion in air does not attach to itself satellite molecules whose number varies with temperature. The electric force of the ion accounts for the facts just as cohesional force accounts for the corresponding facts in the viscosity of gases. I may mention that for the negative ion in air the result at 94° can be expressed as well as all the others by changing (9) to

$$u = \frac{A'T^{\frac{1}{2}}}{1 + C'/(T - T_a)}$$

where

$$T_a = 70^\circ, A' = 0.1764, C' = 150.5.$$

In the further investigation of (8), until experiment informs us definitely as to the atoms or radicals which form the ions, we shall be forced to make assumptions. For the diatomic element gases we can write $m_3 = 2m_1$ if the ions are formed of their atoms, and we may also put $a_3^3 = 2a_1^3$, although we must be on the watch lest the atom when made an ion may undergo change of volume. Possibly the change caused by the positive electron is different from that caused by the negative. In such a compound gas as CO_2 we shall assume an average ion of mass $m_3/2$ and of volume $2\pi a_3^3/3$, and shall consider the average of the nearly equal mobilities of the positive and negative ions. Then (8) becomes

$$e \frac{dE}{dx} = 3.59 N_3 a_3^2 m_3 v_3 [1 + 2\{\phi(a_1 + a_3) + \psi(a_1 + a_3)\}/m_3 v_3^2] u. \quad (10)$$

For air at 0°C . the left side of this equation has the value 10^{-12} because $e = 3 \times 10^{-10}$ and $dE/dx = \text{volt/cm.} = 1/300$. To evaluate the right side we put $N_3 m_3 = 0.001293$, and $N_3 m_3 v_3^2 = 3pV$ in the usual notation of the gas laws, with $V = 1$, and $p = 1,014,000 \text{ dynes/cm.}^2$, and $2a_3 = 2.38 \times 10^{-8}$ (Phil. Mag. Feb. 1909, p. 321), while

$$[1 + 2\{\phi(a_1 + a_3) + \psi(a_1 + a_3)\}/m_3 v_3^2] u = u(1 + C'/T) \\ = A'T^{\frac{1}{2}} = 0.222 \times 273^{\frac{1}{2}} = 3.67$$

for both the positive and the negative small ion in air at 0°C . With these values the right side of (10) is 1.18×10^{-13} , whereas the left side is 10^{-12} . Here we meet, even in a more pronounced form, the discrepancy which has led previous writers to suppose the small ion to be a cluster of molecules, for which $2a_3$ would be large enough to make the

right side equal to the left. It shows that Wellisch's theory does not give the right reason for the difficulty. The frictional resistance expressed by the right side of (10) must be made 8.6 times as great to make the dynamical theory of the small ions in air correct. The additional resistance is introduced if we take account of the two new types of viscosity (Phil. Mag. [6] xiv. p. 1) shown to be fundamental for the motion of ions in liquids. We shall find these to be as important in the dynamical theory of ions in gases. The first of these viscosities, whose coefficient is ζ , has its origin in the mutual potential energy of the opposite electron charges of the two kinds of ion. The second has its origin in the mutual potential energy of an ionic charge, and the polarization which it induces in the neighbouring molecules of the surrounding medium. Its coefficient was denoted by θ . As the causes producing ionization and maintaining it in a liquid solution are different from those acting in a gas, it will be necessary to make a special calculation for ζ and θ in a gas, though similar to that furnished for liquid solutions.

In the paper just mentioned it is shown that if q positive and q negative ions of charge e are uniformly distributed through a cm.^3 , of a medium of dielectric capacity K , they possess a rigidity

$$N = \frac{2\pi}{3K} e^2 q^{4/3}.$$

If they are strained by electric force so that each positive electron is displaced in one direction and each negative in the opposite, a corresponding stress is developed. Now the presence of molecules may cause this strain to relax by forcing the ions back to uniform distribution; they will convert the rigidity N into a viscosity NT , where T is the time required to reduce the stress to $1/e$ of its initial amount, e being the base of natural logarithms. When the ions are those of an electrolytic solution, the relaxing action of the molecules of solvent is due to their ionizing force, that force which pulls the ions of the solute apart and keeps them uniformly distributed. In the paper just cited this ionizing force is taken to be proportional to $q^{2/3}$ as it overcomes the direct electric attraction between neighbour opposite ions. But in a gas we do not regard the molecules as direct ionizers. What then are the actions in a gas tending to keep ions uniformly distributed? The chief one is the following. According to the principle of the electric origin of molecular attraction each molecule behaves like an electrically polarized

or electrized sphere, similar to a uniformly magnetized sphere, and its electric axis tends to set itself very readily along the lines of electric force. In the absence of external electric force the axes of neighbours so adjust their mutual directions as to produce cohesion. But when the powerful central electric force of an electron acts upon neighbour molecules, it forces their electric axes all to pass nearly through itself. Those molecules next to the nearest neighbours have their axes caused to converge towards the electron, but not so accurately, and so on. Thus the cohesive forces of the molecules are deranged, and the surrounding molecules are attracted towards the electron. We shall consider the law of this attraction soon, but just now we must recognise that near the electron the gas will be compressed by this attraction to a greater density than the average. The tendency of this denser gas to diffuse outwards from the central electron is just equilibrated by the inward electric attraction. Let r be the distance between two neighbour electrons, then we may take the average value of the force of diffusion per cm^2 of section to vary as $1/r$. Hence, when two neighbour electrons have r between them changed to $r + dr$ by electric force, the unequilibrated force of diffusion called into action will be proportional to dr/r^2 . But dr/r is the strain to which the associated stress F' is proportional, so the force restoring uniformity is proportional to F'/r ; but in Maxwell's theory of transition from rigidity to viscosity this is proportional to F'/T , therefore in a gas T is proportional to r . It must also be proportional to the resistance offered by the gas to the motion of the whole ion with unit velocity, which we have denoted by F . Thus we have $T \propto Fq^{-1/3}$ and therefore ζ , the viscosity NT derived from N , varies as Fqe^2/K . For a gas we may put $K=1$.

We have now to obtain an expression for θ . The electric polarization consists in the directing of the axes of electrization of the molecules towards the nearest ion. Let R be the distance from an ion to the six nearest molecules. Though this is less than the average distance between two molecules because of condensation due to attractive forces round the ion, it is proportional to that distance. The mutual potential energy of an ion and a neighbour molecule is $\phi(R) + \psi(R)$. The rigidity N of the system ion and molecules is equal to the potential energy per unit volume and is therefore proportional to $\{\phi(R) + \psi(R)\}/R^3$.

We have now to find the time of relaxation T . The force between ion and molecule is

$$d\{\phi(R) + \psi(R)\}/dR.$$

Per unit area it is proportional to

$$R^{-2}d\{\phi(R) + \psi(R)\}/dR,$$

and the change of this for change dR is

$$dR d[R^{-2}d\{\phi(R) + \psi(R)\}/dR]/dR.$$

But the associated stress F' is proportional to dR/R , so in this case the force restoring uniformity when the system of ion and molecules is strained is proportional to

$$F'R d[R^{-2}d\{\phi(R) + \psi(R)\}/dR]/dR$$

and also to F'/T , so that T^{-1} is proportional to

$$R d[R^{-2}d\{\phi(R) + \psi(R)\}/dR]/dR.$$

Now we know that for two electrons the potential varies as $1/R$, for an electron and an electric doublet as $1/R^2$, and for two doublets as $1/R^3$. In the actual case at distance R the form of $\phi(R) + \psi(R)$ is almost that of $1/R^2$, as will appear in Table II. : hence in the product NT we get a result proportional to R^5/R^5 , that is, a constant. Thus in a gas this factor of θ is independent of the density, just as the ordinary viscosity of a gas is. In the previous reasoning we have considered only the molecules which are immediate neighbours of an ion. Similar reasoning applies to the next more remote lot of neighbours and so on. It is important to notice that the argument by which this factor of θ was proved independent of density makes it also independent of the nature of the gas ; it is a universal constant. To the above reasoning we must add the statement that T is proportional to F , so that NT and therefore θ is proportional to F , which varies from one gas to another. In applying these new viscosities ζ and θ in the study of the motion of ions through gases we treat θ as acting just like F , while as regards ζ we consider it to act over area $q^{-2/3}$ with a relative motion $u_1 - u_2$ between two neighbour ions at distance $q^{-1/3}$ apart, u_1 being the velocity of the positive ion and u_2 of the negative, so that the resistance experienced by an electron is

$$\zeta(u_1 - u_2)q^{-2/3}/q^{-1/3}.$$

For the motion of the ions in a field of intensity dE/dx we have the equations

$$\left. \begin{aligned} e \frac{dE}{dx} &= \zeta(u_1 - u_2)q^{-1/3} + (\theta + F)u_1 \\ -e \frac{dE}{dx} &= -\zeta(u_1 - u_2)q^{-1/3} + (\theta + F)u_2 \end{aligned} \right\} \quad (11)$$

The easiest case is that in which q is so small as to make $\xi q^{-1/3}$ negligible. These equations then take the same form as (1) with F replaced by $\theta + F$. We found that in the case of air the right-hand side of (10) must be made 8.6 times as large as it is to give the facts of experiment. Thus for air then we have $\theta = 7.6F$, and in forming the theory of θ we found that this same relation must hold for all gases. Thus then we have found that the small ion in gases does not consist of a cluster of molecules, but that its electric charge causes it to experience a viscosity of electric origin and also to behave as if it had an enlarged radius. Introducing the factor 8.6 into the right-hand side of (10) we get as the general equation for the mobility of a small ion the equation

$$e \frac{dE}{dx} = 30.9 N_3 a_3^2 m_3 v_3 [1 + 2\{\phi(a_1 + a_3) + \psi(a_1 + a_3)\}/m_3 v_3^2] u \quad (12)$$

$$\text{or } 1 = u T^{-1/2} (1 + C'/T) / A'.$$

In establishing this we have confirmed the theory of electrically induced viscosity by finding that it is the chief factor in determining the mobility of ions in gases. We have also found that the potential energy of ion and molecule in contact in gases plays the same part as the mutual potential energy of molecules in contact in the theory of the ordinary viscosity of gases. Investigations of C in the theory of gaseous viscosity and of C' in (12) enable us to study the potential energy of molecules under ideally simple conditions. To find C' directly for any substance experiments like those of Phillips on air will be necessary. But from the known values of u for different substances we can make a preliminary indirect determination of C' and therefore of

$$\phi(a_1 + a_3) + \psi(a_1 + a_3).$$

From (12) we have, when $dE/dx = \text{volt/cm.}$, and the temperature and pressure are fixed,

$$um^{1/2} a^2 (1 + C'/T) = \text{constant.} \quad (13)$$

This is the origin of a roughly approximate law announced by Lenard that $um^{1/2}$ is constant for several gases. For both positive and negative ion in air at 15°C. we have seen that

$$u(1 + C'/T) = A'T^{1/2} = 0.222 \times 288^{1/2}, \quad a = 1.19 \times 10^{-8}.$$

It is rather more convenient at present to use B^{23} instead of a^2 , B being the limiting volume of a gramme-molecule, and to use the ordinary molecular mass M instead of m , namely 28.86 for air. The relation between B and a is

$$B = 4 \times 10^{19} \times 22430 (2a)^3.$$

Thus we transform (13) to

$$uM^{1/2}B^{2/3}(1+C'/288)=106\cdot9 \dots\dots (14)$$

which is the desired equation for finding C' from a single measurement of u for a substance and its known values of M and B . The values of u have been collected by J. J. Thomson in his book on 'The Conduction of Electricity through Gases,' and by Wellisch with new determinations of his own for a number of vapours, in a paper read before the Austr. Assoc. Adv. Sc. Jan. 1909. There is a difficulty in comparing the values of B when these are arrived at by different methods for different substances. This can be seen by comparing the results of two methods for one substance, for example, Cl_2 . For this gas its viscosity gives $2a=3\cdot13 \times 10^{-8}$, and so $B=27\cdot4$, whereas in the inorganic chlorides as well as in the organic B for Cl_2 is 38 (Phil. Mag. [5] xxxix. p. 1). It cannot be assumed that the value of B for Cl in Cl_2 is necessarily the same as that for Cl in compounds, but I think the chief part of the discrepancy between 27·4 and 38 is to be traced to the assumption that molecules of a gas can still be treated as spheres while they are in collision. The kinetic theory of gases gives us $2a$ as the distance between the centres of two molecules while they are in collision, which may be less than the mean diameter of either. For this reason I shall present the data for discussion in two ways, first using only the values of B derivable from the kinetic theory of gases, and second only those derived from the apparent limiting volumes of liquids and solids.

TABLE II.

	H_2 .	He.	N_2 .	O_2 .	Cl_2 .	CO.	CO_2 .	N_2O .
$2a \times 10^{10}$	181	160	246	225	313	228	241	277
100B	528	368	1330	1026	2742	1066	1254	1912
$10u$	73	57	16	16	10	10·7	7·9	8·2
M	2	4	28	32	71	28	44	44
$100C'/288$	241	293	125	133	39·5	290	278	175
$100C'(2a)^2/288$	731	698	702	708	359	1404	1501	1251

The values given for $10u$ are $5(u_1+u_2)$, that for He being obtained by Franck and Pohl (*Ann. d. Ph. Beibl.* xxxi. 1907, p. 1133), namely $u_1=5\cdot09$ and $u_2=6\cdot31$. To this monatomic gas the formula (14) for diatomic and compound

gases has been applied without change. The values calculated for $100C'/288$ belong to an imaginary mean ion. For the positive and negative ions separately they can be found by using u_1 and u_2 , instead of their mean u . In the last line $100C'(2a)^2/288$ has been given. For four out of five element gases it is nearly 710, the exceptional value for Cl_2 being 359. For the three compound gases the values are not far from 1385, which is nearly 2×710 . At 15°C . both CO_2 and N_2O are a few degrees below their critical temperature, so that their actual condition does not comply with the stipulations made in obtaining the theoretical equations. Probably the error introduced is not great. But in the case of the more easily liquefied gases with higher critical temperature like NH_3 , SO_2 , and HCl , and of the vapours of liquids the orbit of the relative motion of ion and molecule after collision will be too often a curve of finite range to allow the theory founded upon infinite range to hold. Nevertheless, for the sake of information I shall apply (14) to the data for easily liquefied gases and vapours. For these the values of B can be obtained from "Further Studies on Molecular Force" (Phil. Mag. [5] xxxix. p. 1).

TABLE III.

	NH_3 .	SO_2 .	HCl .	$\text{C}_2\text{H}_5\text{Cl}$.	C_5H_{12} .	CH_3CO_2 .	$\text{CH}_3(\text{C}_2\text{H}_5)_2\text{O}$.	$\text{CH}_3\text{CO}_2\text{C}_2\text{H}_5$.
B.....	21	35	27	57	93.5	62	82	80
$100u$	73	41	127	31	34	31	29	28
M	17	64	36.5	64.5	72	74	74	88
$100C'/288$...	366	204	53	190	80	156	127	119
$10B^{2/3}C'/288$.	274	218	48	281	165	244	240	221

	CH_3Br .	CH_3I .	CCl_4 .	$\text{C}_2\text{H}_5\text{I}$.	Aldehyde.	Alcohol.	Acetone.
B.....	47	57	84.5	74	39.5	48	56.5
$100u$	28	21	30	16	29	29	29
M	95	142	154	156	44	46	58
$100C'/288$...	201	188	47	203	376	311	229
$10B^{2/3}C'/288$.	262	278	90	358	436	411	337

In this table we notice that when much the greater part of the mass of a molecule consists of chlorine as in HCl and CCl_4 , the potential energy measured by $100C'/288$ is con-

spicuously small, as in the case of Cl_2 , and so also is $10B^{2/3}C'/288$ conspicuously small. But in $\text{C}_2\text{H}_5\text{Cl}$ this effect does not appear. In aldehyde, alcohol, and acetone, which are known to be associative liquids, the potential energy and its product by $B^{2/3}$ are both large. The other substances of the table give values of $10B^{2/3}C'/288$ near to 240, except C_5H_{12} , which has the value 165. This 240 is nearly double 138.5 found for compound gases in the last table. We learn then, that on the whole $B^{2/3}C'$ increases with the liquefiability of a substance. For the substances of Table III. the values of $100C'/288$ are only apparent values of the potential energy, since our theory would apply to them strictly only at temperatures above the critical. Probably the values of $100C'/288$ in Table III. are roughly proportional to the desired potential energies. As stated above, the Table has been drawn up to give only preliminary information. Returning to the more definite facts of Table II., we see that constancy of $B^{2/3}C'$ would imply that the potential energy of ion and molecule in contact varies inversely as the square of the distance between their centres. Now one of the main results found in my studies on molecular attraction is this, that molecules attract one another as though each were a uniformly electrized sphere of total electric moment E . If an ion of charge e and volume half that of the molecule were in contact with the molecule in the position of minimum potential energy, that energy would be proportional to $-Ee/B^{2/3}$. This shows the probable origin of the constancy of $B^{2/3}C'$. But I have shown that in most cases E is proportional to B . So it appears that, if the ion were able to turn the whole field of electrization of the molecule so that its axis passes through the ion in the position of minimum potential energy, C' would be proportional to $B/B^{2/3}$, that is to $B^{1/3}$, instead of to $1/B^{2/3}$, as we have found above to be the case for H_2 , He , N_2 , and O_2 . It appears then that instead of the E which is proportional to B we have to do with another electric moment E' which is the same for these four element gases. An ion does not turn the whole electric field of a molecule so that its axis passes through the ion, but on the average it turns that electric field so that its component along the line joining the centre of ion to centre of molecule is that arising from an electric moment E' which is the same for four out of five element gases in Table II. The absolute magnitude of this moment can be found by means of the following data for H_2 at 15°C ., using 4×10^{19} for the number of molecules

in a cm.³ at 0° C. and 1 atmo. $Nmv^2 = 3pV288/273$ with

$$p = 1,014,000, \quad V = 1, \quad N = 4 \times 10^{19},$$

$$2\{\phi(a_1 + a_3) + \psi(a_1 + a_3)\}/mv^2 = C'/288 = 2.41,$$

$$\therefore \phi(a_1 + a_3) + \psi(a_1 + a_3) = 916 \times 10^{-16}.$$

If E' were due to two opposite electrons of charge e at distance x apart, we should have

$$\phi(a_1 + a_3) + \psi(a_1 + a_3) = e^2x/4a^2,$$

whence

$$x = 0.0352 \times 10^{-8}.$$

I hope to show in a future communication that the fundamental mode of motion determining the structure of all spectra is that of two opposite electrons vibrating within each atom about positions at a mean distance apart of the order 0.05×10^{-8} . It seems very probable then that this electric moment E' , which we have been investigating here, is the moment of this pair of electrons which at the same fixed distance apart in all atoms determine all radiation and the structure of all spectra. The further investigation of C' must be a matter of special experiment, just as in the case of C in the viscosity of gases.

The coefficients of diffusion D of ions in gases are proportional to their mobilities, being a measure of their mobilities under the driving force of a certain space-rate of change of partial pressure, instead of the electric force of a volt/cm. Their values have been investigated chiefly by Townsend, and have been found to be some such fraction as a fifth or a tenth of those for comparable uncharged molecules. This difference has been generally ascribed to the formation of molecular clusters round each ion. From the foregoing it is plain that the difference is due to the operation of the electrically induced viscosity θ on the ions. In making a useful comparison of the two types of diffusion coefficient we must remember that we are assuming the ion in a pure gas to be of half the mass and half the volume of the molecule. Townsend has determined D for the positive ions in CO_2 as 0.023, and for the negative 0.026, mean 0.0245. For comparison with this we must choose the value of the diffusion coefficient for a gas which has about half the molecular mass 44 of CO_2 and half its molecular volume. The nearest we can get to this is to take the case of CO whose molecular mass is 28 diffusing into CO_2 with a coefficient 0.131, or that of O_2 with coefficient 0.136. In these cases the molecular volume of CO and O_2 is nearly

equal to that of CO_2 , a difference which could be approximately allowed for by a short calculation, but it is hardly worth making, as the point under consideration is well enough presented by comparing the mean of 0.131 and 0.136 with 0.0245. Thus we find the resistance to the diffusion of the ion 5.4 times as great as the corresponding resistance to that of the molecule. This result is in general agreement with the demonstration given that the new viscosity θ increases the resistance of ordinary gaseous viscosity to 8.6 times its amount. The hypothesis of molecular clusters is unnecessary to explain the slow diffusion of ions in gases.

We must also discuss another quantity closely related to u , namely α , the coefficient of recombination of ions in gases according to the very simple formula introduced by J. J. Thomson :

$$dN_1/dt = -\alpha N_1^2. \quad . \quad . \quad . \quad . \quad (15)$$

There are certain fundamental objections to the excessive simplicity of this formula which makes the recombination take place according to the chemical law of mass action, notwithstanding the large range of the electric forces of attraction and repulsion amongst ions. Langevin has given (*Ann. de Ch. et de Ph.* [7] xxviii. 1903, pp. 289 & 433) the now generally accepted proof that this simplicity is justified. The essence of his argument is this : that the electric force at the surface of a sphere surrounding an ion being e/r^2 , and the number of opposite ions that can be drawn across the surface of the sphere being proportional to $4\pi r^2 N_1 u_1$, the number entering the sphere in unit time is proportional to $4\pi e N_1 u_1$, and is thus independent of the radius of the sphere and of the range of the electric forces of the ions. If we take account of the mobilities of both sorts of ions we get Langevin's formula

$$\alpha = 4\pi e(u_1 + u_2), \quad . \quad . \quad . \quad . \quad (16)$$

where ϵ is that fraction of the ions drawn into a sphere which combine to form neutral molecules. Langevin has investigated values of ϵ experimentally, and Richardson (*Phil. Mag.* [6] x. 1905, p. 242) has applied considerations of probability to the calculation of ϵ . But it seems to me that, though Langevin's theory gives as a first approximation the justification of J. J. Thomson's equation (15), a second approximation is necessary to bring the theory of ions in gases into agreement with experimental facts and the kinetic theory of gases. Experimental proof of the insufficiency of (15) has been given by Barus (*Ann. der Phys.* xxiv. 1907, p. 225), using J. J. Thomson's condensation method of counting N_1 .

The results of Barus will be discussed soon in connexion with the following attempt to carry the theoretical equations of Thomson and Langevin to a higher degree of approximation. If the ions were always distributed symmetrically, each would be in statical equilibrium as regards the electric forces, and the beginnings of recombination would never arise. Recombination depends chiefly upon the rate at which unsymmetrical positions are formed during the motion of the ions. Let us start, then, with a convenient specification of the symmetrical positions. Suppose the space divided into equal cubes of edge R with an ion placed at each corner which is common to 8 cubes, the ions occurring alternately negative and positive along the straight lines formed by the edges of the cubes according to the specimen plan of fig. 1. Like ions are arranged along diagonals. If any ion is displaced from a diagonal, it will be attracted most strongly towards the nearest diagonal. Let us then replace the electrons by lines of electricity of density $e/2^{1/2}R$ along diagonals running downwards from left to right.

Fig. 1.

#	b	#	b	#
b	#	b	#	b
#	b	#	b	#
b	#	b	#	b
#	b	#	b	#

Then in fig. 2 let us make a section of these parallel lines by a plane at right angles to them, the intersections of the lines and plane being marked # for the positive lines and b for the negative.

Fig. 2.



Again # ranges in diagonal lines, and so does b. These diagonals are at distance $R/3^{1/2}$ apart, while the distance between # and its neighbour # is $R(3/2)^{1/2}$. Spread the lines of density $e/2^{1/2}R$ so that they become planes of surface density $e/3^{1/2}R^2$, these planes being alternately # and b. In this way we have substituted for the original point distribution an equivalent laminar one. We can now treat the problem of recombination of ions as one of leakage in our laminar distribution. Writing the intensity of the electric force between two laminæ as $4\pi e/3^{1/2}R^2$, we consider the rate of leak proportional to this force and to the mobility u_1 or u_2 of the ions, and to N_1 . But $N_1R^3 = 1$, so with A a parameter we have finally

$$dN_1/dt = -AN_1^{5/3}. \quad . \quad . \quad . \quad . \quad (17)$$

This factor of proportionality A is proportional to $u_1 + u_2$,

and to the rate at which unsymmetrical distributions arise to produce the motion which we have treated as a leak. This equation can be written

$$dN_1^{2/3}/dt = -\frac{2}{3}A(N_1^{2/3})^2, \quad . \quad . \quad . \quad (18)$$

and its integral is

$$N_1^{-2/3} = C + 2At/3, \quad . \quad . \quad . \quad (19)$$

where C is a constant. Thus we have $N_1^{-2/3}$ a linear function of the time, whereas the integral of (15) makes N_1^{-1} a linear function of the time. From the tabulated data of Barus for ions in air, I find with second for unit t

$$10^5 N_1^{-2/3} = 52 + 6t \quad . \quad . \quad . \quad (20)$$

with the following comparison :—

Seconds	0	5	10	20	30	60	120
$10^{-3}N_1$ exp.	83	38	27	14	9	5	1
$10^{-3}N_1$ cal.	84	35	27	14	9	4	1.5

Barus gives five other sets of data in graph form, and in these $10^5 \times 2A/3$ has a value about 4.5. The comparison just given covers a range of N_1 large enough to show that (17) is to be preferred to (15). But as (15) has been taken by J. J. Thomson to be proved by various experiments originating in the Cavendish laboratory, it is necessary to look rather more closely into the evidence. In the paper of J. J. Thomson and Rutherford on the passage of electricity through gases exposed to Röntgen rays (Phil. Mag. [5] xlii. 1896, p. 392), the experimental results can be expressed with a slightly smaller average error by the N_1^2 formula than by the $N_1^{5/3}$ one, but they can be expressed with a still smaller average error by a formula using $N_1^{5/2}$. These early experiments are therefore not suitable for deciding between the two formulæ. In Rutherford's paper on the rate of recombination (Phil. Mag. [5] xliv. 1897, p. 422) the experimental results are represented with the following average errors per cent. :—

	On page	423	427	429
With the N_1^2 formula	5.7	6.0	6.7	
„ „ $N_1^{5/3}$ „	2.2	3.5	7.7	

The theoretical handling of McClung's experiments (Phil. Mag. [6] iii. 1902, p. 283) on the rate of recombination of
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ions in gases under different pressures has been subjected to some criticism in the *Phil. Mag.* (see G. W. Walker, viii. 1904, p. 206, Robb, x. 1905, p. 237), and by Langevin (*Journ. de Phys.* [4] iv. 1905, p. 322), who shows that on the basis of the N_1^2 formula it can be proved that in the experiments at a pressure of one atmo diffusion would cause one-tenth of the effect recorded as recombination by McClung,

ile at one-eighth of an atmo at least eight-tenths of the recorded effect is to be credited to diffusion. Diffusion must also have produced a large effect in the large apparent variation of α with temperature in later experiments of McClung's. We can take McClung's experiments at 1, 2, and 3 atms to be complicated with not more than 10 per cent. of diffusion effect. I find that his results can be represented as well by the $N_1^{5/3}$ formula as by the N_1^2 . The reason for this fact is that the residual experimental error in these difficult measurements is such that when N_1 is graphed as ordinate with t as abscissa, the points lie on an area which is traversed just as well by the curve which makes $N_1^{-2/3}$ linear in t as that which makes N_1^{-1} linear in t . Thus I think I have shown that the experimental evidence upon which J. J. Thomson relies for verification of the N_1^2 formula (15) is on the whole a little more favourable to the $N_1^{5/3}$ formula (17). But (15) fails to apply to the experiments of Barus given previously in verification of (17), which is therefore the better approximation to the truth. In this connexion it would be important to make a full review of Langevin's own experiments, which, on account of wide variations of the conditions, he considers to verify the two laws that ionic velocity is proportional to strength of electric field, and that the rate of recombination of the two sorts of ions is proportional to the product of their numbers per c.c., or the two electrical densities which he denotes by p and n . Thus Langevin holds that his experiments verify the N_1^2 formula. The theory of his method of experimenting is rather elaborate, so he contents himself with giving only final results and samples of his data which are not sufficient to allow me to investigate how the $N_1^{5/3}$ formula would apply to his experiments directly. But the theory by which the $N_1^{5/3}$ formula is established receives indirect support from Langevin's results, if we work it out in greater detail. We must investigate A in (17) more closely. Imagine the two sorts of ions for an instant uniformly distributed, alternately at the corners of a uniform cubical subdivision of the volume which they occupy. On account of its thermal velocity each ion will move away from this imaginary position, and the amount

by which it moves away will determine the degree of irregularity which causes the recombination which we suppose to go on as a sort of leak. To determine A as a function of the density of the gas containing the ions is a problem in the calculus of probabilities known as that of the random walk. A man walks a distance l in any direction from an origin O , then he walks the same distance in any other direction, and so on till he has taken n walks. It is required to find the probability that his final distance from O lies between r and $r+dr$. Evidently this is the same as our ionic problem, for the ion after n free paths of average length l is at some required average distance from O , its original position in the initial imaginary uniform distribution. The solution has been given by Rayleigh ('Nature,' lxxii. 1905, p. 318; Phil. Mag. [5] x. 1880, p. 73, xlvii. 1899, p. 246) for the case when n is large. The required probability is, when $l = 1$,

$$\frac{2}{n} e^{-r^2/n} r dr. \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

and the probability that the distance from O is greater than r is $e^{-r^2/n}$. If we introduce l explicitly the exponent becomes $-r^2/l^2n$. In comparing results for different substances or for the same substance at different pressures, we ought to take n to be the number of paths described in unit length, because, though the departures from uniformity recur with a frequency proportional to the molecular velocity, each lasts a time inversely proportional to the velocity. For this reason the element of time does not enter into the comparison. Thus nl is to be constant for the same substance at different pressures and also for different substances. We have now to find how r is to be chosen in order that the comparison may be statistically correct. The agencies maintaining uniformity are proportional to l the mean free path, for instance, D the coefficient of diffusion is proportional to l . But the same agencies produce departures from uniformity temporarily, while they are maintaining average uniformity, just as the thermal velocities, which keep the average pressure of a gas constant and also its average density constant, at the same time cause those temporary local variations of pressure and density which occur when two molecules collide. As regards departures from uniformity, then the proper unit for measuring its amount linearly is the mean free path of a molecule. Thus r must be proportional to l . Accordingly r^2/l^2n takes the form cl where c is a constant. Let l_0 be the value of l when the

pressure is $p_0 = 760$ mm. of Hg, then $l = l_0 p_0 / p$, and we now have A in (17) proportional to $(u_1 + u_2)e^{-cl_0 p_0 / p}$,

$$\therefore \log A/(u_1 + u_2) \text{ is linear in } 1/p. \quad (22)$$

If we compare (15) and (17), and suppose them applied to the same set of experiments over a certain range of values of N_1 , we see that for the average value of N_1 we must have approximately $A = \alpha N_1^{1/3}$. But in Langevin's experiments N_1 was the number of ions generated by a single flash from a Röntgen bulb, so that on the average for gas at different pressures we may take N_1 proportional to p . Thus then A is proportional to $\alpha p^{1/3}$, and so from (16) $A/(u_1 + u_2)$ is proportional to $\epsilon p^{1/3}$, and so from (22) $\log \epsilon p^{1/3}$ is linear in $1/p$. Thus from Langevin's experiments I find with p in mm. of Hg

$$\left. \begin{array}{l} \text{for air} \quad \log_{10} \epsilon p^{1/3} = 1.35 - 745/p \\ \text{for CO}_2 \quad \log_{10} \epsilon p^{1/3} = 1.35 - 509/p \end{array} \right\}, \quad (23)$$

which furnish the following comparison with experiment:—

Air.						
p	152	375	760	1550	2320	3800
ϵ exper.....	·01	·06	·27	·62	·80	·90
ϵ calcul.	·00005	·03	·26	·64	·81	·91

CO ₂ .						
p	135	352	550	758	1560	2380
ϵ exper.....	·01	·13	·27	·51	·95	·97
ϵ calcul.	·0007	·11	·32	·50	·91	1·02

It is to be remembered that the whole theory and working out of Langevin's experiments is based upon (15) and (16) with the condition that ϵ must be less than 1. In these circumstances the very simple law e^{-cl} , which has been worked out above for comparing departures from uniformity of distribution favourable to recombination, is verified as well as possible.

From the values found for α in (15) for various gases it is easy to obtain A in (17). For instance, Townsend (Phil. Trans. exciii. 1900, p. 129), by an electrical method, in which during 0·93 second N_1 diminished in the ratio 77 to 46 for air, found $\alpha = 3420e$, where e is the electron charge 3×10^{-10} . So by this electrical method $A = 0\cdot0000832$.

We have already seen that the experiments of Barus by the entirely independent condensation method yield 6 and 4.5 for $10^5 \times 2A/3$, so that the mean value of A is 0.0000787, with which the result obtained from Townsend's work is in excellent agreement. With the N^2 formula Barus found that in his experiments α ranged from 3 to 10 times that given by Townsend's and similar experiments. McClung and Langevin found values for α in air in close agreement with that of Townsend, though they used entirely different electrical methods. The experiments of McClung when used to give A yield for air the value 0.000176, which is about double those just calculated. The data of Langevin do not contain all the particulars necessary for a calculation of A absolutely. We shall now consider the values of A for the four gases of Townsend's experiments, obtained from his data

	Air.	O ₂ .	CO ₂ .	H ₂ .
α/e	3420	3380	3500	3020
$10^7 A$	832	769	800	922

It is remarkable that each parameter has a value that changes but little from one gas to another. McClung and Langevin by their independent methods get nearly the same value as Townsend for α for CO₂, and McClung's value in the case of H₂ agrees with that of Townsend. We have now to see how the values just given for A accord with the theory of it. In (23) the coefficient of $1/p$ is to be proportional to the mean free path of an ion through the gas at a pressure of 760 mm., which is inversely proportional to the square of the molecular diameter with appropriate allowance for the attraction between ion and molecule. Thus we have the coefficient of $1/p$ in (23) inversely proportional to

$$(2a)^2(1 + C'/288),$$

which can be calculated from the data of Table II., which yield the following value for the coefficient:—air 745, O₂ 860, CO₂ 462, and H₂ 908. The value 462 thus obtained for CO₂ is to be compared with 509 obtained in (23) from Langevin's experiments. These enable us to calculate for each gas at 760 mm. this coefficient divided by 760 which is the part of $\log \epsilon p^{1/3}$ we require. The corresponding factor of $\epsilon p^{1/3}$ multiplied by $u_1 + u_2$ or by u from Table II., according to the theory of A , is to be proportional to A . Here are the values of A divided by this product:—air 50, O₂ 65, CO₂ 41,

and H_2 20. If with CO_2 we use the coefficient 509, the result is 47. If in $(2a)^2(1+C'/288)$ the effective diameter of H_2 were increased by 16 per cent., the low value 20 for H_2 would be brought up to 50. Such uncertainties must be cleared up by new experiments and by fresh interpretations of those already on record, which have been worked out by their authors on the basis of the insufficient N_1^2 formula. The foregoing theory of A may be summarised by its results in the following way. Let l be the mean free path of an ion in the gas, being $\cdot 677/N_3\pi(2a_3)^2(1+C'/288)$ which for nitrogen (say, air) has the value 0.000004, then

$$A = \cdot 0002484(u_1 + u_2)e^{-2\cdot 2565l/l_0} \quad \dots \quad (24)$$

where $l_0/2\cdot 2565$ is the constant length 0.000001754, which seems to be a fundamental constant in the physics of ions, related to a similar one in the statistics of molecules. In concluding the present discussion of the recombination of ions it may be helpful to notice that the formula (16) of Langevin can be derived from our method of laminæ in the following way. Suppose the ions in cubical order at distance R apart. In a laminar distribution we get density e/R^2 and force $4\pi e/R^2$ between the laminæ. A pair of oppositely charged ions experiencing this force would acquire relative velocity $4\pi e(u_1 + u_2)/R^2$ and would travel R in time

$$R^3/4\pi e(u_1 + u_2).$$

The ions begin to move so that N per unit volume would disappear in this time. In the actual process the N positive and the N negative ions do not suddenly disappear by coming together simultaneously, but their instantaneous rate dN/dt is $4\pi e(u_1 + u_2)N/R^3$ and since $NR^3=1$, this is (16) with $\epsilon=1$. This brings out the assumption involved in (15) and (16).

2. *The Large Ion without Liquid or Solid Nucleus.*

If the small ion investigated in the previous section is placed in a gas whose molecules it attracts so strongly that it carries its immediate neighbours with it as satellites, some profound changes in the conditions are brought about. The induced viscosity θ almost disappears, because the central small ion has but little relative motion to its immediate neighbour molecules in which it has induced electrization, and the molecules relative to which it is moving are so far away that the induced electrization in them is small. We have here a beautifully simple instance of the way in which a rigidity may merge into a viscosity. It is probable that a

small ion in a moist gas tends to attach molecules of H_2O to itself as satellites and to surround itself with an envelope of vapour of H_2O . To such an ion we shall now apply equations (11) with the supposition that θu can be neglected and also Fu as a separate term, though Fu , as the cause of ζu , will be investigated immediately.

In large ions it has been found that $u_2 = -u_1 = -u$, so that the two equations reduce to

$$e \frac{dE}{dx} = 2\zeta u q^{-1/3} \dots \dots \dots (25)$$

ζ is proportional to q , so that when $dE/dx = \text{volt/cm.}$, the last equation makes $uq^{2/3}$ constant for different values of q at a given temperature. This is the result discovered by Moreau in his experiments on the cooled gases of flames sprayed with electrolytic solutions (*Ann. de Ch. et de Ph.* [8] viii. 1906, p. 201, *Compt. Rend.* cxli. 1905, p. 1225). He found that in the cooled gases the number of ions per cm.^3 varies as the square root of the concentration of the electrolyte in the solution, in agreement with the law of Arrhenius for uncooled gases of flames at ordinary flame temperatures. H. A. Wilson has pushed the temperature up till the ionic dissociation in gases sprayed with electrolyte is complete.

If in Moreau's experiments we denote the concentration of the sprayed electrolytic solution by n , we have $q \propto n^{1/2}$ and so from (25) we get $un^{1/3}$ constant at a given temperature. He found $u_1 = -u_2$ and q of the order 10^6 . We shall take his results with solutions of KCl as typical, their strengths being normal N , $N/4$, and $N/16$, which we shall express by putting $n=1$, $1/4$, and $1/16$, giving his values of u in cm./sec. for $dE/dx = \text{volt/cm.}$

TABLE IV.

Temp. C. 170°			110°		100°		70°		30°		15°	
100u 100un ^{1/3}			100u	100un ^{1/3}	100u	100un ^{1/3}	100u	100un ^{1/3}	100u	100un ^{1/3}	100u	100un ^{1/3}
$n=1$	39	39	16	16	16	16	10	10	4.5	4.5	1.5	1.5
$n=1/4$	65	41	27	17	24	15	15	9.4	—	—	2.4	1.5
$n=1/16$	90	36	47	19	40	16	24	9.5	7.2	2.9	5.1	2.0

The departures from constancy in $un^{1/3}$ are irregular. Moreau sought to give a different explanation for the origin of the constancy of $un^{1/3}$ in his various solutions.

The main feature of (25) being verified, we shall now apply it to investigate the remarkable variation of u with temperature. So we must consider ζ more closely. The first business is to find the number of H_2O molecules kept captive by an electron, or rather by our nucleolus. We have already seen that an electron in air increases the density of the air round it. In the same way, but to a greater extent, it increases the density of the H_2O round it, because H_2O has an exceptionally large electric moment es . For this reason an electron can constrain molecules of H_2O to describe orbits of finite range round it. In the sequel, as before, let suffix 1 refer to the nucleolus, 3 to the gas such as air in which the ion is moving, and 2 to H_2O . The electric moment of H_2O is es_2 , and if an electron e is at distance r along the axis from the centre of H_2O , their mutual potential energy is e^2s_2/r^2 . Now when the nucleolus has captured one or two H_2O molecules, its velocity of translation may be neglected in comparison with that of the air molecules and of the free H_2O . So we treat the electron as at rest, and the free H_2O molecules as if moving with velocity v_2 past it. The dynamical condition that a molecule of H_2O with velocity v_2 at distance r should just be able to travel to infinity is $m_2v_2^2/2 = e^2s_2/r^2$. Within this radius r the electron gathers a number of H_2O molecules whose axes all tend to pass through it, so that radially these molecules will attract one another, while each will repel its lateral neighbours which are at the same distance from the electron. Thus we have a highly characteristic field of force in our cluster of H_2O molecules. The lateral repulsions will nearly equilibrate one another, so that we can treat all the H_2O molecules as revolving round the centre with constant average linear velocity v_2 . So it is necessary to take account of the radial cohesion. The cohesive potential energy of a molecule of H_2O just retained on the outer surface of radius r by the combined effect of attraction from electron and of cohesion may be assumed to be approximately constant and be written in kinetic form $m_2w_2^2/2$. Hence to determine r we have the more complete equation

$$m_2(v_2^2 - w_2^2)/2 = e^2s_2/r^2. \quad . \quad . \quad . \quad (26)$$

Now that we have taken account of the cohesive energy, we can make a correct enough average case by treating all the water molecules as gathered on the surface of this sphere of radius r round the nucleolus and all moving with the average velocity of the ion. We shall take the number of H_2O molecules per unit surface of this sphere to be propor-

tional to N_2 the number per unit volume in the air. The total number on the sphere is proportional to $N_2 r^2$, say equal to $AN_2 r^2$. We wish to find the resistance F to the motion of this sphere through the air with unit velocity. Just as in the theory of the viscosity of gases we may say that on account of the electric force of the nucleolus and the cohesive force of the sphere, the number of molecules of air encountered by the sphere will be increased in the proportion $1 + (2e^2 s_2 / r^2 m_2 + w_2^2) / v_2^2 : 1$. Unless the number of H_2O molecules in the sphere is great, the tendency will be for each molecule to experience as much resistance as if the others were absent. It is not a case of a compact sphere of radius r , but of $AN_2 r^2$ spheres of radius a_2 . If the H_2O liquefies into something like water, we shall have to deal rather with a single large sphere formed by the liquid. For the resistance to one molecule of H_2O moving with unit velocity through the air we get from the kinetic theory of diffusion

$$\frac{4}{3} N_3 (a_2 + a_3)^2 \left\{ 2\pi (v_2^2 + v_3^2) / 3 \right\}^{\frac{1}{2}} m_2 m_3 / (m_2 + m_3) \quad (26 a)$$

Strictly we ought to write down the resistance offered to the nucleolus, but, as it would be quite similar to this, we shall merge it in the resistance offered to the $AN_2 r^2$ molecules. Thus

$$F = \frac{4}{3} AN_2 N_3 r^2 \left\{ 1 + (2e^2 s_2 / r^2 m_2 + w_2^2) / v_2^2 \right\} (a_2 + a_3)^2 \\ \times \left\{ 2\pi (v_2^2 + v_3^2) / 3 \right\}^{\frac{1}{2}} m_2 m_3 / (m_2 + m_3).$$

Replacing $m_2 v_2^2 / 2$ by T the temperature and $m_2 w_2^2 / 2$ by T_a a constant temperature, we find from (26) that $r^2 \propto 1 / (T - T_a)$ and $1 + (2e^2 s_2 / r^2 m_2 + w_2^2) / v_2^2 = 2$, while $(v_2^2 + v_3^2)^{\frac{1}{2}} \propto T^{\frac{1}{2}}$. Returning to (25) remembering that ξ is proportional to Fq , we get

$$e \frac{dE}{dx} = C q^{2/3} N_2 N_3 T^{\frac{1}{2}} u / (T - T_a) \quad . \quad . \quad (27)$$

where C is constant. This is the general equation for the motion of an ion consisting of nucleolus and H_2O molecules in an electric field. To apply it to the case of Moreau's experiments we put $n^{1/3}$ for $q^{2/3}$, and have N_2 , N_3 , and n all varying inversely as T , so that u the mobility when $dE/dx = \text{volt/cm.} \propto T^{11/6} (T - T_a)$. From Table IV. the mean value of $un^{1/3}$ at each temperature gives a useful mean value of u for the case in which the solution sprayed into the flame was normal, and from these I find that $T_a = 270$. In the next Table the first row gives temperatures, the second

mean values of u in cm./sec. for $dE/dx = \text{volt/cm.}$, and in the third $10^{10}u/T^{11/6}(T-270)$ which by (27) is to be constant.

TABLE V.

Temp. C.	170°	110°	100°	70°	30°	15°
100u	39	17	16	10	3.7	1.7
	317	276	300	308	317	292

This theory of the slower ion in gases leads to the revelation of an error in the ordinary experimental method of measuring their mobilities, the true values in the limit being only half of those assigned. Let AB and CD be two

electrodes giving an electric field from AB to CD between which a stream of ionized gas is led in the direction of the arrow. It is assumed that, when all the ions are just caught as the stream flows uniformly, a positive ion entering at A travels along AD, and a

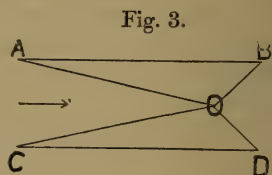


Fig. 3.

negative ion entering at C travels along CB. But, if this were so, the positive ion after it had crossed the intersection of AD and CB would have no neighbour negative ions, and would be free from the viscous resistance ξ of electric origin, it would finish its path more swiftly under the resistance F only. The paths of electrons entering at A and C are AOD and COB. If u_a is the mobility along AO and u_d along OD at right angles to AB, we have $AC/u = AC/2u_a + AC/2u_d$ or $2/u = 1/u_a + 1/u_d$. Thus when u_d is large, u_a the desired mobility is only half of u which is usually assigned as the experimental measure of u_a . Moreau's values of u quoted in this paper should be divided by 2, as should also Langevin's mobility for the very slow ion in air, namely about $1/3000$, which should be nearly $1/6000$, and a similar remark applies to many other determinations of u . Since, so far, we have been considering only relative values of u this correction is not needed for the purposes of the previous part of this section. It is important in absolute calculations concerning these ions. This error does not affect the velocities of the small ions considered in section 1.

3. The Large Ion with Liquid or Solid Nucleus.

To account for the very small mobility of the large ion of Langevin I have imagined the structure already described,

namely a nucleus of $(\text{H}_2\text{O})_2$ or $(\text{H}_2\text{O})_3$ or both in a state very similar to that of a liquid surrounded by an envelope of H_2O vapour which is kept highly concentrated close to the nucleus. This envelope is similar to the surface film of vapour of H_2O deposited on the grains of fine powders. The number of H_2O molecules per cm^3 close to the nucleus will have a value N_{2s} like a saturation value, and the number will diminish with increasing distance from the nucleus till it becomes N_2 where its influence has ceased. The thickness of this transition layer may be taken to be constant for a given value of N_3 , or we can replace it approximately by a constant length l such that $4\pi r^2 l$ is the volume of the transition layer, in which the mean value of N_2 is not $(N_{2s} + N_2)/2$ but $E'N_{2s} + H'N_2$ where E' and H' are constants. The resistance to the motion of all the vapour with unit velocity through the air will be $4\pi r^2 l(E'N_{2s} + H'N_2)$ times (26 a). We can use (26 a) also to give the resistance to the motion of the nucleus by replacing $a_2 + a_3$ by r . We neglect the resistance of the nucleolus and write the total resistance to unit velocity in the form

$$r^2 N_3 T^{1/2} (E'N_{2s} + G' + H'N_2) \quad . \quad . \quad . \quad (28)$$

where G' is the constant for the nucleus. This is F , and neglecting it in comparison with $\zeta q^{-1/3}$ which is proportional to $Fq^{2/3}$ we obtain from (25)

$$e \frac{dE}{dx} = r^2 q^{2/3} N_3 T^{1/2} (EN_{2s} + G + HN_2) u \quad . \quad . \quad (29)$$

where E , G , and H are constants. Thus the reciprocal of the mobility for $dE/dx = \text{volt/cm.}$ is linear in N_2 which is proportional to the humidity. This is one of Pollock's results for the Langevin ion. He finds $1/u = 1200 + 107.5h$, where h is the humidity in grams/m^3 . The constant term 1200, being not far from half the 2500 to 3000 usually found for the Langevin ion in ordinary air, shows that the resistance associated with N_{2s} is about equal to that associated with N_2 , a result which necessitates the stipulation above that the mean value of N_2 takes the form $E'N_{2s} + H'N_2$. Equation (29) shows that the mobility of a Langevin ion in a gas should be inversely proportional to the density, so that refined observations in the atmosphere ought to be reduced to values for standard atmospheric pressure. But the most striking result in (29) is that $1/u$ should be proportional to $q^{2/3}$, varying rapidly with change of concentration of the large ions. Moreover, as before in (27), r^2 is probably proportional to $1/(T - T_b)$, so that (29) gives a fairly complicated dependence of u upon the natural variables.

Further experiments must elucidate many of the points raised in the foregoing sketch of a theory. According to the theory the experimental values of the velocities found hitherto for the large Langevin ion ought to be doubled for the same reason as that for doubling the ionic experimental mobilities discussed in section 2. Very interesting questions present themselves as to the equilibrium of these three orders of the ion in the atmosphere. Radioactive substances generate the small ions, most of which disappear through recombination in such a way as nearly to keep the number of small ions constant under a given set of conditions. But some of the small ions will gather envelopes round them and so build up the ion of the intermediate type. This will be liable to destruction through neutralization of its charge by a small ion. The difficulties of building up the large Langevin ion must be great, but once it is formed, the following principle will act favourably to its permanence. The ionizing power of water comes into play. Consider a very small drop of water in which there is one molecule of NaCl . This will be ionized into Na^+ and Cl^- , which will be driven as far apart as possible by the ionizing action of the water, whose tendency is favourable to the splitting of the drop into two parts charged with the opposite ions. Thus then the nucleus of a large Langevin ion by its ionizing action will tend to keep its charge unneutralized by opposite ionic charges. So we can account for the permanent existence of the Langevin ion in the atmosphere. These large ions sometimes carry 50 times as much electricity as the small ions in the atmosphere at the same time. Lusby has shown that when the Langevin ions are removed from air, 22 minutes are required for the formation of the same number. If the large ion contains a nucleus of quasi-liquid H_2O , its structure merges into that of the drop of water in cloudy condensations. Something similar to the large ion ought to appear in the condensation of H_2O on small ions, and in the evaporation of a drop surrounding an ion. The theory of large ions is closely connected with that of cloudy condensation. Now J. J. Thomson in 'Conduction of Electricity through Gases' and Langevin (Bloch, *Ann. de Ch. et de Ph.* [8] iv. 1905, p. 25) have given a theory of cloudy condensation upon ions, which in both cases seems to furnish a remarkably successful explanation of the experimental fact that fourfold saturation is needed in vapour of H_2O to produce cloudy condensation on small ions. But they use quite different expressions from those developed here concerning the potential energy of an electron and surrounding molecules. It seems to me that

their expressions are not the right ones for the purpose to which they apply them. For example J. J. Thomson writes $e^2/2Ka$ for the electrical potential energy of an electrified drop of radius a surrounded by a medium of dielectric capacity K , which is put $=1$, and Langevin also uses $e^2/2a$. They then apply this to an electron surrounded by water. But, unless the electron spreads itself over the surface of the drop in much the same way as a large number of like electrons would, this expression is no longer valid. The electrical energy of an electron and an envelope of molecules is a mutual affair, tending to take the form e^2s/a^2 for a central electron and a molecule at distance a , so the total electric energy of electron and drop must be expressed by a form quite different from the $e^2/2a$ used by both authors. Then again they carry the conception of surface tension and surface energy even into the dynamical specification of the smallest drops. The directive effect of the central electron on the few molecules of the smallest drop must profoundly modify their cohesional forces. For these reasons the theory of Thomson and Langevin appears to me to be quite illusory in its details and to give its remarkable result concerning fourfold saturation by a concealed compensation of errors. They give a theory of what is essentially a molecular affair by means of purely molar considerations and data. Their theory applies to drops which are large enough to have a surface tension like that of ordinary water and which carry a surface charge of electricity. It can give no account of the real happenings when vapour of water begins to gather round a small ion.

SUMMARY.

In a dynamical theory of the ions of gases the two new types of viscosity previously investigated for ions in electrolytic solutions are found to be of paramount importance. The induced viscosity, namely that which arises from the electric action of the ion upon surrounding molecules, produces in all gases about 7.6 times as much resistance to the motion of the small ion as ordinary gaseous viscosity does. The other type of viscosity, namely that which arises from the mutual energy of oppositely charged ions, is not of large enough amount to make its appearance in the experimental study of small ions at the low concentrations in which these are usually employed. In the case of the small ion there is another result of the electric action between it and surrounding molecules, for this causes collisions to occur more frequently, as cohesional force does in the theory of the effect of temperature on the viscosity

of gases. In a gas above the critical temperature and at ordinary pressures the mobility of a small ion varies with temperature according to the formula $u = A'T^{1/2}/(1 + C'/T)$ in which A' is a parameter characteristic of each gas, and C' is proportional to the mutual potential energy of ion and molecule when in contact during a collision, being similar to C , the corresponding quantity in the theory of the viscosity of gases. On the principles of the kinetic theory of gases A' is found for any gas in terms of the number, diameter, mass, and velocity of its molecules, by an analysis which leads finally to (12). The theory is verified by means of the experiments of Phillips on the effect of temperature on the mobility of small ions in air, and by means of the data so far obtained experimentally for the mobility of small ions in gases and vapours. It leads to values of the potential energy of ion and molecule in contact which are given in Tables II. and III., and these furnish evidence of the presence in H_2 , He, N_2 , and O_2 of an electric doublet of constant electric moment. By taking account of the new induced type of viscosity and also of C' it is shown that the small ion has no molecules attached to it. It is like the ion of electrolytic solutions. The smallness of the coefficients of diffusion of ions is traced to the induced viscosity, the hypothesis of molecular clusters being unnecessary. As regards the rate of recombination of small ions in gases it is necessary to replace the first approximation expressed in the formula of J. J. Thomson, namely $dN/dt = -\alpha N^2$, by a second approximation $dN/dt = -AN^{5/3}$, which is deduced theoretically by regarding recombination as a leak in laminar distribution of the ionic charges. This formula is verified over a wide range of values of N by the experiments of Barus and also by the experiments adduced by J. J. Thomson in support of the N^2 formula. From statistical considerations formula (24) is established for A and verified by the experiments of Langevin on the effect of pressure on the rate of recombination of ions in air and CO_2 .

But although the small ion is not associated with molecular clusters, there are cases in which the small ion attaches molecules to itself and becomes a large ion. These are divided into two classes, the large ion consisting of an envelope of vapour, such as that of H_2O , surrounding a small ion which is the central nucleolus, and the larger ion in which a liquid or solid nucleus forms round the ionic nucleolus, the whole being surrounded by an envelope. With these large ions the induced viscosity becomes of negligible importance, because the moving electric charge is too far from the molecules of the surrounding gas. But the direct electric

viscosity arising from the rigidity belonging to the evenly mixed ions becomes of paramount importance because of the greatly increased time of relaxation for large ions. The theory of the large ion without nucleus leads to (27), an equation connecting the mobility of this kind of large ion with concentration and temperature. This is verified in Tables IV. and V. by means of the experiments of Moreau on the slow ions formed by spraying solutions into flames and cooling the resulting gaseous mixture. For the large ion of Langevin, on the supposition that it has a nucleus of water, equation (29) is worked out to show the connexion of mobility with concentration of the ions, density of the air, humidity and temperature. This shows general agreement with the results of Pollock as to the effect of humidity on the mobility of the large ion of Langevin. Experimental data are lacking to test it otherwise. To account for the permanence of this large ion it is pointed out that the ionizing power of water tends to keep it intact. Certain objections are raised to the theory of J. J. Thomson and Langevin to account for cloudy condensation of H_2O vapour on small ions at fourfold saturation.

During the working out of the foregoing I have had the advantage of correspondence with Prof. Pollock while engaged in his experimental inquiries on large ions.

Melbourne, June, 1909.

XLIV. *The Echelon Spectroscope, its Secondary Action, and the Structure of the Green Mercury Line.* By HERBERT STANSFIELD, D.Sc., Demonstrator in Physics, Manchester University*.

Thesis approved for the Degree of Doctor of Science in the University of London †.

[Plates X.-XII.]

Part I.—THE ECHELON SPECTROSCOPE.

The Echelon and its Mounting.

THE echelon spectroscope described in this paper was constructed by Messrs. Adam Hilger for Professor Schuster, a modification, which has proved to be very valuable, being made in the usual design. A front elevation,

* Communicated by the Physical Society: read June 11, 1909.

† The following alterations have been made:—Equations 2, 3, 4, 9, 9A, and the section on the spectrum given by a hot lamp have been added; also the faint lines previously described as doubtful are shown to have their origin in the echelon.

plan, and two end elevations of the echelon are reproduced on Plate X., the plan being drawn with the cover removed.

There are 33 glass plates. The smallest plate is 13 mm. wide, and they increase in width by steps of 1 mm. until the last plate is reached, which is 12 mm. wider than the last but one, the aperture being reduced to 1 mm. by the screen S. The effective aperture of the first plate is similarly reduced in width to 1 mm. by the block B. All the plates are 40 mm. high, and the common thickness is 9.48 mm. The plates are pressed together by the two nickel steel rods marked T, which are intended to have the same coefficient of expansion as the glass. The plates are in very close contact, in most cases the greater part of an interface being taken up by a patch of definite but irregular outline that appears "black" by reflected light. The remaining part of the interface generally shows white of the first order, but here and there the film of air may be thick enough to show the yellow or even the red of the first order.

The common thickness of the glass plates is 9.48 mm.; their refractive index, deduced by a Hartmann formula from the values given by the makers of the glass, is 1.5802 for the green line 5461; and the dispersive power, $\frac{d\mu}{d\lambda}$, is -918 per cm. for this wave-length.

Optical Effects due to clamping the Plates.

The echelon produces a slight cylindrical convergence in a beam of light, altering the focus of the observing telescope by 1 mm. (the focal length of the object-glass being 53 cms.). This effect is probably due to the clamping, as Twyman* states that when the clamping pressure is applied a change of focus is produced. He attributes the effect to a uniform increase in the compression of the plates from the largest to the smallest; but there is direct evidence (see p. 378) that the echelon plates become slightly prismatic, and this effect, increasing towards the smaller end, would also produce convergence.

The focus is altered by rotating the echelon about a vertical axis. The convergence is increased, as would be expected, by turning the echelon so as to reduce the width of the emergent beam (see fig. 3 A), and diminished when it is turned so as to make the beam broader (fig. 3 B). Changes in the focus are also produced by covering some of the step-faces at either end of the echelon.

* Twyman, Proceedings of the Optical Convention, 1905, p. 53.

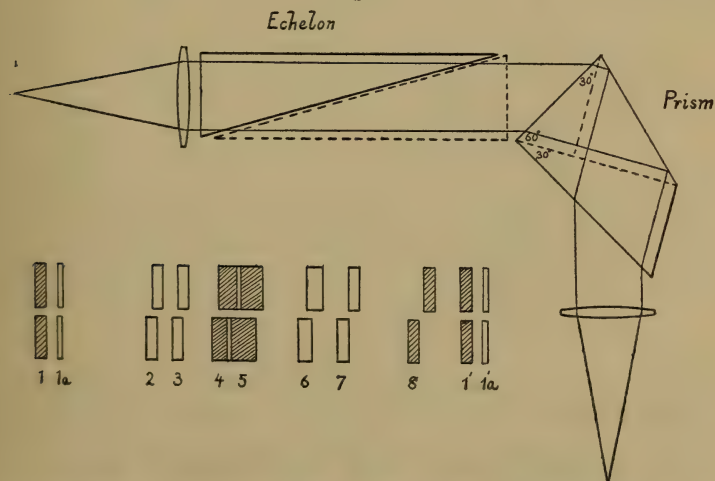
Use of Auxiliary Prism.

Auxiliary spectroscopes have generally been employed to pick out the particular line in the spectrum to be examined by the echelon, the slit of the echelon spectroscope being illuminated with the selected line; but a modification in the design of this instrument was made for Professor Schuster so that the prism from the auxiliary spectroscope could be mounted next to the echelon, as shown in fig. 1, the prism being made larger than usual in order to take the full width and height of the echelon beam. This arrangement, which appears to have been originally contemplated by Professor Michelson, has been found to have important advantages.

The dispersion produced by the prism, which is 2 per cent. of that given by the echelon, is subtracted from the echelon dispersion when the echelon is in the usual position shown by full lines, and is added to it when the echelon is in the reversed position shown by the dotted lines.

The change of 4 per cent. in the dispersion obtained in this way produces the alteration in the spectrum shown in fig. 1. The distance apart of successive orders of the same wave-length is not altered; but when the dispersion is reduced by the prism, all the lines belonging to the same order approach one another and draw away from the neighbouring orders. It is evident from fig. 1 that all but two of the lines

Fig. 1.



represented belong to the same order, and that these two lines, marked 1' and 1'a, belong to the next higher order.

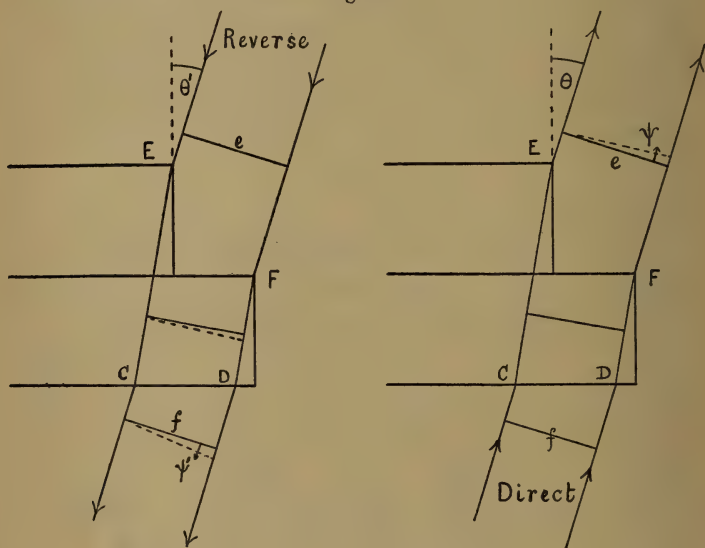
With the prism on an auxiliary spectroscope, no evidence of this kind is obtained; and it has simply been assumed in some cases that the companion lines belonged to the bright central line nearest to them.

PRIMARY ACTION OF THE ECHELON IN THE DIRECT AND REVERSED POSITIONS.

Equations for the Principal Maxima.

The theory of the echelon given by Michelson* for the case of light falling normally on the larger end, has been extended by Galitzin†, who has investigated, both by calculation and experiment, the motion and changes in the dispersion of the spectra when the echelon is rotated through small angles within the limits of two or three degrees on either side of the normal position. As the theory of the echelon in the reversed position has not, as far as I know, hitherto been considered, a comparison will be made below with the theory of the ordinary action.

Fig. 2.



In fig. 2 the reversed and direct cases are represented diagrammatically. The rays and wave-fronts drawn with full

* A. A. Michelson, *Astrophysical Journal*, viii. pp. 36-47 (1898). *American Journal* [4] v. pp. 215-217 (1898). *Journ. de Phys.* [3] viii. pp. 305-314 (1899).

† Fürst B. Galitzin, *Bulletin de l'Académie Impériale des Sciences de St. Pétersbourg*, 5 série, t. xxiii. pp. 67-118 (1905).

lines are those regularly refracted, the dotted wave-fronts are drawn perpendicular to the directions in which a maximum of order n is formed. The angle between this direction and the normally refracted rays is called ψ in the direct case and ψ' in the reversed case. The distance apart in the air of regularly refracted rays passing through corresponding points E and F of neighbouring step-faces is called f at the end of the echelon and e on the step side. The condition for a principal maximum is that the sum of the distances of E from the incident wave-front on one side and the dotted wave-front on the other, shall be n wave-lengths greater than the sum of the distances of the corresponding point F from the same wave-fronts.

When the angles ψ and ψ' are sufficiently small, we may employ Fermat's principle and measure the optical paths along the regularly refracted rays. This gives the general equations in the form

$$R - e\psi = n\lambda, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$R - f\psi' = n\lambda, \quad . \quad . \quad . \quad . \quad . \quad (1A)$$

where R is the retardation produced in a regularly refracted ray by its passage through a single plate. Here the equations referring to the reversed case are distinguished by a letter A.

The values of R , e , and f are given below, and are plotted in fig. 4 (p. 380).

$$R = t \left(\mu \sqrt{1 - \frac{\sin^2 \theta}{\mu^2}} - \cos \theta \right), \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$e = s \cos \theta + t \sin \theta, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$f = s \cos \theta + \frac{t}{\mu} \sin \theta \frac{\cos \theta}{\sqrt{1 - \frac{\sin^2 \theta}{\mu^2}}}, \quad . \quad . \quad . \quad (4)$$

Here s is the width of the step-faces, t the thickness of the plates, and θ the angle of incidence of the light on the plates. This angle is in practice generally kept within the narrow limits of $\pm 2^\circ$, and it is sufficient to retain only the lowest powers of θ . These expressions then become

$$R = R_0 \left(1 + \frac{\theta^2}{2\mu} \right), \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$e = s + t\theta, \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$f = s + t \frac{\theta}{\mu}, \quad . \quad . \quad . \quad . \quad . \quad (7)$$

R_0 (the value of R when $\theta=0$) is $(\mu-1)t$. The parabolic expression for $R - R_0$ in equation (5) gives a very close

approximation to the value given by (2), the difference, which depends on θ^4 , being only 1 part in 800 when $\theta = 5^\circ$. The slight deviations of e and f from the linear expressions (6) and (7) may be detected in fig. 4.

Substituting the approximate expressions for R , e , and f in equations (1) and (1 A), we obtain the general equations in the form:

$$R_0 \left(1 + \frac{\theta^2}{2\mu} \right) - s\psi \left(1 + \frac{t}{s} \theta \right) = n\lambda, \quad . \quad . \quad . \quad (8)$$

$$R_0 \left(1 + \frac{\theta'^2}{2\mu} \right) - s\psi' \left(1 + \frac{t}{s} \frac{\theta'}{\mu} \right) = n\lambda. \quad . \quad . \quad . \quad (8A)$$

Equation (8) is in agreement with Galitzin's calculated values and has the support of his measurements which were made on orders close to the position of greatest brightness so that the angle ψ was always small.

If the principal maxima several places removed from the direction of the regularly refracted rays are considered, it becomes necessary to take into account the terms depending on ψ^2 , which were neglected in applying Fermat's principle. When the path of the diffracted light is measured along the diffracted rays, the equation for the direct case may be written

$$R - e \sin \psi + (t \cos \theta - s \sin \theta)(1 - \cos \psi) = n\lambda. \quad . \quad (9)$$

If we now suppose the light to retrace its path, the angle of incidence, θ' , on the step-faces is equal to $\theta - \psi$ (see fig. 2), and the angle $\theta' + \psi'$ at which the diffracted rays leave the end plate is equal to θ . Hence the equation for the reversed case may be found by substituting $\theta' + \psi'$ for θ and ψ' for ψ in equation (9) after substituting for R its value from equation (2). The equation thus obtained may be written in the form:

$$t(\sqrt{\mu^2 - \sin^2(\theta' + \psi')}) - \cos \theta' - s\{\cos \theta' \sin \psi' - \sin \theta'(1 - \cos \psi')\} = n\lambda. \quad . \quad (9A)$$

If the terms whose order in θ and ψ is higher than the second are neglected, equations (9) and (9 A) reduce to

$$R_0 \left(1 + \frac{\theta^2}{2\mu} \right) - s\psi \left(1 + \frac{t}{s} \theta - \frac{1}{2} \frac{t}{s} \psi \right) = n\lambda. \quad . \quad (10)$$

$$R_0 \left(1 + \frac{\theta'^2}{2\mu} \right) - s\psi' \left(1 + \frac{t}{s} \frac{\theta'}{\mu} + \frac{1}{2} \frac{t}{s} \frac{\psi'}{\mu} \right) = n\lambda. \quad (10A)$$

These equations only differ from (8) and (8 A) by the addition of small terms depending on ψ^2 and ψ'^2 , whose values are given in Table I. below.

Position of the Orders in the Field of View and Dispersion.

According to the first approximation formulæ (1) and (1A)

$$\psi = \frac{R - n\lambda}{e} \quad \text{and} \quad \psi' = \frac{R - n\lambda}{f}.$$

R , e , and f depend on θ but not on ψ ; so the orders are equally spaced and the dispersions, given by

$$\frac{d\psi}{d\lambda} = - \frac{n - t \frac{d\mu}{d\lambda}}{e} \quad \text{and} \quad \frac{d\psi'}{d\lambda} = - \frac{n - t \frac{d\mu}{d\lambda}}{f},$$

do not vary along the spectrum.

The second approximation formulæ (10) and (10 A) give

$$\psi = \frac{R - n\lambda}{s \left(1 + \frac{t}{s} \theta - \frac{1}{2} \frac{t}{s} \psi \right)} \quad \text{and} \quad \psi' = \frac{R - n\lambda}{s \left(1 + \frac{t}{s} \frac{\theta}{\mu} + \frac{1}{2} \frac{t}{s} \frac{\psi'}{\mu} \right)}$$

If the echelon is rotated a little, so that one order, say the m th, is in the position of maximum brightness where ψ or $\psi' = 0$,

$$\psi = \frac{m - n}{1 + \frac{t}{s} \theta - \frac{1}{2} \frac{t}{s} \psi} \left(\frac{\lambda}{s} \right) \quad \text{and} \quad \psi' = \frac{m - n}{1 + \frac{t}{s} \frac{\theta}{\mu} + \frac{1}{2} \frac{t}{s} \frac{\psi'}{\mu}} \left(\frac{\lambda}{s} \right)$$

Table I. gives the values of the small terms in the denominators depending on ψ and ψ' for the first five orders on either side of the central order.

TABLE I.

$m \sim n.$	$\frac{1}{2} \frac{t}{s} \psi.$	$\frac{1}{2} \frac{t}{s} \frac{\psi'}{\mu}.$
1	·003	·002
2	·005	·003
3	·008	·005
4	·010	·006
5	·013	·008

The Wave-Length Intervals between the Orders.

The repetition of a line in a number of orders provides an echelon spectrum with a wave-length scale of nearly equal divisions, and the wave-length value of these divisions, $\Delta\lambda$, is a constant of the echelon which can be calculated for any wave-length from the thickness of the plates and the refractive indices given for the glass. The expressions for $\Delta\lambda$ for the direct and reversed cases may be obtained from the general equations. Neglecting the terms depending on θ^2 , which do not affect the values by more than one part in ten thousand, they both reduce to

$$\Delta\lambda = \frac{\lambda}{n-t} \frac{d\mu}{d\lambda}.$$

The changes that are made in n by rotating the echelon are so small compared with its whole value (10,070 for this echelon and the green mercury light) that $\Delta\lambda$ is sensibly constant for all positions. On the other hand, $\Delta\lambda$ may be increased or decreased by 2 per cent. by means of the auxiliary prism, as described on p. 373.

NOTES ON CHANGES PRODUCED BY ROTATING THE ECHELON
ABOUT A VERTICAL AXIS.

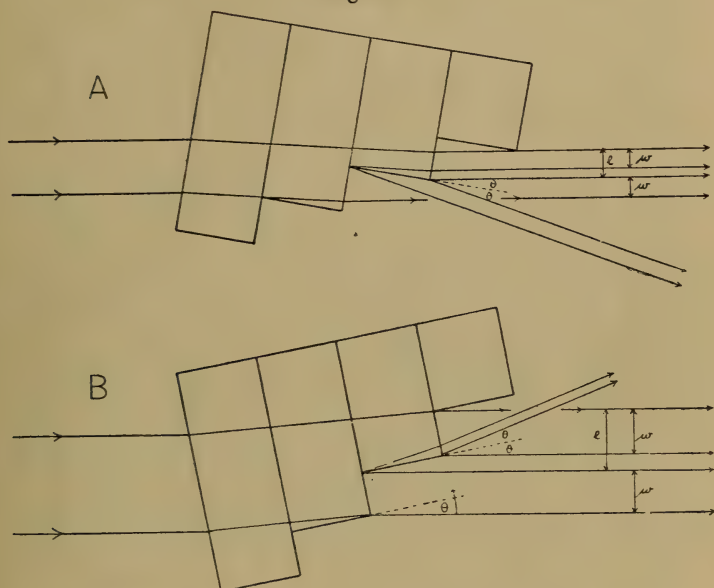
Changes in brightness, and the position of greatest brightness.—As an order is moved across the field of view by rotating the echelon, it crosses the lateral maxima of the distribution of light due to the individual step apertures, disappearing as it crosses the diffraction minima, and having its greatest brightness as it crosses the central diffraction maximum. This position of greatest brightness corresponds to the direction of the regularly refracted rays, and is the origin from which the angle ψ is measured. It does not quite coincide with the position of the image of the slit when the echelon is removed, the position of greatest brightness being displaced towards the side on which the step-faces of the echelon lie. This displacement indicates that the echelon-plates are slightly prismatic.

Changes in dispersion and retardation.—Some of the effects produced by rotating the echelon about a vertical axis are represented in fig. 3. The echelon is shown at A rotated so as to make θ negative; in this case e is smaller and therefore (see p. 377) the dispersion is larger than in

the normal position. B shows the echelon rotated in the opposite direction so that e is increased and the dispersion decreased.

The way in which R , e , and f change with θ is represented in fig. 4. The scale for R is given in wave-lengths, m being

Fig. 3.



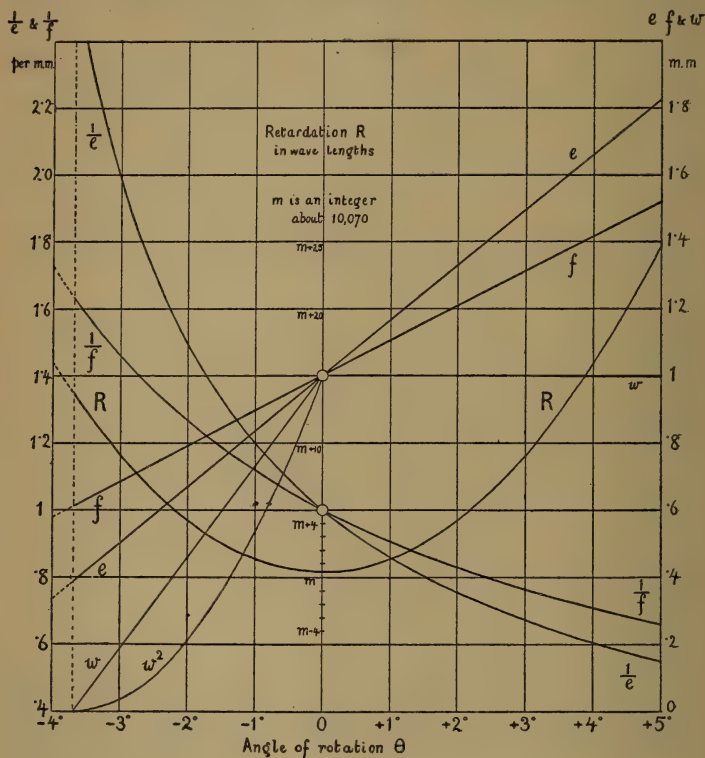
the number of wave-lengths in the retardation of the order which is nearest to the position of greatest brightness on the lower retardation side when $\theta=0$. The positions of the row of marks on the $\theta=0$ line represent, by their distances from the apex of the curve, the distances of the orders near the m th from the position of greatest brightness when $\theta=0$, and the abscissæ of the curve passing through the marks which lie within it, give the angle through which the echelon must be turned in either direction in order to bring orders higher than the m th up to the position of greatest brightness.

The reciprocals of e and f are also plotted to show how the dispersion is changed by the rotation in the direct and reversed cases. These changes in dispersion are simply changes in magnification, as they are not accompanied by any sensible change in the wave-length intervals between the orders.

The way in which the effective width, w , of the individual step apertures is reduced when the echelon is rotated so as

to make θ negative, is represented in figs. 3 and 4; w^2 is also plotted in fig. 4, to indicate the falling off in the intensity of the light on this account.

Fig. 4.



Width of the diffraction-bands.—It will be seen from fig. 4 that both e and f are greater than w , except when $\theta=0$.

Hence, the angular interval, $\frac{\lambda}{w}$, between the equally spaced diffraction minima is in general greater than $\frac{\lambda}{e}$ or $\frac{\lambda}{f}$, the angular intervals, in the direct and reversed cases, between successive orders.

Light reflected from the ends of the plates.—The step ends of the echelon plates, which are finely ground, scatter some of the light falling upon them and also give regularly reflected beams, represented in fig. 3. The reflected beams produce echelon spectra; but as the ends are not polished, the spectra

are poorly defined and the spreading of the light brings into greater prominence the broad diffraction-bands corresponding to the narrow sources represented by the illuminated end-faces of the plates. When the echelon and prism are used together, as in fig. 1, the reflected spectrum of the green mercury-line may be made to overlap the direct spectrum of the violet line by giving θ a small negative value. I think it would be an advantage to have the ends of the plates polished or blackened.

The position of minimum deviation.—As the echelon is rotated from one side of the normal position to the other, the orders first move across the field of view in the direction of decreasing deviation (measured by ψ) and then turn round and go back again. If the value of R happens to be a whole number of wave-lengths for normal incidence, say $m\lambda$, then the m th order will come to its position of minimum deviation in the position of greatest brightness when $\theta=0$, but higher and lower orders will not then be quite in their positions of minimum deviation. Consider, for example, a lower order: ψ (or in the reversed case ψ') will be positive, and its value when $\theta=0$ can be reduced a little by increasing θ (in the positive direction), as at first the value of R is almost stationary, while e is increasing, and therefore the dispersion is decreasing.

By differentiating the general equations (1) and (1 A), it will be found that the conditions for the turning-points in the direct and reversed cases are,

$$\frac{dR}{d\theta} = \psi \frac{de}{d\theta} \quad \text{and} \quad \frac{dR}{d\theta} = \psi' \frac{df}{d\theta}.$$

Substituting the values of the differential coefficients and calling the minimum values of the deviation in the two cases ψ_M and ψ'_M and the corresponding angles of incidence θ_M and θ'_M ,

$$\theta_M = \frac{\mu}{\mu-1} \psi_M \quad \text{and} \quad \theta'_M = \frac{1}{\mu-1} \psi'_M.$$

Hence as the echelon is rotated, so as to increase θ , the orders higher than the m th come to their minimum deviation positions before the echelon is normal to the light, and the lower orders have their minimum deviation after the normal position is passed. The central orders are very nearly in their positions of minimum deviation when $\theta=0$. If ψ_0 and ψ'_0 are written for the values of ψ and ψ' in this case,

it may be shown that

$$\frac{\psi_0 - \psi_M}{\psi_M} = \frac{1}{2} \frac{t}{s} \frac{\mu}{\mu - 1} \psi_M \quad \text{and} \quad \frac{\psi'_0 - \psi'_M}{\psi'_M} = \frac{1}{2} \frac{t}{s} \frac{1}{\mu - 1} \psi'_M,$$

and the values calculated from these formulæ show that, when this echelon is in the direct position, $\psi_0 - \psi_M$ is .7 per cent. of ψ_M for an order one place from the position of greatest brightness, 1.4 per cent. of ψ_M for an order two places away, and so on. The corresponding values for the reversed position are .4 per cent. and .9 per cent.

Effects produced by Temperature Changes.

The position of the various orders in the field of view when the echelon is in a definite position, such as the normal position, depends on the temperature of the echelon and on the refractive index of the surrounding air. Records of the temperature of the echelon and micrometer readings of the position of the orders in the field of view, taken from day to day, show that a rise of temperature of 8.6° C. moves the spectrum through a distance equal to the interval between neighbouring orders, indicating an increase of one wavelength in the retardation, R , for this rise of temperature.

Curving of the Spectrum Lines.

The echelon spectrum lines, like those of a prism spectrum, are curved with the concave side toward the violet end. The curving is due to the variation in the angle of incidence, and therefore in the retardation, R , of light from different points along the length of the slit. The theory has been given by Laue*.

Consider the simplest case, in which the slit is vertical and the plates are normal to the axis of the collimator: then light from points above and below the centre of the slit will have a vertical plane of incidence on the plates. Writing i instead of θ for the angle of incidence in equation (3),

$$R - R_0 = \frac{1}{2} t \frac{\mu - 1}{\mu} i^2 ;$$

but from equation (1),

$$R - R_0 = e(\psi - \psi_0) ;$$

* *Physikalische Zeitschrift*, vi. pp. 283-285 (1905).

and in this case $e = s$, so that

$$\psi - \psi_0 = \frac{1}{2} \frac{t}{s} \left(\frac{\mu - 1}{\mu} \right) i^2. \quad . \quad . \quad . \quad (8)$$

Hence, for small values of i , the spectrum lines are parabolas. The curvature is very small. If, for example, the extreme values of i are ten times the angular separation of the orders, $\Delta\psi$, then the top and bottom of the curved image of the slit, representing one order of the spectrum, will be 2 per cent. of $\Delta\psi$ to the violet side of the centre of the image.

Effects produced by Rotating the Echelon about a Horizontal Axis parallel to the Plates.

The echelon can readily be tilted in this way by placing a block under the foot at the small end. The block which I employ tilts the echelon through an angle of nearly 3° , and the photographs Nos. 1 to 3 in Plate XI. of the green mercury line were taken with this angle of tilt. The horizontal axis of the parabolic lines, which passes through the normal to the plates, has been raised much above the field of view of these photographs, so that the lines where they cross the field of view are considerably inclined to the vertical. The reproductions in Plate XI. have the centre of the curves below them, as they have been turned round in order to put the shorter wave-lengths on the left.

Part II.—SECONDARY ACTION OF THE ECHELON.

Secondary Bands in the Primary Spectrum.

The inclined spectrum lines of photograph No. 1, Plate XI. are broken up and have a ropy or screw-like appearance because they are crossed by a secondary system of bands. This screw-like structure of echelon spectrum lines was observed by Gehrecke*. He does not explain how the new bands are produced, but shows that the appearance can be imitated by tilting an echelon with only two apertures, so that the echelon bands slope across the central vertical diffraction band.

When the echelon is in the ordinary position the secondary bands are parallel to the spectrum lines, and so their effects, though very important, are not so easily recognized as they are in this photograph taken with the echelon tilted.

* *Annalen der Physik*, xviii. p. 1074 (1905).

Character of the Secondary Bands.

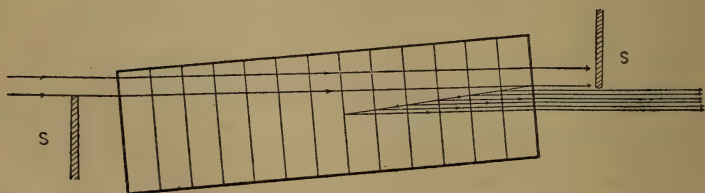
The secondary bands are superposed on the echelon lines and resemble them in appearance; they are also affected in the same way, but to a greater extent, by adjustments of the echelon. When the echelon is rotated, for example, the secondary bands move faster than the spectrum lines in the same direction and so move across them. When the echelon plates are vertical, the secondary bands, like the echelon lines, are vertical in the centre of the field; they are also curved in the same direction but more strongly. Their width is about the same as that of the finer spectrum lines, and so, in the ordinary position of the echelon, they are not easily recognized; but when the echelon is tilted they become more inclined than the spectrum lines in the same direction and can then be seen as in photograph No. 1, Plate XI., intersecting all the spectrum lines.

The behaviour of the secondary bands suggested the idea that they might be spectrum lines of a higher order, such as might be produced by the reflexion of light in the echelon.

Fabry and Perot Spectrum produced by the Secondary Action.

The secondary light, which has been twice reflected in the echelon, is by no means negligible. The echelon was tilted as shown in fig. 5, screens SS being arranged to cut out the

Fig. 5.



primary light, and it was found that the echelon was bright with secondary light coming out below the second screen, the brightness extending down to the bottom of some of the step apertures.

Suppose that each interface reflects a very small proportion of the light falling upon it and leave out of account for a moment the step structure of the echelon. There will be a secondary beam produced by the combination of the n faint secondary beams which have gone back through one plate, n being the number of plates coming into action; let the

retardation of this beam be Δ . There is another secondary beam whose retardation is 2Δ , made up of $n-1$ faint beams which have each gone back through two plates, and so on. Hence the secondary action of the pile of plates in the echelon is similar to that of a Fabry and Perot spectroscope, and, under suitable conditions of illumination, the secondary light by this action would be thrown into a ring spectrum of a high order. The retardation is $2\mu t$ for normal incidence, so it is $\frac{2\mu}{\mu-1}$ times as great, in this case about five and a half times as great, as that of the primary spectra.

The secondary light also undergoes the ordinary echelon treatment as it leaves by the step-faces, and it is therefore confined to the primary spectrum lines.

The photograph No. 2, Plate XI., which shows short portions of the rings of the Fabry and Perot spectrum crossing the echelon spectrum, was obtained by stopping the primary light in the way described above, and making the echelon lines broad by widening the slit. The spectrum lines produced by the primary action of the echelon upon the secondary light are inclined because the echelon is tilted as shown in fig. 5, and the centre of the ring system is above the field of view for the same reason. The echelon has also been rotated (in the positive direction of θ), so the centre of the ring system is displaced laterally as well as vertically.

The Fabry and Perot spectrum lines are not dependent for their definition, like the echelon spectrum lines, on the narrowness of the slit; their want of clearness in this photograph is partly due to the overlapping of lines belonging to different orders, which is produced by the orders overlapping four deep.

The Secondary Point Spectrum.

If the slit, instead of being opened wide to show portions of the rings, is made narrow enough to give good definition to the primary echelon action, the secondary light which has undergone both actions is confined to dots indicating the position of the points of intersection of the spectrum lines in one system with corresponding lines, that is lines representing the same wave-length, in the other system.

The horizontal dispersion of the dots, given by the ordinary echelon action, now prevents overlapping, as the wave-length interval between the orders in this system is a little greater than the length of the spectrum, and the secondary light

gives in this point spectrum, therefore, the advantages of a very high order without the usual overlapping.

Gehrcke and Baeyer* obtained similar spectra which they call interference points, by crossing plane parallel plates, and have pointed out the advantages of combining two independent high dispersions.

A photograph of the secondary point spectrum, obtained with an exposure of one hour, is reproduced in Plate XI., No. 3. The echelon had the usual tilt, about 3° , and the echelon table was rotated about 2° in the positive direction of θ from the normal position. The dispersion in the Fabry and Perot system is, in this case, twelve times that in the echelon system, so the wave-length intervals can be best determined by the position of the dots in the former system, while the dots can be recognized and their wave-lengths can be roughly fixed by their position in the latter system.

There was no difficulty in recognizing the dots marked 1, 2, 3, 4, 5, 6, and 7 in the photograph, which represent well-known lines in the green line spectrum.

In order to determine the wave-length intervals, it is necessary to calculate $\Delta\lambda$, the wave-length interval between neighbouring orders of the Fabry and Perot spectrum. It was found from the formula

$$\Delta\lambda = \frac{\lambda}{p - 2t \cos r \frac{d\mu}{d\lambda}},$$

where p , the order of the spectra, is given by

$$p = \frac{2\mu t \cos r}{\lambda},$$

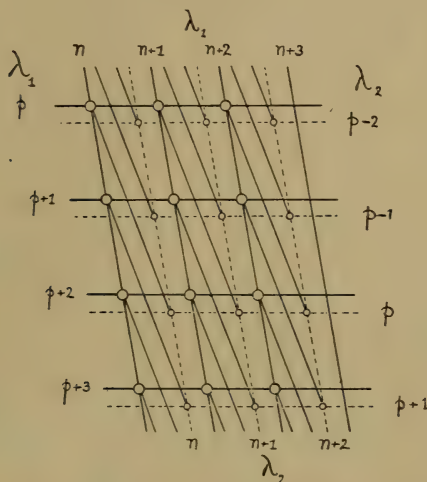
r being the angle of refraction of the light in the plates, and t their thickness. The value found in this way for the centre of the photograph is 96.5 m.Å. The wave-length intervals between the components deduced from the measurements of the photograph and this value of $\Delta\lambda$ are given in Table III., p. 393; most of them agree closely with the values which have been found by other methods.

* E. Gehrcke and O. v. Baeyer, *Annalen der Physik*, xx. p. 269 (1906).

Character of the Point Spectra produced by the Secondary Light.

Fig. 6 is a diagram representing the type of spectrum produced by the secondary light when the echelon is in the direct position and tilted about a horizontal axis, that is, the

Fig. 6.



type represented in the photograph of the secondary point spectrum No. 3, Plate XI.; but in order to make the diagram clearer, the ratio of the dispersion in the Fabry and Perot system to the dispersion in the primary echelon system has been made much smaller than it is in the photograph.

In this case all the rays of light from a given point of the slit are parallel to one another during their passage through the plates, as the lateral diffraction does not take place until the light leaves the echelon by the step-faces. The spectra, or lines of constant retardation, in the Fabry and Perot system are therefore drawn in the diagram as horizontal lines. When the photograph was taken the echelon had been rotated (from the normal position about a vertical axis) as well as tilted, so the plane parallel to the axis of the collimator and the diffracting apertures would not be quite vertical, but the deviation from the vertical is a small angle of the second order (equal to the product of the small angles of tilt and rotation), and it will be seen that lines joining two dots representing the same wave-length and of the same order in the Fabry and Perot system, would be sensibly horizontal,

although if the slit had been opened wide so as to show portions of the Fabry and Perot rings, they would have been inclined about 45° , as in photograph No. 2.

When the echelon is reversed, the lateral diffraction takes place as the light enters the echelon and the lines of equal retardation in the Fabry and Perot system will be represented by circles whose centre is the point in the image plane corresponding to the direction of the normal to the plates.

The four long inclined lines in the diagram (fig. 6) represent spectra in the primary echelon system; they are inclined to the vertical because the echelon is tilted. The orders of a wave-length λ_1 are represented in the two systems by full lines, the dotted lines representing similarly the spectra of a second wave-length λ_2 greater than λ_1 . The order of the spectra in the two systems is indicated at the ends of the lines. The points where the full lines of the two systems intersect give a series of points (marked by the larger circles in the diagram) which represent λ_1 in the joint system, each point being defined by two orders. The top left-hand point in fig. 6 may be described for example as the np order of wave-length λ_1 . In the same way the intersections of the dotted lines give the positions where λ_2 is represented in the joint system, and the shorter inclined lines joining the λ_1 and λ_2 points of the same denomination represent the appearance of the joint spectra corresponding to a spectrum continuous between these limits.

It will be seen that there is no chance of spectra of different denominations overlapping in the joint system as long as there is no overlapping in one of the two systems which are combined.

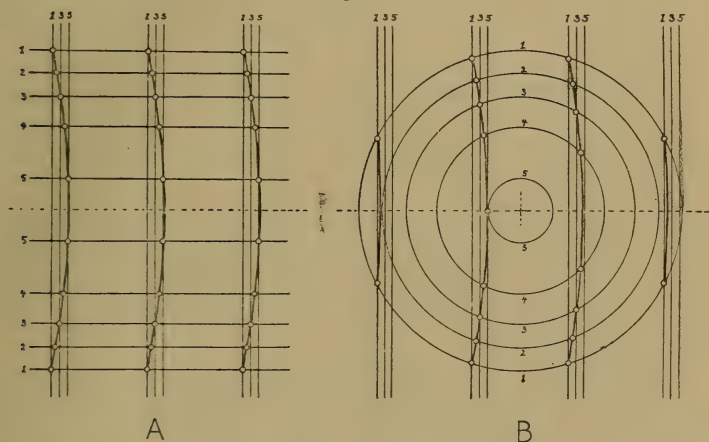
The spectra in the two systems may be regarded as forming an oblique system of coordinates; the echelon system giving horizontal dispersion may be called the X system and the Fabry and Perot system the Y system. Then the slope of the spectra in the joint system, $\frac{dy}{dx}$, is the ratio of $\frac{\partial \lambda}{\partial x}$, the dispersion in the X system, keeping y constant, to $\frac{\partial \lambda}{\partial y}$, the dispersion in the Y system, keeping x constant.

One important feature of these point spectra is that they give a system of lines whose definition depends in general on the defining power of the two systems which are combined, but does not depend on the smallness of the range of wave-length in the light examined, so long as that range does not exceed a certain relatively large limit.

If the definition is poor in one of the two systems, a monochromatic radiation would be represented by dots elongated in the direction of the spectrum lines of the other system, and this would in general spoil the sharpness of the lines representing in the joint system a spectrum continuous between narrow limits; but if these lines are nearly parallel to the spectrum lines of one system, the want of definition in the other will not spoil the definition of the lines, as each dot representing a single wave-length will be drawn out in a direction nearly parallel to the length of the joint spectra. This special case is realized when the echelon is in the direct or reversed normal positions.

The diagrams A and B in fig. 7 represent the conditions

Fig. 7.



in these cases. They are similar to fig. 6, but spectrum lines have been drawn in the Fabry and Perot system for a series of five wave-lengths whose values increase by equal increments from λ_1 to λ_5 . In the primary echelon system the lines representing alternate wave-lengths have been omitted. B represents the case in which the echelon is in the reversed position, the radii of the circles, representing the spectra in the Fabry and Perot system, being chosen so that their squares increase by equal increments. The horizontal lines in diagram A represent the same spectra when the echelon is turned round into the direct normal position.

For convenience in drawing the diagrams, the dispersion in the Fabry and Perot system has been represented as only about ten times as great as that in the primary echelon

system. It should be several hundred times as great even for those parts of the diagram farthest from the centre.

It will be seen that in both diagrams the secondary point spectra, drawn through the points of intersection of lines representing the same wave-length in the two systems which are combined, are curved so as to be concave towards the side of shorter wave-length in the primary echelon system; the curvature has been much exaggerated by taking the ratio of the dispersions so much smaller than it actually is.

Origin of the Secondary Bands.

There is no doubt that the secondary bands which are observed when the echelon is employed in the usual way, that is, with the secondary light superposed on the primary, are very closely connected with the point spectra produced by the secondary light. All the characteristics of the secondary bands described on page 384 may be explained by supposing that they are, like the point spectra of the secondary light, the loci of the points of intersection of lines representing the same wave-length in the primary echelon and Fabry and Perot spectra.

Another characteristic which may be explained in the same way, with the help of fig. 7, is that when the echelon is in the direct normal position the secondary bands affect each order of the spectrum in the same way, while, when the echelon is reversed, there are marked differences between the different orders.

The secondary bands have other characteristics which may find their explanation in interference taking place between the secondary light and the primary. The dark secondary bands appear to cut through the bright primary spectra. This is shown to some extent in photograph No. 1, Plate XI., but it is much more marked when the secondary light is made relatively stronger by covering half the echelon apertures so as to stop all the light which has passed through less than half the plates in the echelon. The brightness of the primary lines is no doubt increased above and below the dark bands, making them appear darker by contrast, but I think there is little doubt that there has been an actual reduction in the brightness of the primary light in the dark secondary bands.

Another feature of the secondary bands, which may be seen in photograph No. 1, Plate XI., where they cross the line 1, is that they always appear in pairs, fainter and stronger bands alternating with one another. This may be connected

with the difference in phase between the primary and the secondary light produced by the two reflexions of the latter. Apart from these phase changes at reflexion, the primary and secondary light would be in phase at the points of intersection of their maxima, as the retardations in both systems are whole numbers of wave-lengths in the direction in which the maxima are formed. There are two cases to be considered for the secondary light, according as it has undergone both reflexions at interfaces or one reflexion at an interface and the other at an external surface. In the former case, as the air-films are very thin, it is possible that a change of phase of $\frac{\pi}{2}$ may be introduced at each reflexion which would give the best phase conditions for interference between the maxima. The latter case may account for the second set of bands shifted relatively to the first.

PART III.—STRUCTURE OF THE GREEN MERCURY LINE. (5461.)

Description of Spectrum given by an Arons Lamp, and Comparison with the Results obtained by other Observers.

This spectrum consists of a bright principal line, which is a close double, with six companion lines, three on either side. A photograph of the primary echelon spectrum is reproduced in Plate XII. together with a diagram of the spectrum. The photograph shows, in addition to the genuine components, a number of faint lines which have their origin in the echelon. The genuine components are numbered from 1 to 8, and the false lines are marked 1 *a*, 1 *b*, 3 *a*, &c. The lines 1 *a*, 1 *b*, 1 *c*, &c., mark the positions of the secondary diffraction maxima on the longer wave-length side of the principal diffraction maximum 1, and the lines 3 *a*, 5 *a*, 6 *a*, 7 *a*, and 8 *a* represent the first secondary maxima on the longer wave-length side of the lines 3, 5, 6, 7, and 8. When the aperture of the echelon is reduced by covering the first ten step-faces at the smaller end, the faint lines move away from their parents into the new positions of the secondary maxima corresponding to the reduced number of apertures.

The numbers in Table II. and Table III. give the distances of the various components from the component of shortest wave-length. I adopted this method of measuring the positions because the bright central line appeared to be the most variable component, while the component of shortest wave-length is a good reference line. The results of the other observers, given in the Tables, are not convenient for

comparison in their original form, because Gehrcke and Baeyer*, Janicki†, and Galitzin and Wilip‡, give the distances from the centre of the principal line, while Fabry and Perot§, Baeyer|| (in a later paper), and Nagaoka¶, divide the principal line into two components and measure the distances from the brighter one.

Another difficulty in the comparison is that the values obtained by Gehrcke and Baeyer, and later by Baeyer, by the use of crossed plane-parallel plates, are systematically higher than the values obtained with echelon spectroscopes, which agree fairly well amongst themselves. I have reduced all Baeyer's intervals 5 per cent., and the earlier values of Gehrcke and Baeyer (the means of three sets of measurements given in their paper) by 3 per cent.; and it will be seen that, apart from these differences in the constants, the two independent methods are in close agreement.

TABLE II.—Measurements of the Green Mercury-line Spectrum.

The distances of the component lines from the component of shortest wave-length are given in milli-Ångström units.

Fabry & Perot.	Crossed Plates.		Echelon Spectroscopes.		Reference Numbers.
	Gehrcke & Baeyer.	Baeyer.	Janicki.	Galitzin & Wilip.	
·01 Å. units	reduced 3 p.c.	reduced 5 p.c.			
0	0	0	0	0	1
15	126	136	133	137	2
17	169	169	166	168	3
...	...	189	...	189	...
<u>22</u>	} <u>234</u> {	214	} <u>232</u> {	<u>236</u> {	4 {
23		<u>238</u>			5 } Central Band.
31	318	320	320	321	6
36	364	363	365	365	7
...	...	449	...	...	8

The brightest component in each case is underlined.

* F. Gehrcke and O. Von Baeyer, *Annalen der Physik*, vol. xx. p. 269 (1906). † L. Janicki, *Annalen der Physik*, vol. xix. p. 36 (1906).

‡ Fürst B. Galitzin und J. Wilip, *Mémoires de l'Académie Impériale des Sciences de St. Pétersbourg*, sér. 8, vol. xxii. no. 1 (1906).

§ Astrophysical Journal, xv. p. 218 (1902).

|| O. v. Baeyer, *Verhandlungen der Deutschen Physikalischen Gesellschaft*, ix. no. 4, p. 84 (1907). ¶ Nagaoka, 'Nature,' vol. lxxvii. p. 582 (1908).

TABLE III.—Measurements of the Green Mercury-line Spectrum (*continued*).

Nagaoka.	Author.			Reference Numbers.
Echelon Spectroscope.	Primary Spectrum.	Secondary Point Spectrum.	Width.	
0	0	0	17	1
31	...	...	...	...
72	...	...	...	...
105	...	...	...	...
137	135	141	13	2
163	164	167	13	3
189	...	...	...	...
223	232 { (217) (243)	217	16	4
<u>247</u>		<u>243</u>	24	5 } Central Band.
280	...	...	...	...
315	319	322	17	6
356	363	365	12	7
390	...	...	...	...
448	448	...	14	8
477	...	...	...	...

Fabry and Perot's values* given in the Table are those published by Zeeman† in 1902. They help to confirm the accuracy of the constants employed in the echelon calculations.

The results obtained by several other observers are given by Janicki‡.

The series of faint lines given by Nagaoka resembles the series of false lines in my primary echelon photographs. I published my measurements for comparison with his before discovering that the faint lines in my photographs were not genuine §.

* M. Perot informs me (in a letter dated Oct. 13th, 1908) that they have not published any particulars of this line since then.

† Astrophysical Journal, xv. p. 218 (1902).

‡ Loc. cit. p. 61.

§ 'Nature,' vol. lxxviii. p. 8 (1908).

Width of the Components.

The mean values of the widths of the photographic images of the components are given in Table III. If the echelon acted perfectly, the width of a principal light maximum constituting one order of the primary echelon spectrum of a monochromatic radiation, with a narrow slit, would be $2/33$ of the interval between the orders, which corresponds in this case to a width of 30 m.Å. The narrowest lines in the photographs which still have a measurable width, are about 10 m.Å. wide. The widths of the brighter lines vary considerably with their exposure, while the width of the companion line 8, which is too faint to be overexposed, is the same on each of the three plates on which its width could be measured.

Measurement of Secondary Spectrum.

The results obtained by measuring the secondary point spectrum are also entered in Table III. The photograph measured was exposed for an hour, and is the one reproduced in Plate XI., No. 3. It will be seen that the agreement of these results, obtained by an independent method, with the ordinary echelon values, is fairly close, except for the component 2. The methods agree very closely as to the position of the central line. They both give the centre of the double line at 232 and the dividing dark line at 228. The two components of the central line were not measured separately in the primary spectrum photographs, but the position of the dark dividing line was sometimes recorded. The positions of the centres of the components given in brackets in the second column of Table III., are deduced from the position and width of the whole line and the position of the dark dividing line (neglecting its width).

Spectrum given by a Bastian Lamp.

A striking variation in the spectrum of the green line was observed with a Bastian mercury arc-lamp. The glass tube through which the discharge passes is bent nearly into the form of an S in a horizontal plane, so that when one part of the discharge is parallel to the slit plate, another part may be normal to it. When the image of the "end-on" portion of the discharge is put on the slit, the change in the spectrum is so great that it is difficult at first to recognize the components. On measuring a photograph of the "end-on" spectrum, however, it was found that the companion lines keep their relative positions, although some become

broader and brighter, but the dark space between 4 and 5, the two components of the principal line, is greatly increased (from 6 to 26 m.Å.) and, being no longer brighter or broader than the rest, they look like ordinary companion lines.

The "side-on" spectrum of the Bastian lamp resembles more nearly that given by the Arons lamp.

The separation of the components of the principal line in the "end-on" radiation has been investigated by Galitzin and Wilip*, who observed the phenomenon first in the case of a Geissler tube arranged so that the axis of the discharge was normal to the slit plate.

*Spectrum of the Green Mercury line given by a hot
Mercury lamp.*

Janicki† describes the broadening of the components of the green and yellow mercury lines which takes place when a mercury lamp is allowed to become sufficiently hot, and a peculiar system of five equidistant bands which he observed when the original components of the lines were lost in a continuous spectrum. Galitzin and Wilip‡, who give measurements of the bands, suggest that they may be due to a reversal of the lines, or perhaps to some peculiar property of the echelon spectroscope.

The theory of the secondary echelon spectra (see page 388) indicates that the secondary bands would be well defined in a short continuous spectrum, and it seemed probable that they were the bands which Janicki and Galitzin & Wilip observed.

I have tested this point with a quartz-lamp fitted with an air-manometer, similar to that described by Galitzin and Wilip§.

When an arc is started with the lamp cold, the central line (4 and 5 together) and all the companion lines are at first plainly visible, but the pressure in the lamp increases rapidly and the broadening and coalescing of the lines soon takes place. The position occupied by the bright central line in the various orders is now marked by a dark line, probably due to absorption. As the companion lines become merged in the general brightness the fine secondary bands become clearly visible in all the bright parts of the

* Fürst B. Galitzin und J. Wilip, "Ueber die Eigenschaften einiger Emissionslinien des Quecksilberdampfes," *Mémoires de l'Académie Impériale des Sciences de St. Pétersbourg*, sér. 8, vol. xxii. no. 1 (1907).

† *Loc. cit.* pp. 49-55.

‡ *Loc. cit.* pp. 34 & 76.

§ *Loc. cit.* p. 4.

field, the fact that they are secondary bands being shown by their motion relative to the primary spectrum when the echelon is given a slight rotation. The secondary bands show clearly on the green line when the pressure in the lamp is about one atmosphere.

This research was commenced at the request of Professor Schuster, and I have much pleasure in acknowledging the help I have received from the interest he has taken in its progress.

I wish to thank Mr. E. Marsden for assistance in my first experiments on the secondary effects, and Mr. W. A. Harwood, B.Sc., for measuring several of the photographs. My thanks are also due both to Professor Schuster and to Professor Rutherford, for placing the resources of the Physical Laboratories at my disposal.

XLV. *On the Radium-Content of Sea Water.*

*By J. JOLY, F.R.S.**

IN his interesting contribution to the subject of the radium-content of sea water †, Dr. Eve quotes the results given in a former paper of mine ‡. The general results of my subsequent experiments have appeared elsewhere §, but not under conditions permitting their statement in detail or any discussion of their claims. Although incomplete they may serve to define more fully the nature of the difference between Dr. Eve's observations and mine. But first I would wish to call attention to certain points of agreement.

The fact of very considerable differences in the radium-content of sea water from one locality to another, seems to find increased confirmation as experiments multiply. In my paper referred to by Eve these variations range from 8 to 39.3 (I adopt Eve's notation, and express the values in billionths of a gram per litre). This variation is found among only 11 samples. It is true that they are from very various localities and include harbour water, coastal and oceanic waters, and extend to the Arabian Sea. Eve finds even wider differences. Here I must point out that by an oversight Eve

* Communicated by the Author.

† Phil. Mag. July 1909.

‡ Phil. Mag. May 1908.

§ 'Radioactivity and Geology,' p. 46.

appears to have misquoted his own earlier results. In his paper of Feb. 1907 * he gives his result on Atlantic water as 0.3 and on sea-salt as 0.6. These figures he interchanges in his last paper. When correction is made for this we find that among seven experiments on Atlantic water only he obtains variations ranging from 0.3 to 1.5. Thus all reported results agree in showing that wide differences in radium-content are observed in samples of water from various localities. I have presently to quote instances of even more striking variations.

Early in my observations upon this subject I sought to find if the variations also obtaining from one test to another of the same sample, might not throw light upon the differences from sample to sample; the possibility suggesting itself to me that these latter might only be apparent and really dependent upon the conditions attending the liberation of the emanation. The fact of variations in successive tests and the conditions controlling these variations became, therefore, of special interest, and I sought by altering the conditions of experiment to investigate the circumstances affecting the liberation of the emanation.

I have found in the course of my experiments that where conditions of acidification by hydrochloric acid vary, considerable variations in the amount of liberated emanation may arise, more especially where there has been concentration of the sample. But where the acidification and other treatment remain unchanged the variation from one test to another is, in general, relatively small. The effect of the acidification is to increase the quantity of emanation liberated, but beyond a certain degree of acidification small additions of acid seem ineffective in producing further change. Dr. Eve's mode of tabulating my earlier results does not bring out the change of conditions. Thus under the heading "With HCl" he cites a difference of reading of 2.0 and 14.6 obtained on the same sample; there is nothing to show that here the first reading was obtained on a sea water concentrated from 2500 ccs. and acidified with only 10 ccs. of HCl while the second reading was obtained after the acidification had been increased to 60 ccs. So also in the case of the variation from 19.3 to 27.3 obtained upon another sample. The greatest variation which I have as yet noticed in successive boilings, where there has been no difference of treatment, is from 4.4 to 8.0. This was on an unconcentrated sea water. The general fact of variations from test to test, even under similar

* *Phil. Mag.* Feb. 1907.

conditions, seems fully brought out by Eve's recent experiments, which read from 0·75 to 1·85 &c., &c.

The following table contains all the results which I have up to the present obtained upon sea water. Those given in my earlier paper are summarized in the entries (1)–(5) and (6)–(10). (19) has also been already dealt with. An arithmetical error in computing the constant of a recently constructed electroscope affected the original values given for (15) and (16). A misprint ('Radioactivity and Geology,' p. 46) slightly affected (23). The last experiment (25), has not hitherto been published. In the table they are not generally listed in the order in which they have been made; (15) and (16) are two of the latest.

TABLE I.

	Radium per litre.
(1)–(5) Coast round Ireland (5)	34
(6)–(10) Atlantic Ocean; Madeira to Bay of Biscay (5)	17
(11) Atlantic, New York to Ireland, Arctic Current	14
(12) " " " Gulf Stream	14
(13) " " " Mid Atlantic	11
(14) " " " 260 m. W. of Ireland	8
(15) " lat. 0° 0', long. 31° 26' W.	23
(16) " lat. 16° 54' S., long. 37° 29' W.	4
(17) Mediterranean	14
(18) Black Sea	7
(19) Arabian Sea, lat. 10° 40' N., long. 58° 0' E.	27
(20) Indian Ocean to Mediterranean; Sandheads to Madras.	4
(21) " " " off Colombo	7
(22) " " " Minicoy to Sokotra ..	4
(23) " " " Red Sea off Jiddah..	6
(24) " " " Mediterranean	2
(25) Valentia	4

As regards the precautions taken in carrying out these experiments the following remarks apply to all alike. Whenever evaporation was required it was carried out in the recently opened Botanical Laboratory of Trinity College. In this laboratory all the apparatus and fittings are new and no radium work has ever been done in the building. The acid used was a 're-distilled' HCl which I tested at intervals. Allowance was made for a very small trace of radium—almost negligible. The final testing was carried out in flasks of (R) glass, which were procured for the purpose and not used for any other. The rubber stoppers were also new and reserved for this special use. In effecting the experiment the increased rate of discharge of the electroscope was

observed about 15 minutes after the introduction of the emanation and again in three hours. The characteristic gain in rate is an assurance that the effect was indeed due to radium and not to any loss of insulation, &c. The constant of the electroscopes was from 0.5 to 0.7 : in other words this was the radium-value in billionths of a gram for a gain of one scale-division per hour. It will be seen that working with so low a constant resulted in quite unmistakable increments of rate even in the case of the lowest values obtained. Ebullition is in every case carried on under reduced pressure and for from fifty minutes to one hour. A quantity of powdered talc (tested for radium) averaging 50 milligrams is added to improve the ebullition.

With regard to the nature of the bottles in which the water was collected, their proper cleansing, and the time in which the water was retained in them. In the case of experiments (20)–(24) the bottles were new, of white glass, and stoppered. They were carefully washed and were filled from the canvas buckets. The water was stored in them for periods varying from six weeks downwards, according as the homeward journey of the friend who acted for me was accomplished. The bottles for (15) and (16) were supplied, and specially washed, by me. No pains were spared in their thorough cleansing. A third, similar bottle, was sent out on the same voyage. This brought back water from the Rio de la Plata. The three bottles were in appearance alike, and were new, closed with rubber stoppers. The collector in this case was a gentleman of considerable scientific training who quite understood the importance of care in guarding against contamination. Owing to some negligence in their delivery they did not reach me till about three months after they were filled. Sample (25) was also in a bottle supplied and washed by me. It reached me in a few hours after filling. In all the other cases the bottles were purchased by the collectors or supplied on board ship. I have received every assurance as to the precautions taken in obtaining these samples. The Atlantic series were in black glass bottles. The water was enclosed for from twelve to five or six days before reaching me.

The details of the experiments are contained in the table which follows. Owing to the method adopted in boiling off the emanation about 200 ccs. of distilled water are added upon each experiment (to expel all residual gas from the flask), and, hence, between each experiment this is again removed by evaporation.

TABLE II.

No.		Concentration.	HCl. ccs.	I.	II.	III.
11.	Arctic Current	2090 to 1600	50	6.0	5.7	14.1
12.	Gulf Stream	2200 „ 1500	50	spoiled	2.6	13.6
13.	Mid Atlantic	2170 „ 1300	70	10.0	10.6	
14.	260 m. W. of Ireland.	2120 „ 1350	50	8.0	8.0	
15.	Atlantic, lat. 0° 0', long. 31° 26' W. ...	1625	50	22.8		
16.	Atlantic, South	1790	50	4.0		
17.	Mediterranean	3315 to 1200	72	1.7	..	14.0
18.	Black Sea	2500 „ 1600	50	4.7	...	7.2
20.	Sandheads to Madras.	2950 „ 1700	80	3.7	...	3.6
21.	Off Colombo ..	2500 „ 1500	84	2.9	...	6.8
22.	Minicoy to Sokotra ..	2400 „ 1700	50	4.4	...	4.0
23.	Red Sea	2500 „ 1700	50	2.5	...	6.0
24.	Mediterranean	2500 „ 1600	50	2.2	...	2.2
25.	Valentia	2800 „ 1000	60	4.0	4.0	

Column I. contains the result of the first experiment, column II. that of the second, when a second under the same conditions has been made, and column III. the result obtained after a treatment which requires some explanation.

When dealing with (12) it was noticed that after ebullition the water had quite lost its original limpidity. The appearance was plainly not due to the trace of talc which is inserted in all experiments alike, to promote free ebullition. Held close to the eye and regarded against a dark background the flask appeared filled with semi-translucent particles. I concluded for want of better explanation that this appearance was probably due to the coagulation, and perhaps clumping, of organic substances which had escaped decomposition. They were evidently insoluble. Always mindful of the fact that we seek in these experiments to expel a quantity of gas of the order of the billionth of a cubic millimetre per litre of sea water, I thought it desirable to bring this suspension into solution. Accordingly when the trace of talc had settled, the water was passed three times through a thick filter-paper. This treatment rendered it once again limpid. The paper was then dried over the water-bath, incinerated, and the ash fused in a platinum crucible with a couple of grams of the anhydrous carbonates of sodium and potassium. The melt was taken up with distilled water, acidified, and the resulting clear solution returned to the flask of sea water. The result given in column III. was obtained when in due course the water was again examined for radium. I may add that a separate experiment in which the carbonates were fused in

the same crucible revealed only the usual trace of radium in the carbonates : in this case negligible.

This treatment was then applied to many of the sea waters where any haziness appeared to have developed. It will be noticed that an increase in the evolution of emanation has not always occurred. But in one other case, in which the haziness was considerable, there was a very marked increase observed (17). The suspension from the Atlantic water (11) was tested separately for radium, and the filtered water also independently dealt with. The quantity in column III. is from the added results of these tests. About one-third of the quantity of radium contributing to the final result was found in the suspension. More particularly : the water in its final reading showed 18.9×10^{-12} gram radium and the whole of the separated suspension 10.7×10^{-12} .

If my interpretation of these experiments is correct it would appear that in some cases, where much organic matter is present, the emanation may be retained to a considerable extent even after prolonged boiling. I can offer no opinion as to the ultimate origin of the effect observed : whether it arises owing to conditions of decay of organic substances progressing in the flask or whether it originates in the living protoplasm. I would point out, however, that if it was found that the minute organisms of the plankton entangled or abstracted radioactive substances from the water, and in this way held them in suspension, then a satisfactory explanation of the remarkable variations in the radioactivity of different parts of the ocean is forthcoming. Whereas chemical conditions in general are fairly constant over the extent of the ocean it is quite otherwise with organic life. It is well-known that in bacterial distribution, for instance, the numbers are very much greater near shore and are extremely variable over the surface of the sea. But, of course, the present experiments are too few in number to justify any discussion of the question ; further experiments may throw light on the matter. I hope shortly to have surface and sub-surface waters at my disposal : a comparison of these will be of interest. In connexion with this subject I think it is instructive to recall an interesting experiment of Eve and McIntosh * (and one which I have confirmed in different forms and degrees), as to the concealment of the emanation in solutions wherein any trace of precipitate may have developed. Eve and McIntosh found that a little sulphuric acid added to a rock solution reduced the evolution of the emanation to one-fifth of what it had previously been. The

* *Phil. Mag.* Aug. 1907.

fact is that the limiting volume attained by the emanation is so small that its genesis within even the smallest solid or quasi-solid particle may render its subsequent abstraction impossible.

The mean of the above additional experiments is 8.6, and if the results obtained before filtering are accepted the mean is 5.6. The waters are, all but one, oceanic. This single exception is of interest. It was taken at flood tide at Valentia. It remained in the flask a considerable time before testing and then showed a very consistent yield of emanation on successive experiments. It appears to indicate that coastal waters are not invariably of high radioactivity.

The foregoing figures enable us to estimate more particularly the discrepancy between Eve's results and my own. Eve makes comparison between the mean of all my determinations as given in my British Association Address (16.0) and that of his own recent observations (0.9). This, as he points out, gives a ratio of about 17 to 1. Among the results included in my mean are several high coastal waters: omitting these the mean is 11.3.

There is, however, no improbability in the assumption that Eve is dealing with waters more aptly compared with those which enter largely into my results and in which the radium-content is expressed as less than 10. Thus the mean of my observations for across-Atlantic water is—if filtration results be excluded—6.7. Many of my measurements range about 4 to 7. One is as low as 2.2. Such may be regarded as from 3 to 7 times those of Eve. Mr. Strutt's pioneer observation upon sea-salt gives 2.4. It is very probable that sea-salt in course of preparation loses some of the more readily precipitated substances, among which is radium. With this probability in view Mr. Strutt's observation is in fair agreement with my lower results.

With regard to the higher results embodied in my list, there is, I believe, difficulty in any explanation but that which admits their essential veracity. A comparison of (15) and (16) is instructive in this connexion. Here every circumstance as regards precautions, the nature of the bottles and the conditions of collecting are alike. Upon their receipt the water was transferred without any further manipulation into the flasks; equal quantities of acid added from the same bottle, and in due time the radium determined under the same experimental conditions. If the difference in the results is not due to a difference in the original radium-content of the water, we have to assume either that the water which reads lowest is really as rich in radium as the other,

and, without any appearance to suggest concealment, can retain its emanation; or that the higher-reading water picked up its radium from the bottle. It is true that I have found salt water to be a considerably more active solvent of silicates than fresh water *, but in this present case we must suppose a large quantity—some grams—of glass brought into solution from the one bottle and practically none from the other. Again the results of (15) and (16) find support in quite unconnected experiments recorded from other localities and from some in the same localities, but carried out in a different manner, as will presently be seen.

Referring to the general considerations which Eve has frequently and so effectively raised, might we not regard this subject from another aspect and gather support for the considerable radioactivity of sea water from the ionization and penetrating radiation observed in the overlying atmosphere? It does not seem probable that thorium will prove responsible for any very large part of these effects. From a single determination of the thorium-content of water from the Indian Ocean (20) *ante*, I measured the quantity as 0.9×10^{-5} gram per litre. Taking the radium as 3.6, the uranium equivalent of this will be about the same in mass as the thorium. This is inferred, indeed, from one measurement only; but three other observations, on lesser quantities of water, seemed to show a major limit but little above the quantity of thorium found in the water from the Indian Ocean (Phil. Mag. July 1909).

It remains to notice a recent and important contribution to the subject of oceanic radioactivity. In the course of a voyage across the Atlantic and through the Straits of Magellan to Chili, W. Knoche (*Phys. Zeit.* March 1909) in twelve experiments measured the emanation content of sea water. He used the Engler and Sieveking apparatus as modified by Kohlrausch and Löwenthal. Eight of the observations are on Atlantic water, from $12^{\circ} 28' \text{ N.}$ to $42^{\circ} 12' \text{ S.}$ In these experiments a litre of the sea water is shaken violently with air in a closed vessel for two minutes, and the air then immediately introduced into the electroscope. Expressed in electrostatic units the current established in the electroscope, as obtained from observation of the fall of potential during the first thirty minutes, ranges in the several experiments from 0.00000 to 0.00029. This current is due to the emanation; the normal current, estimated over the previous thirty minutes, being deducted from the reading. The mean of the twelve measurements is 0.00012. After remarking

* Proc. Royal Irish Academy, xxiv. A. p. 21.

upon the large increase of conductivity observed in every case but one, the author states that the mean activity shown by the twelve measurements is about one-twentieth of that of a feebly active spring. The experiments were made in calm weather.

The interest of the individual results is so considerable that I quote the entire series below; the last column being added by me.

			Current. E. S. Units.	Equivalent Radium per litre.
17th Sept. 1908.	12° 28' N. 24° 19' W.		0·00009	12·6
18	" "	8° 24' N. 26° 38' W.	0·00015	21·0
19	" "	4° 24' N. 28° 18' W.	0·00023	32·2
20	" "	0° 27' N. 30° 52' W.	0·00016	22·4
21	" "	3° 54' S. 32° 32' W.	0·00029	40·6
23	" "	12° 54' S. 35° 40' W.	0·00000	00·0
28	" "	31° 48' S. 49° 40' W.	0·00006	8·4
2 Oct.	" "	42° 12' S. 59° 59' W.	0·00008	11·2
5	" "	Entrance Straits of Magellan.	0·00010	14·0
6	" "	In Straits of Magellan	0·00009	12·6
13	" "	Bay of Corral	0·00010	14·0
15	" "	Off the Bay of Corral	0·00010	14·0

Assuming that radium in the water is principally responsible for the observed effect, we may arrive at an approximate estimate of the radium in one litre of sea water as shown by these experiments. Apparently no deduction is made by the author for the effects of the active deposit from the emanation, a large part of which would be generated in the electroscope during the thirty minutes of observation and the influence of which must enter with increasing effect into the potential-fall observed. In order not to risk exaggeration of the radium I shall assume that the whole of the active deposit is concerned in ionizing the air. On Rutherford and Geiger's result that in one gram of radium $3\cdot4 \times 10^{10}$ atoms break up per second the corresponding emanation, in equilibrium with its products of rapid change, should emit about 10^{11} alpha rays per second. Assuming that a total of $1\cdot5 \times 10^{16}$ ions (the maximum number) are generated by these rays and that the charge on the ion is $4\cdot6 \times 10^{-10}$ unit, we find that a saturation current of 0·00012 electrostatic unit involves the quantity of radium 17×10^{-12} gram. And this is the amount of radium associated with a litre of sea water according to the mean of Knoche's results, on the assumption that the radium accompanies the emanation. I have made this calculation for each of the observations; for the last column in

the table I am, therefore, responsible. Although some part—presumably a small part—of the ionization may be ascribable to thorium emanation, the assumptions made in my estimate of the radium are all on the side of reducing the final result.

The author offers no opinion as to the origin of the emanation save that it can only in the very smallest degree be ascribed to the atmosphere, and that his highest result being obtained near the volcanic island of Fernando Noronha, in such a locality the solution of active materials and the absorption of gases emitted beneath the sea may be of influence. A little consideration will, I think, show that the greater part of the emanation must come from radium contained in the water. We arrive at this result by a process of exclusion. If we assume the emanation comes from the atmosphere then we not only greatly intensify the difficulties upon which Dr. Eve has been dwelling but we find the absence of emanation, as determined by McLennan, from the waters of Lake Ontario, inexplicable*. The anomaly would be the more remarkable in view of Kofler's recent researches (*Phys. Zeit.* Jan. 1, 1908) showing that the emanation of radium is more soluble in fresh than in salt water. If we ascribe the emanation to the globigerina ooze about two miles beneath, difficulties of a quantitative character, which I believe are insurmountable, present themselves. The quantity of emanation near the equator, as represented by the equivalent radium, is about 300 times what has been observed in the atmosphere. But on the ocean floor the porosity, and the convective circulation of gases, such as facilitate its liberation from the soils, cannot exist. The emanation, presumably, can only move outwards on slow diffusive movements, and when it has been liberated from the mud its further motion must be regulated by the leisurely creeping of deep-sea currents. However, in order to account for the sustained radioactivity of the surface waters we evidently must assume a steady flow of emanation-rich water from beneath; and thus we have to find some source from which the emanation, continually perishing throughout the entire depth of the ocean, may be supplied. Taking the depth at 2000 fathoms and the radium-content of the ooze as 7×10^{-12} gram per gram, the continuous liberation of all the emanation from a depth of not less than 5 metres of ooze, and its distribution without loss throughout the overlying water, must be assumed—conditions which do not merit discussion. Unless there exist bottom currents having a velocity adequate to concentrate the emanation escaping from some

* C. S. Wright, *Phil. Mag.* Feb. 1909, p. 316.

thousands of square miles into the equatorial region and there bring it to the surface within the lapse of a very few days, the view that the emanation is for the larger part unattended by radium seems quite untenable.

It is otherwise if we assume that the carriage of radium-rich water, from beneath upwards, takes place. The time element does not then enter the question. It may well be the case that, among other factors, vertical circulation is largely concerned in the distribution of the surface radioactivity of the ocean. Such a view involves no difficulties; but in waters far from the land we are not entitled to assume the absence of much of the equivalent radium without adequate direct evidence to support such a difficult view.

There are many features of interest in these recent observations. The fact of a very wide range of radioactivity is strikingly brought out. It must be remembered, however, that a null result such as appears among Knoche's observations may be due to temporary de-emanation where the quantity of radium is small. Allowing that some part of the emanation may be free or exotic, the higher values are in fair agreement with my higher readings, but somewhat in excess. The mean is a little above the mean which, on the strength of a couple of high oceanic readings, I thought most probable when writing my Address to Section C. A comparison of the results given in the earlier part of this paper with the radium-equivalent of the emanation measured by Knoche shows many points of resemblance. Knoche's determinations show a very decided maximum about the Equator. The one observation which I was able to make on these waters is in good agreement. Going south the radioactivity diminishes and Knoche's lowest results are from waters between 12° and 30° S. Correspondingly one of the lowest readings obtained by me was from the sample taken in latitude 17° S. The waters examined by me were taken in the same year, 1908, but at a different season—February.

I see no reason to ascribe these coincidences to chance. The vertical circulation beneath the Equator, which so many facts go to support, is probably responsible for the main features of the Atlantic observations. On this view examination of the bottom waters should reveal a radioactivity generally in excess of that which has been observed at the surface of the more northern parts of the ocean.

Considerations arising from the denudative history of the continents show that vast quantities of uranium have entered the ocean in the course of geological time and have subsequently been removed into the sediments. On an evaluation

of the mass of the sub-oceanic sediments, and on the assumption of a mean radioactivity for these comparable with that of globigerina ooze, I recently arrived at a total of about 10^6 tonnes as approximating to the radium rejected by the ocean. This calculation may be made in another form in which we use as basis the quantity of rock which has probably been denuded from the primary to the secondary condition and the estimated difference between the radium-content of igneous and sedimentary rocks. The result arrived at is very similar. Presumably all this radioactive material has been at one time in solution or suspension in the ocean, whose waters, for all that we could have anticipated, might have possessed a content of radium some fifty times greater than the figures appear to indicate.

*XLVI. Anomalous Dispersion by Metallic Vapours.**By P. V. BEVAN, M.A., Royal Holloway College*.*

[Plate XIII.]

THE experiments of Wood on sodium vapour and the anomalous dispersion of a beam of light passing through it have provided a beautiful example of this effect and have yielded measurements which are of importance for the general theory of the transmission of light through absorbing media. From Wood's experiments it appears as if sodium behaves in some peculiar way, the metallic vapour retaining its form even when it is in a high vacuum. This statement of Wood's is perhaps the reason why, as far as I am aware, no experiments of a similar nature have been made with other metals. The author of this communication has shown† that, in a vacuum, sodium vapour ceases to have the peculiar property of not diffusing into the surrounding space and that it behaves just as one would expect, diffusing practically instantaneously to the colder parts of the tube and condensing just as any other volatile substance would do. It seems that the sodium cloud of Wood cannot exist in a high vacuum and that it probably retains its definite form owing to temperature effects, the boundary being a layer at which condensation takes place. There seeming to be no special property appertaining to sodium in this way, the author has tried by a similar arrangement to detect anomalous dispersion in the cases of other metals and this communication is to give the results with two other alkali metals, lithium and potassium.

* Communicated by the Author.

† Camb. Phil. Soc. Proc. xiii. p. 129.

The method of Wood is well known and needs no detailed description. A piece of steel bicycle tubing is used for heating the metal. The tube is closed by glass ends and brass jackets are fixed close to the ends through which water circulates to keep the sealing-wax which fixes the glass plates from melting. The under side of the tube is heated, and the upper side is kept cool by a water-trough fitting and resting on the tube through which a stream of water can be kept flowing. It was found that this method of cooling the top of the tube was very satisfactory and kept the conditions more steady, leading to better definition in the photographs of the effect. The light used was from a Nernst lamp. The light, after passing through the steel tube, was focussed by a lens forming a horizontal image of the filament across the vertical slit of the spectroscope, so that when the steel tube was cold, a spectrum was produced consisting of a line of colour. When the tube is heated, metallic vapour is formed, and this being in layers of unequal density, produces dispersion in a vertical plane, giving a vertical displacement in the line of colour finally observed. The effect is easily obtained with both lithium and potassium and, though not so striking as in the case of sodium, gives a very beautiful picture in each case of the anomalous dispersion curves. In the case of lithium (Pl. XIII. fig. 1) a considerably higher temperature is required than for sodium, and it was found that a good result could be obtained by heating the tube with an ordinary foot blow-pipe so that the under side of the tube was at a red heat. The figure is from the original negative, the apparatus at the author's disposal only permitting a small negative to be taken. The anomalous dispersion takes place in the region of the red lithium line w.-l. 6705, and there is no trace of anomalous dispersion in the region of the other lithium lines in the visible spectrum. This fact seems of importance when taken into consideration with the regions of anomalous dispersion in sodium and potassium vapours, as in all three cases the region of anomalous dispersion appears to be only near the lines of the primary series of Kayser and Runge. At present the author has not determined whether anomalous dispersion does take place at the other lines of the primary series for lithium, as these lines are in the ultra-violet, and so far no apparatus has been available for obtaining photographs in this region. Further experiments are, however, in progress to investigate this region. The photographs of the effects are not so clear and well defined as one could wish, but it is difficult to keep the temperature steady over a long exposure, and long exposures

of the photographic plates are necessary as the light intensity is small and the plates sensitive in this region are not rapid.

In the case of potassium the temperature required is not so high as in the case of sodium, and a burner consisting of a brass tube with small holes at intervals of about a centimetre was used. This heated twelve to fifteen centimetres of the tube and was found satisfactory when quite small flames about 1 cm. high just reached the tube. The chief region of anomalous dispersion for potassium is about the first pair of the primary series (w.-ls. 7699, 7665). Figure 2 is a photograph of the effect when small dispersion is used. Here the anomalous dispersion appears as if only one absorption line were concerned. If a larger dispersion in the spectroscope be employed, then the double effect, as in fig. 3, appears. It is difficult to photograph this condition, as the absorption rapidly increases with the density of the vapour and the light is not intense enough to produce a good negative. Figure 3 was taken with a pair of heavy glass prisms in the spectroscope. With the only grating available the dispersion was too great and the loss of light too much for a satisfactory negative to be obtained, the region between the two lines being specially difficult to photograph. If the temperature be too high and the potassium vapour in consequence too dense, absorption takes place, the region with longer wave-lengths disappearing first, the dispersion in the meantime increasing, so that dispersion may be observed extending almost down to the D lines. At the same time some beautiful absorption bands appear which seem to possess some regularity (fig. 4). These the author hopes to be able to investigate later, but there appears to be no relation between them and the tabulated lines of the emission spectrum of potassium. And further, there appears to be no anomalous dispersion exhibited at these bands. Anomalous dispersion also appears at the pair of lines 4047, 4044 with potassium. These are the second pair of the primary series. Again, the ultra-violet region contains the other primary series lines for potassium and as yet these have not been investigated.

The conditions in the tube as to the amount of gas present do not seem to matter very much as regards the anomalous dispersion due to the metallic vapour. In the case of lithium and potassium the effect shows very clearly both at a low pressure and at a pressure somewhat greater than an atmosphere. In each case the gas in the tube was hydrogen. In the case of lithium, hydrogen is rapidly absorbed when the metal is heated and the pressure becomes low. In any

case, however, there is, under the conditions of the experiment, a considerable amount of hydrogen present even when the tube is exhausted initially. A certain amount of hydroxide is inevitably introduced with the metal, and this on heating with the free metal evolves hydrogen; so that presumably a steady condition is reached when the pressure of the hydrogen reaches the value corresponding to the equilibrium pressure of the metallic hydride.

In the case of sodium vapour Wood has shown that anomalous dispersion takes place at the first three pairs of the principal series*, and further that the absorption spectrum consists of lines of the principal series†, together with a channelled spectrum. If the absorption spectrum be observed as the temperature of the sodium vapour is raised and its density increased, absorption is observable of gradually increasing amount near the D lines and soon the whole of the green disappears, but there is no apparent connexion between this absorption and the position of the green lines of sodium; the maximum absorption in the bands which appear does not coincide even approximately with the green line. The behaviours of sodium and potassium are thus very much alike. In the case of lithium, no absorption except in the region of the red line has as yet been detected. This may simply be due to the fact that a small quantity of lithium was used and a much higher temperature may be required to show effects analogous to those observed in the cases of sodium and potassium. The absolute amount of metallic vapour present to produce the dispersions observed must be very small, as a potassium tube can be used over and over again without apparent change in the amount of potassium remaining at the bottom of the tube. The potassium does slowly distil to the cooler parts of the tube, but the distillation is extremely slow and the tube may be used for many hours without any diminution of effect.

It can be seen that the amount of dispersion due to the potassium vapour is different in the neighbourhood of the two red lines, as we should expect, seeing that one of the lines is more intense than the other in the emission spectrum. The dispersion spectrum is thus unsymmetrical about the mean position of the two lines. In the case of sodium the same phenomenon occurs, and apparently the same is true of the dispersion due to lithium. The question is one not easy to decide definitely as the dispersion is small and the apparent effect is masked by the differences in absorption that occur;

* Phil. Mag. 1904, viii. p. 296.

† Phil. Mag. Dec. 1908.

but it does appear as if there is a want of symmetry in the lithium dispersion spectrum, which indicates that the lithium line is a double line with one component stronger than another.

Ebert * has observed the anomalous dispersion of potassium vapour by a method in which the vapour was kept in more or less prismatic form by means of two currents of hydrogen: the position of the absorption bands given in the diagram in Kayser's *Handbuch*, however, does not seem to agree with that shown in fig. 4. The method does not seem to promise such steady conditions as may be obtained with the simple method of Wood, and further, there must always be some moisture carried over with the hydrogen which must make the conditions not so permanent as in the simpler method. The author hopes to continue the experiments in the ultra-violet region. This has been made possible by a generous grant from the Royal Society.

XLVII. *On the Calculation of the Atomic Weight of Radium from Spectroscopic Data.* By W. MARSHALL WATTS, D.Sc.†

THE existence of exact relationships between the atomic weights of allied elements and their spectra does not admit of doubt. If we knew the precise law of this relationship, the measurement of the wave-lengths of the lines in the spectrum of a new or rare element, particularly of one difficult to obtain in a state of purity, would afford much more exact data for the calculation of the atomic weight, than the chemical method. If we consider that even in the case of an element whose atomic weight has been determined repeatedly with the utmost possible care—say silver or chlorine—the result is not certain to one part in 3000, whereas the wave-length of a line can be measured with an accuracy of one part in 100,000, we see how important the spectroscopic method might become.

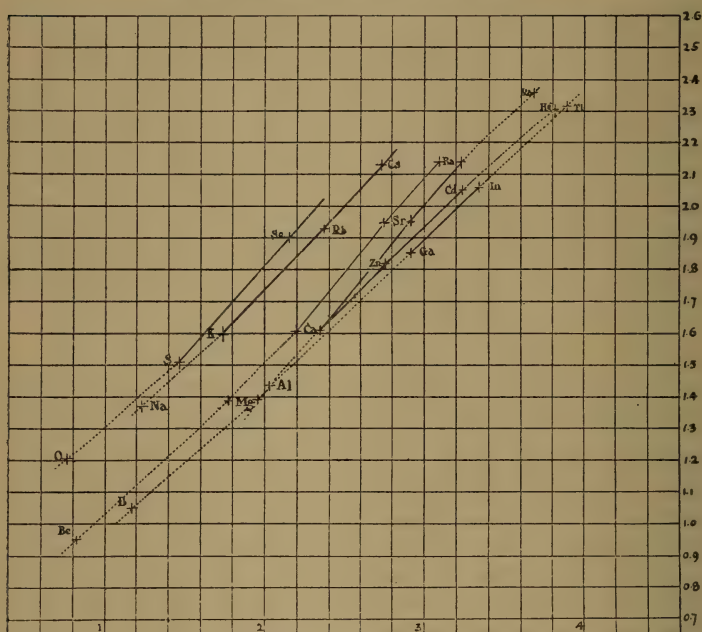
In the *Phil. Mag.* for Aug. 1904, in discussing the results obtained by Runge and Precht, I have pointed out that the number 257·8 obtained by them for Radium, from the atomic weights of Mg, Ca, Sr, and Ba, must be too high, judging from the similar calculation of the atomic weight of mercury from the atomic weights of zinc and cadmium, which would give 222 instead of 200. It appears that Runge and Precht's rule—that the separations of homologous pairs of lines are

* *Phys. Zeitschr.* 4, p. 473; Kayser, *Handbuch der Spectroscopie*, vol. iv. p. 563.

† Communicated by the Author.

proportional to some power of the atomic weight—though nearly true in many families of allied elements, is not *strictly* true in any one case. The diagram, in which the logarithms of atomic weights are plotted as ordinates and logarithms of

Fig. 1.—Runge and Precht's Law.



separations as abscissæ, shows that the lines joining the points in each family of elements are not *exactly* straight, but are slightly curved lines of similar shape, curving downwards at the top to pass through the elements of higher atomic weight, and curving upwards at the bottom for the elements of small atomic weight. The lines shown upon the diagram are:— (1) O, S, Se; (2) Na, K, Rb, Cs; (3) Be, Mg, Ca, Sr, Ba, plotted from the differences between the first and third lines in the triplet-series; (4) Mg, Ca, Sr, Ba, and Ra plotted from the intervals given in Runge and Precht's paper [Phil. Mag. [6] vol. v. p. 476 (1903); *Phys. Zsch.* iv. p. 285]; (5) Zn, Cd, Hg; (6) B, Al, Ga, In, Tl.

The intervals employed by Runge and Precht are:— Mg 91.7, Ca 223, Sr 801, Ba 1691, and Ra 4858.5. This interval occurs three times in the spectrum of radium (4858.3, 4858.5, and 4858.6); the interval 1691 occurs three times in the spectrum of barium; 801 occurs four times in strontium; 223 four times in calcium; and 91.7 three times

in magnesium : and it is not possible that we have to deal with an accidental coincidence.

An absolutely straight line cannot be drawn through more than two of these points in the diagram. The six possible lines give the following values for the atomic weights:—

	Mg.	Ca.	Sr.	Ba.	Ra.
Mg Ca line ...	[24·32]	[40·09]	82·30	125·28	226·83
Mg Sr line ...	[24·32]	41·13	[87·62]	136·30	254·43
Mg Ba line ...	[24·32]	41·23	88·13	[137·37]	257·14
Ca Sr line ...	23·28	[40·09]	[87·62]	138·36	263·81
Ca Ba line ...	23·36	[40·09]	87·22	[137·37]	260·93
Sr Ba line ...	23·78	40·89	[87·62]	[137·37]	259·26

The straight line chosen by Runge and Precht corresponding to the formula

$$\log \text{at. wt.} = 0\cdot2005 + 0\cdot5997 \log x$$

gives the following values :—

Mg	23·84	instead of	24·32
Ca	40·6	„	40·09
Sr	87·5	„	87·62
Ba	136·9	„	137·37

and for Ra 257·8.

The following formula represents a line of slight curvature which passes almost exactly through Mg, Ca, Sr, and Ba, and which gives 226·6 for the atomic weight of Radium :—

$$\begin{aligned} \log \text{at. wt.} = & 1\cdot886242 + \cdot623179 (\log x - 2\cdot813121) \\ & - \cdot080391 (\log x - 2\cdot813121)^3 \\ & - \cdot0374175 (\log x - 2\cdot813121)^4, \end{aligned}$$

giving

Mg	24·32,
Ca	40·08,
Sr	87·62,
Ba	137·41,
Ra	226·56.

Such a formula can only be considered as an interpolation-formula ; but it seems desirable to place it upon record, since it shows that the spectroscopic data are consistent with the atomic weight obtained for Radium by chemical methods.

I am indebted to my friend, Rev. P. H. Jones, M.A., R.N., for assistance in calculating this formula.

XLVIII. *On the Resistance due to Obliquely Moving Waves and its dependence upon the Particular Form of the Fore-part of a Ship.* By Lord RAYLEIGH, O.M., F.R.S.*

I SUPPOSE that everyone is familiar with the system of oblique waves advancing in *echelon* from the bow of a ship which travels through smooth water. What is not so easily observed from on board is the corresponding wave-profile, *i. e.* the deviation of the water-surface at the side of the ship from the position which it would occupy in a state of rest. Sketches, both of the whole system of waves and of various wave-profiles have been given by W. and R. E. Froude †, and the influence of the various components of the wave-system in contributing to the aggregate *wave-resistance* has been discussed. Attention has perhaps tended to concentrate upon the directly advancing waves—those whose crests are perpendicular to the ship's motion—and upon the remarkable interaction between the systems originating at the bow and stern. But, apart from its interesting geometrical features, the oblique part of the wave-system also impresses an observer with its mechanical importance as probably contributing in no mean degree to the total wave-making resistance.

From the time of my first acquaintance with drawings of wave-profiles I have been struck with their significance as indicating that the usual form of bow (and perhaps of stern) is not well adapted to minimise the forces of resistance. At the stem and immediately behind, the water is raised above the normal level, and this elevation is undoubtedly the principal feature. The additional pressure thence arising operates normally upon the skin of the ship, and the component in the fore-and-aft direction acting sternwards holds the ship back. It is, I suppose, in order to diminish the force here operative that the bows of many ships are made *hollow*, *i. e.* with a curvature concave outwards. But the question does not stop here. If we follow the wave-profile, we find that somewhat further back the water falls to the normal level and further back still to a level below the normal, that in fact there is a succession of waves of elevation and *depression* of gradually diminishing importance as we recede from the bow. As regards the force acting upon the ship in the fore-and-aft direction, we know that at any point it is the

* Communicated by the Author.

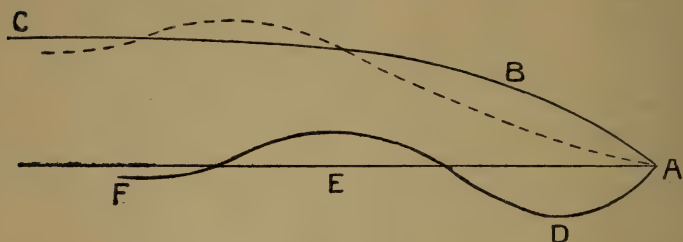
† Naval Arch. Trans. 1881; or see Kelvin, Proc. Mech. Eng., Aug. 1887; also Lamb's Hydrodynamics, 3rd ed. p. 414.

product of two factors, one the pressure acting perpendicularly upon the skin of the ship and secondly the sine of the angle (θ) between the skin and the direction of travel. If, as usual in the fore part of a ship, the skin faces everywhere forwards so that θ is positive, we recognize that if we estimate the pressures from the normal condition of rest, there is a force of retardation from the water-surface downwards in the region of an elevation, but on the other hand a force of acceleration in the region of a depression. And the question at once arises whether we cannot, at least in some degree, accommodate the angle to the pressure so as to diminish the sum total of the fore-and-aft components. In order to attain this end it is evident that θ should be diminished (perhaps even to the extent of becoming negative) in the region of an elevation, and should be increased in the region of a depression.

Several years ago (I think it was in 1902) with the late Mr. Gordon, I attempted some experiments in illustration of this theory. The water, to a depth of 4 or 5 inches, was contained in large flat sponge-bath mounted upon a turntable and maintained in uniform rotation. Near the circumference the revolving water flowed past a fixed model which represented the ship. The model was of wood with vertical sides and flat bottom, and was curved along its length in conformity with the circular motion of the water. No attempt was made to measure the forces of resistance, but observations were taken of the wave-pattern as affected by changes in the form of the bow. Starting from a somewhat blunt figure, the wood was gradually cut away from the rear of the region of the waves of elevation, thus for one thing sharpening the bow, care being taken not to continue this operation into the rear of the region of the waves of depression. In this way with frequent trials verifying the actual positions of the waves at the various stages, an undulating form with vertical sides, and (it must be admitted) of unprepossessing appearance, was gradually developed, which seemed to have the property of originating a much less pronounced wave-system, with promise of a diminished resistance.

So far as these experiments went, they tended to confirm me in the idea that advantage might arise from a similar figuring of an actual ship, but it was evident that much better trials upon models in a suitable tank was the next step, and my object in writing the present note is to suggest the execution of such trials. The natural course would be to start from a paraffin model of an actual ship, and after

observation of the wave-system and resistance to add and remove material at and under the places indicated (see figure), and then after each moderate change to redetermine



ABC is the original form of one side of the ship; ADEF the wave-profile, D and F being elevations and E a depression. The dotted line then indicates the direction of the proposed alterations.

the form of the waves and the force of resistance. It is possible of course that experiments in this direction with discouraging results have been made already, but I have seen no suggestion of them; and it may be that the prepossessions of naval architects would be too unfavourable to allow such an idea to be entertained.

One or two questions are touched upon in conclusion. It may be asked how far above and below the normal water level should the undulating form be carried? Observations of the actual pressures at various points with suitable gauges would be the best guide, but in general the variation of pressure due to waves does not extend far down in comparison with the wave-length. But here it is to be remembered that most actual ships are exposed to variations of loading and to encounters with long waves originating in wind. Another thing that might have to be borne in mind is the danger of eddy making if the undulations of form were made too pronounced, but this consideration can hardly be prohibitive of a moderate application of the principle. Again, it may be noted that strictly speaking the wave-system is a function of the speed, so that the adjustment could be complete only for a single speed. But this objection does not seem very serious. To discuss these questions further at the present stage would be premature.

Terling Place, Witham.

XLIX. *Inductance and Resistance in Telephone and other Circuits.* By J. W. NICHOLSON, M.A., D.Sc.; Trinity College, Cambridge*.

I. EFFECTIVE INDUCTANCE.

A GENERAL formula for the effective inductance of a circuit consisting of two long parallel wires has been given by the author †, and is suitable for cases in which the current distribution in either wire is greatly affected by the frequency of alternation. In its general form, although its limitations are clearly defined, the result is not well adapted, in the absence of tables, to rapid calculation. The main object of the present paper is to examine certain important cases in detail, and to obtain formulæ capable of immediate use. A calculation of the effective resistance is also made in each case. A problem to which attention has been mainly directed, which includes several practical cases of great interest, is that of the simple telephone circuit, in which the leads are not twisted round each other in order to annul the inductive effects of the earth and of neighbouring circuits.

In the proof of the general formula, the influence of electrostatic capacity was ignored. This imposes a great limitation upon the types of circuit for which the expression may be used. An estimation of the maximum capacity causing no alteration to a given order of accuracy is given in this paper.

Throughout the investigation, only iron and copper wires, as the two extreme cases, are considered. The large permeability of iron completely changes the character of the effect of frequency on its self-induction, as compared with other metals. To all metals except iron greatly used in practice, the formulæ developed for copper wires may be applied with a nearly identical order of accuracy.

Let a be the radius of either wire, c their distance apart, and (μ, σ) their permeability and resistivity. They are equal in all respects.

The value of the inductance per unit length is then

$$L = 4 \log \frac{c}{a} + \frac{4\mu}{x} \cdot \frac{\text{ber } x \text{ ber}' x + \text{bei } x \text{ bei}' x}{(\text{ber}' x)^2 + (\text{bei}' x)^2} + M, \quad (1)$$

where, if $n/2\pi$ be the frequency,

$$x = 2a(\pi\mu n/\sigma)^{\frac{1}{2}}, \quad \dots \dots \dots (2)$$

* Communicated by the Physical Society: read June 11, 1909.

† Phil. Mag. Feb. 1909; Proc. Phys. Soc. vol. xxi.

and, M and N being real,

$$c^2(M + \iota N) = \frac{4a^2(\log a/c - \mu J_0/kaJ_0')(1 - \mu J_1/kaJ_1')}{(\log ha - \mu J_0/kaJ_0')(1 + \mu J_1/kaJ_1')}. \quad (3)$$

The Bessel functions have an argument $ka = x\iota^{\frac{3}{2}}$, and if V be the velocity of propagation of electromagnetic disturbances in the outer medium,

$$h = na/V. \quad (4)$$

M may be described as the correction for closeness of the wires.

The effective resistance per unit length is given by

$$R = \frac{4\pi n}{x} \cdot \frac{\text{ber } x \text{ bei}' x - \text{bei } x \text{ ber}' x}{(\text{ber}' x)^2 + (\text{bei}' x)^2} - Nn. \quad (5)$$

Now the functions $\text{ber } x$, $\text{bei } x$ are usually defined by

$$J_0(x\iota^{\frac{3}{2}}) = \text{ber } x + \iota \text{ bei } x,$$

so that

$$J_1(x\iota^{\frac{3}{2}}) = -\iota^{-\frac{3}{2}}(\text{ber}' x + \iota \text{ bei}' x),$$

$$J_1'(x\iota^{\frac{3}{2}}) = -\iota(\text{ber}'' x + \iota \text{bei}'' x),$$

and

$$\frac{\mu}{ka} \cdot \frac{J_1}{J_1'} = \frac{\mu}{x} \cdot \frac{\text{ber}' x + \iota \text{bei}' x}{\text{ber}'' x + \iota \text{bei}'' x},$$

whence on reduction

$$(1 - \mu J_1/kaJ_1')/(1 + \mu J_1/kaJ_1') = (E + \iota F)/D, \quad (6)$$

where

$$\left. \begin{aligned} Ex^2 &= (x \text{ber}'' x)^2 + (x \text{bei}'' x)^2 - (\mu \text{ber}' x)^2 - (\mu \text{bei}' x)^2, \\ Fx &= 2\mu(\text{ber}' x \text{bei}'' x - \text{bei}' x \text{ber}'' x), \\ Dx^2 &= (x \text{ber}'' x)^2 + (x \text{bei}'' x)^2 + (\mu \text{ber}' x)^2 + (\mu \text{bei}' x)^2 \\ &\quad + 2\mu x(\text{ber}' x \text{ber}'' x + \text{bei}' x \text{bei}'' x). \end{aligned} \right\} \quad (7)$$

But if accents denote differentiations with respect to x ,

$$y = \text{ber } x + \iota \text{ber } x$$

is a solution of

$$xy'' + y' = \iota xy,$$

so that

$$x \text{ber}'' x + \iota x \text{bei}'' x + \text{ber}' x + \iota \text{bei}' x = \iota x(\text{ber } x + \iota \text{bei } x),$$

and therefore

$$\left. \begin{aligned} x \text{ber}'' x &= -\text{ber}' x - x \text{bei } x, \\ x \text{bei}'' x &= x \text{ber } x - \text{bei}' x, \end{aligned} \right\} \quad (8)$$

from which may be deduced

$$\left. \begin{aligned} \text{ber}' x \text{ bei}'' x - \text{bei}' x \text{ ber}'' x &= \text{ber } x \text{ ber}' x + \text{bei } x \text{ bei}' x, \\ \text{ber}' x \text{ ber}'' x + \text{bei}' x \text{ bei}'' x &= \text{ber } x \text{ bei}' x - \text{bei } x \text{ ber}' x \\ &\quad - x^{-1}(\text{ber}' x)^2 - x^{-1}(\text{bei}' x)^2, \\ (x \text{ ber}'' x)^2 + (x \text{ bei}'' x)^2 &= (\text{ber}' x)^2 + (\text{bei}' x)^2 + (x \text{ ber } x)^2 \\ &\quad + (x \text{ bei } x)^2 - 2x(\text{ber } x \text{ bei}' x - \text{bei } x \text{ ber}' x). \end{aligned} \right\} \quad (9)$$

Writing

$$\left. \begin{aligned} P &= \text{ber}^2 x + \text{bei}^2 x, \\ \alpha P &= (\text{ber}' x)^2 + (\text{bei}' x)^2, \\ \beta P &= \text{ber } x \text{ ber}' x + \text{bei } x \text{ bei}' x, \\ \gamma P &= \text{ber } x \text{ bei}' x - \text{bei } x \text{ ber}' x. \end{aligned} \right\} \quad \cdot \quad \cdot \quad \cdot \quad (10)$$

Then

$$Ex^2/P = \alpha(1 - \mu^2) + x^2 - 2\gamma x,$$

$$Fx/P = 2\mu\beta,$$

$$Dx^2/P = \alpha(1 - \mu)^2 + x^2 - 2x\gamma(1 - \mu),$$

where

$$D\lambda_1 = E + \iota F. \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (11)$$

For the special case $\mu = 1$; $D = P$, so that

$$\lambda_1 = 1 - 2(\gamma - \iota\beta)/x, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (12)$$

a very simple result, which is true in practice for all but iron wires. Again, on reduction,

$$\log a/c - \mu J_0/kaJ_0' = \log a/c - \mu(\beta - \iota\gamma)/\alpha x,$$

$$\log ha - \mu J_0/haJ_0' = \log ha - \mu(\beta - \iota\gamma)/\alpha x,$$

and their quotient becomes

$$(A + \iota B)/C,$$

where

$$\left. \begin{aligned} x^2 A &= \alpha x^2 \log ha \cdot \log a/c - \mu\beta x \log ha^2/c + \mu^2, \\ xB &= \mu\gamma \log hc, \\ x^2 C &= \alpha x^2 \log^2 ha - 2\mu\beta x \log ha + \mu^2, \end{aligned} \right\} \quad (13)$$

and it has been noticed that $\beta^2 + \gamma^2 = \alpha$ identically.

With these values, it is found that, M and N being given by (3),

$$c^2 CD(M + \iota N) = 4a^2(E + \iota F)(A + \iota B).$$

Thus

$$\left. \begin{aligned} M &= 4a^2(AE - BF)/c^2 CD, \\ N &= 4a^2(AF + BE)/c^2 CD. \end{aligned} \right\} \quad \cdot \quad \cdot \quad \cdot \quad (14)$$

Moreover,

$$\frac{CD}{P} = \left(\alpha \log^2 ha - \frac{2\mu\beta}{x} \log ha + \frac{\mu^2}{x^2} \right) \left(\frac{\alpha}{x^2} \overline{1-\mu^2} + 1 - \frac{2\gamma}{x} \overline{1-\mu} \right). \quad (15)$$

$$\frac{AE-BF}{P} = \left(\alpha \log \frac{a}{c} \log ha - \frac{\mu\beta}{x} \log \frac{ha^2}{c} + \frac{\mu^2}{x^2} \right) \left(\frac{\alpha - \alpha\mu^2}{x^2} + 1 - \frac{2\gamma}{x} \right) - \frac{2\mu^2\beta\gamma}{x^2} \log hc. \quad (16)$$

The succeeding reduction depends upon the particular circumstances of the wires. We now write $\mu=1$, and suppose that the wires are of copper. The only case needing separate treatment is that of iron, where μ is very large.

Inductance when $\mu=1$. High frequency.

Quoting the value of M in (14), the inductance becomes, when $\mu=1$,

$$L = 4 \log_e \frac{c}{a} + \frac{4\beta}{\alpha x} + \frac{4\alpha^2(AE-BF)}{c^2 CD}, \quad \dots \quad (17)$$

where

$$\left. \begin{aligned} \frac{AE-BF}{P} &= \left(\alpha \log \frac{a}{c} \log ha - \frac{\beta}{x} \log \frac{ha^2}{c} + \frac{1}{x^2} \right) \\ &\quad \left(1 - \frac{2\gamma}{x} \right) - \frac{2\beta\gamma}{x^2} \log hc, \\ \frac{CD}{P} &= \alpha \log^2 ha - \frac{2\beta}{x} \log ha + \frac{1}{x^2}. \end{aligned} \right\} \quad (18)$$

Asymptotic formulæ for the functions (α, β, γ) have been developed by Dr. A. Russell*.

When the argument x is not less than 8, then to four significant figures, if $\lambda = x \sqrt{2}$ †, so that, in terms of the characteristics of the two wires,

$$\lambda = 2a \left(\frac{2\pi n}{\sigma} \right)^{\frac{1}{2}}, \quad \dots \quad (19)$$

we may write

$$\left. \begin{aligned} \alpha &= 1 - \frac{1}{\lambda} + \frac{1}{2\lambda^2} + \frac{3}{4\lambda^3}, \\ \beta \sqrt{2} &= 1 - \frac{1}{\lambda} - \frac{1}{4\lambda^2}, \\ \gamma \sqrt{2} &= 1 + \frac{1}{4\lambda^2} + \frac{1}{2\lambda^3}, \end{aligned} \right\} \quad \dots \quad (20)$$

* Proc. Phys. Soc. vol. xxii.

† This modification is more convenient for purposes of printing.

and after reduction

$$\left. \begin{aligned} \frac{AE-BF}{P} &= \left(1 - \frac{3}{\lambda} + \frac{5}{2\lambda^2} - \frac{3}{4\lambda^3}\right) \log \frac{a}{c} \log ha \\ &\quad - \frac{2}{\lambda} \left(1 - \frac{3}{\lambda} - \frac{1}{4\lambda^2}\right) \log ha + \frac{1}{\lambda} \left(1 - \frac{5}{\lambda} \right. \\ &\quad \left. + \frac{15}{4\lambda^2}\right) \log hc + \frac{2}{\lambda^2} \left(1 - \frac{2}{\lambda}\right), \\ \frac{CD}{P} &= \left(1 - \frac{1}{\lambda} + \frac{1}{2\lambda^2} + \frac{3}{4\lambda^3}\right) \log^2 ha - \frac{2}{\lambda} \left(1 - \right. \\ &\quad \left. \frac{1}{\lambda} - \frac{1}{4\lambda^2}\right) \log ha + \frac{2}{\lambda^2}. \end{aligned} \right\} \quad (21)$$

We proceed to examine the limiting frequency for which the four-figure accuracy of these results is preserved. For a copper wire of low resistance as ordinarily used, the resistivity may be taken as 1696 C.G.S. If f be the frequency of alternation, $n=2\pi f$, leading to $x = \frac{1}{5} af^{\frac{1}{2}}$ approximately.

Thus $x=8$ leads to the determining relation

$$af^{\frac{1}{2}} = 40. \quad . \quad . \quad . \quad . \quad . \quad (22)$$

If $af^{\frac{1}{2}}$ is less than 40, the four-figure accuracy, in so far as it depends on x , is lost. When f is only 400 per second, the limiting diameter of a wire becomes so large as 4 centimetres, so that the formulæ are unsuited for practice until the frequency is really great. The usual range of diameter for wires applied to such purposes as telephonic communication is from 1 to 3 millimetres in the case of cables, and to 5.7 millimetres in that of overhead wires. The frequency in cases of transmission of speech will not be greater than 2000. Taking $f=1600$ as perhaps the real maximum, the limiting radius of a wire becomes 1 centimetre. The results therefore cannot have a great accuracy in such a case, and the formula (43) below must be used. But it will appear that the corresponding formula for iron leads can be so employed.

The present results have a two-figure accuracy even when $x=3$, leading to a radius of 3.75 millimetres, and therefore will apply to many cases even of telephony when thicker wires are used. In such cases, their use is preferable to that of (43), in that the calculation is more rapid.

It is necessary not to overlook the other sources of error. Firstly, in the proof of (1)*, h^2a^2 was neglected in com-

* Phil. Mag. Feb. 1909.

parison with unity. In the most unfavourable case furnished by the telephone, $f=1600$, $a=3$ millimetres, leading to $h^2a^2=10^{-14}$ approximately.

Thus this source of error needs no further consideration in such a case. But in fact, the neglect of this quantity is always legitimate. For if f be the frequency, $h^2a^2=4f^210^{-20}$ for a wire of a centimetre radius. It can only therefore affect the fourth figure if $f=20$ million, and the second if $f=200$ million, and such a frequency is never used in combination with so thick a wire.

Secondly, a^4/c^4 was neglected in comparison with unity. Now even in ordinary telephone construction there is usually about $2\frac{1}{2}$ millimetres of paper and air between the wires, so that in the case of the cable, taking an extreme radius of 1.3 millimetres, $c=4a$, and only the third figure is affected. For overhead wires, the distance apart is about 30 centimetres.

The limiting practical closeness of the wires, when a knowledge of the self-induction is required, is, I believe, attained in Mr. A. Campbell's experiments on variable mutual inductances*. The error may then be so great as one or two per cent.

Effect of a Small Capacity.

Heaviside † has divided circuits into five classes, with the following determining properties:—

(1) Submarine cables proper, in which the capacity is the main factor, and whose treatment must follow the electrostatic theory.

(2) Short lines of low frequency, in which self-induction and resistance determine the effects. This class includes the majority of short telephone circuits.

(3) A class in which capacity and self-induction are equally important. These circuits are very difficult in theory.

(4) Circuits of high frequency, but small capacity and resistance, and with the inductance so chosen that signals may travel without distortion. For this class the electrostatic theory cannot be applied however long the circuit.

(5) Circuits in which distortionless propagation is obtained by allowing an electric leakage.

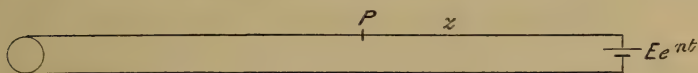
The effect of leakage is small in all but the fifth case, and will be ignored.

Let an impressed force Ee^{int} act at one end of a circuit consisting of two parallel wires, with terminal apparatus

* Phil. Mag. Jan. 1908.

† 'Electrical Papers,' vol. ii.

whose inductance, capacity, and resistance are neglected. Let z denote distance of a point P from the end at which the force acts. If R , L , C be the resistance, inductance,



and capacity per unit length, and (ι_1, V) the current and potential at a point P; then

$$\left. \begin{aligned} C \partial V / \partial t &= -\partial \iota_1 / \partial z \\ L \partial \iota_1 / \partial t + R \iota_1 &= -\partial V / \partial z \end{aligned} \right\}, \quad \cdot \cdot \quad (23)$$

where $\partial / \partial t \equiv in$.

The solution of these equations is found to be

$$\iota_1 = (E/I) e^{\iota(n t - \alpha)}, \quad \cdot \cdot \cdot \quad (24)$$

where

$$\left. \begin{aligned} I &= L^{\frac{1}{2}} C^{-\frac{1}{2}} (1 + R^2/n^2 L^2)^{\frac{1}{2}} (\cosh 2lQ_1 - \cos 2lQ_2)^{\frac{1}{2}} \\ Q_1, Q_2 &= n(\frac{1}{2}LC)^{\frac{1}{2}} \{ (1 + R^2/n^2 L^2)^{\frac{1}{2}} \mp 1 \}^{\frac{1}{2}} \end{aligned} \right\} \quad (25)$$

With the value of α we are not concerned. For the effect of leakage, reference may be made to Heaviside's paper*.

$$\text{If } C = 0, \quad I = l(R^2 + L^2 n^2)^{\frac{1}{2}}, \quad \cdot \cdot \cdot \quad (26)$$

which is the ordinary impedance assumed to be valid in the previous paper. In the electrostatic theory, we write $L=0$, which is obviously unjustifiable if R is small or n very great, so that circuits of the first kind lose their character in these cases.

We proceed to estimate the correction to be made in the value of L , to take account of a small capacity, regarding only the first order term. Writing

$$\lambda_1^2, \lambda_2^2 = (1 + R^2/n^2 L^2)^{\frac{1}{2}} \mp 1,$$

then on reduction,

$$\begin{aligned} \cosh 2lQ_1 - \cos 2lQ_2 &= LC n^2 l^2 (\lambda_1^2 + \lambda_2^2) \{ 1 + \frac{1}{6} LC n^2 l^2 (\lambda_1^2 - \lambda_2^2) \} \\ &= 2C n l^2 (R^2 + L^2 n^2)^{\frac{1}{2}} (1 - \frac{1}{3} l^2 n^2 LC), \end{aligned}$$

so that

$$I = l\sqrt{2} \cdot (R^2 + L^2 n^2)^{\frac{1}{2}} (1 - \frac{1}{6} l^2 n^2 LC).$$

* *Loc. cit.*

If L' be the equivalent induction when this small capacity is taken into account,

$$R^2 + L'^2 n^2 = (R^2 + L^2 n^2)(1 - \frac{1}{3} l^2 n^2 LC)$$

or $(L' - L)/L = -l^2 C(R^2 + L^2 n^2)/6L$ approximately. (27)

This equation serves to limit the types of circuit to which the uncorrected inductance formulæ may be applied, in so far as error due to capacity is concerned. For example, if a three-figure accuracy is required, it is necessary that

$$l^2 C(R^2 + L^2 n^2)/6L \nless 10^{-4}, \quad . \quad . \quad . \quad (28)$$

where l is the length of either wire of the circuit.

We have also neglected

$$\frac{1}{90} l^4 n^4 L^2 C^2 (\lambda_1^4 + \lambda_2^4 - \lambda_1^2 \lambda_2^2) = \frac{1}{90} l^4 n^4 L^2 C^2 (4 + R^2/L^2 n^2)$$

in comparison with unity. This restricts the capacity for which the correction (27) may be used. Thus, for low resistance,

$$4l^4 n^2 C^2 \nless 9 \cdot 10^{-5}.$$

This equation will ordinarily supply the lower limit for C . Thus for frequencies such that

$$nC \nless (5l^2)^{-1} 10^{-2} \quad . \quad . \quad . \quad (29)$$

the uncorrected (for capacity) formulæ may be used provided that C is also so small as to make

$$Cn^2 L \nless 6l^{-2} 10^{-4}, \quad . \quad . \quad . \quad (30)$$

again neglecting R .

For a capacity of one microfarad per kilometre, $C = 10^{-20}$. In a case like that of the Atlantic cable the capacity is about a quarter of this amount, and the limiting frequency, with l in centimetres, is about $l^{-2} 10^{18}$. This excludes such a cable altogether from the investigation, although a shorter cable of the same radius of wire and capacity gradient can be within its scope. For example, a frequency of so much as 10 million can be treated if l is not greater than 1.2 kilometres, for a suitable range of inductance given by (30). The range of inductance is of course dependent mainly on the distance between the wires.

Obviously all short telephone circuits satisfy both (29) and (30), and their capacity needs no consideration.

Copper Wires with High Frequency.

With the above restrictions as regards capacity and frequency (the latter being of little practical import), writing in (17),

$$\begin{aligned}\frac{\beta}{\alpha x} &= \frac{1}{\lambda} \left(1 - \frac{1}{\lambda} - \frac{1}{4\lambda^2} \right) \left(1 - \frac{1}{\lambda} + \frac{1}{2\lambda^2} \right) \\ &= \frac{1}{\lambda} \left(1 - \frac{2}{\lambda} + \frac{5}{4\lambda^2} \right). \quad . \quad . \quad . \quad . \quad . \quad (31)\end{aligned}$$

Then with the values of (21),

$$L = 4 \log_e \frac{c}{a} + \frac{4}{\lambda} \left(1 - \frac{2}{\lambda} + \frac{5}{4\lambda^2} \right) + \frac{4a^2}{c^2 CD} (AE - BF),$$

where $\lambda = 2a(2\pi\mu n/\sigma)^{\frac{1}{2}} = 4\pi a(8/\sigma)^{\frac{1}{2}},$

with a four-figure accuracy so far as λ is concerned, if $\lambda > 8\sqrt{2}$. But a further simplification may be introduced. Since ha is a very small magnitude in actual cases, its logarithm is large and negative. Thus if $\rho = \log_e ha$, the functions may be expanded in a descending series of powers of ρ , and

$$\frac{CD}{P\rho^2} = 1 - \frac{1}{\lambda} \left(1 + \frac{2}{\rho} \right) + \frac{1}{\lambda^2} \left(\frac{1}{2} + \frac{2}{\rho} + \frac{2}{\rho^2} \right) + \frac{1}{\lambda^3} \left(\frac{3}{4} + \frac{1}{2\rho} \right)$$

or

$$\frac{P\rho^2}{CD} = 1 + \frac{1}{\lambda} \left(1 + \frac{2}{\rho} \right) + \frac{1}{\lambda^2} \left(\frac{1}{2} + \frac{2}{\rho} + \frac{2}{\rho^2} \right) - \frac{1}{\lambda^3} \left(\frac{3}{4} + \frac{1}{2\rho} \right), \quad (32)$$

whence, after considerable reduction

$$\begin{aligned}\frac{\rho^2}{CD} (AE - BF) &= \rho \log \frac{a}{c} - \frac{1}{\lambda} \left(\rho + 4\rho \log \frac{a}{c} + 3 \log \frac{a}{c} \right) \\ &\quad + \frac{1}{4\rho^2\lambda^3} \left\{ 8\rho - 8\rho^2 + 19\rho^3 - (8 - 8\rho + 3\rho^2) \log \frac{a}{c} \right\}.\end{aligned}$$

The vanishing of the coefficient of λ^{-2} is curious.

Finally, for a pair of copper wires, the main error being of relative magnitude a^4/c^4 when they are close together, if $\lambda > 8\sqrt{2}$,

$$\lambda = 4\pi a(f/\sigma)^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad (33)$$

if f is the frequency, then

$$L = 4 \log_e \frac{c}{a} + \frac{4}{\lambda} \left(1 - \frac{2}{\lambda} + \frac{5}{4\lambda^2} \right) + \frac{4a^2}{c^2} \log \frac{a}{c} \\ - \frac{4a^2}{c^2 \lambda \rho^2} \left(\rho + 4\rho \log \frac{a}{c} + 3 \log \frac{a}{c} \right) \\ + \frac{a^2}{c^2 \lambda^3 \rho^4} \left\{ 8\rho - 8\rho^2 + 19\rho^3 - (8 - 8\rho + 3\rho^2) \log \frac{a}{c} \right\}, \quad (34)$$

where $\rho = \log_e ha$, and the limiting capacity is given by (29). For most useful purposes it may be greatly shortened, according to the value of λ .

Iron Wires with High Frequency.

When iron is used, there is no object to be served in obtaining so accurate a formula, on account of the vagaries in the magnetic behaviour of the metal. But the approximate value of μ is known, for Lord Rayleigh* has shown that for feeble magnetizing forces of periodic character, μ usually lies between 90 and 100, and becomes much greater as the forces are increased. It is therefore lawful to take μ as of order 10^2 . In the formulæ (20), (α, β, γ) never become large, and therefore whatever the value of x (or λ), the approximation may be conducted by expanding the functions in inverse powers of μ .

Thus if $\rho = \log ha$ as before,

$$\frac{CD}{P} = \frac{\alpha \mu^4}{x^4} \left(1 - 2\beta \rho \frac{x}{\mu} + \alpha \rho^2 \frac{x^2}{\mu^2} \right) \left(1 - \frac{2}{\mu} \left(1 - \frac{\gamma x}{\alpha} \right) + \frac{1}{\mu^2} \left(1 - \frac{2\gamma x}{\alpha} + \frac{x^2}{\alpha^2} \right) \right) \\ = \frac{\mu^4 \alpha}{x^4} \left\{ 1 - \frac{2}{\mu} \left(\beta \rho x + 1 - \frac{\gamma x}{\alpha} \right) + \frac{1}{\mu^2} \left(1 + \alpha \rho^2 x^2 - \frac{2\gamma x}{\alpha} \right. \right. \\ \left. \left. + \frac{x^2}{\alpha} + 4\beta \rho x - 4\beta \gamma \rho \frac{x^2}{\alpha} \right) \right\} \\ \text{or} \\ \frac{P \mu^4 \alpha}{x^4 CD} = 1 + \frac{2}{\mu} \left(1 + \beta \rho x - \frac{\gamma x}{\alpha} \right) + \frac{1}{\mu^2} \left(3 + 4\beta^2 \rho^2 x^2 + 4\beta \rho x + \frac{4\gamma^2 x^2}{\alpha^2} \right. \\ \left. - 4\beta \gamma \rho \frac{x^2}{\alpha} - \frac{x^2}{\alpha} - \frac{6\gamma x}{\alpha} - \alpha \rho^2 x^2 \right). \quad (35)$$

This neglects the ratio x^3/μ^3 . Taking a wire of gauge $\frac{1}{2}$ cm., an approximate practical limit for overhead wires, and

* Scientific Papers; iii.

writing, as an ordinary value for iron, $\sigma = 10,000$ c.g.s.,

$$x = \frac{3}{20}f^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad (36)$$

if $\mu = 100$. Thus even when $f = 10,000$ per second, neglect of x^3/μ^3 only affects the third figure of CD, and therefore at most the fourth of L, when the ratio a^2/c^2 is noticed. This process is justifiable whether the ascending or descending series are used for (α, β, γ) .

In a similar manner,

$$-\frac{(AE-BF)}{P} = + \frac{\alpha\mu^4}{x^4} \left(1 - \frac{\beta x}{\mu} \log \frac{ha^2}{c} + \frac{\alpha x^2}{\mu^2} \log ha \log \frac{a}{c} \right) \left(1 - \frac{\alpha + x^2 - 2\gamma x}{\alpha\mu^2} \right) \\ + 2\mu^2 \frac{\beta\gamma}{x^2} \log hc ;$$

and after some reduction,

$$-\frac{AE-BF}{CD} = 1 + \frac{1}{\mu} \left(2 + \beta\rho x - \frac{2\gamma x}{\alpha} - \beta x \log \frac{a}{c} \right) \\ + \frac{1}{\mu^2} \left(2 + 2\beta\rho x - \alpha\rho^2 x^2 + 2\beta^2\rho^2 x^2 + \frac{4\gamma^2 x^2}{\alpha^2} - \frac{4\gamma x}{\alpha} - \frac{2x^2}{\alpha^2} \right) \\ + \frac{1}{\mu^2} \log \frac{a}{c} \cdot (\alpha x^2 \rho - 2\beta^2 x^2 \rho - 2\beta x).$$

In the extreme case above, $\frac{x}{\mu^2} \nmid \frac{1}{600}$, and the final error consequent upon its neglect is of order $(6000)^{-1}$ relative to unity even when the wires have limiting practical closeness.

Accordingly,

$$-\left(\frac{AE-BF}{CD}\right) = 1 + \frac{1}{\mu} \left(2 + \beta\rho x - \frac{2\gamma x}{\alpha} - \beta x \log \frac{a}{c} \right) \\ + \frac{1}{\mu^2} \left(2\beta^2\rho^2 x^2 - \alpha\rho^2 x^2 + \frac{4\gamma^2}{\alpha^2} x^2 - \frac{2x^2}{\alpha} + x^2\rho(\alpha - 2\beta^2) \log \frac{a}{c} \right).$$

Consider now the case in which $x > 8$. In the coefficient of μ^{-2} , the first order values of $(\alpha\beta\gamma)$ may be used, with the second order in that of μ^{-1} .

Under these conditions, the coefficient of μ^{-2} is found to vanish, leading to the very simple result, to order μ^{-3} ,

$$-\left(\frac{AE-BF}{CD}\right) = 1 + \frac{1}{\mu} \left\{ 2 + \frac{1}{2}\lambda \left(1 - \frac{1}{\lambda} \right) \log hc - \lambda \left(1 + \frac{1}{\lambda} \right) \right\} \\ = 1 + (\lambda - 1)(\log hc - 2)/2\mu,$$

and thus for iron wires with $\lambda > 8\sqrt{2}$,

$$L = 4 \log_e \frac{c}{a} + \frac{4}{\lambda} \left(1 - \frac{2}{\lambda} + \frac{5}{4\lambda^2} \right) - \frac{4a^2}{c^2} - \frac{2a^2}{c^2\mu} (\lambda - 1)(\log hc - 2), \quad (37)$$

where

$$\lambda = 4\pi a \left(\frac{f\mu}{\sigma} \right)^{\frac{1}{2}}, \quad \text{and} \quad \frac{a^2}{c^2} \frac{\lambda}{\mu^2} \text{ is neglected.}$$

On account of μ , this formula may be applied, for any given frequency, to a wire of only about a quarter the corresponding limiting radius in the case of copper. For example, for a frequency 800, and radius 2 millimetres, the value of x becomes 5, with $\mu = 100$. Thus a three-figure accuracy is obtained for the most unfavourable case of the telephone circuit. Accordingly, for a telephone circuit of iron wires not twisted together, this result is of universal application.

Iron Wires with Small Frequency.

This case is not very important, but the result may be given at once. Thus if x is not greater than 2, we may neglect the term involving μ^{-2} , and write, to three significant figures,

$$L = 4 \log \frac{c}{a} + \frac{4\beta}{x\alpha} - \frac{4a^2}{c^2} - \frac{4a^2}{c^2\mu} \left(\beta x \log hc + 2 - \frac{2\gamma x}{\alpha} \right). \quad (38)$$

The values of (α, β, γ) suited to this case have been given by Russell*.

$$\text{If} \quad z = \frac{x}{2} = a\pi \left(\frac{2\mu f}{\sigma} \right)^{\frac{1}{2}},$$

$$\left. \begin{aligned} \alpha &= z^2 \left(1 - \frac{5}{12} z^4 + \frac{143}{720} z^8 - \frac{7661}{4^2 \cdot 7!} z^{12} \right) \\ \beta &= \frac{1}{2} z^3 \left(1 - \frac{11}{24} z^4 + \frac{473}{3 \cdot 6!} z^8 - \frac{304107}{4 \cdot 12^2 \cdot 7!} z^{12} \right) \\ \gamma &= z \left(1 - \frac{1}{3} z^4 + \frac{19}{5!} z^8 - \frac{687}{7 \cdot 6^4} z^{12} \right) \end{aligned} \right\}. \quad (39)$$

Moreover,

$$\left. \begin{aligned} \frac{\beta}{\alpha} &= \frac{1}{2} z \left(1 - \frac{1}{24} z^4 + \frac{13}{4320} z^8 - \frac{647}{12^2 \cdot 360 \cdot 56} z^{12} \right) \\ \frac{\gamma}{\alpha} &= z^{-1} \left(1 + \frac{1}{12} z^4 - \frac{1}{180} z^8 + \frac{11}{12 \cdot 28 \cdot 30} z^{12} \right) \end{aligned} \right\} \quad (40)$$

* *Loc. cit.*

Heaviside* has also given the value of β/α , and Lord Rayleigh† that of γ/α , by different modes of proof. Thus finally, when

$$z = \alpha\pi(2\mu f/\sigma)^{\frac{1}{2}} \gg 1, \quad . \quad . \quad . \quad (41)$$

$$\begin{aligned} L = 4 \log \frac{c}{a} + 1 - \frac{1}{24} z^4 + \frac{13}{4320} z^8 - \frac{647}{12^2 \cdot 360 \cdot 56} z^{12} - \frac{4a^2}{c^2} \\ - \frac{4z^4 a^2}{c^2 \mu} \log_e hc \left\{ 1 - \frac{11}{24} z^4 + \frac{473}{3 \cdot 6!} z^8 - \frac{304107}{4 \cdot 12^2 \cdot 7!} z^{12} \right\} \\ + \frac{8a^2}{c^2 \mu} \left(1 + \frac{1}{6} z^4 + \frac{1}{90} z^8 - \frac{11}{28 \cdot 180} z^{12} \right). \quad . \quad . \quad . \quad (42) \end{aligned}$$

Small Frequency, Copper Wires.

In the notation of (18), with $\rho = \log_e ha$,

$$x^2 \frac{CD}{P} = 1 - 2\beta x \rho + \alpha x^2 \rho^2.$$

The frequencies for which this formula will ordinarily be required lie between 200 and 1600, and the radii between 1 and 3 millimetres. These values cause ρ to vary between about 13 and 16 only, and z between $\frac{1}{4}$ and unity, so that P/CD cannot be expanded in ascending or descending powers of ρ or z in general. The result may be most simply written in the form

$$\begin{aligned} L = 4 \log \frac{c}{a} + \frac{2}{z} \frac{\beta}{\alpha} + \frac{4a^2}{c^2} \left(1 - 2\beta \rho z - \frac{\gamma}{z} \right) / (1 - 4\beta \rho z + 4\alpha \rho^2 z^2) \\ + \frac{8a^2}{c^2} \log \frac{a}{c} \cdot (2\alpha \rho z^2 - 2\alpha \rho \gamma z - \beta z + 2\beta \gamma) / (1 - 4\beta \rho z + 4\alpha \rho^2 z^2), \quad (43) \end{aligned}$$

where $(\alpha \beta \gamma z)$ are given by (39), and $\rho = \log_e ha$.

The two final terms in the last numerator are only about $\frac{1}{50}$ of the other two, and may be ignored.

* Elec. Papers, ii. p. 64.

† Phil. Mag. xxii. p. 381 (1886); Scientific Papers, vol. ii.

II. EFFECTIVE RESISTANCE.

Copper Wires with High Frequency.

The limitations of all the formulæ below, except when otherwise stated, are the same as in the corresponding inductance formulæ. With the previous notation, the effective resistance per unit length of the two wires is

$$R = \frac{4n\mu}{x} \frac{\text{ber } x \text{ bei}' x - \text{bei } x \text{ ber}' x}{(\text{ber}' x)^2 + (\text{bei}' x)^2} - nN$$

$$= \frac{4n\mu}{x} \cdot \frac{\gamma}{\alpha} - \frac{4a^2n}{c^2 CD} (AF + BE). \quad (44)$$

where CD/P has the value in (15), and on reduction

$$\frac{AF + BE}{P} = \frac{2\mu\beta}{x} \left(\alpha \log ha \log \frac{a}{c} - \frac{\mu\beta}{x} \log \frac{ha^2}{c} + \frac{\mu^2}{x^2} \right)$$

$$+ \frac{\mu\gamma}{x} \log hc \left(1 - \frac{2\gamma}{x} + \frac{\alpha}{x^2} \overline{1 - \mu^2} \right). \quad (45)$$

When $\mu = 1$, and the frequency is high, so that, if $x\sqrt{2} = \lambda$, the formulæ (20) may be employed, then after further reduction,

$$\frac{AF + BE}{P} = \frac{2}{\lambda} \log \frac{a}{c} \log ha \left(1 - \frac{2}{\lambda} + \frac{5}{4\lambda^2} \right) - \frac{4}{\lambda^2} \log ha \left(1 - \frac{2}{\lambda} \right)$$

$$+ \frac{1}{\lambda} \log hc \left(1 - \frac{15}{4\lambda^2} \right) + \frac{4}{\lambda^3},$$

and with the help of (32), we ultimately obtain

$$\frac{\lambda\rho^2}{CD} (AF + BE) = 2\rho \log \frac{a}{c} + \rho - \log \frac{a}{c} - \frac{1}{\lambda\rho} \left\{ \rho^2 + \log \frac{a}{c} (2 - 3\rho + 2\rho^2) \right\}$$

$$+ \frac{1}{4\lambda^2\rho^2} \left\{ 8\rho - 8\rho^2 + 3\rho^3 - \log \frac{a}{c} (8 - 8\rho + 3\rho^2 + 2\rho^3) \right\}$$

where $\rho = \log_e ha$.

Finally, for a pair of copper wires, with high frequency, and with the limitations of (34),

$$R = \frac{4n}{\lambda} \left(1 + \frac{1}{\lambda} + \frac{3}{4\lambda^2} \right) - \frac{4a^2n}{\lambda c^2 \rho^2} \left\{ \rho + (2\rho - 1) \log \frac{a}{c} \right\}$$

$$+ \frac{4a^2n}{\lambda^2 c^2 \rho^3} \left\{ \rho^2 + (2 - 3\rho + 2\rho^2) \log \frac{a}{c} \right\}$$

$$- \frac{a^2n}{\lambda^3 c^2 \rho^4} \left\{ 8\rho - 8\rho^2 + 3\rho^3 - (8 - 8\rho + 3\rho^2 + 2\rho^3) \log \frac{a}{c} \right\}. \quad (46)$$

Iron Wires with High Frequency.

When μ is large, we may write from (45),

$$\frac{AF+BE}{P} = \frac{\mu^3}{x^3} \left(2\beta - \alpha\gamma \log hc - 2\beta^2 \frac{x}{\mu} \log \frac{ha^2}{c} \right), \quad (47)$$

and by (35),

$$\frac{P\mu^4\alpha}{x^4CD} = 1 + \frac{2}{\mu} \left(1 + \beta\rho x - \gamma \frac{x}{\alpha} \right).$$

so that

$$\begin{aligned} \frac{AF+BE}{CD} &= \left(\frac{2\beta}{\alpha} - \gamma \log hc \right) \\ &\quad - \frac{2x}{\alpha\mu^2} \left\{ \left(1 + \beta\rho x - \gamma \frac{x}{\alpha} \right) (2\beta - \alpha\gamma \log hc) + \beta^2 x \log \frac{ha^2}{c} \right\}. \end{aligned}$$

Neglecting x/μ^2 as before, and writing the first approximations to $(\alpha\beta\gamma)$ in the second term, and the second in the first, we finally obtain

$$\frac{AF+BE}{CD} = \frac{\lambda}{2\mu} \left(2 - \rho + \log \frac{a}{c} \right) - \frac{\lambda^2}{2\mu^2} \left(4\rho - 2 - \rho^2 + \rho \log \frac{a}{c} \right),$$

the term in $\lambda^0\mu^{-1}$ vanishing identically.

Thus the effective resistance becomes

$$\begin{aligned} R &= \frac{4n\mu}{\lambda} \left(1 + \frac{1}{\lambda} + \frac{3}{4\lambda^2} \right) - \frac{2a^2n\lambda}{c^2\mu} \left(2 - \rho + \log \frac{a}{c} \right) \\ &\quad + \frac{2a^2n\lambda^2}{c^2\mu^2} \left(4\rho - 2 - \rho^2 + \rho \log \frac{a}{c} \right). \quad (48) \end{aligned}$$

This result is much more accurate than the corresponding formula (37) for inductance, owing to the presence of μ in the numerator of the first term. Moreover, λ here denotes the previous λ multiplied by $\mu^{\frac{1}{2}}$. It is applicable to all iron circuits of importance in practice, for a frequency greater than about a hundred with a radius of 1 millimetre.

Iron Wires with Low Frequency.

This result has little interest, but may be obtained at once. For $(\alpha\beta\gamma)$ are all comparable with unity in this case, and x is not greater than 2. We therefore ignore x^2/μ^2 , and obtain

$$\frac{AF+BE}{CD} = \frac{x}{\mu} \left(\frac{2\beta}{\alpha} - \gamma \log hc \right). \quad (49)$$

This term also may be ignored unless a very high order of

accuracy be desired, which can rarely happen owing to the uncertainty as to the value of μ . But if it be retained, the resistance becomes

$$R = \frac{2n\mu}{z^2} \left(1 - \frac{1}{12} z^4 - \frac{1}{180} z^8 \right) - \frac{8na^2 z^2}{\mu c^2} \left(1 - \frac{1}{24} z^4 + \frac{13}{4320} z^8 \right) \\ + \frac{8na^2 z^2}{\mu c^2} \left(\rho - \log \frac{a}{c} \right) \left(1 - \frac{1}{3} z^4 + \frac{19}{120} z^8 \right). \quad (50)$$

where z has the value in (39), and the brackets may be shortened except when z is nearly unity.

When the frequency is smaller, this makes $R = 2\sigma/\pi a^2$, the value appropriate to steady currents.

Copper Wires with Low Frequency.

The formula suited to this case is, writing $\beta^2 + \gamma^2 = \alpha$, if $(z \alpha \beta \gamma)$ are defined as in (39), and under the same limitations,

$$R = \frac{2n}{z} \cdot \frac{\gamma}{\alpha} - \frac{4na^2}{zc^2} (\beta - 2\alpha\beta z + 2\gamma\rho z^2)/(1 - 4\beta\rho z + 4\alpha\rho^2 z^2) \\ - \frac{8na^2}{c^2} \log \frac{a}{c} \left(\gamma^2 - \beta^2 - \gamma z + 2\alpha\beta\rho z \right)/(1 - 4\beta\rho z + 4\alpha\rho^2 z^2), \quad (51)$$

again reducing to $2\sigma/\pi a^2$ for steady currents.

Trinity College, Cambridge,
April 21, 1909.

L. *On Vaschy's or Pirani's Method of comparing the Self-Inductance of a Coil with the Capacity of a Condenser.*
By CHARLES H. LEES, D.Sc., F.R.S., Professor of Physics
in the East London College, University of London*.

IN a recent number of *L'Eclairage Electrique*, M. O. de A. Silva † examined in detail the validity of Vaschy's or Pirani's method of comparing the self-inductance of a coil with the capacity of a condenser for the case in which the

* Communicated by the Author.

† O. de A. Silva, *L'Eclairage Electrique*, 50. p. 113 (1907).

discharge of the condenser was non-oscillatory. In the June number of this Magazine Mr. E. C. Snow* completed the examination by dealing with the oscillatory case.

In treating the problem Messrs. Silva and Snow write down the Kirchhoff equations for the currents in each branch of the resistance-bridge, and from them deduce a differential equation of the third order for the current through the galvanometer. This equation they solve and find the quantity of electricity discharged through the galvanometer by integrating the expression for the current as a function of the time between the limits 0 and ∞ . On equating this quantity to zero, there results the well-known equation for the method $L = Kr^2$. The investigations are therefore very detailed, and if the object were to determine, for example, the time which must elapse before the quantity of electricity which has passed through the galvanometer amounts to say .999 of its final value (a question which might arise in connexion with the condition that the discharge must have passed through the galvanometer before its moving part has moved appreciably) such detail would be unavoidable. But if we start, as do Messrs. Silva and Snow, with the assumptions that the galvanometer satisfies the above condition, and that the needle starts from a symmetrical position†, the investigation may be simplified considerably.

The method of using the Electro-Kinetic Energy and Rayleigh's Dissipation Function‡ for the treatment of problems of this kind, first introduced by Maxwell§, and extended by Fleming|| and Niven¶, has proved so powerful, and it is so much in keeping with modern dynamical methods** that a brief statement of it may not be out of place here.

If a network of conductors consist of branches having resistances R_1, R_2, R_3 , &c., self-inductances L_1, L_2, L_3 , &c., mutual inductances M_{12}, M_{23}, M_{34} , &c., and capacities K_1, K_2, K_3 , &c., and if the quantities of electricity which have flowed through the various branches up to a given time t are

* E. C. Snow, *Phil. Mag.* vol. xvii. p. 849 (1909).

† See Russell, *Phil. Mag.* vol. xii. p. 202 (1906).

‡ Lord Rayleigh, *Proc. Lond. Math. Soc.* iv. p. 357 (1873), and *Scientific Papers*, i. p. 176.

§ Clerk Maxwell, 'Electricity and Magnetism,' 2nd edit. vol. ii. p. 365 (1881).

|| J. A. Fleming, *Phil. Mag.* vol. xx. p. 242 (1885).

¶ J. C. Niven, *Phil. Mag.* vol. xxiv. p. 225 (1887).

** See E. T. Whittaker, 'Analytical Dynamics,' pp. 226, 228 (1904).

x_1, x_2, x_3 , &c., respectively, then if we write down the Electrokinetic Energy

$$T = \Sigma \frac{1}{2} L_n \dot{x}_n^2 + \Sigma M_n M_{nn'} \dot{x}_n \dot{x}_{n'}, \quad . \quad . \quad . \quad (1)$$

the Dissipation Function

$$D = \Sigma \frac{1}{2} R_n \dot{x}_n^2, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and the Electrostatic Energy

$$V = \Sigma \frac{1}{2} \frac{x_n^2}{K_n}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

the equations for the flow of electricity through the various branches of the network may be written in the form :

$$\frac{\partial}{\partial t} \left(\frac{\partial T}{\partial \dot{x}_n} \right) + \frac{\partial D}{\partial \dot{x}_n} + \frac{\partial V}{\partial x_n} = 0. \quad . \quad . \quad . \quad (4)$$

In this equation there is no reference to electromotive forces due to cells or other causes present in the system. To extend the method to cover such cases, we make use of a device well known to readers of Heaviside*, that is we consider a constant electromotive force E as due to the presence of a condenser of large capacity K possessing an initial charge $X = EK$. The Electrostatic Energy of such a condenser when it has given up a finite quantity of electricity x is equal to $\frac{1}{2}(X-x)^2/K$, *i. e.* to $\frac{1}{2}KE^2 - Ex$ since x/X is small. As we are only concerned with changes of Energy, the term contributed to the Electrostatic Energy by the cell reduces to $-Ex$. The extended form of the Electrostatic Energy becomes therefore

$$V = \Sigma \frac{1}{2} x_n^2 / K_n - \Sigma E_n x_n. \quad . \quad . \quad . \quad (3')$$

Although in stating these propositions it has been convenient to take a simple symbol for the quantity of electricity which has flowed through each branch of the network, it is more convenient in applying the method to the solution of a problem to follow Maxwell's plan of assigning a simple symbol to the quantity which has flowed round a mesh of the network. Kirchhoff's first law for the distribution of currents in networks, *i. e.* that the currents leaving a node have a sum equal to zero, is then fulfilled automatically.

The following figure gives the arrangement of the circuit

* See for example, O. Heaviside, *Electrical Papers*, ii. p. 216 (1892).

Rearranging equations (8), (9), and (10) we have

$$(B + P + Q)\ddot{x} - (P + Q)\dot{y} - P\dot{z} = E, \quad . \quad . \quad . \quad (8')$$

$$-(P + Q)\dot{x} + (P + Q + R + S)\dot{y} + (P + R)\dot{z} = r\dot{u} - L\ddot{y}, \quad . \quad (9')$$

$$-P\dot{x} + (P + R)\dot{y} + (R + G)\dot{z} = -\lambda\ddot{z}. \quad . \quad . \quad (10')$$

In the steady state when $\dot{u}$, $\ddot{y}$, and $\ddot{z}$ are each $=0$, the current $\dot{z}$ will be $=0$ if the minor of E in the determinant for z is zero, that is if $P/Q = R/S$.

Integrating the above equations with respect to the time between the limit 0, at which the currents and quantities are zero, and the time t_1 , at which the currents are steady and the quantities of electricity which have passed have attained the values x_1 , y_1 , z_1 , we have :—

$$(B + P + Q)x_1 - (P + Q)y_1 - Pz_1 = \int_0^{t_1} E dt, \quad . \quad (12)$$

$$-(P + Q)x_1 + (P + Q + R + S)y_1 + (P + R)z_1 = (Kr^2 - L)\dot{y}_1, \quad (13)$$

$$-Px_1 + (P + R)y_1 + (R + G)z_1 = 0. \quad . \quad . \quad (14)$$

From the second of which the relation $u_1 = Kr\dot{y}_1$ given by (11) has been used to eliminate u_1 .

From these equations it is seen by inspection that $z_1 = 0$ if $Kr^2 - L = 0$, and the minor of $\int_0^{t_1} E dt$ in the determinant for z_1 is zero, that is if $P/Q = R/S$, *i. e.* the condition for a steady balance previously obtained.

Since the only conditions assumed to hold in the above proof are that the current in each mesh of the network is initially zero and finally steady, the question whether oscillations take place in the interval or not, does not influence the result.

Whether t_1 can be so chosen that the currents throughout the network have become sufficiently steady, without the condition that there has been no motion of the galvanometer-needle or coil during that time being violated, is quite another question. With a modern ballistic galvanometer of the type recently constructed by Prof. B. O. Peirce*, of Harvard, having a period of 10 minutes, there will be very few cases in which there is any doubt that both conditions are satisfied.

It is well to remember that even then, the needle or coil should start from a symmetrical position if the absence of reflexion is to be taken as a proof that the time integral of the current throughout the instrument is zero †.

* B. O. Peirce, Proc. Amer. Acad. xlv. p. 283 (1909).

† A. Russell, Phil. Mag. xii. p. 202 (1906).

LI. *Proceedings of Learned Societies.*

GEOLOGICAL SOCIETY.

[Continued from vol. xvii. p. 772.]

February 10th, 1909.—Prof. W. J. Sollas, LL.D., Sc.D., F.R.S.,
President, in the Chair.

THE following communications were read:—

1. 'Note on some Geological Features observable at the Carpalla China-Clay Pit in the Parish of St. Stephen's (Cornwall).' By Joseph Henry Collins, F.G.S.

An east-and-west fault traverses this pit near its southern wall, with a downthrow to the south of more than 50 feet. North of the fault there is china-clay rock or 'carclazyte,' at one point underlying granite not sufficiently altered to yield china-clay, and sometimes containing embedded lenticles or irregular masses of partly-kaolinized granite. The carclazyte is often traversed by veins of secondary quartz, in most instances associated with schorl. It also contains lepidotite, gilbertite, topaz, fluor, and schorl. South of the fault there is nearly horizontal tourmaline-schist, at one point 50 feet thick, and thinning off southwards and eastwards. This, like the rock of the north side, is overlain by subsoil or 'growan,' covered in turn by soil or 'meat-earth.' Underlying the schist there occurs also china-clay rock to a distance of many fathoms from the fault. This occurrence of china-clay under a thick schistose overburden is unique in Cornwall, although the other features of the pit are reproduced elsewhere. The author considers that this example is strongly in favour of the pneumatolytic origin of carclazyte, the gases producing the change being possibly in part carbonic acid, but probably to a more important degree chlorine, fluorine, and boron.

2. 'Some Recent Observations on the Brighton Cliff-Formation.' By Edward Alfred Martin, F.G.S.

The author records in his paper certain features presented by the face of the cliffs between successive falls at Black Rock, Brighton, during the past eighteen years. As the cliffs have worn back, the base-platform of Chalk grows in height, and the layer of sand which Prestwich found above the Chalk grew thinner and thinner until finally it completely disappeared. At the same time, the

Phil. Mag. S. 6. Vol. 18. No. 105. Sept. 1909. 2 G

raised beach has grown in thickness from $1\frac{1}{2}$ to 12 feet. In 1890 there were six feet of sand, with a foot and a half of beach above it. There was practically no protection at this date in the shape of groynes. In 1892 the sand had decreased to between 3 and 4 feet, but the beach remained as in 1890. Many falls of cliff took place between 1892 and 1895, and at the latter date the beach had increased to between 4 and 5 feet. The eastern limit of the beds had become more clearly defined, the trough in the Chalk in which they had been defined taking an upward direction about 300 yards east of the Abergavenny Inn. Many blocks of red sandstone had become dislodged, and were lying on the modern beach. In 1897, 10 feet of chalk formed the lower portion of the cliff, with 8 feet of raised beach above it in places, but there was a mere trace of sand left. The rubble-drift above was seen to be distinctly stratified. Many masses of red sandstone had fallen out of the cliff, the largest measuring 5 feet in its greatest dimension. In 1899 the raised beach had reached a thickness of 10 feet. Great masses of moved and reconstructed chalk were observed on the eastern boundary embedded in the beach. Two rounded lumps of granite were extracted from the beach. In 1903, the beach was but a little over 8 feet thick in the exposed parts, but the platform of Chalk was 14 feet thick. The upper portions of the beach, which were the least consolidated, had fallen away in such a manner as to leave cave-like gaps beneath the rubble. The number of red sandstone blocks which lay on the modern beach was remarkable, forty such blocks being counted in a space of 50 yards square. In 1906, the raised beach had increased from 15 to 20 feet: farther west, however, the thickness was not so great. In 1908, there were 17 feet of Chalk, 12 feet of beach. It is noteworthy that as the degradation of the cliff proceeds, the material is rapidly carried away by the sea. No talus remains for any length of time, and if the material is to be prevented from disappearing into deep water, some such contrivance as chain-cable groynes seems to be demanded, fixed somewhere between low and high tide-marks. The only organic remains observed in the cliffs were some fragments of shells, found at the top of the raised beach.

February 24th, 1909.—Prof. W. J. Sollas, LL.D., Sc.D., F.R.S.,
President, in the Chair.

The following communications were read :—

1. 'Palæolithic Implements, etc., from Hackpen Hill, Winterbourne Bassett, and Knowle-Farm Pit (Wiltshire).' By the Rev. Henry George Ommanney Kendall, M.A.

2. 'On the Karroo System in Northern Rhodesia, and its relation to the General Geology.' By Arthur John Charles Molyneux, F.G.S.

In 1903 (Quart. Journ. Geol. Soc. vol. lix.) the author described the occurrence of deposits, that have since been recognized as of Karroo age, in Southern Rhodesia. The present communication traces their extension across the Zambesi, where their boundary follows the foot of the remarkable line of escarpments that divide the plateau, nearly 4000 feet in altitude, from the low-lying (1500 feet) regions of the Zambesi Valley. Karroo deposits also form the floor of the peculiar trench-like valleys of the Luangwa, Lukasashi, and Lusenfwa (or Luano), the walls of which are similarly steep, and of metamorphic rock-gneiss, schist, and granite.

The Luano Valley is described. Its northern precipice is known as the Machinga (native meaning 'a fence'), and the Lusenfwa and Molongushi rivers are followed in their mature courses across the flat plateau-plains. They reach the Luano by waterfalls into deeply incised gorges, cutting back 15 and 8 miles respectively into the plateau. Rivers that join the Luano from the south, on the contrary, descend into open valleys of Karroo floors, that are divided one from the other by tongues from the southern highlands, and suggest 'rolled-out' folds.

Between the Kafue junction and Feira, the Zambesi River also occupies a trough-valley, lined by steep escarpments; that on the south side rises 2000 feet in less than a mile, being formed of a flexure of altered sediments. The Danda flats show Karroo beds.

The Lufua River runs parallel with the strike of the gneiss of the locality, and crosses two synclinal basins of elastic deposits, separated by Archæan ridges. The Losito has also a deep strike-channel.

The Karroo deposits are grouped into basal conglomerates, coal-measures, Upper Matobola Beds, and Escarpment Series. No effusive basalts were seen, but there is an area of Forest Sandstones near the Losito-Zambesi confluence. In the Luano Valley, the conglomerates are made up of resisting quartz-quartzite boulders and pebbles—all having dimpled or concave depressions on one or more sides; they possess no orientation, are unsorted, and exhibit a varying matrix. Though they form the base of the Karroo System there is no certain evidence of glaciation, but the beds seem to have originated as scree-deposits on an uneven floor. The grinding of the pebbles into one another is accounted for by the soft nature of the schists and limestones, which would have been removed in the internal movement before consolidation.

In the Lukasashi and the Luano there is a general dip of the strata north-westwards, that is, towards the escarpment, and evidence of minor anticlinal and synclinal folds along east-north-east axes. By a combination of these the Karroo deposits become

lowered from plateau-level on the south, towards the north-west culminating in a great downthrow fault along the foot of the Machinga.

Nowhere on the plateau in the immediate vicinity of the valley-walls have Karroo beds been found, and if they did once extend there, it is remarkable that they should have disappeared. But it is certain that the valleys were at one time filled almost to plateau-level, as the rivers pass through Archæan inliers by deep clefts, and must thus have laid out their courses before such hard masses rose from the Karroo beds by erosion of the latter. Also the comparatively late times in which the Machinga escarpment was laid bare, and the rejuvenation of the Lusenfwa River, etc., suggest a complete filling of the valleys.

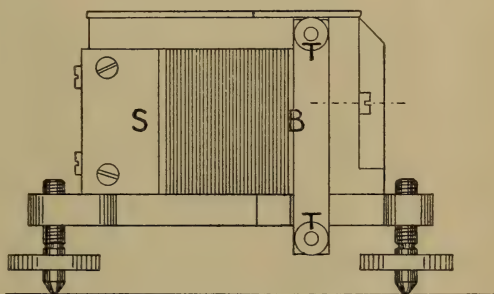
It is thus possible that the Karroo beds extended over a part of the plateau, and were included in the folding and faulting movements already mentioned. Subsequently, by some continuous agency the whole surface was planed off to a plateau of remarkable monotony; and on a further radical change of conditions taking place, erosion of the softer Karroo strata set in by which the present valleys are again reaching a plane of denudation.

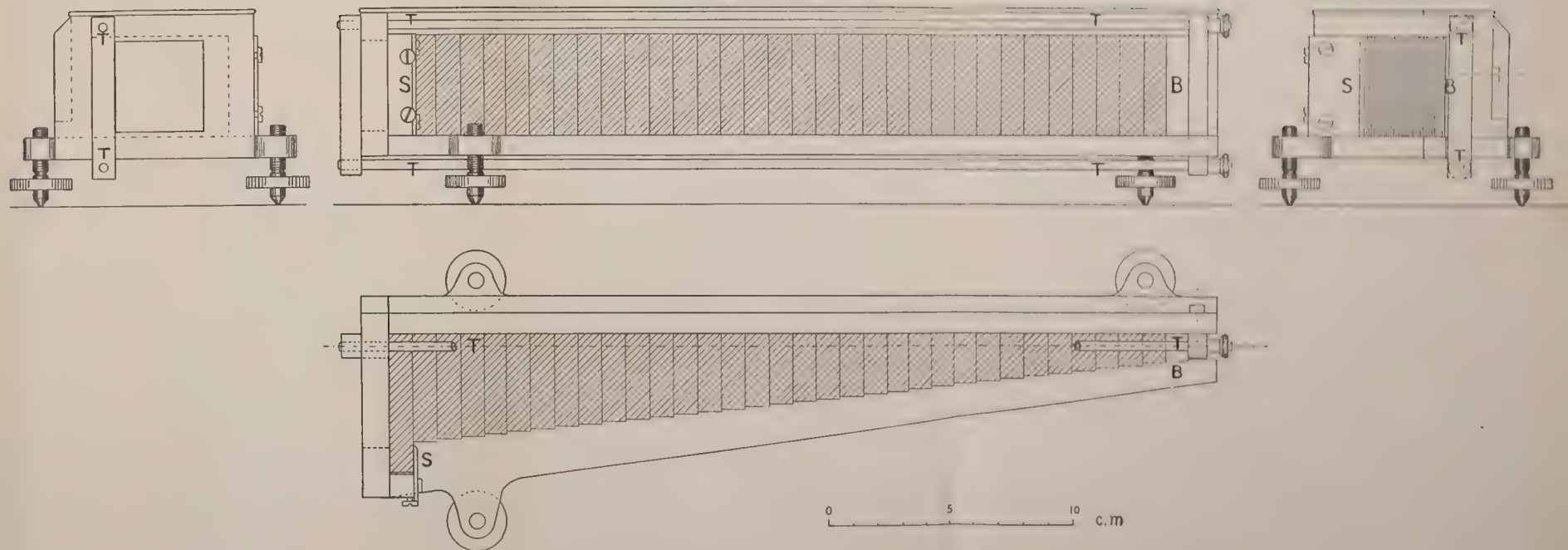
The facility with which atmospheric waters and acids attack the sediments is notable, and decomposition extends to 100 feet in depth over the flat regions of the plains.

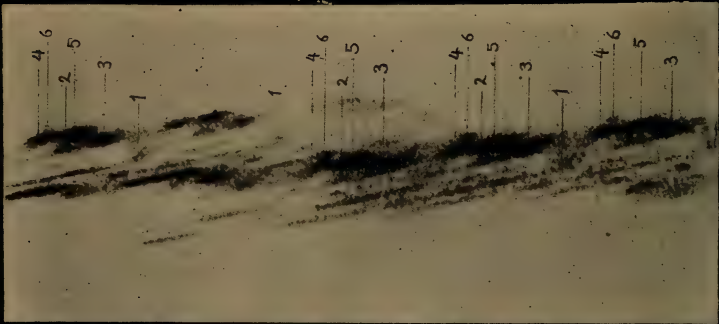
The author suggests that the trough-valleys of elastic rocks merely follow the axis of pre-Karoo and post-Karoo movements—trending in three directions. The Luano and part of the Zambesi course agree with that of the folds and cleavage of the complex, certain ranges of hills, and the Machinga Fault (east-north-east); a second (south-easterly) trend is that of the Kafue, Losito, Inyanga range, and Lufua, and the folds and cleavage of the complex in these regions; while the third follows the dominant direction of the great tectonic movements of South Africa (north-east). Mr. L. A. Wallace has noted that the Luangwa and mid-Zambesi are on this strike, and Mr. G. W. Lamplugh has suggested a northerly extension of his Deka Fault. A distance of 800 miles thus displays movements that commenced in pre-Karoo periods, and have repeated themselves since the Karroo time.

Fossils from the areas described support the previous allocation of the deposits to the Permo-Carboniferous, and to the Karroo System of South Africa. Notable specimens are members of the *Glossopteris*-flora, including pith-casts of *Schizoneura*; carapaces of *Estheria*; ostracods; and fragments of bone, fish-scales, and teeth.

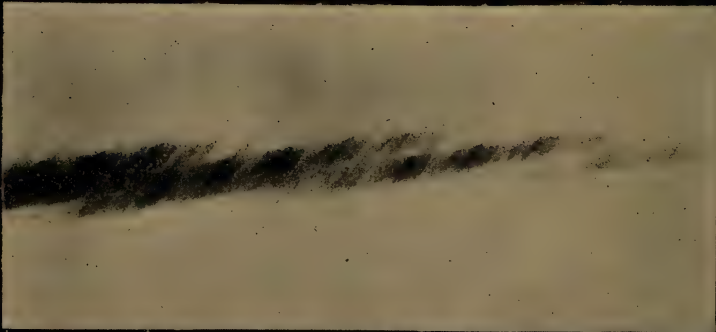
Palæolithic stone-implements (axe-heads) were found at separate localities on the surface, about the latitude of $14^{\circ} 50' S$.



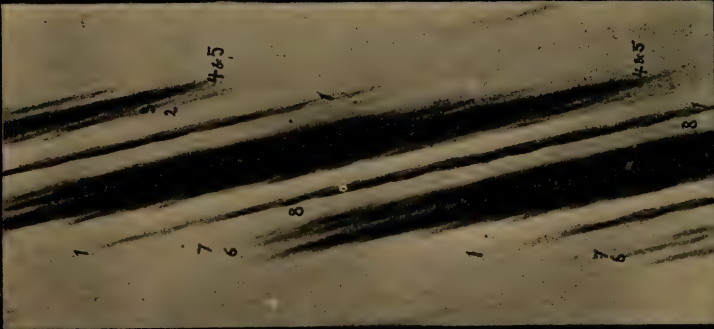




3.



2.



1.

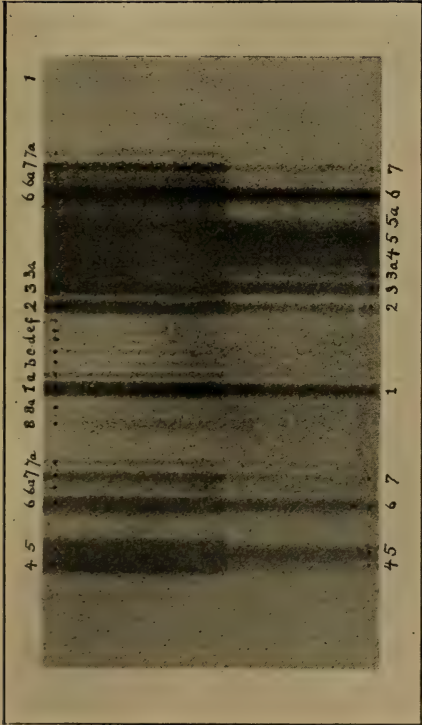


Diagram of Spectrum

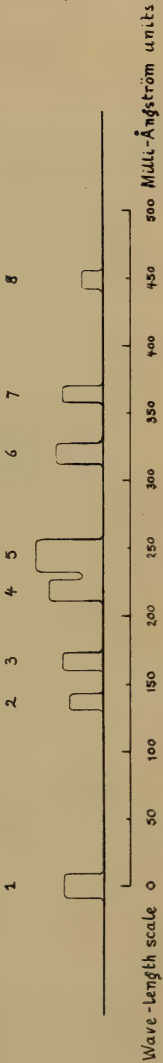


FIG. 1.



FIG. 2.



FIG. 3.

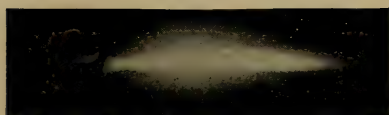


FIG. 4.



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THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[SIXTH SERIES.]

OCTOBER 1909.

LII. *On Striations in the Electric Discharge.*

By Sir J. J. THOMSON, M.A., F.R.S.*

ONE of the most conspicuous features of the electric discharge through gases, when the pressure is within certain limits, is the exceedingly well-marked alternations of light and darkness which occur in the positive column. These alternations, which are called striations, are so varied and beautiful that since their discovery by Abria in 1843 they have attracted the attention of many physicists. Grove, Gassiot, Spottiswoode and Moulton, De la Rue and Müller, Crookes, Wood, Skinner, H. A. Wilson, and Willows have published important researches on the conditions under which the striations are produced; on the influence upon them of such things as the nature and pressure of the gas, the size of the tube, the current passing through it; and on the distribution of the electric force in the neighbourhood of a striation. The investigations described in the following paper relate for the most part to the last of these questions, and were made with the object of testing a theory of the striations which I gave in my Treatise on the Conduction of Electricity through Gases. For these experiments I used tubes fitted with Wehnelt cathodes, *i. e.* the cathode was a

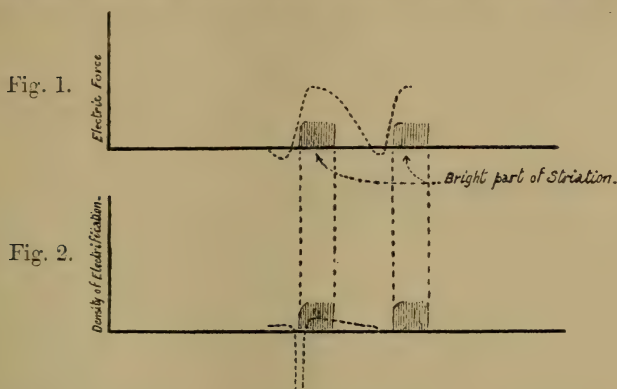
* Communicated by the Author. A Discourse given at the Royal Institution on Friday evening, April 2, 1909.

strip of platinum-foil heated to redness, and having on it a spot of lime or barium oxide*.

With these cathodes large currents can be sent through the tube, and remarkably bright and steady striations obtained at lower pressures and with smaller potential-differences than with the ordinary type of discharge. The pressure has, however, to be low, considerably less than 1 mm. of mercury, to get the full advantages from these cathodes. The first point to which attention was directed was the distribution of electric force along the line of the discharge. Investigations on this point have already been made by Skinner and H. A. Wilson, but it seemed to me that the steadiness of the striations with the Wehnelt cathode made this method of investigation particularly suited for these investigations. The first method used to measure the variations in the electric force along the discharge was to find the variation in the difference of potential between two platinum wires 1 mm. apart as the wires were moved from the cathode to the anode. Several devices were used for this purpose: in some the platinum wires (surrounded up to about a millimetre from their tips with glass rods) were carried on a sort of railroad and moved from cathode to anode. The electrodes in the discharge-tube in this case were fixed. The measurements of the potential-differences made by this method at low pressures gave the very remarkable result that just on the cathode side of the bright part of a striation the electric force was negative (*i. e.* that the force on a positive charge was in the direction from *cathode to anode*): on crossing over the bright boundary to the anode side the electric force at once became positive, and rose to a high value. It soon, however, began to diminish, and went on diminishing up to the cathode side of the bright part of the next striation on the anode side. The distribution of the electric force in the striation is represented in fig. 1, and the corresponding distribution of positive and negative electricity in fig. 2, the ordinates representing the distribution of the electrification both as to magnitude and sign. Thus if these measurements of the electric field can be relied on we have intense negative electrification at the bright head of a striation (by head is meant the side next the cathode) and a weak positive electrification through the rest of the field. The

* My assistant, Mr. Everett, has found that these cathodes can be very easily made by letting a drop of sealing-wax fall on the foil, and then burning away the combustible matter by heating the foil to incandescence. Sealing-wax seems to contain large quantities of some salt of barium.

transition from positive to negative force was very abrupt and well marked, so much so indeed that the position of the

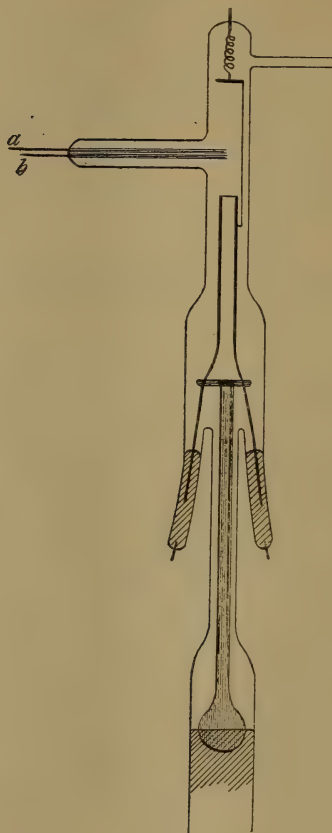


platinum wires in the striation could be ascertained with great accuracy without looking at the discharge, by observing the deflexions of the electrometer by which the potential-difference between the wires was measured.

Many changes were made in the way in which the wires used as detectors were arranged: thus, to prevent any screening of the one wire by the other, an apparatus was used in which the two platinum wires were brought in from opposite sides of the tube, so that there should be no overlapping; exactly the same results were obtained with this apparatus as with the other. Again, another arrangement, similar to one previously used by Professor H. A. Wilson, was tried, in which the exploring electrodes were kept fixed, and the anode and cathode kept at fixed distance apart were, by means of a float, moved relatively to the exploring electrodes *a, b*, so that these could occupy all positions from the anode to the cathode. The arrangement is represented in fig. 3 (p. 444). The exploring electrodes in some of the experiments protruded about a millimetre beyond the ends of the glass tubes into which they were sealed; in other experiments very fine hollow glass tubes were used to cover the wires, and the wires instead of protruding beyond the glass stopped short at about a millimetre from the end of the tube; this arrangement was adopted with the idea of protecting the wires against streams of corpuscles coming down the tube: these by giving up their charges to the wire might cause this to acquire potentials different from those of the gas at the tips of the wire. The results obtained with all these modifications were exactly the same as those obtained with the first type of apparatus, *i. e.* there was always when the pressure of the gas was low a

negative electric force just in front, *i. e.* on the cathode side of the bright part of a striation; this changed to a large

Fig. 3.



positive force as soon as the bright boundary of the striation was passed; at a short distance from the front of the striation this force began to diminish and went on diminishing until the front of the next striation on the anode side was reached.

Though the indications of all these wire explorers agreed in pointing to the existence of a negative force in front of these striations, yet I felt that the existence of a negative force could never be proved by the use of wire detectors.

For let us consider what the existence of a negative force implies. The electric current is always in the same direction throughout the tube, and therefore the average

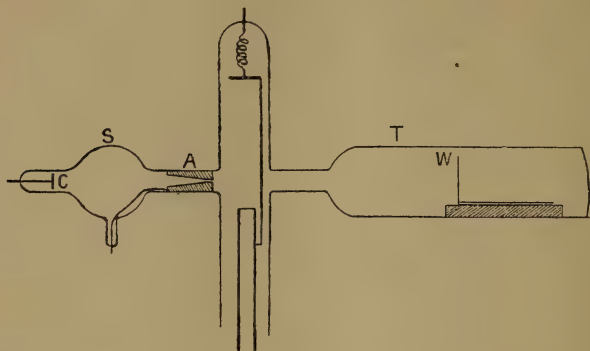
movement of the ions is in the same direction at all parts of the tube; thus, whenever the electric force is negative, there must be ions moving against the electric force instead of with it. Now the validity of the method of the wire electrodes depends upon the assumption that the ions in the neighbourhood of the tip of these electrodes follow the lines of force, that if, for example, the tip were at a higher potential than the gas so that the force on a positive ion were away from the tip, negative ions would follow the direction of the force acting upon them, and run into the tip and lower its potential until it became the same as that of the gas in its neighbourhood. But if the ions do not follow the electric force, and the existence of a negative force implies that some of them at any rate do not, we have no right to assume that the potential of the wire is the same as that of the gas. In some simple cases it is evident that it would not be so. Thus suppose the wire were exposed to a stream of cathode rays, and that there were no positive ions in its neighbourhood, then it is evident that the wire would acquire the potential of the cathode from which the rays started and not that of the gas around the wire.

For these reasons I felt that the existence of a negative force could not be established by means of wire electrodes, and I adopted an entirely different method of measuring the electric force along the discharge-tube. The principle of this method is as follows: imagine a very fine pencil of cathode rays, travelling at right angles to the line joining the cathode and anode, to pass through the discharge-tube. As it crosses the discharge at any place it will be acted upon by the electric force at the point of the discharge, and will be deflected by an amount proportional to the electric force. The deflexion will be from the cathode of the discharge-tube if the force is positive, towards it if the force is negative. If very small pencils of cathode rays are used the disturbance of the electric field in the discharge-tube due to the negative charge on the rays is quite insignificant, and there is none of that distortion of the striations which, to a greater or less extent, always occurs when exploring metallic electrodes are used.

The arrangement by which this principle is carried out in practice is shown in fig. 2. The cathode and anode are fastened together by a piece of glass-rod and fastened to a float, floating on the top of a mercury column. By raising or lowering the column the anode and cathode can be moved up and down the discharge-tube. This arrangement is the same as that used with the wire detectors and shown in fig. 3.

The wires *a*, *b* (fig. 3) were replaced by cathode rays generated in the side tube S (fig. 4) by a small induction-coil:

Fig. 4.



the cathode C is at the end of the tube, the anode is the metal plug A connected with the earth; a very fine hole was bored in this plug and through it a pencil of rays passed across the discharge and then entered the side tube T. In this tube there was a screen W, covered by a phosphorescent substance, in some cases willemite; in others the screen was a zinc sulphide one procured from Mr. Glew. The place of impact of the cathode rays against the screen is marked by a luminous spot, and by measuring with a cathetometer the deflexion of this spot the magnitude and direction of the electric force acting on the cathode rays as they pass across the discharge-tube can be determined. Tinfoil was wrapped round the outside of the discharge-tube to neutralize the effect of electric charges on the glass walls of the tube. The use of cathode rays not only avoids the disturbance due to the presence of the wires, but inasmuch as the cathode rays are negatively electrified particles it enables us to measure the effect of the field on such particles, and as it is the corpuscles which carry practically all the current in the discharge, the method enables us to observe in a very direct way the most important factor in the discharge.

The method is, however, limited to the case where the pressure in the discharge-tube is low, as it is only at low pressures that the cathode rays produce a well-defined spot on the screen.

Observations made with this method showed unmistakably the existence of negative forces in certain parts of the discharge, and in fact the distribution of electric force along

the tube as determined by this method agreed remarkably well with that determined by the method of the exploring wire. When the discharge was striated it was generally found that where the cathode rays passed underneath a striation, *i. e.* on the cathode side of the bright part of the striation, there was a small deflexion of the cathode rays towards the cathode of the discharge-tube, showing that in this part of the discharge the electric force is negative, while when the path of the cathode rays passed through the bright part of a striation there was a large deflexion of the cathode rays from the cathode of the discharge-tube, showing that in this part of the discharge the electric force was strongly positive. The change from the small negative deflexion to the strong positive one was exceedingly abrupt, so much so that when the anode and cathode were moving downwards, owing to the sinking of the float supporting them, and one striation after another was thus being brought across the path of the cathode rays, the phosphorescent spot moved as abruptly as if it had been struck by a blow when the bright head of a striation crossed its path. At the low pressures at which these observations are made the potential-difference between the electrodes when the current is large enough to produce striations is exceedingly small, often not exceeding 60 or 70 volts. Under these circumstances the negative forces although unmistakable are small: when, however, the current through the tube is reduced until the discharge is no longer striated, the potential-difference between the electrodes is very much increased, and now large negative forces can be observed in the neighbourhood of the anode. Sometimes the region in which the force is negative extends a considerable distance from the anode: in one case I observed a negative force for two-thirds of the distance from the anode to the cathode.

As the corpuscles in the cathode rays have an exceedingly small mass they are able to follow very rapid variations in the electric field; by means of them we can observe the gradual establishment of the steady state of the discharge and the change in the direction of the electric force at certain places from positive to negative. Thus suppose the steady current through the tube is small and the potential-difference is considerable, and that the pencil of cathode rays is passing through the discharge near the anode, then if we watch the behaviour of the phosphorescent spot in the interval immediately following the application of the potential-difference to the tube, we shall find that when the current first starts through the tube the spot is repelled from the cathode,

showing that at this stage the electric force is positive throughout the tube. This repulsion of the cathode rays is however only momentary: the spot jumps back, and after a very short interval the spot is *attracted* towards the cathode, showing that the force in this region is now negative. Thus during this interval the ions in the gas and those clinging to the walls of the tube have rearranged themselves in such a way as to reverse the force in the field. This momentary deflexion is much more perceptible near the anode than some distance away from it; the rearrangement seems to spread from the cathode, and to be established so rapidly close to that electrode that there is no time to observe it, while as we travel away from the cathode the steady state is reached after longer and longer intervals, and there is time to observe the initial distribution of the electric field.

We see that the result of the experiments with the cathode rays is to confirm the indications of the wire detector, even when the main current is travelling against the electric force. That the wires in this case should indicate the potential is very remarkable, and must be due I think to the presence in the discharge of slowly moving ions in addition to the swiftly moving ones which carry the main portion of the current, having acquired in other parts of the field sufficient impetus to carry them for some distance against an opposing electric force. The slowly moving ions would be produced by the collisions of the quick ones, and those produced near the tips of the wire electrodes would follow the lines of electric force near the wire and equalize the potentials between the wire and the gas.

The great change in the electric force which occurs at the bright fronts of the striations shows that in these regions we have a great accumulation of negative electricity, while the distribution of the electric force in other parts of the striations and in the dark parts between two striations shows that in those regions there is a slight excess of positive electricity. The magnitude of the charges in the electric force is shown by the following numbers which indicate the electric force in volts per centimetre at different parts of the striation. x in the following table is the distance in millimetres from the bright head of the striation: x is taken positive when measured towards the anode, negative towards the cathode, thus $x = -1$ denotes a place 1 millimetre from the bright head of the striation on the cathode side. X is the electric force in volts per centimetre at x . The gas was hydrogen at a low pressure.

x .	X .
— .5	— 9
+ .5	+ 67
+ 1.5	+ 33
+ 3.5	+ 30
+ 7	+ 10
+ 9	— 10

The last reading was at a point just in front of a second striation.

The distance between the bright heads of successive striations was 9 mm. and the thickness of the dark space 2 mm.

From the preceding table we see that in the space of 1 mm. at the head of a striation we have a change in the electric force of 76 volts per cm. By means of the equation

$$\frac{dX}{dx} = 4\pi\rho$$

we see that the density of the negative electricity at the head of the striation is about $\frac{1}{5}$ of an electrostatic unit per c.c. The density of the positive electricity in the other portions is very much less than this. With Wehnelt electrodes there is frequently only a small potential-difference between corresponding points in adjacent striations: in some cases this difference was only 2.7 volts.

The changes in the electric force are much more abrupt at low pressures than at high ones; though there is always a large increase in the force at the bright head of the striation. I have not observed the existence of the negative forces when the pressure was more than a fraction of a millimetre of mercury.

I have found other cases in which the negative forces are even more pronounced than those I have already considered; perhaps the most striking of these is one where the anode and cathode are connected together and with earth by stout metallic connexions, so that the two are at the same potential, and therefore the average negative force between them is as great as the average positive force. The anode is perforated by a very fine hole, and through this hole a stream of Canal rays, *i. e.* positively electrified particles, passes into the tube: this produces when the pressure is suitable a fully developed discharge, with striations, Faraday dark space, a well-developed negative glow and dark space; and in spite of the anode and cathode being at the same potential there is in this case the normal cathode fall of about 300 volts at the cathode: the negative forces in a tube of this kind must be very considerable, as they have to balance the cathode fall.

The heaping up of the negative electricity at the head of the striations seems to me to be the most important factor in the production of striations.

This concentration of the negative electricity at regular intervals along the discharge may be explained as follows. Consider a stream of negative corpuscles projected from the neighbourhood of the cathode with considerable velocity: they will collide against the molecules of the gas, and thereby lose velocity: if the electric field acting on them is not sufficiently intense to restore the velocity lost by the collisions, the corpuscles will lose velocity as they travel through the gas, thus the corpuscles in the rear will gain on those in front, and therefore the density of the corpuscles and therefore of the negative electricity will be greater in the front, and by the equation $\frac{dX}{dx} = 4\pi\rho$, when X is the electric force,

x the distance from the cathode, and ρ the density of the electricity, the electric force will increase rapidly in consequence of this concentration. This increase in the force will increase the velocity of the particles in front. If the increase in velocity is not sufficient to make the corpuscles ionize the gas by collision, the congestion will be relieved by the gradually increasing velocity of the corpuscles in front, and there will be no periodicity either in the density of the electricity or the electric force. If, however, the force increases so that the corpuscles produce ions by collision quite a different state of affairs will occur; suppose that when the corpuscles get to a place P , their velocity is sufficient to produce ionization. On the anode side of P positive and negative ions will be produced, the positive ones will crowd towards P , the negative ones will move away from it; the consequence will be that there will be an excess of positive electrification on the anode side of P : now positive electrification implies a diminution in the electric force as we move towards the anode, thus the electric force will fall. When it has fallen below the value required for ionization the negative electricity will as before begin to accumulate in the front of the stream, and the electric force will again increase to the value required for ionization when the process will be repeated. We shall in this way get a periodicity in the electric force such as is observed in the striated discharge. Thus on this view the concave side of the bright head of the striation acts as a cathode, the corresponding anode being the convex side of the bright head of the adjacent striation on the anode side. Between these two places we have a complete discharge, forming a unit by the combination of which the whole discharge is built up. The ions which carry the current through

any unit are for the most part manufactured in the units themselves, so that these units will behave, as Goldstein and Spottiswoode and Moulton have observed the striations to behave, as if they were to a considerable extent independent of each other. The effect of pressure on the distance between the striations can easily be understood from this point of view, for the lower the pressure the greater will be the distance which particles projected with high velocity will travel before their velocity is destroyed. Again, the variations in the electric field are due to the accumulation of electrical charges in the tube. These accumulations may be regarded as electrified disks whose cross-section is that of the tube; the distance from the disk at which these forces fall to a certain fraction of their maximum value will depend upon the diameter of the disk; the larger the diameter the greater this distance, so that when the diameter of the tube is small the fluctuations in the intensity of the electric force will be much more rapid than when it is large, and thus we should expect the striations to be much nearer together in a narrow tube than in a wide one.

To explain the variations in the luminosity which accompany these fluctuations in the electric field we must consider the variation in the kinetic energy possessed by the positive ions when they recombine. The recombination of ions does not in general seem to be accompanied by luminosity, unless the ions possess a definite amount of kinetic energy. We certainly can have a gas with great electrical conductivity, and in which a large number of ions are recombining without any visible luminosity; it seems as if the ions must have a definite amount of kinetic energy for visible light to be developed on their recombination. Now in the space between two striations the electric field in the part near the cathode side of the bright head of a striation—the dark part—is weak; here the ions have not got the minimum amount of energy requisite for them to be luminous when they recombine; in the bright part of the striation the electric field is strong, and here the ions get sufficient kinetic energy to enable them to give out light when they combine. If the energy required for an ion to give out visible light is greater for light at the blue end of the spectrum than at the red, we might get blue light at one part of the striation, red at another, an effect often observed when we have a mixture of mercury vapour and hydrogen in the tube; a similar separation of the spectra of the two gases in a striation might also be produced if one of the gases were more easily ionized than the other.

I wish to thank my assistant, Mr. Everett, for the assistance he has given me in these investigations.

LIII. *On the Lateral Deflection and Vibration of "Clamped-Directed" Bars.* By JOHN MORROW, M.Sc. (Vict.), D. Eng. (L'pool), Lecturer in Engineering, Armstrong College (University of Durham) *.

Section I.—*Introduction and Contents.*

§ 1. THE vibrations of a bar under the terminal conditions which are probably most frequent in engineering practice, appear to have hitherto attracted but little attention. These conditions occur when one end of the bar is clamped, and the other is constrained to retain its original direction. Such conditions may be realized by having two initially parallel bars, CA and DB (fig. 1), each clamped at one end, with the otherwise free ends connected by a rigid bar AB.

Figs. 1 and 2 show two distinct ways in which such a

Fig. 1.

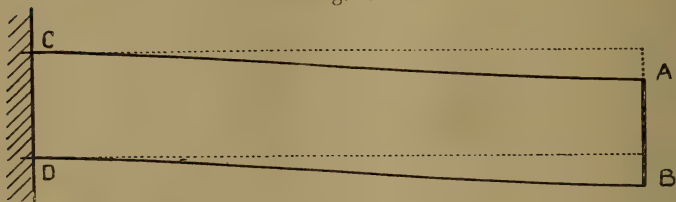
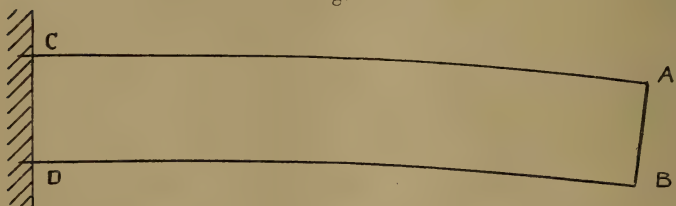


Fig. 2.



system may vibrate. In the former case we have pure lateral vibration, whereas in the latter there is a necessary accompaniment of longitudinal vibrations in each bar. The former case is the more important since in it the frequency of the fundamental type will, in general, be the lower.

I propose to refer to a bar under the end conditions of fig. 1 as a "Clamped-Directed" bar.

Although there exists a considerable literature on the theory of the lateral vibration of thin rods, the practical application of the results is extremely limited. The explanation of this is to be sought, not in any lack of enterprise on the part of

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the technologist, but rather in the fact that the problems that have been solved are not necessarily those of which the solutions are most required in practice.

§ 2. As an important example, in which the terminal conditions here considered occur, one might mention the case of the cylinder of a steam-engine supported on two or more mild steel standards. It will be recognized that the upper ends of the standards must be treated as "directed."

It is worthy of notice that these terminal conditions are mentioned by Lord Rayleigh in his 'Theory of Sound'* but that the directed end is at once dismissed from consideration with the remark that "there are no experimental means by which the contemplated constraint could be realized."

§ 3. *Notation and End Conditions.*—

Let y, y_z = deflexions at points x and z in the length of the bar ;

ω, I = area and moment of inertia of cross-section, the cross-dimensions being supposed small compared with the length ;

l = length of bar ;

ρ = density of the material ;

E = Young's Modulus for the material (when there is an axial pull P in the bar, E stands for $P/\omega + \text{Young's Modulus}$) ;

t = time, measured from any instant at which y is everywhere zero ;

N = number of complete vibrations per second.

The symbol $\ddot{y}$ is written for d^2y/dt^2 ; and if y_1 is the instantaneous deflexion at some particular point in the length of the bar, we have, for simple harmonic vibrations,

$$-\ddot{y}/y = -\ddot{y}_1/y_1 = k^2 \text{ (say), } \quad . \quad . \quad . \quad (1)$$

where

$$N = k/2\pi.$$

The curve assumed by the elastic central line at any instant is given in terms of y_1 . If a is the amplitude at the point in the length at which y_1 is the instantaneous deflexion, the value of y_1 is given by

$$y_1 = a \sin kt. \quad . \quad . \quad . \quad (2)$$

The origin will always be taken at the clamped end. At $x=0$ we have, therefore, $y=dy/dx=0$; whilst at the directed

* See Rayleigh's 'Sound,' vol. i. p. 259, 1894 edition.

end, $x=l$, $dy/dx=0$, and, if there be no concentrated load there, $d^3y/dx^3=0$.

Section II.—Unloaded Massive Bar.

§ 4. In the ordinary case of a bar vibrating under the effect of its own mass only, the form of the elastic central line and the frequency of the lateral vibrations are to be determined from the well-known equation

$$\frac{d^4y}{dx^4} = \mu^4 y,$$

in which

$$\mu^4 = -\frac{\rho\omega}{EI} \frac{\ddot{y}_1}{y_1},$$

and of which the general solution is

$$y = A \sin \mu x + B \cos \mu x + C \sinh \mu x + D \cosh \mu x.$$

The end conditions give four equations whose compatibility requires that

$$\tan \mu l = -\tanh \mu l, \quad . \quad . \quad . \quad (3)$$

of which the least root is

$$\mu l = 2.36502 \dots \dots \dots (4)$$

The frequency of the fundamental is therefore

$$N = \frac{5.5933}{2\pi} \sqrt{\frac{EI}{\rho\omega l^4}}.$$

§ 5. It can be shown that the values of θ satisfying

$$\cos \theta \cosh \theta = 1, \quad . \quad . \quad . \quad . \quad (5)$$

are the same as those which satisfy

$$\tan \frac{\theta}{2} = \pm \tanh \frac{\theta}{2},$$

and hence that the admissible roots of equation (3) are each one half of the alternate roots of (5). Accurate solutions of (5) have been provided by Rayleigh*, and from these the values of μl which satisfy (3) are

$$\mu_1 l = 2.365020$$

$$\mu_2 l = 5.497804$$

$$\mu_3 l = 8.639380,$$

after which

$$\mu_n l = (n - \frac{1}{4})\pi,$$

to more than six decimal places, n being an integer.

* 'Theory of Sound,' Rayleigh, vol. i. p. 278, 1894 edition.

§ 6. The values of μl given above for the lower harmonics may be readily calculated by the method explained below for $\mu_2 l$.

We know that $\mu_2 l = (2 - \frac{1}{4})\pi$ approximately. Hence

$$\tanh \mu_2 l = .999967, \quad . \quad . \quad . \quad . \quad (5a)$$

to the limits of accuracy of the tables at my disposal. By equation (3) therefore

$$\tan \mu_2 l = -.999967 \text{ approximately.}$$

$$\therefore \mu_2 l = 2\pi - .7853815 = 5.4978038,$$

which is more than sufficiently accurate*. Even when this method is used for the fundamental, the error is less than 0.01 per cent.

§ 7. To examine the curve assumed by the centre line of the bar, we obtain a further relation by putting $y = y_1$ when $x = l$; whence it can easily be shown that

$$y = \frac{1}{2}y_1[(\sinh \mu l + \sin \mu l)(\sin \mu x - \sinh \mu x) - (\cosh \mu l - \cos \mu l)(\cos \mu x - \cosh \mu x)] \div (1 - \cos \mu l \cosh \mu l). \quad (6)$$

The values of μl to be used in this equation are those given in § 5.

§ 8. When dealing with harmonics the positions of the nodes are to be found from (6) by putting $y = 0$. We thus get, at each node,

$$\frac{\sin \mu x - \sinh \mu x}{\cos \mu x - \cosh \mu x} = \frac{\cosh \mu l - \cos \mu l}{\sinh \mu l + \sin \mu l}. \quad . \quad . \quad (7)$$

The fundamental is free from nodes. For the first harmonic the value of the right-hand side of (7) is 1.000033. For higher tones it may be taken as unity.

Solving (7) by trial, we find in the case of the first harmonic that the distance of the node from the clamped

* This method depends on the fact that a slight difference between two numbers of the magnitudes involved makes no appreciable difference in their hyperbolic tangents. If h is small

$$\tanh(x+h) - \tanh x = h(1 - \tanh^2 x)$$

approximately. In the example in the text the result shows that

$$\tanh \mu_2 l = \tanh(2 - \frac{1}{4})\pi$$

to at least eight decimal places. There can be no hesitation, therefore, in accepting equation (5a). The method appears more natural than the normal one, which would be to put $\mu_2 l = (2 - \frac{1}{4})\pi + x$ in $\tan \mu_2 l$ and $\tanh \mu_2 l$ (where x is small), and to proceed by approximations.

end is given by

$$x = .7166 l.$$

For higher harmonics, the i th tone has $i-1$ nodes ; and of these the first node, that is the one nearest the clamped end, is at

$$\frac{x}{l} = \frac{5.0175}{4i-1};$$

the second occurs at

$$\frac{x}{l} = \frac{8.9993}{4i-1},$$

and beyond this the j th node is given with sufficient accuracy by

$$\frac{x}{l} = \frac{4j+1}{4i-1}.$$

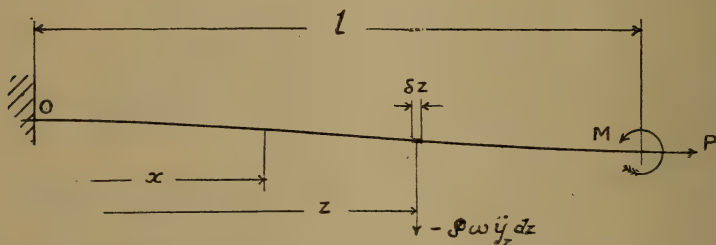
Section III.—Unloaded Massive Bar, Longitudinal Tension.

§ 9. When the bar is subjected to an axial tensile force P as shown in figure 3, the differential equation, based on the ordinary Bernoulli-Eulerian theory, may be written

$$EI \frac{d^2 y}{dx^2} + \rho \omega \int_x^l \ddot{y}_z (z-x) dz + P(y_1 - y) + M = 0. \quad (8)$$

in which M is the couple required to direct the end and y_1 is the deflexion there.

Fig. 3.



Differentiating twice with respect to x ,

$$EI \frac{d^4 y}{dx^4} + \rho \omega \ddot{y} - P \frac{d^2 y}{dx^2} = 0.$$

which is the differential equation common to all unloaded bars of constant flexural rigidity. It may be written

$$\frac{d^4 y}{dx^4} + \frac{\rho \omega}{EI} \ddot{y} - \frac{P}{EI} \frac{d^2 y}{dx^2} = 0, \quad (9)$$

for the case of stationary simple harmonic vibrations. The

general solution is then of the form

$$y = A \cosh \alpha x + B \sinh \alpha x + C \cos \beta x + D \sin \beta x,$$

in which, whatever the end conditions,

$$\left. \begin{aligned} -\frac{\ddot{y}_1}{y_1} &= \frac{\alpha^2}{\rho\omega} (EI\alpha^2 - P) \\ -\frac{\ddot{y}_1}{y_1} &= \frac{\beta^2}{\rho\omega} (EI\beta^2 + P) \end{aligned} \right\} \dots \dots \dots (10)^*$$

The terminal conditions give four equations for which it is necessary that

$$\alpha \tanh \alpha l = -\beta \tan \beta l. \quad \dots \dots \dots (11)$$

Elimination of α and β from the equations (10) and (11) would give an expression for $-\frac{\ddot{y}_1}{y_1}$ and enable the frequency to be calculated.

§ 10. The form of the centre line of the bar at any instant is then given by

$$y = y_1 (\tanh \alpha l \sinh \alpha x - \cosh \alpha x - \frac{\alpha}{\beta} \tanh \alpha l \sin \beta x + \cos \beta x) \\ \div \left(\frac{1}{\cos \beta l} - \frac{1}{\cosh \alpha l} \right),$$

whilst the couple M at the directed end can be obtained from

$$\frac{M}{y_1} + P = -\rho\omega \frac{\ddot{y}_1}{y_1} \left(\frac{1}{\beta^2} \cosh \alpha l + \frac{1}{\alpha^2} \cos \beta l \right) \div (\cosh \alpha l - \cos \beta l).$$

§ 11. Failing an exact solution of equations (10) and (11), a useful approximate one can be obtained by limiting the longitudinal force to such values as may occur in practice.

Writing $\phi = \alpha l$, $\theta = \beta l$, equation (11) is

$$\phi \tanh \phi = -\theta \tan \theta.$$

$$\therefore \frac{d\phi}{d\theta} = -\frac{\theta + \tan \theta + \theta \tan^2 \theta}{\phi + \tanh \phi - \phi \tanh^2 \phi}.$$

Also (10) gives

$$\theta^2 = \phi^2 - \frac{Pl^2}{EI} \dots \dots \dots (12)$$

and when P is zero (equation (4))

$$\phi = \theta = 2.36502$$

for vibrations of the fundamental type.

* Cf. Rayleigh's 'Theory of Sound,' vol. i. 1894, p. 299.

Now, when Pl^2/EI is small, θ differs little from 2.36502, and we have, to the first order in this difference

$$\phi = \left[\phi \right]_{\theta=2.365} + \left[\frac{d\phi}{d\theta} \right]_{\theta=2.365} (\theta - 2.365),$$

that is

$$\alpha l = 10.5083 - 3.4432 \beta l. \quad . \quad . \quad . \quad (13)$$

Eliminating α from (12) and (13), we find, after an expansion by the Binomial as far as the second term

$$\beta^2 l^2 = 5.5933 - .2251 \frac{Pl^2}{EI},$$

by means of which the second of (10) gives

$$k^2 = 31.285 \frac{EI}{\rho \omega l^4} \left\{ 1 + .09831 \frac{Pl^2}{EI} - .00557 \left(\frac{Pl^2}{EI} \right)^2 \right\}. \quad (14)$$

§ 12. For harmonics we may in all cases take

$$\tanh \phi = 1;$$

thus, reasoning as in the last paragraph,

$$\alpha l = (1 - \eta) \beta l + \frac{1}{2} \eta^2,$$

where

$$\eta \equiv (2i - \frac{1}{2})\pi,$$

whence

$$\beta^2 l^2 = \frac{1}{4} \eta^2 - \frac{1}{\eta} \frac{Pl^2}{EI}$$

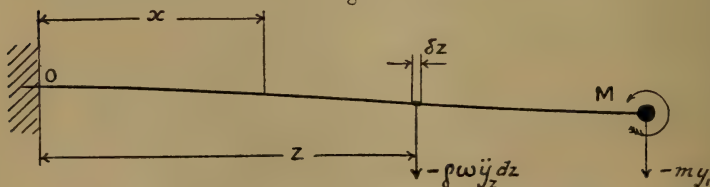
and

$$k^2 = \left(\frac{\eta}{2} \right)^4 \frac{EI}{\rho \omega l^4} + \frac{\eta}{2} \left(\frac{\eta}{2} - 1 \right) \frac{P}{\rho \omega l^2} - \frac{\eta - 1}{\eta^2} \frac{P^2}{\rho \omega EI}. \quad (15)$$

Section IV.—Massive Bar, Load at Directed End.

§ 13. When a massive clamped-directed bar carries a load concentrated at its directed end, the frequency and type of lateral vibration can be found to any required degree of accuracy by the method of continuous approximation*.

Fig. 4.



In figure 4, let the origin be at the clamped end and

* See Phil. Mag., July 1905, pp. 113-125, and March 1906, pp. 354-374.

assume for the first approximation to the vibration curve

$$y = y_1 \{3(x/l)^2 - 2(x/l)^3\}, \quad . \quad . \quad . \quad (16)$$

which, at the clamped end, satisfies $y = \frac{dy}{dx} = 0$, and, at the directed end, $y = y_1$, $\frac{dy}{dx} = 0$.

Let m = the mass concentrated at the end,

M = the couple required to maintain the direction at that end.

Then the ordinary approximate theory leads to

$$EI \frac{d^2 y}{dx^2} = -\rho \omega \ddot{y}_1 \int_x^l y_z(z-x) dz - m \ddot{y}_1(l-x) + M. \quad (17)$$

Inserting the value of y_z from (16) and performing the integrations,

$$\begin{aligned} -EI y = \rho \omega \ddot{y}_1 \left(\frac{13}{120} l^2 x^2 - \frac{1}{12} l x^3 + \frac{1}{120} \frac{x^6}{l^2} - \frac{1}{420} \frac{x^7}{l^3} \right) \\ + m \ddot{y}_1 \left(\frac{l x^2}{4} - \frac{x^3}{6} \right), \quad . \quad (18) \end{aligned}$$

the constants of integration and the value of M having been determined by the end conditions. This expression is the second approximation to the vibration type, and is often sufficiently accurate.

Putting $x = l$ we have

$$k^2 = \frac{EI}{\frac{13}{420} \rho \omega l^4 + \frac{1}{12} m l^3}. \quad . \quad . \quad . \quad (19)$$

§ 14. Proceeding to a still closer approximation we can insert in (17) the value of y_z given by (18) and again integrate. In the resulting expression for y , we can again put $x = l$ and get

$$k^2 = \frac{EI}{\left(\frac{.3832 \rho \omega l + m}{.3714 \rho \omega l + m} \right) \frac{13}{420} \rho \omega l^4 + \frac{m l^3}{12}}. \quad . \quad . \quad (20)$$

The portion shown in brackets is the correction which this result gives to that first obtained. When the mass of the load is equal to that of the bar, (20) reduces to

$$k^2 = \frac{8.730 EI}{\rho \omega l^4}.$$

Section V.—Loaded Bars; Mass of Bar Neglected.

§ 15. When the mass of the bar is neglected and vibrations occur under the action of a concentrated load only, the exact solutions can be determined directly.

If there be no longitudinal force and the mass m be at the directed end, the equation of equilibrium is

$$EI \frac{d^2 y}{dx^2} = -m\ddot{y}_1(l-x) - M.$$

$$\therefore EI y = -m\ddot{y}_1 \left(\frac{1}{4} l x^2 - \frac{1}{6} x^3 \right),$$

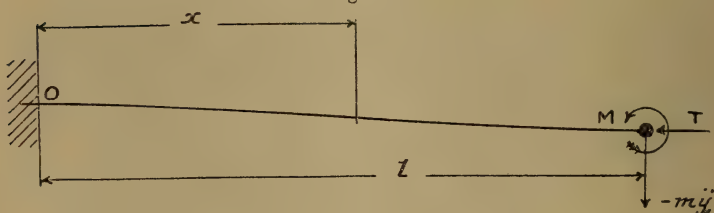
and

$$k^2 = \frac{12 EI}{ml^3} \dots \dots \dots (21)$$

§ 16. *Longitudinal Thrust*.—If in addition to the load m there be a longitudinal thrust T acting on the massless bar as represented in figure 5,

$$EI \frac{d^2 y}{dx^2} = -m\ddot{y}_1(l-x) + T(y_1 - y) - M.$$

Fig. 5.



By virtue of the conditions that at $x = 0$, $y = \frac{dy}{dx} = 0$, and, at $x = l$, $\frac{dy}{dx} = 0$, the solution may be written

$$y = \frac{m\ddot{y}_1}{nT} \left\{ \frac{\cos nl - 1}{\sin nl} (1 - \cos nx) + nx - \sin nx \right\}$$

in which $n^2 = T/EI$; and putting $x = l$,

$$k^2 = \frac{T}{ml} \frac{nl \sin nl}{2 - 2 \cos nl - nl \sin nl} \dots \dots \dots (22)$$

This result might have been deduced from that for a bar clamped at each end with a load at the centre*.

* Phil. Mag. September 1906, p. 243.

Equation (22) shows that $k^2 = 0$ and vibration ceases when $nl = \pi$, that is, when

$$T = EI \pi^2 / l^2.$$

After expansion, (22) becomes

$$k^2 = \frac{12EI}{ml^3} \left(1 - \frac{4}{15} n^2 l^2 + \dots \right),$$

agreeing with (21) when T , and therefore n , is zero.

When $nl = 2i\pi$ (i being an integer) the right-hand side of (22) is indeterminate but has the limit $-T/ml$, vibration being impossible if T is positive.

§ 17. The result may also be written

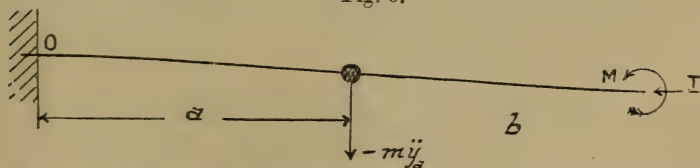
$$k^2 = \frac{T}{ml} \frac{\frac{1}{2} nl}{\tan \frac{1}{2} nl - \frac{1}{2} nl},$$

from which (nl being positive) the expression for the frequency is real when $\tan \frac{1}{2} nl > \frac{1}{2} nl$, that is, for values of nl between 0 and π , and for decreasing intervals in the neighbourhood of 3π , 5π , &c.; vibration always ceasing when $nl = (2i-1)\pi$.

The first of these intervals is from $nl = 8.9868 \dots$ to $nl = 3\pi$, and this corresponds to the first harmonic.

With the massless bar harmonics are impossible with low values of T . As T is increased the frequency falls, becoming zero when $nl = \pi$, $\ddot{y}$ being then zero. Between π and 8.9868 vibration is impossible, the deflexion increasing with the time. If this region be safely passed that between 8.9868 and 3π is reached, during which vibration may again occur, T being now sufficient to enable the bar to assume the curve of the first harmonic type.

Fig. 6.



§ 18. If the mass be situated at any point in the length, let it divide the bar into segments a and b , as indicated in fig. 6. Then for $x < a$ the differential equation becomes

$$EI \frac{d^2 y}{dx^2} = -m \ddot{y}_a (a-x) + T(y_1 - y) - M,$$

of which the solution is

$$y = A_1 \sin nx + B_1 \cos nx + \frac{m \ddot{y}_a}{T} (x-a) + y_1 - \frac{M}{T}.$$

For $x > a$ the equation is

$$EI \frac{d^2 y'}{dx^2} = T(y_1 - y') - M,$$

and the solution

$$y' = A_2 \sin nx + B_2 \cos nx + y_1 - \frac{M}{T},$$

where the dashed symbols refer to values of y in the region b .

The conditions at $x=0$ and $x=l$ lead respectively to

$$\left. \begin{aligned} y &= -\frac{c}{n^3} \sin nx + \left(\frac{m \ddot{y}_a}{T} a + \frac{M}{T} - y_1 \right) (\cos nx - 1) + \frac{m \ddot{y}_a}{T} x \\ y' &= \frac{M}{T} (\sin nl \sin nx + \cos nl \cos nx) + y_1 - \frac{M}{T} \end{aligned} \right\} \quad (23)$$

The conditions of continuity of y and $\frac{dy}{dx}$ when $x=a$ give an expression for the couple at the directed end, namely

$$M = m \ddot{y}_a \frac{\cos na - 1}{n \sin nl}.$$

Putting $x=a$ in the expressions for y and y' we find

$$k^{-2} = \frac{m}{nT} \left\{ \sin na (2 - \cos na) - \cot nl (1 - \cos na)^2 - na \right\}. \quad (24)$$

When $a=l$, this reduces to the result of § 16.

It can be shown, from equations (23), that the amplitude is a maximum at the directed end.

Section VI.—*Deduction of the Results of some Statical Problems.*

§ 19. The calculations in Section V. are similar to those required for the corresponding statical problems. Thus if a clamped-directed bar be subjected to a force W at and perpendicular to the directed end, the centre-line assumes the form (see § 15)

$$y = \frac{W}{EI} \left(\frac{1}{4} lx^2 - \frac{1}{6} x^3 \right).$$

Similar transformations may be made in §§ 16 and 18, and important results obtained. In each case the maximum deflexion is at the directed end.

When the bar is subjected to a force w per unit length acting normally to the x -axis, its deflexion curve is

$$y = \frac{w}{EI} \left(\frac{1}{6} l^2 x^2 - \frac{1}{6} l x^3 + \frac{1}{24} x^4 \right) \dots$$

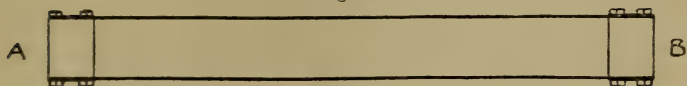
We notice that, for a force concentrated at the end, the maximum deflexion of a clamped-directed bar is one-quarter of that for a clamped-free bar; whilst when the force is uniformly distributed the ratio is one-third.

Section VII.—*Some Experimental Results.*

§ 20. The experiments described in this section were made merely to compare, in a few cases, the frequency calculated by the methods given in this paper with that obtained by direct observation. Care was taken that the accuracy should reach certain limits.

The composite beams used for the experiments consisted of two flat wrought-iron bars, each 3.00 cms. broad by 0.315 cm. thick, the latter dimension being in the plane of

Fig. 7.



vibration. The bars were rigidly connected together at their ends by being securely screwed to massive cast-iron blocks, as shown in fig. 7. The effective length was measured between the blocks, and this, as well as the distance between the bars, was varied.

A striking feature of the apparatus is the certainty with which the end conditions can be relied upon. When this is the case, the distance between the bars is immaterial; some of the experiments may, in this respect, be taken as a verification of the absolute rigidity of the ends.

§ 21. The flexural rigidity of a piece of one of the bars was determined by supporting it on knife-edges at each end of a span of 58.42 cms. (23 inches), and applying loads at the centre. The observed deflexions are given in the following table.

Load in lbs.	Deflexion. Inches.	Difference. Inches.
0	·627	·047
1	·580	·048
2	·532	·049
3	·483	·050
4	·433	·050
3	·483	·048
2	·531	·049
1	·580	·049
0	·629	
Average difference per pound ·0488		

The average deflexion at the centre was thus found to be 0·0488 inch per pound, corresponding to a flexural rigidity of $15\cdot20 \times 10^6$ grammes weight-cms.² The value of E here involved is the static modulus, but the difference between it and the kinetic modulus would be too small to affect the calculations.

§ 22. The results of the experiments on the compound bars are given in the following table. In each case vibrations took place in a horizontal plane, that is, in the plane of fig. 7.

The clamped end was very securely bolted to a large lathe-bed in such a manner as to leave no room for doubt as to the applicability of the terminal conditions at that end.

No. of Experiment.	Effective Length. cms.	$\frac{1}{2}$ Weight at Directed end, gms.	Distance apart of bars. cms.	Observed Frequency.	Calculated Frequency.
1	114·3	1604	16·5	1·27	1·26
2	114·3	780	16·5	1·66	1·67
3	91·5	1604	16·5	1·78	1·78
4	91·5	780	16·5	2·40	2·40
5	114·3	1604	6·0	1·26	1·26
6	114·3	802	6·0	1·65	1·65

The last column is calculated from equation (20); the values are, however, practically identical with those given

by (19). It will be seen that the agreement between the observed and calculated frequencies is very satisfactory.

I should like to take this opportunity of making a slight correction in my paper "On the Lateral Vibration of Bars subjected to Forces in the Direction of their Axes." On page 236 of the *Philosophical Magazine* for September 1906, and on page 227 of the *Proceedings of the Physical Society*, vol. xx., instead of $\tanh \phi$ I have used its reciprocal. The error makes very little difference to the final equation, the correct expression being

$$-\frac{\ddot{y}_1}{y_1} = 500.56 \frac{EI}{\rho \omega l^4} + 12.91 \frac{P}{\rho \omega l^2} - .167 \frac{P^2}{\rho \omega EI}.$$

Armstrong College,
January 1908.

LIV. *On the Electric Discharge through the Gases* HCl, HBr, and HI. By L. VEGARD, *Universitets stipendiat of Christiania University**.

[Plate XIV.]

§ 1. IN some recent experiments made by Matthies † on the discharge through the vapours of HgCl, HgBr, HgI, as well as through the elements Cl₂, Br₂, and I, some very striking results are found regarding the potential gradient in the positive column. When compared for equal currents and pressures the gradients are found to be greatest for Br and smallest in the case of I. This remarkable result led me, at the suggestion of Sir J. J. Thomson, to undertake an investigation of the distribution of potential in the gases HCl, HBr, and HI.

As the properties of discharge depend on the form and size of the tube, it is difficult to reduce the numbers to universal units. In order therefore to compare our results with those found for other gases, measurements have been made with the same tube for oxygen.

Apparatus and Mode of Procedure.

§ 2. Fig. 1 gives the general arrangement of the tube-system. The air could be pumped out with a mercury-pump and the vacuum tested with a McLeod gauge. The chemically active gases were removed by blowing a current

* Communicated by Sir J. J. Thomson.

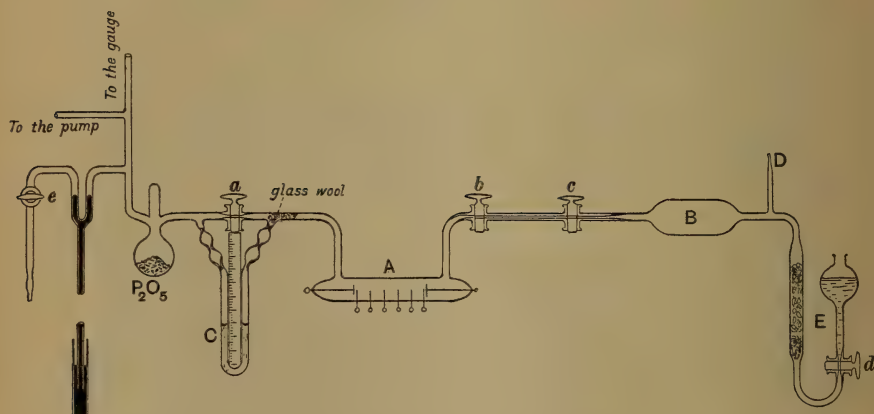
† W. Matthies, *Ann. d. Phys.* xvii. p. 675 (1905); id., *ibid.* xviii. p. 473 (1905).

of air through the tube system from *e* to D (fig. 1). The gas was introduced into the discharge-tube (A) by means of two taps (*b*) and (*c*) separated by a capillary tube. The U-tube C containing concentrated sulphuric acid (sp. g. 1.84) served for measuring the pressure. The level difference was read with a cathetometer; in that way the pressure could be found with an accuracy of $\frac{1}{100}$ mm. Hg.

§ 3. The gases were produced in somewhat different ways; in all cases, however, a method was employed allowing the gases to be formed in an exhausted vessel.

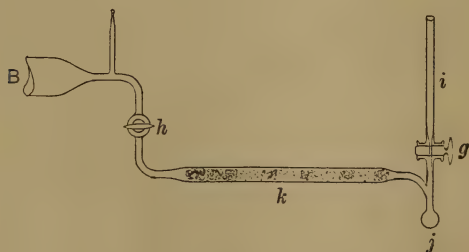
The HCl gas was produced from NH_4Cl and concentrated H_2SO_4 in the apparatus shown in fig. 1 (E). The somewhat

Fig. 1.



wide tube contains the ammonium salt, the small bulb the acid. Before exhausting, the acid was made to fill the boring of the tap. The HBr and HI gases were produced from their

Fig. 2.



concentrated aqueous solutions in an apparatus shown in fig. 2. The air from the boring of the tap *g* was removed

by sealing the tube *i* and evacuating. Then the acid was introduced into the bulb *j*. The gas given off by the liquid had to pass through the tube *k* containing a number of layers of glass wool and phosphorus pentoxide, and finally a plug of phosphorus. Thus water-vapour and traces of bromine and iodine vapours were removed from the gas.

The discharge-tube was similar in form to that used by Matthies, the chief difference being that the electrodes consisted of elliptical-shaped platinum plates fixed to platinum rods (fig. 3), while he used thick platinum rods. The distribution of potential was measured with fixed explorers of platinum surrounded with narrow glass tubes. The tube had the following dimensions :—

Diam. of the tube.	Area of the Cathode surface.	Area of Anode surface.	Distance between electrodes and explorers.					
			A—1.	A—2.	A—3.	A—4.	A—5.	A—K.
3.3 cm.	3.2 cm. ²	1.3 cm. ²	mm. 3.1	mm. 25.3	mm. 46.1	mm. 67.5	mm. 87.2	mm. 91.6

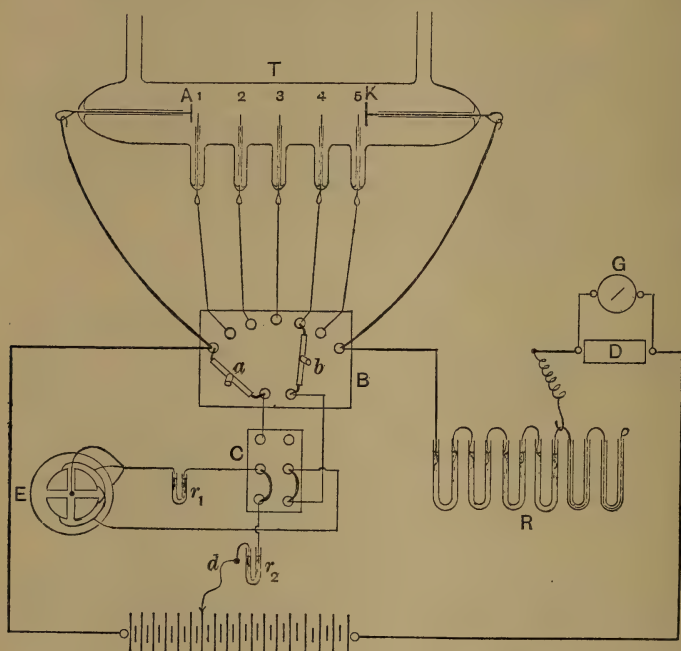
The potential for producing the discharge was taken from a number of storage-cells, the potential of which was measured before and after each series of measurements.

The arrangement for current and P.D. measurements is shown in fig. 3. The P.D.'s were measured with a quadrant electrometer (Dolezalek type). This instrument being too sensitive for the purpose, the following way of measuring was used:—The anode and the positive end of the battery being connected through a very small resistance had the same potential. To measure the P.D., say between the anode and the explorer (4), the latter is put in connexion with the terminal leading to the electrometer and the other electrometer terminal is connected to the wire (*d*) coming from the battery. By taking out a suitable voltage from the battery the P.D. (A—4) can be so balanced that the electrometer gives a suitable deflexion.

Wires from the electrodes and explorers were connected to a set of mercury cups cut in a piece of paraffin wax and placed on the periphery of a circle, so that by the key *b* they can easily be put in connexion with the electrometer.

The current was measured with a d'Arsonval galvanometer (G) put as a shunt to the head circuit. In order to vary the

Fig. 3.



current and in order to keep it constant during the discharge, a series of high resistances had to be used. The resistances used consisted of a number of U-shaped glass tubes filled with a solution of cadmium iodide in amyl alcohol (fig. 3, R), and with resistances varying from about $\frac{1}{2}$ to 30 megohm.

The large resistances (r_1 , r_2) were put into the electrometer connexions to avoid any short circuit through the electrometer.

The whole system was insulated, only at one point the circuit was put to earth through a very big resistance so as to fix the zero point of the potential; for if the explorers require some time to take up the surrounding potential, fluctuations in the absolute value of the potential in the circuit might cause unsteadiness and faults in the determination of the P.D.

Results of Measurements.

§ 4. Measurements were made for a series of pressures varying from about 0.25 to 2.4 mm. Hg, and for each pressure the distribution of potential was measured for a series of currents. In general the same voltage from the cells was used for measurements at the same pressure; the current was changed by changing the resistance.

The discharges to be compared for different gases must have the same structure, and for comparison of the potential gradients of the positive column the latter must be uniform.

It is found that for the greatest part of the current interval that gives a uniform discharge the electric force is very nearly constant all along the positive column. For very small currents it is found (especially for HCl, HBr, and O₂) that the P.D. curve is slightly curved, and in such a way that the electric force increases towards the anode. This is in agreement with results found by Matthies for discharge through vapours of HgCl, HgBr, and HgI.

Fig. 4 (Pl. XIV.) gives some P.D. curves for HBr at pressure of 0.55 mm., showing the general character of these curves for varying currents.

In the case of this variation in the force we shall for the comparison use the average force, which in our case can be put equal to the potential-difference between the explorers (1) and (4) divided by their distances.

Discharge through HCl.

§ 5. The general character of the discharge through HCl is the same as for an ordinary gas. Three forms of discharge have been observed, the uniform, the striated, and one form which is got when the positive column is driven into the anode and which we shall call the dark discharge.

Some results of the measurements of potential distribution in HCl made with the tube described are given in the following table:—

Gm is the mean P.G. (potential gradient).

U means uniform discharge.

S, striated discharge.

D, dark discharge.

TABLE I.

HCl.

<i>p.</i>	10 ⁵ I.	Gm.	A—1.	A—2.	A—3.	A—4.	A—5.	A—K.	Type.
mm.	am.	volt/cm.	volt	volt	volt	volt	volt	volt.	
0.26	0.30	35.5	80	242	300	309	243	542	U
	9.9	36.1	66	186	239	299	349	781	„
0.22	29.2	22.5	40	87	129	172	227	782	S
0.50	1.3	36.0	80	219	263	311	318	642	U
	22.0	41.6	39	128	175	307	380	859	„
0.43	115	14.1	36	73	112	127	139	984	S
0.75	0.41	40.8	98	207	235	361	492	755	U
	12.6	49.8	56	177	273	377	396	804	„
0.60	90	20.2	34	104	159	165	172	734	S
	1.2	9.3	45	80	95	105	174	438	D
1.6	1.15	75.0	129	367	538	613	704	1024	U
	35.8	93.0	63	287	477	666	804	1171	„

For the uniform discharge is given the distribution corresponding to the biggest and smallest currents which for each pressure are observed for this form of discharge.

On the Stability of the Different Forms of Discharge.

§ 6. In order to get the greatest possible current-interval for the uniform discharge we must take care not to begin the discharge with too large a current, but it is advisable to undertake the measurements in the order of increasing currents and begin with the lowest that can be made to pass.

When the current becomes somewhat large the uniform type will be unstable. Striations set in generally accompanied by a rapid increase in the current, which means a fall in the potential-difference between the electrodes.

For very small pressures the change sometimes takes place with diminution of current. This is due to an increase in the cathode fall, which is caused partly by the diminution in pressure that generally accompanies this change at low pressures.

Also the striated form becomes unstable for very large currents, the positive column is gradually "driven into the anode," leaving the positive column dark behind or rather

with a feeble green phosphorescent light that seems to radiate from the cathode.

The stability of the different forms of discharge depends on pressure and the size of the electrodes. When other circumstances are the same the interval for a uniform discharge is greater for a greater pressure. With a tube of exactly the same shape and size, but with electrode-surfaces about 0.4 cm.^2 , it was found that the uniform as well as the striated type for pressures below 1.5 mm. was very unstable. The uniform discharge could be got when fresh gas was introduced or when the tube had been "resting" over night; but if the current was larger than about $2 \cdot 10^{-5} \text{ amp.}$ striations set in, which, however, soon disappeared, and the dark type became stable for nearly all currents; only for the very smallest currents that could be made to pass was the uniform discharge sometimes obtained. The gradients got with this tube for the different types are given in Table II.

TABLE II.

HCl.

	$p=0.2.$		$p=0.4.$			$p=1.0.$	
10^5 I	0.26	9.0	0.35	11.6	7.7	1.8	25.0
Gm.	25.2	0.4	25.6	6.6	0.3	34	2.3
	U.	D.	U.	S.	D.	U.	D.

We see from Tables I. and II. that the average P.G. in the positive column is greatest for the uniform and smallest for the dark discharge. Thus when the discharge by applying a large current is made to change type, the change is *always accompanied by a fall in the average potential gradient.*

Reversibility of the Discharge.

§ 7. As long as the discharge was maintained in the unstriated state the conditions were found to be reversible with regard to current, or the distribution of potential does not depend on the order of operation. Within these limits the tube will possess a certain characteristic curve and the potential gradient is expressible as a function of current.

When the discharge in the way described is made to assume the striated or dark type, the conditions are no longer

reversible. In both cases the characteristic curve gets a permanent change and the interval for uniform discharge is reduced to such a degree that generally this type is obtained only for the very smallest currents that can be made to pass. A tube giving a striated or dark column when left to itself has a tendency to recover or to increase the interval of uniform discharge. This will make the characteristic curve change with time. The recovery of a tube brought into the state of dark column was also observed by Skinner for nitrogen.

Fig. 5 (Pl. XIV.) shows how the characteristic curve was changed when the discharge was brought from the uniform to the striated state.

The reversibility of the uniform discharge is a very important point in connexion with the discharge through HCl, for it shows that at least within these limits the conditions for discharge are not altered by the current through it to such an amount as to give any appreciable change in the distribution of potential. As a possible decomposition necessarily would be accompanied by a change in the character of discharge the reversibility shows that the *measured uniform discharge corresponds to the undecomposed hydrochloric acid gas.*

Discharge through HBr.

§ 8. The luminosity in the positive column compared for equal currents was much weaker in HBr than in HCl and showed a darker blue colour. The discharge could be kept perfectly steady and the interval of stability for the uniform column proved to be much greater than for HCl, and as a matter of fact within the current limits that could be got by using the lowest resistance and largest voltage at hand the striated state was not obtained, and as the measurements in a uniform column were my chief object no further effort was made to obtain possible limits for the uniform discharge.

If there was no change of pressure the conditions were found to be reversible with regard to current, which shows that no decomposition took place which was able to cause any appreciable influence on the discharge.

In Table III. is given the distribution of potential in HBr for a series of pressures. It contains at each pressure the measurements corresponding to smallest and largest current for which observations are taken.

TABLE III.—HBr.

<i>p.</i>	10 ⁵ I.	Gm.	A-1.	A-2.	A-3.	A-4.	A-5.	A-K.
mm.	am.	volt/cm.						
0.135	0.36	27	44	120	202	218	254	691
	2.1	24	53	125	177	208	236	829
0.26	0.66	40	68	175	283	327	355	650
	24.2	29	45	125	173	229	248	845
0.55	0.85	51	126	351	409	457	491	809
	95	52	48	177	275	387	458	983
1.03	0.65	65	154	389	482	574	676	997
	43.3	89	80	282	451	655	771	1154
2.35	1.2	133	200	523	857	1090	1273	1530
	59.2	146	124	486	746	1068	1239	1591

Discharge through HI.

§ 9. For very small currents the discharge through HI could be kept fairly constant, and gave under these conditions an unstriated column with feeble blue light. With decrease of resistance the larger current would not keep constant, but increased gradually, the pressure diminished considerably, and the discharge became striated. The potential distribution for different pressures is given in Table IV.

TABLE IV.—HI.

<i>p.</i>	10 ⁵ I.	Gm.	A-1.	A-2.	A-3.	A-4.	A-5.	A-K.
mm.	am.	volt/cm.						
0.51	0.18	64	141	356	431	551	663	980
0.54	0.33	84	137	357	485	680	825	1030
0.65	0.95	91	176	473	634	762	880	1203
1.09	0.62	136	166	522	813	1044	1230	1520
1.16	0.62	154	162	581	906	1157	1390	1660

The Potential Gradients.

§ 10. For HCl and HBr, where the conditions within the intervals of uniform discharge were reversible, the potential
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gradient for each pressure is a function of current. These relations are represented in figs. 6 and 7, and show that except for very small currents the variation of P.G. obeys Herz's law, and especially in such a way that the P.G. is very nearly independent of current.

The relation between P.G. and pressure for currents of about $0.8 \cdot 10^{-5}$ am. is given in fig. 8. For pressures less than 0.3 mm. there is a peculiar difference between the curves for HCl and HBr. The P.G. for HBr falls rapidly with diminution of pressure while for HCl it keeps nearly constant. We see that at least for pressures above 0.25 mm. the potential gradients of HCl, HBr, and HI follow in the same order as the molecular weights of the gases, when compared for equal pressures. The value of the P.G. for oxygen is between that of HCl and HBr and nearest to the latter.

Table V. gives the P.G. for the four gases for a series of pressures.

TABLE V— $10^5 I = \text{ca. } 0.8$.

<i>p.</i>	HCl.	HBr.	HI.	O ₂ .
mm.	volt/cm.	volt/cm.	volt/cm.	volt/cm.
0.4	36	46	70	41
0.7	41	57	98	53
1.0	51	68	130	64
1.4	67	86	182	82
1.8	85	106	...	...
2.2	104	127	...	...

The results got with respect to the potential gradients for HCl, HBr, and HI as far as the I compound is concerned is contrary to those found by Matthies for the similarly constituted gases. Further measurements with the halogen elements and their mercury compounds would be desirable to find the cause of this discrepancy. In these experiments special attention ought to be paid to the structure of the discharge, for the possibility is not excluded that the small P.G.'s for the iodine compounds may be due to a striated structure, and the fact that the tube had to be surrounded with an electrical oven may have made an examination of the discharge difficult.

If the values found by Matthies really correspond to a uniform structure it seems natural to seek the cause for the small P.G. in the case of the iodine compounds in some kind

of secondary process which is most marked in the iodine vapours. The great diminution in the P.G. with increase of current which is found by Matthies for the iodine compounds would point to a secondary effect, for if it existed it ought to be more marked for larger currents. Then to make the secondary effect the least possible it would be advisable to make the comparison for very small currents, as actually has been done for HCl, HBr, and HI.

The Cathode Fall.

§ 11. The normal cathode fall corresponding to a partly covered cathode is determined in a number of measurements for different pressures and currents. As the mean of the observed values we get :

For HCl.....	329.6	volts	platinum	electrodes.
„ HBr ...	350.3	„	„	„
„ O ₂	364	„	„	„

Fig. 9 gives the cathode fall above the state of saturation of the cathode for the tube described. We notice the characteristic difference between the curves for high and low pressures. The horizontal line gives the normal cathode fall, and where the curves cut this line we have the biggest current that gives the normal cathode fall for the tube and pressure considered. The relation between saturation current and pressure is shown in Pl. XIV. fig. 10. These curves cut the p axis, and for all three gases at the same point. Thus for each gas there is a certain pressure, below which no current that can be made to pass can give the normal cathode fall, and which is the same for the three gases. This result suggests that for the same tube the minimum pressure for a normal cathode fall is independent of the nature of the gas. For the tube used the minimum pressure = 0.25 mm.

The Anode Fall.

§ 12. To get a strict definition of the anode fall we shall let the P.D. curves (see Pl. XIV. fig. 4) continue in their direction past the explorer (1) and their points of intersection with the vertical axis through the anode will give the anode fall.

In order to find the influence of current and pressure on the anode fall other conditions must be unaltered. Thus the discharge must be reversible, which in our case requires a uniform structure. Further, the tube must be run for some time in the gas to be tested before any measurements are taken. It is found that the anode fall when a new gas is introduced will begin with a too large value. This is probably due to changes in the surface-conditions of the electrodes, for

when constant conditions are reached they are not changed by introduction of a new portion of the same gas.

The anode fall (A) for HCl and HBr, found under the conditions stated above, is given in Table VI. for a series of different currents and pressures.

TABLE VI.

HCl.						HBr.							
$p=0.5.$		$p=0.8.$		$p=1.6.$		$p=0.27.$		$p=0.55.$		$p=1.03.$		$p=2.35.$	
10 ⁵ I.	A.	10 ⁵ I.	A.	10 ⁵ I.	A.	10 ⁵ I.	A.	10 ⁵ I.	A.	10 ⁵ I.	A.	10 ⁵ I.	A.
0.5	58	0.41	81	1.2	96	0.63	56	0.85	94	0.65	125	0.88	169
3.3	52	1.5	60	1.3	95	2.2	51	2.8	64	2.2	105	1.2	157
7.9	36	2.3	60	4.4	83	6.0	48	6.9	60	11.0	70	4.0	120
15.8	30.5	6.6	50	9.8	62	15.6	39	20.2	44	27.6	58	12.6	80
22	28.5	12.6	41	35.8	36	24.0	37	95	35	43	51	54	71

The anode fall can be expressed by either of the two functions :

$$(1) \quad A = A_0 + a_1 p + p(a_2 - b p) e^{-\kappa I},$$

$$(2) \quad A = A_0 + \frac{1}{\beta + \frac{1}{p(a_1 + a_2 e^{-\kappa I})}}.$$

Within the pressure limits of my experiments both equations satisfy equally well the observations: it is first for higher pressures that the difference would be significant. The constants of the two functions for HCl and HBr are given in the table.

TABLE VII.

Equation 1.			Equation 2.		
	HCl.	HBr.		HCl.	HBr.
$A_0 \dots$	23.8	35.0	$A_0 \dots$	23.8	35.0
$a_1 \dots$	8.2	15.0	$a_1 \dots$	9.0	17.2
$a_2 \dots$	62	112.5	$a_2 \dots$	61.2	110
$b \dots\dots$	12	28	$\beta \dots$	420	250
$\kappa \dots\dots$	1.1×10	1.9×10^4	$\kappa' \dots$	1.1×10^4	2.0×10^4

The observations are represented in figs. 11 and 12 (Pl. XIV.). The curves are determined from equation (1), and we see that they fit in with the observed values to a degree that is almost astonishing when the character of the phenomenon is taken into consideration.

Among the properties of the anode fall expressed in the two formulæ we shall notice the following two:

1. *By diminution of pressure the anode fall approaches a minimum value (A_0) independent of current.*

2. *For somewhat large currents (when κ is not too small) the anode fall will be nearly independent of current and follow a linear law with regard to pressure.*

Absorption of Gas during the Discharge.

§ 13. It is a well-known phenomenon that gas is absorbed in a vacuum-tube when a discharge is made to pass through it. It is a common phenomenon in Röntgen tubes where it causes the hardening of the rays with age. Considerable quantities can be absorbed in this way: thus J. J. Thomson* observed that several cubic cm. of gas at atmospheric pressure were absorbed without indication of any diminution of the absorbing-power. The absorption has been studied by Riecke and Skinner, and more systematic experiments made by Willows†.

In the course of my experiments I have had the opportunity of examining the absorption for HCl, HBr, and O₂ under conditions of various pressures and currents, and at the same time been able to make comparisons with the distribution of potential. In this way I have met with some characteristic properties of absorption, which I think will throw some light on these phenomena that up to the present have been very little understood.

Before describing the observations it is good to make clear the following conceptions:—

(1) *Velocity of absorption (μ)* equal to number of cm.³ absorbed per sec. = $\frac{V}{760} \frac{dp}{dt}$ (V = volume of tube). For the tube used $\mu = 0.31 \frac{dp}{dt}$.

(2) *The quantity (q)* absorbed when 1 coulomb has passed through is equal to $\frac{1}{I} \frac{V}{760} \frac{\Delta p}{\Delta t} = \frac{\mu}{I}$.

* J. J. Thomson, 'Conduction of Electricity through Gases,' p. 552.

† R. S. Willows, Phil. Mag. [6] i. pp. 503-517 (1901).

Absorption in Oxygen.

§ 14. *As long as the pressure is fairly high the absorption is extremely small even for very large currents.*—A current of $157 \cdot 10^{-5}$ am. with a pressure so low as 0.8 mm. gave $q=0.006$. With a pressure of 0.56 mm. and a current of $116 \cdot 10^{-5}$ am. the discharge could no longer be kept constant. Current and pressure decreased rapidly, until after 25 min. the current had fallen to $0.38 \cdot 10^{-5}$ and the pressure diminished to 0.13 mm. The average current in the interval was $42.5 \cdot 10^{-5}$ am., which gives the average value of $q=0.21$, or more than thirty times as large as the value at the somewhat higher pressure. The rapid absorption set in with a cathode fall of about 650 volts.

Another experiment was tried with a pressure 0.27 mm., but in this case the discharge was commenced with small currents. In spite of the smaller pressure it was found that *for currents below a certain value the discharge could be kept constant and the absorption was inappreciable*. Currents from $0.5 \cdot 10^{-5}$ to $14 \cdot 10^{-15}$ am. were allowed to pass for about 15 min. each, but no change in pressure was observable; with a current of $40 \cdot 10^{-5}$ the pressure and current began to decrease. The variation of the properties of discharge during the "running down" of the tube is given in Table VIII., and the variation of current, pressure, and q is represented in fig. 13.

TABLE VIII.

Time.	$10^5 I.$	$p.$	Gm.	Cathode-fall.	$\mu.$	$q.$
0	40	0.27	14.0	640		
2.5	39.5	0.26	...	660	$4.0 \cdot 10^{-5}$	0.10
5	38.5	0.22	...	800	7.5 "	0.20
8	35	0.16	...	...	9.5 "	0.27
12	12	0.11	...	...	3.1 "	0.26
18	5.0	0.094	...	1220	1.34 "	0.27
23	2.7	0.088	...	1260	0.64 "	0.24
29	2.0	0.075	1.0	1265	0.56 "	0.28

We see that the quantity (q) shows a very simple variation in spite of the rather complicated manner in which current and pressure vary. The value of q increases to a maximum value, after which it keeps constant, and the maximum value is of the order of the electrochemical equivalent.

Absorption in HBr.

§ 15. The characteristic abrupt occurrence of the absorption found for oxygen is also observed for HBr. With a pressure of 0.55 mm., currents up to $95 \cdot 10^{-5}$ gave very little absorption, the value of q for the largest current being only 0.05. For higher pressure the absorption was still less. For a pressure of 0.27 and currents up to $20 \cdot 10^{-5}$ the absorption was almost inappreciable. A current of $24 \cdot 10^{-5}$ gave $q=0.015$; raising it to $34 \cdot 10^{-5}$, however, a sudden absorption took place. The properties of discharge varied as shown in Table IX.

TABLE IX.

Time.	$10^5 I.$	$p.$	Cathode fall.
0	34	0.243	650
4.0	6.7	0.156	—
22	0.55	0.148	880

The mean value of I in the time interval is $4.7 \cdot 10^{-5}$ am., which gives for q the average value 0.48, or about double the value found for oxygen.

Absorption in HCl.

§ 16. The absorption effect observed for O_2 and HBr has also been found for HCl; but in those cases examined the effect has not been so abrupt. With a pressure so low as 0.46 the discharge was maintained with currents of $16 \cdot 10^{-5}$ and $22 \cdot 10^{-5}$ am. each for about a quarter of an hour without noticeable absorption; but with a current of $98 \cdot 10^{-5}$ the absorption became very marked, the current decreased, and within 14 min. the pressure went down from 0.460 to 0.350 mm., the absorption per coulomb $q=0.052$. With pressure 0.27 a current of $29 \cdot 10^{-5}$ gave considerable absorption. As regards the interpretation of this somewhat smaller absorption effect in HCl we must keep in mind that there is always a possibility for a contra-effect in the evolution of hydrogen from the cathode.

Concerning the Cause of the Absorption.

§ 17. A very important property of this kind of absorption is that it is always found to stop immediately the

discharge is broken. This fact makes it very improbable that the absorption can be due to any secondary effects, but leads us to find the cause in some property of the discharge itself. The peculiar abruptness in the occurrence of the absorption, as well with regard to current as pressure, must show that the absorption is not a necessary accompaniment to the discharge, but sets in when peculiar conditions occur.

We have seen that the great absorption is always accompanied with a large cathode fall, and if we suppose that the absorption per coulomb (q) is mainly a function of the cathode fall, and suppose further that this function has the character that it increases very abruptly when the cathode fall is raised above a certain value, then the character of the absorption phenomenon will be immediately understood. This will be apparent by looking at the curves (fig. 9), giving the variation of the cathode fall. For somewhat high pressures the cathode fall after "saturation" increases very slowly with increase of current, reaches a maximum value and falls again. If, now, the cathode fall for which the great absorption sets in is greater than this maximum value, the large absorption cannot be got at all for that pressure even for very large currents. First, when the pressure becomes somewhat low there will be a current for which the cathode fall is big enough to produce the great absorption.

Our assumption also explains the fact that even for the same pressure the absorption first sets in for a certain strength of current.

The cathode fall, for which the great absorption sets in, we might call the critical cathode fall. The value of this may depend on the quality and form of the electrodes and the quality of the gas, and it may also be somewhat different at different pressures. For O_2 and HBr with the tube used it is about 650 volts.

The rapid absorption with diminution of pressure is also observed by Willows for air, nitrogen, and hydrogen; but he comes to the conclusion that the absorption is due to a chemical combination between the gas and the walls of the tube. Such a chemical combination seems *a priori* very unlikely; for it would mean that the discharge in a mysterious way imparted to the walls a chemical activity *which should only be possessed by the walls as long as the discharge was running*. Another objection to Willows's explanation, which he also mentions himself, is that the absorption seems not to depend essentially on the chemical character of the gas, but is found even for inert gases.

The only experiment which in the case of ordinary discharge should indicate an influence of the substance of the tube, is that a tube of lead-glass under equal conditions gave 10 per cent. less absorption than a similar one of soda-glass. But this may equally well be explained by differences in the size of the cathode surfaces. Silvering the inside, but keeping electrodes unaltered, gave no change in absorption.

Some of his experiments, for which he has not been able to give any satisfactory explanation, are easily explained by the assumed connexion between cathode fall and absorption.

With a tube where one electrode was a small rod, the other a cylindrical vessel surrounding the first one, the absorption for the current used ($108 \cdot 10^{-5}$) was exceedingly small when the cylinder was made a cathode, but had the usual large value when it was made an anode. This is immediately explained by the difference that must exist between the cathode falls for the two directions. Another experiment (*loc. cit.* fig. 7), where he also used electrodes of different size, is explained in the same way. In an experiment where there was dissymmetry with regard to tube, but symmetry as regards electrodes, a reversion of current produced no change in absorption.

From this it seems beyond doubt that the cathode fall is the essential property for the absorption. The next question will be in what way the cathode fall is able to produce absorption. The assumption that almost forces itself upon us is that the absorption is carried out by the ions which by the large cathode fall are given such velocities as to be shot as it were into the solid matter. This is in agreement with the fact that the maximum absorption is of the same order as the electrochemical equivalent.

From the view we have at present as to the mechanism of the discharge, it is further most likely that the greatest absorption effect is produced by the positive ions falling into the cathode. We must remember that the essential thing is not that the ions acquire a certain velocity, but that they have it the moment they strike the solid matter. *Now the positive ions get their maximum velocity just when they strike the cathode.* The negative electricity will within the space of the cathode fall mostly exist as corpuscles. Some of these may pass through the gas without collision, in which case they can produce no absorption, some will stick to molecules and communicate to them their kinetic energy acquired at the cathode fall; but these molecules may now make collisions, and thus gradually lose kinetic energy. Thus it should at least only be a small fraction of the negative

ions that would meet the walls or anode with the velocity necessary for absorption. This is also in agreement with the experiment that silvering of the inner surface had no effect on absorption. And if the absorption was brought about quite as much by negative as by positive ions, it would be difficult to understand why the potential fall between the electrodes had to be concentrated near the cathode in order to produce absorption.

This hypothesis, that the absorption is caused by the positive ions being shot into the cathode, also explains the rapid increase of absorption when the cathode fall exceeds a certain value, for it only means that a certain velocity is required for ions to be shot into solid matter.

It is worth noticing that it is in the state of the rapid absorption that the most rapid disintegration of the cathode takes place, which suggests, what from our view seems quite natural, that these two phenomena are closely connected.

Summary of Results.

(1) The general character of the discharge through HCl, HBr, and HI is the same as for elementary gases.

(2) In HCl and HBr the discharge could be kept constant and with a uniform structure within wide current intervals; the conditions were "reversible." For HI a constant discharge could only be got for very small currents.

(3) Within the interval of reversibility the decomposition is too small to effect the discharge.

(4) The characteristic curve depends on the "history" of the tube, and a change in structure caused by current is always accompanied by a fall in the P.G. in the positive column.

(5) Under comparable conditions the potential gradients in HCl, HBr, HI follow the order of the molecular weights.

(6) The variations of P.G. with current for HCl and HBr follow Herz's law except for very small currents.

(7) The normal cathode fall is found for HCl and HBr; the smallest pressure that can give a normal cathode fall is the same for O₂, HCl, and HBr.

(8) The anode fall in the interval of observation is found to be expressible by either of the two functions :

$$A = A_0 + a_1 p + p(a_2 - bp)e^{-\kappa T},$$

$$A = A_0 + \frac{1}{\frac{1}{\beta} + \frac{1}{p(\alpha_1 + \alpha_2 e^{-\kappa T})}}.$$

(9) The "electric absorption" of gas is found to be closely connected with the size of the cathode fall, which leads to the assumption that the absorption is chiefly caused by positive ions being shot into the cathode.

In conclusion, I wish to thank Sir J. J. Thomson for suggesting the work to me.

I have also much pleasure in thanking Mr. E. Everett for his assistance with some of the glasswork.

Cavendish Laboratory,
May 1st, 1900.

LV. *The Decay of Waves in a Canal.* By W. J. HARRISON, B.A., Fellow of Clare College, Isaac Newton Student in the University of Cambridge*.

§1. **I**N a paper published recently Dr. R. A. Houston gave the results of some experiments on the damping of long waves in a rectangular trough †. The observed damping is from two to three times as great as that calculated from the formula obtained by him. In this paper an explanation of the discrepancy is attempted. It has been shown by the present author that under certain circumstances the air exerts a great damping influence on water waves ‡. It may be as well to emphasize here the cause of this influence, as it might appear strange that a fluid of such slight density as air could affect the decay of water waves. The modulus of decay τ of wave-motion at the surface of a single fluid of depth large compared with the wave-length λ of the motion depends on λ^2 ; when there is another fluid superposed τ depends on $\lambda^{\frac{5}{2}}$; and thus when λ is large the difference is considerable, despite the small density of air. Now the waves considered by Dr. Houston in his paper are of long wave-length, and it was supposed that the air might have some considerable effect. But it might have been surmised *à priori* that this would not be the case, as the effect of the finite depth is itself to produce a first approximation of the order $\lambda^{\frac{5}{2}}$. On obtaining the analytical expression for the damping due to the air, there was found to be not the slightest ground for supposing that we have here the explanation of the discrepancy. Although a consideration of the influence of the air is of no use in our present investigation, the expressions for the

* Communicated by the Author.

† Phil. Mag. [6] vol. xvii. pp. 154-164.

‡ Proc. Lond. Math. Soc. ser. 2, vol. vi. p. 396.

period and modulus of decay will be given below §3, on account of their intrinsic interest, and, also, in order that they may be recorded on account of the possibility of their usefulness in some other problem.

It is believed that the real cause of the discrepancy must be looked for along the following lines. Dr. Houstoun in his paper treats of waves of the type $\sin qx \sin ry$, the axes of x and y being horizontal, but his analysis is only suitable for waves of the type $\sin qx$. This is clearly brought into evidence by the fact that he obtains for the period of his waves to a first approximation the expression $2\pi/\{gq^2h\}^{\frac{1}{2}}$, whereas the correct expression is $2\pi/\{g(q^2 + r^2)h\}^{\frac{1}{2}}$, as will be shown later. The same applies to the modulus of decay; he obtains it as a function of q^2 , whereas it is the same function of $(q^2 + r^2)$. But as will be seen later, this in itself does not wholly explain the discrepancy. It will be shown that we cannot adequately take account of the influence of the sides of the trough in damping the motion. Further discussion will be deferred till the question has been analytically considered.

The damping of long waves at the surface of a single liquid.

§2. This problem was considered by Hough*, and he obtained an expression for the modulus of decay τ , which is equivalent to

$$\tau^{-1} = 2k^2\nu + \frac{g^{\frac{1}{4}}k^{\frac{5}{4}}\nu^{\frac{1}{2}}}{2\sqrt{2} \cosh^{\frac{5}{4}}kh \sinh^{\frac{3}{4}}kh},$$

where ν is the kinematical coefficient of viscosity, $k=2\pi/\lambda$, λ being the wave-length, and h is the depth of the liquid. The motion under consideration is two-dimensional. The term depending on ν in the above expression is not correct; it should be

$$k^2\nu \left[2 + \frac{\cosh^2 kh + \sinh^2 kh}{4 \cosh^2 kh \sinh^2 kh} \right].$$

This makes an appreciable difference in the case of *long* waves, since we now have

$$\frac{1}{\tau} = \frac{\nu}{4h^2} + \left\{ \frac{g^{\frac{1}{2}}k\nu}{8h^{\frac{3}{2}}} \right\}^{\frac{1}{2}}.$$

This additional term $\nu/4h^2$ makes a difference of from 6 to 10 per cent. in the theoretical damping of waves such as those

* Proc. Lond. Math. Soc. vol. xxviii. p. 276 (1897).

considered by Dr. Houstoun. This will be seen by comparing the third column in Table II. with the sixth column in the table on page 159 of his paper.

The period equation for this motion is given by Bassett*. For convenience it will be repeated here in the notation of this paper.

$$\left| \begin{array}{cccc} m \cosh mh, & -\sinh mh, & 0 & , & 2vkma \\ -m \sinh mh, & \cosh mh, & a + 2k^2\nu, & , & gk \\ -k \sinh kh, & \cosh kh, & 2k^2\nu & , & gk \\ k \cosh kh, & -\sinh kh, & 0 & , & a^2 + 2\nu k^2 a \end{array} \right| = 0,$$

where $m^2 = k^2 + a/\nu$.

As regards x and t the motion is of the type e^{ikx+at} .

In the approximation it is assumed that mh is large, on account of the smallness of ν , and with this assumption the equation becomes

$$m\{a^2 \cosh kh + gk \sinh kh\} - k\{a^2 \sinh kh + gk \cosh kh\} + 4k^2\nu ma \cosh kh = 0.$$

We obtain by successive approximations

$$a = \pm \{gk \tanh kh\}^{\frac{1}{2}} - \frac{1 \pm i}{2\sqrt{2}} \frac{g^{\frac{1}{2}} k^{\frac{5}{2}} \nu^{\frac{1}{2}}}{\cosh^{\frac{5}{2}} kh \sinh^{\frac{3}{2}} kh} - 2k^2\nu - \frac{k^2\nu(\cosh^2 kh + \sinh^2 kh)}{4 \sinh^2 kh \cosh^2 kh}.$$

The additional term obtained by this more rigorous method of approximation is negligible compared with $2k^2\nu$ when the wave-length is small; it has only an appreciable value when the wave-length is large compared with the depth. This is probably the reason why it was omitted from Hough's approximation.

The influence of a superposed fluid on the damping of long waves.

§3. The results given in this paragraph have little bearing on the problem under consideration for reasons already given, but it may be as well to record the solution of the period equation to a second approximation.

* Treatise on Hydrodynamics, vol. ii. p. 314.

The period of the motion is $2\pi/(\beta - \gamma)$, and the modulus of decay is γ^{-1} , where

$$\beta = \left\{ \frac{gk(\rho - \rho') \sinh kh}{\rho \cosh kh + \rho' \sinh kh} \right\}^{\frac{1}{2}},$$

$$\gamma = \frac{g^{\frac{1}{2}}(\rho - \rho')^{\frac{1}{2}}k^{\frac{5}{2}}\rho}{2\sqrt{2}\{\rho \cosh kh + \rho' \sinh kh\}^{\frac{5}{2}}\sinh^{\frac{3}{2}}kh(\rho\sqrt{\nu'} + \rho'\sqrt{\nu})} \cdot Q,$$

where

$$Q = \rho\{\nu\nu'\}^{\frac{1}{2}} + 2\rho'\nu \cosh^2 kh + \rho'(\nu + \nu') \sinh kh \cosh kh + \rho'(\nu' - \nu) \sinh^2 kh;$$

ρ and ν refer to the lower fluid of depth h ; ρ' and ν' to the superposed fluid.

I have calculated the solution to a third approximation, but the result is too complicated to be given here, and the additional effect derived from it is negligible, even when the second approximation gives an appreciable additional effect.

Some idea of the damping due to air can be derived from a comparison of Tables II. and III.

Waves in a rectangular canal.

§4. There is one difficulty which Dr. Houstoun escapes by his use of *long* waves; it is that the lapping at the sides of the box is virtually neglected, since the vertical motion is not considered. If the problem be attacked in the ordinary way, and the usual conditions of no motion at a fixed boundary be written down, it will be found that the equations are only satisfied by a state of no motion everywhere. The same applies to motion in which slipping at the boundary is allowed, but which is resisted by a traction proportional to the velocity. Hence to discuss the problem at all we are forced to adopt the assumption of a canal with *smooth* sides. The only alternative is to generalize the method for *long* waves.

Suppose the canal to be of depth h and breadth b . Take the origin of coordinates in the undisturbed surface at one side, the axis of z perpendicular to the side and in the surface, and the axis of y vertically upwards.

The equations of motion are

$$\frac{Du}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \nabla^2 u,$$

$$\frac{Dv}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - g + \nu \nabla^2 v,$$

$$\frac{Dw}{Dt} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 w,$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}.$$

If we neglect the squares of velocities, these equations are satisfied by

$$u = \frac{\partial \phi}{\partial x} + \frac{\partial H}{\partial y} - \frac{\partial G}{\partial z},$$

$$v = \frac{\partial \phi}{\partial y} + \frac{\partial F}{\partial z} - \frac{\partial H}{\partial x},$$

$$w = \frac{\partial \phi}{\partial z} + \frac{\partial G}{\partial x} - \frac{\partial F}{\partial y},$$

where

$$\frac{p}{\rho} = -\frac{\partial \phi}{\partial t} - g\eta,$$

provided

$$\nabla^2 \phi = 0,$$

$$\nu \nabla^2 F = \frac{\partial F}{\partial t}, \text{ \&c.}$$

and

$$\frac{\partial \eta}{\partial t} = v_{y=0}.$$

Consider

$$\phi = (A_1 \cosh my + A_2 \sinh my) \cos \frac{n\pi z}{b} e^{ikx+at},$$

$$F = (F_1 \cosh m'y + F_2 \sinh m'y) \sin \frac{n\pi z}{b} e^{ikx+at},$$

$$G = (G_1 \cosh m'y + G_2 \sinh m'y) \sin \frac{n\pi z}{b} e^{ikx+at},$$

$$H = (H_1 \cosh m'y + H_2 \sinh m'y) \cos \frac{n\pi z}{b} e^{ikx+at},$$

where

$$m^2 = k^2 + n^2\pi^2/b^2,$$

$$m'^2 = k^2 + n^2\pi^2/b^2 + a/\nu$$

$$= m^2 + a/\nu.$$

These are not the most general expressions of the type which might have been written down, but they are the only ones which satisfy the conditions of the problem.

The conditions are

$$(1) \quad u=v=w=0 \quad (y=-h),$$

$$(2) \quad w=0 \quad (z=0, z=b),$$

$$(3) \quad \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0, \quad \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 \quad (z=0, z=b),$$

$$(4) \quad \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0, \quad \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0 \quad (y=0),$$

$$(5) \quad \frac{\partial \phi}{\partial t} + g\eta + 2\nu \frac{\partial v}{\partial y} = 0 \quad (y=0).$$

Some of these conditions are satisfied identically ; from the remainder we derive the six equations

$$a(a+2\nu m^2) A_1 + gm A_2 + g U + 2vm' a V = 0, \quad (1)$$

$$2i km A_2 + ik U + m' W = 0, \quad (2)$$

$$2mn \pi/b A_2 + n \pi/b U + m' X = 0, \quad (3)$$

$$-m A_1 \sinh mh + m A_2 \cosh mh + U \cosh m'h - V \sinh m'h = 0, \quad (4)$$

$$ik A_1 \cosh mh - ik A_2 \sinh mh - W \sinh m'h + Y \cosh m'h = 0, \quad (5)$$

$$-n\pi/b A_1 \cosh mh + n\pi/b A_2 \sinh mh + X \sinh m'h - Z \cosh m'h = 0, \quad (6)$$

where

$$U = n\pi/b F_1 - ik H_1,$$

$$V = n\pi/b F_2 - ik H_1,$$

$$W = m' H_1 - n\pi/b G_2,$$

$$X = m' F_2 - ik G_2,$$

$$Y = m' H_2 - n\pi/b G_1,$$

$$Z = m' F_2 - ik G_1.$$

Among these six quantities there exist the two relations

$$m' U + ik W - n\pi/b X = 0, \quad (7)$$

$$m' V + ik Y - n\pi/b Z = 0. \quad (8)$$

From equations (1) to (8) we eliminate $A_1, A_2, U, V, W, X, Y, Z$, and obtain the period equation.

Approximating on the assumption that $m'h$ is large by reason of the smallness of ν , we find that, if we replace k by m in the results for waves propagated in one direction, we obtain the corresponding solution for this problem. This

result could doubtless be obtained much more simply by generalizing Dr. Houstoun's method. But Dr. Houstoun, although he allows no motion over the sides of the box, yet obtains the same period and damping as for waves propagated in one direction. Now, in the case of wave-motion in the rectangular trough which he used, $m^2 = \frac{p^2\pi^2}{a^2} + \frac{q^2\pi^2}{b^2}$, for waves of the type $\sin \frac{p\pi x}{a} \sin \frac{q\pi y}{b}$, where $a=152.4$ cms., $b=20.3$ cms. Hence, for the mode $p=1, q=1$ which he takes as the fundamental one in his solution, $m^2 = \pi^2/b^2$, approximately. This leads to quite different results.

The Table below gives the periods of the different modes and for the same depths as given by Dr. Houstoun.

TABLE I.

Depth in cms.	Observed period.	$p=1,*$ $q=0.$	$p=0,$ $q=1.$	$p=1,$ $q=1.$	$p=3,$ $q=1.$	$p=1,$ $q=3.$
	secs.	secs.	secs.	secs.	secs.	secs.
1	10.1	10.25	1.323	1.310	1.229	.451
2	7.12	7.02	.938	.929	.872	.345
3	5.80	5.68	.779	.771	.725	.313
5	4.49	4.39	.634	.629	.596	.297
7	3.68	3.71	.573	.569	.549	.295
10	3.12	3.11	.534	.531	.509	.294

In this table the observed periods are those given on page 159 of Dr. Houstoun's paper; it will be noticed that they are slightly different from those on page 161, but the difference is quite immaterial for our purpose.

Thus there can be no doubt that in the motion experimentally observed by Dr. Houstoun there existed the mode $p=1, q=0$. The periods of the other modes are so much shorter that they probably passed unnoticed.

Now, if there is no slipping at the sides and there are no surfaces of slip in the liquid, this mode cannot enter into the motion. But it is evident from the fact that the water rises

* This difference between the quantities in this column and the sum of those in the 3rd and 5th columns on p. 159 of Houstoun's paper is due partly to the fact that the quantities in the 5th column are sometimes miscalculated.

and falls at the sides that there must be either slipping at the sides, or else there must exist surfaces of slip near the sides in the liquid, most probably both coexist. This fact shows that we have been unable to analyse the motion correctly. The sides must influence the damping of the motion in any actual case, and this has been neglected in the analysis, the sides having been taken as *smooth*. It will be seen from what has been said above, that we do not know sufficient about the modes in which the water actually vibrates to say that one predominates to such an extent that the damping calculated for that mode should agree with the observed (*cf.* page 158 of Dr. Houstoun's paper). But even if we admit that part of the observed damping may be due to the existence of higher modes of appreciable amplitude, we have only found an explanation in the case of the depths 1, 2, 3 cms. The discrepancy is still unexplained for the depths 5, 7, 10 cms., as will be seen from Table II. Hence the only rational assumption is that the increase of damping is due to the sides. The existence after a time of a surface film such as suggested by Dr. Houstoun would only increase the damping over those values tabulated by him (compare data on pages 159 and 160 of his paper).

Table II. exhibits the reciprocal (κ) of the modulus of decay for the same cases as Table I. gives the periods.

TABLE II.

Depth in cms.	Observed κ .	$p=1,$ $q=0$.	$p=0,$ $q=1$.	$p=1,$ $q=1$.	$p=3,$ $q=1$.	$p=1,$ $q=3$.
1	·048	·0324	·0826	·0849	·0853	·1271
2	·032	·0183	·0481	·0463	·0472	·0521
3	·028	·0133	·0309	·0313	·0316	·0239
5	·023	·0090	·0165	·0168	·0163	·0075
7	·022	·0070	·0093	·0093	·0089	·0052
10	·011	·0053	·0040	·0040	·0037	·0047

With the last row in this table may be compared the values of κ when the damping effect of air is taken into account. These quantities are calculated on the assumption that in the formulæ in §3, we may replace k by m , as for waves in a single liquid.

TABLE III.

10	·011	·0053	·0042	·0042	·0039	·0055
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The quantities in Table II. seem to exhibit no regular increase or decrease from one column to another, but this is due to the conflict between the decay due to the internal friction and the decay due to the bottom of the trough.

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LVI. *Some Relations in Capillarity.* By R. D. KLEEMAN, D.Sc., B.A., Research Student of Emmanuel College, Cambridge, and McKinnon Student of the Royal Society*.

THE potential energy of the surface of a liquid due to the attractions between its molecules has been calculated in general terms by Einstein. Einstein's line of reasoning is as follows. The potential energy of a mass of liquid is first considered. It is assumed that the potential energy of two molecules is given by the expression $P_{\infty} - c_1 c_2 \psi(r)$ where $\psi(r)$ is some function of the distance between the molecules and c_1 and c_2 are constants depending upon the molecules. For n molecules the second term is replaced by

$$\frac{1}{2} \sum_{\alpha=1}^{a=n} \sum_{\beta=1}^{\beta=n} c_{\alpha} c_{\beta} \psi(r_{\alpha}, \beta).$$

If the molecules are identical this becomes

$$\frac{1}{2} c^2 \sum_{\alpha=1}^{a=n} \sum_{\beta=1}^{\beta=n} \psi(r_{\alpha}, \beta).$$

It is next assumed that the potential is the same as if the matter were distributed evenly in space. The potential energy may now be written

$$P = P_{\infty} - \frac{1}{2} c^2 N^2 \iiint d\tau d\tau' \psi(r_{d\tau, d\tau'}),$$

where $d\tau$ and $d\tau'$ are elements of volume occupied by matter and N is the number of molecules in unit volume. If each molecule consists of several atoms, we may put $c = \sum c_a$ as is

* Communicated by the Author.

usually done in gravitational investigations. Thus

$$P = P_{\infty} - \frac{1}{2} \frac{(\Sigma c_a)^2}{v^2} \iint d\tau, d\tau' \psi(r_{d\tau, d\tau'}),$$

where v denotes the molecular volume. This expression, on transforming the integral to rectangular coordinates, becomes

$$P = P_{\infty} - \frac{1}{2} \frac{(\Sigma c_a)^2}{v^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx dy dz \psi(\sqrt{x^2 + y^2 + z^2}),$$

or, if we denote the integral by κ

$$P = P_{\infty} - \frac{\kappa}{v^2} (\Sigma c_a)^2.$$

Einstein then expresses the potential energy of a liquid of volume V and surfaces S as

$$P = P_{\infty} - \frac{\kappa}{v^2} (\Sigma c_a)^2 V - \frac{\kappa'}{v^2} (\Sigma c_a)^2 S, \quad . \quad . \quad . \quad (1)$$

where

$$\kappa' = \int_{x_1=0}^{x_1=1} \int_{y_1=0}^{y_1=1} \int_{z_1=0}^{z_1=1} \int_{x=-\infty}^{x=\infty} \int_{y=-\infty}^{y=\infty} \int_{z=-\infty}^{z=\infty} dx_1 dy_1 dz_1 dx dy dz \psi \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2}.$$

The potential energy E per unit surface is the coefficient of S in (1), or

$$E = \frac{\kappa'}{v^2} (\Sigma c_a)^2 = \frac{\kappa' \rho^2}{m^2} (\Sigma c_a)^2, \quad . \quad . \quad . \quad (2)$$

since $v = \frac{m}{\rho}$, where ρ is the density and m the molecular weight of the liquid.

The potential energy of the surface-film of a liquid is also given by $\left(\lambda - T \frac{d\lambda}{dT} \right)$ where λ is the surface-tension at the temperature T . This follows from thermodynamics. When the surface is increased by 1 cm.² the external work done on the film is λ and the heat absorbed, according to the Helmholtz free-energy equation, to keep the temperature constant, is $-T \frac{d\lambda}{dT}$, and thus the potential energy is increased by

$$\left(\lambda - T \frac{d\lambda}{dT} \right).$$

Consequently,

$$\lambda - T \frac{d\lambda}{dT} = \frac{\kappa' \rho^2}{m^2} (\Sigma c_a)^2. \quad (3)$$

Einstein then proceeded to find the values of c_a for a number of atoms on the assumption that κ' is a constant, and that $(\lambda - T \frac{d\lambda}{dT})$ is independent of the temperature. The values of c_a for a few atoms were calculated from the physical constants of a large number of compounds involving these atoms. That the values of c_a should agree well although the number of compounds whose constants were employed is far larger than the minimum number necessary to obtain c_a , would be strong evidence in favour of the validity of the formula and the underlying assumptions.

Table I. contains in the third and sixth vertical columns the values of Σc_a for a number of substances calculated by means of equation (2). The fourth and seventh vertical columns give the values calculated by substituting the values of c_a in Σc_a from the first column in the table. The values of c_a were obtained from the third and sixth columns by the method of least squares. It will be seen that on the whole the values in the two columns agree fairly well.

TABLE I.*

C_a .	Formula of liquid.	Experimental ΣC_a .	Calculated ΣC_a .	Formula of liquid.	Experimental ΣC_a .	Calculated ΣC_a .
H = -1.6	C ₁₀ H ₁₆	510	524	C ₂ OHCl ₃	358	335
C = 55.0	CO ₂ H ₂	140	145	C ₇ H ₅ OC ₁	462	484
O = 46.8	C ₂ H ₄ O ₂	193	197	C ₇ H ₆ Cl ₂	492	495
Cl = 60	C ₃ H ₆ O ₂	250	249	Br ₂	217	304
Br = 152	C ₄ H ₈ O ₂	309	301	C ₂ H ₅ Br	251	254
I = 198	C ₅ H ₁₀ O ₂	365	352	C ₃ H ₇ Br	311	306
	C ₄ H ₆ O ₃	350	350	C ₃ H ₇ Br	311	306
	C ₆ H ₁₀ O ₄	505	501	C ₃ H ₅ Br	302	309
	C ₃ H ₈ O ₂	494	520	C ₄ H ₅ Br	353	354
	C ₉ H ₁₀ O ₂	553	562	C ₅ H ₁₁ Br	425	410
	C ₆ H ₁₀ O ₂	471	454	C ₆ H ₅ Br	411	474
	C ₇ H ₉ O	422	419	C ₇ H ₇ Br	421	526
	C ₈ H ₁₀ O	479	470	C ₂ H ₄ Br ₂	345	409
	C ₈ H ₁₀ O ₂	519	517	C ₃ H ₆ Br ₂	395	461
	C ₅ H ₄ O ₂	345	362	C ₂ H ₅ I	288	300
	C ₅ H ₁₅ O	348	305	C ₃ H ₇ I	343	352
	C ₁₀ H ₁₄ O	587	574	C ₃ H ₇ I	357	352
	C ₆ H ₅ Cl	385	379	C ₃ H ₅ I	338	355
	C ₇ H ₇ Cl	438	434	C ₄ H ₉ I	428	403
	C ₇ H ₇ Cl	440	434	C ₅ H ₁₁ I	464	455
	C ₃ H ₅ OC ₁	270	270			

* The repetition of a formula indicates an isomeric compound.

The assumption made by Einstein that the potential energy of the surface-film of a liquid is independent of the temperature is, however, not true. The potential energy for different substances over a wide range of temperature has been calculated by Whittaker* from surface-tension values. He finds that the potential energy decreases with increase of temperature, the decrease being about 20 to 25 per cent. over a range of 100° . If the potential energy E in equation (2) is assumed constant, then κ' cannot be constant, for there is no other quantity in the equation to balance the variations in ρ as m and Σc_a are from their nature absolute constants. It is very unlikely then that κ' should be a constant.

It is now necessary, from the above considerations, to modify Einstein's formula. Let us assume that κ' in equation (2) is, if not a constant, a function which is the same for all liquids at corresponding states. We shall then compare the results of this assumption with experimental results. Integrating the equation, we have

$$\lambda = CT - \frac{T}{m^2} (\Sigma c_a)^2 \int \frac{\rho^2 \kappa'}{T^2} dT,$$

where C is an arbitrary constant. Let us write $\rho = \alpha \rho_c$ and $T = \beta T_c$ where, as usual, ρ_c and T_c denote the critical density and the critical temperature.

On substitution we have

$$\lambda = CT - \frac{T}{m^2} (\Sigma c_a)^2 \frac{\rho_c^2}{T_c} \int \frac{\alpha^2 \kappa' d\beta}{\beta^2}.$$

Making use of the fact that $\lambda = 0$ when $T = T_c$ we obtain the value of the constant

$$C = \frac{(\Sigma c_a)}{m^2} \frac{\rho_c^2}{T_c} k,$$

where k is another constant.

Therefore

$$\lambda = \frac{(\Sigma c_a)^2}{m^2} \left\{ \frac{T \rho_c^2}{T_c} k - \frac{T \rho_c^2}{T_c} \int \frac{\alpha^2 \kappa'}{\beta^2} d\beta \right\}.$$

With the aid of $\rho = \alpha \rho_c$ and $T = \beta T_c$,

$$\lambda = \frac{(\Sigma c_a)^2 \rho^2}{m^2} \left\{ \frac{d\beta}{\alpha^2} k - \frac{\beta}{\alpha^2} \int \frac{\alpha^2 \kappa'}{\beta^2} d\beta \right\} = \frac{(\Sigma c_a)^2}{m^2} \rho^2 \kappa'', \quad (4)$$

where κ'' involves only quantities relating to corresponding states. For different liquids at corresponding states κ'' is therefore the same. The expression for λ given by equation (4) is, it will be observed, similar to the expression for E given by equation (2).

* Proc. Roy. Soc. A. vol. lxxx. p. 21 (1908).

Combining equations (2) and (4) we have

$$\frac{E}{\lambda} = \frac{\kappa'}{\kappa''}.$$

If the assumption that κ' , if not a constant, is a function which has the same value for all liquids at corresponding states is true, then $\frac{E}{\lambda}$ should be the same for liquids at corresponding states. Values of $\frac{E}{\lambda}$ at various corresponding temperatures are given in Table II. $E_1, E_2 \dots E_5$ and $\lambda_1, \lambda_2 \dots \lambda_5$ are the values of the potential energy and the surface-tension at the corresponding temperatures $\frac{2}{3}T_c, \frac{1}{2}\frac{1}{4}T_c, \frac{3}{4}T_c, \frac{5}{7}\frac{9}{2}T_c$, and $\frac{8}{9}T_c$. Table III. gives the values of E_1 &c., λ_1 &c., and ρ_1 &c. at

TABLE II.

Name of liquid.	$\frac{E_1}{\lambda_1}$	$\frac{E_2}{\lambda_2}$	$\frac{E_3}{\lambda_3}$	$\frac{E_4}{\lambda_4}$	$\frac{E_5}{\lambda_5}$
Ether	3.48	4.03	4.76	6.73	11.45
Methyl formate	3.39	3.90	4.67	6.70	
Carbon tetrachloride	3.45	3.96	4.61	6.51	11.35
Benzene	3.41	3.93	4.69	6.53	10.80
Chloro benzene.....	3.36	3.96	4.66	6.22	

TABLE III.

Name of liquid.	$T_c 2/3$	λ_1	E_1	ρ_1	$T_c 17/24$	λ_2	E_2	ρ_2
Ether	311.7	14.19	49.36	.6907	331.1	12.02	48.50	.6633
Methyl formate.....	324.7	19.80	67.22	.9283	344.9	17.20	67.09	.8937
Carbon tetrachloride .	371.0	16.69	57.64	1.4385	393.9	14.23	56.33	1.3702
Benzene	374.3	17.87	60.93	.7826	397.7	15.18	60.42	.7634
Chloro benzene	422.0	17.78	59.80	.9611	448.3	15.14	60.00	.9289
	$T_c 3/4$	λ_3	E_3	ρ_3	$T_c 59/72$	λ_4	E_4	ρ_4
Ether	350.6	9.92	47.27	.6432	383.2	6.61	44.5	.5938
Methyl formate	365.2	13.98	65.30	.8598	399.1	9.38	62.83	.7942
Carbon tetrachloride .	417.0	11.82	54.44	1.3356	455.7	8.00	52.07	1.2395
Benzene	421.1	12.57	58.88	.7336	460.3	8.43	54.93	.6798
Chloro benzene	474.7	12.56	58.51	.8924	518.8	8.42	52.34	.8263
	$T_c 8/9$	λ_5	E_5	ρ_5				
Ether	415.5	3.55	40.65	.5333				
Methyl formate	432.9	5.06	...	.7139				
Carbon tetrachloride	494.2	4.36	49.5	1.1183				
Benzene	499.1	4.68	50.52	.6139				
Chloro benzene.....	562.7	4.57						

these temperatures. These values were obtained by linear interpolation from Whittaker's tables*. Table II. shows that the ratio $\frac{E}{\lambda}$ at corresponding temperatures is very approximately constant.

Equation (2) affords another test of the validity of the assumption that κ' is the same for all liquids at corresponding states. If the assumption is true then $\frac{E_1\rho_2^2}{E_2\rho_1^2}$ should be a constant for liquids at corresponding temperatures. Substituting from Table III. we obtain Table IV.

TABLE IV.

Name of liquid.	$\frac{E_1\rho_2^2}{E_2\rho_1^2}$	$\frac{E_1\rho_3^2}{E_3\rho_1^2}$	$\frac{E_1\rho_4^2}{E_4\rho_1^2}$	$\frac{E_1\rho_5^2}{E_5\rho_1^2}$
Ether	·939	·906	·820	·721
Methyl formate	·956	·884	·842	
Carbon tetrachloride ...	·928	·910	·847	·703
Benzene	·959	·909	·837	·742
Chloro benzene	·931	·991	·845	

which shows that the relation is true for several sets of corresponding temperatures. The ratio is not the same, however, for each set of corresponding temperatures, as would be the case if Einstein's assumption of a constant κ' were true. From equation (4) we obtain another test of the assumption. $\frac{\lambda_1\rho_2^2}{\lambda_2\rho_1^2}$ should be a constant for all liquids at corresponding temperatures. The agreement of this with experimental results is shown in Table V. Though the ratio is constant

TABLE V.

Name of liquid.	$\frac{\lambda_1\rho_2^2}{\lambda_2\rho_1^2}$	$\frac{\lambda_1^2\rho_3^2}{\lambda_3^2\rho_1^2}$	$\frac{\lambda_1^2\rho_4^2}{\lambda_4\rho_1^2}$	$\frac{\lambda_1\rho_5^2}{\lambda_5\rho_1^2}$
Ether	1·09	1·15	1·59	2·38
Methyl formate	1·07	1·13	1·54	2·31
Carbon tetrachloride ...	1·06	1·13	1·55	2·31
Benzene	1·12	1·17	1·60	2·55
Chloro benzene	1·10	1·13	1·56	

* *Loc. cit.*

for each set of corresponding temperatures it varies considerably from set to set.

The relation $\frac{E}{\lambda} = \text{constant}$ for corresponding temperatures which has just been proved, can be deduced from a well-known relation connecting surface-tension with other quantities. The deduction will also bring some other important relations to light.

Ramsay, following Eötvös, has shown that

$$\lambda v^{\frac{2}{3}} = \lambda \frac{m^{\frac{2}{3}}}{\rho^{\frac{2}{3}}} = A (T_c - T - a), \quad . \quad . \quad . \quad (5)$$

where A is a constant whose value is about 2.1 for all liquids, and a is another constant whose value is about 6. Let ρ_0 and λ_0 denote the density and fictitious surface-tension at absolute zero. Then

$$\lambda_0 = \frac{\rho_0^{\frac{2}{3}} A}{m^{\frac{2}{3}}} (T_c - a) = \frac{\rho_0^{\frac{2}{3}}}{m^{\frac{2}{3}}} A T_c,$$

since a is small in comparison with T_c . If ρ_0 is put equal to $b\rho_c$ we have

$$\lambda_0 = \frac{\rho_c^{\frac{2}{3}}}{m^{\frac{2}{3}}} b^{\frac{2}{3}} A T_c.$$

For any temperature other than absolute zero equation (5) may be written

$$\mu \lambda_0 = \frac{(\rho_c \alpha)^{\frac{2}{3}}}{m^{\frac{2}{3}}} A (T_c - T_c \beta - a).$$

We may neglect a except when we are dealing with temperatures quite close to the critical temperature. Omitting a , and combining this equation with the one preceding, we obtain

$$\mu = \frac{\alpha^{\frac{2}{3}}}{b^{\frac{2}{3}}} (1 - \beta). \quad . \quad . \quad . \quad (6)$$

For different liquids α and β are the same at corresponding states, and since absolute zero is a corresponding state for all liquids b is the same, hence μ is the same for all liquids under the same circumstances. This leads to the conclusion

that at corresponding states the surface-tensions of different liquids are equal to the same fraction of their surface-tension at absolute zero. Then the ratio of the surface-tensions of a liquid at two different temperatures is the same for all liquids, if corresponding temperatures are taken. Table VI. bears

TABLE VI.

Name of liquid.	$\frac{\lambda_1}{\lambda_2}$	$\frac{\lambda_1}{\lambda_3}$	$\frac{\lambda_1}{\lambda_4}$	$\frac{\lambda_1}{\lambda_5}$
Ether	1.153	1.430	2.146	3.996
Methyl formate	1.152	1.421	2.111	3.913
Carbon tetrachloride ...	1.173	1.412	2.086	3.828
Benzene	1.150	1.422	2.119	3.818
Chloro benzene	1.174	1.416	2.112	3.891

this out exceedingly well. The variation of the surface-tension of liquids with the temperature is more simply and intelligibly expressed in this way than by equation (5).

The equations

$$E = \lambda - T \frac{d\lambda}{dT}, \quad \lambda = \mu \lambda_0, \quad T = \beta T_c,$$

when combined yield

$$E = \lambda_0 \left\{ \mu - \beta \frac{d\mu}{d\beta} \right\}.$$

This shows that

$$\frac{E}{\lambda} = \frac{1}{\mu} \left\{ \mu - \beta \frac{d\mu}{d\beta} \right\}. \quad . \quad . \quad . \quad . \quad . \quad (7)$$

At corresponding temperatures the right-hand side is the same for all liquids, a result which has been shown to be true directly (Table II.).

We can now find an expression for κ' from equations (2) and (7). We have

$$\kappa' = \frac{m^2}{\rho^2} \frac{E}{(\Sigma c_a)^2} = \frac{m^2 \lambda_0}{\rho_c^2 (\Sigma c_a)^2} \frac{1}{\alpha^2} \left\{ \mu - \beta \frac{d\mu}{d\beta} \right\}.$$

We have seen that κ' is a function which has the same value for different liquids at corresponding temperatures. It

follows from this that $\frac{\lambda_0 m^2}{\rho_c^2 (\sum c_a)^2}$ must be a constant k for all liquids in order that

$$\kappa' = \frac{k}{\alpha^2} \left\{ \mu - \beta \frac{d\mu}{d\beta} \right\},$$

which makes κ' assume the same value for all liquids at corresponding states.

Similarly the function represented by κ'' can be determined. From equations (4) and (6) and the relation $\lambda = \mu \lambda_0$ we have

$$\kappa'' = \frac{m^2}{\rho^2} \frac{\lambda}{(\sum c_a)^2} = \frac{m^2}{\alpha^2 \rho_c^2} \frac{\lambda_0 \mu}{(\sum c_a)^2} = \frac{m^2 \lambda_0}{\rho_c^2 (\sum c_a)^2} \frac{1 - \beta}{\alpha^{\frac{4}{3}} b^{\frac{2}{3}}}.$$

Since κ'' must be a function which has the same value for different liquids at corresponding states, then $\frac{m^2 \lambda_0}{\rho_c^2 (\sum c_a)^2}$ must be equal to some constant k which is the same for all liquids.

Thus we have

$$\kappa'' = k \frac{(1 - \beta)}{\alpha^{\frac{4}{3}} b^{\frac{2}{3}}}, \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and this is the same for all liquids at corresponding states.

From equation (2) we see that any change in κ'' is due to a change in $\frac{E}{\rho^2}$, since m and $(\sum c_a)^2$ are constants which do not vary under any conditions. In Table VII. (p. 500) the values of $\frac{E}{\rho^2 T}$ are calculated for five liquids at temperatures differing by 10° over a range of about 140° C. The constancy of $\frac{E}{\rho^2 T}$ shows that $\frac{E}{\rho^2}$ is proportional to T , or the function κ' is proportional, at least approximately so, to the absolute temperature. This is a more convenient expression for the variation of κ' with temperature than that given above.

The values of $(\kappa')^{\frac{1}{2}} c_a$ for a number of atoms have been calculated from the equation $\kappa' (\sum c_a)^2 = \frac{E m^2}{\rho^2}$, and are given

TABLE VII.

Ether.				Chlorobenzene.			
T.	ρ .	E.	$\frac{E}{\rho^2 T}$.	T.	ρ .	E.	$\frac{E}{\rho^2 T}$.
313	·6894	49·1	·330	423	·9599	59·8	·153
323	·6764	48·9	·333	433	·9480	59·9	·154
333	·6658	48·4	·328	443	·9354	60·0	·155
343	·6532	47·8	·327	453	·9224	60·0	·156
353	·6402	47·1	·326	463	·9091	59·3	·155
363	·6250	46·1	·326	473	·8955	58·6	·154
373	·6105	45·5	·327	483	·8802	58·1	·155
383	·5942	44·7	·331	493	·8672	57·4	·155
393	·5764	43·8	·336	503	·8518	56·7	·155
403	·5580	42·6	·339	513	·8356	55·9	·156
413	·5385	41·1	·343	523	·8196	55·1	·157
423	·5179	39·3	·346	533	·8016	53·8	·157
Carbon tetrachloride.				Benzene.			
T.	ρ .	E.	$\frac{E}{\rho^2 T}$.	T.	ρ .	E.	$\frac{E}{\rho^2 T}$.
363	1·4554	57·9	·753	353	·8145	59·5	·254
373	1·4343	57·5	·750	363	·8041	60·2	·256
383	1·4124	56·8	·744	373	·7927	60·9	·260
393	1·3902	56·4	·743	383	·7809	61·1	·262
403	1·3680	55·6	·737	393	·7692	60·7	·261
413	1·3450	54·8	·734	403	·7568	60·1	·260
423	1·3215	53·9	·727	413	·7440	59·4	·260
433	1·2982	53·1	·727	423	·7310	58·8	·260
443	1·2734	52·6	·733	433	·7185	58·0	·260
453	1·2470	52·2	·741	443	·7043	57·2	·260
463	1·2192	51·7	·752	453	·6906	56·1	·260
473	1·1888	50·8	·759	463	·6758	55·0	·260
483	1·1566	50·3	·778	473	·6605	54·0	·262
493	1·1227	49·8	·801	483	·6432	52·9	·266
503	1·0857	47·3	·797	493	·6255	51·5	·267
				503	·6065	49·9	·269
				513	·5851	48·1	·274
Methyl formate.							
T.	ρ .	E.	$\frac{E}{\rho^2 T}$.	T.	ρ .	E.	$\frac{E}{\rho^2 T}$.
303	·9598	69·4	·249	373	·8452	64·6	·242
313	·9447	68·8	·246	383	·8264	63·8	·244
323	·9294	68·2	·244	393	·8070	62·9	·246
333	·9133	67·5	·243	403	·7860	61·8	·255
343	·8968	67·3	·244	413	·7638	60·4	·251
353	·8803	66·2	·242	423	·7403	58·8	·254
363	·8634	65·5	·242				

in the fourth column of Table VIII., the values of $\frac{E^{\frac{1}{2}}m}{\rho}$ used being given in the second column. The first four liquids supplied the data for the calculation of $(\kappa')^{\frac{1}{2}}\Sigma c_a$ for the four atoms involved. Then the $(\kappa')^{\frac{1}{2}}\Sigma c_a$ for chlorobenzene was calculated from the $(\kappa')^{\frac{1}{2}}c_a$ of its constituent atoms, and is given in the third column of the table, while the actual

TABLE VIII.

Name of liquid.	$\frac{E^{\frac{1}{2}}m}{\rho}$.	$(\kappa')^{\frac{1}{2}}\Sigma c_a$.	$(\kappa')^{\frac{1}{2}}c_a$.	c_a .
Ether.....	752.6		H= 19.71	H=1
Methyl formate	530.1		C=109.96	C=5.581
Carbon tetrachloride	813.0		O=112.66	O=6.224
Benzene.....	778.0		Cl=175.76	Cl=8.919
Chloro benzene	905.0	935.6		

TABLE IX.

Name of liquid.	$\frac{\lambda^{\frac{1}{2}}m}{\rho}$.	$(\kappa'')^{\frac{1}{2}}\Sigma c_a$.	$(\kappa'')^{\frac{1}{2}}c_a$.	c_a .
Ether $C_4H_{10}O$	404.2	404.2	H=10.9	H=1
Methyl formate $C_2H_4O_2$	287.7	287.7	C=59.4	C=5.45
Carbon tetrachloride CCl_4 ...	437.5	437.5	O=57.6	O=5.284
Benzene C_6H_6	421.6	421.6	Cl=94.5	Cl=8.67
Chloro benzene C_6H_5Cl	493.7	505.4		
Ethyl acetate $C_4H_8O_2$	435.1	440.0		
Propyl formate $C_4H_8O_2$	428.0	440.0		
Methyl propionate $C_4H_8O_2$...	428.8	440.0		
Propyl acetate $C_5H_{10}O_2$	502.5	521.2		
Ethyl propionate $C_5H_{10}O_2$...	500.8	521.2		
Methyl butyrate $C_5H_{10}O_2$...	500.0	521.2		
Methyl isobutyrate $C_5H_{10}O_2$	497.6	521.2		

experimental value is given in the second column. The agreement is fairly good. It should be added that this table refers to a temperature of $\frac{2}{3}T_c$. The last column of the table gives the values of c_a , the c_a for H being put equal to unity. Since the values of c_a for different liquids can only be compared if they are calculated from data referring to corresponding states, we are limited to a few liquids since the

surface-tension has been determined over a limited range of temperatures in some cases so that a corresponding temperature referring to each liquid cannot be found, and the values of c_a for a number of atoms cannot therefore be obtained under this restriction. A formula involving Σc_a will be developed at a later stage which is not restricted to the same extent in its application.

Similarly, the values of $(\kappa'')^{\frac{1}{2}} c_a$ for a number of atoms have been calculated from the equation $(\kappa'')(\Sigma c_a)^2 = \frac{\lambda m^2}{\rho^2}$, and are given in the fourth column of Table IX. (p. 501). As before, the first four liquids supplied the data which were taken at $\frac{2}{3}T_c$.

The second column gives the values of $\frac{\lambda^{\frac{1}{2}} m}{\rho}$ obtained from experiment, while the third column gives the theoretically equivalent values of $(\kappa'')^{\frac{1}{2}} \Sigma c_a$. The two columns agree fairly well. The last column of the table gives the values of c_a , that for H being put equal to unity. These values agree fairly well with those obtained in Table VIII., except for oxygen. If the values of c_a were in both cases determined from a larger number of liquids by a method of least squares the agreement would probably be much better.

The equation (4) $\lambda = \frac{\kappa'' \rho^2}{m^2} (\Sigma c_a)^2$ is further interesting since it can be connected with other investigations in capillarity. An expression for the surface-tension of a liquid is usually given in text-books on physics deduced from the work necessary to produce a new liquid surface.

Consider the work required to separate two portions of a liquid A and B, the resulting surfaces to be plane. Let B be divided up into slices parallel to the interface, then the work done in removing the slice whose thickness is dz , and whose height above the plane is z , is per unit of area equal to

$$\rho^2 dz \int_z^\infty \psi(x) dx,$$

where $\rho\psi(x)$ denotes the attraction due to a mass of liquid of density ρ bounded by a flat surface on unit mass placed at a distance x above the surface. The work required to remove the whole of the liquid B standing on unit area is

$$\int_0^\infty \rho^2 v dz, \quad \text{where} \quad v = \int_z^\infty \psi(x) dx.$$

Integrating by parts we have

$$[\rho^2 z v]_0^\infty - \int_0^\infty \rho^2 z \frac{dv}{dz} dz.$$

The term within the brackets vanishes at both limits and

$$\frac{dv}{dz} = -\psi(z).$$

Therefore the work required is

$$\rho^2 \int_0^\infty z \psi(z) dz.$$

The expenditure of this amount of work has resulted in the production of two surfaces each of unit area. Hence the v work expended per unit area (*i. e.* the surface-tension) is

$$\lambda = \frac{1}{2} \rho^2 \int_0^\infty z \psi(z) dz.$$

Comparing this with equations (4) and (8) we see that

$$\frac{1}{2} \int_0^\infty z \psi(z) dz = \frac{\kappa''}{m^2} (\Sigma c_a)^2 = k \frac{(1-\beta)}{\alpha^{\frac{4}{3}} b^{\frac{2}{3}}} \frac{(\Sigma c_a)^2}{m^2}.$$

The latter expression may be conveniently divided into two parts:—

$$\frac{k(1-\beta)}{\alpha^{\frac{4}{3}} b^{\frac{2}{3}}} \quad \text{and} \quad \frac{(\Sigma c_a)^2}{m^2}.$$

The first part depends only upon the temperature, and is the same for different liquids at corresponding temperatures. The latter part involves only quantities which are fundamental and do not depend upon temperature or anything else except the nature of the atom. The first part therefore expresses the effect of the change in the geometrical configuration of the molecules accompanying rise of temperature on the surface-tension of the liquid.

Since the function κ' is approximately proportional to the absolute temperature it may be put equal to $W\beta$ where W is an appropriate constant. Hence

$$E = \frac{W\beta\rho^2}{m^2} (E c_a)^2. \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Whittaker has obtained a relation between the potential energy, the internal latent heat of evaporation, and the absolute temperature. He shows that

$$LKT = E,$$

where K is a constant which is different for different liquids.

The writer * has shown that the value of the constant K is

$$\frac{m^{\frac{1}{3}}\rho_c^{\frac{2}{3}}}{T_c} 5.57 \times 10^{-1}.$$

Hence Whittaker's relation becomes

$$E = \frac{TLm^{\frac{1}{3}}\rho_c^{\frac{2}{3}}}{T_c} 5.57 \times 10^{-1}. \quad . \quad . \quad . \quad (10)$$

With the aid of (9) we get

$$L = \frac{\rho^2 W (\Sigma c_a)^2}{m^{\frac{2}{3}} \rho_c^{\frac{2}{3}} 5.57 \times 10^{-1}} = \rho^2 M. \quad . \quad . \quad . \quad (11)$$

M involves constants only, hence L/ρ^2 must be a constant. The agreement of this conclusion with experiment is shown in Table X. The agreement is not very good, the value rising slightly with increase of temperature.

TABLE X.

Ether.				Chloro benzene.			
T.	L.	ρ .	$M = \frac{L}{\rho^2}$.	T.	L.	ρ .	$M = \frac{L}{\rho^2}$.
273	86.16	.7362	15.90	403	67.07	.9836	69.31
323	73.01	.6764	15.96	453	61.46	.9224	72.24
373	60.33	.6105	16.18	503	55.29	.8672	73.52
423	44.38	.5179	16.54	553	47.17	.7834	76.85
Methyl formate.				Carbon tetrachloride.			
T.	L.	ρ .	$M = \frac{L}{\rho^2}$.	T.	L.	ρ .	$M = \frac{L}{\rho^2}$.
273	113.2	1.0032	112.6	273	48.35	1.6327	18.11
323	99.51	.9294	115.2	373	39.68	1.4343	19.31
373	82.43	.8452	115.3	423	34.42	1.3215	19.70
423	64.03	.7403	116.9	473	38.22	1.1888	19.95
473	33.18	.5658	103.6	523	19.85	.9980	19.93

* Phil. Mag. July 1909.

Benzene.			
T.	L.	ρ .	$M = \frac{L}{\rho^2}$.
273	100.10	.9001	123.6
373	81.98	.7927	130.5
423	71.93	.7310	134.6
473	59.95	.6605	137.5
523	43.39	.5609	137.9

The deduction of (11) involves the relation that κ' is proportional to the absolute temperature or $\kappa' = W\beta$ which is only approximately true. Hence a better constancy in the value of M or L/ρ^2 could scarcely be expected.

Table XI. contains in the second column the mean value

TABLE XI.

Name of liquid.	M.	m .	ρ_c .	$\frac{Mm^{\frac{7}{3}}\rho_c^{\frac{2}{3}}}{(\Sigma c_a)^2}$.
Ether.....	161.4	74	.2604	1018
Methyl formate	112.7	60	.3489	1032
Carbon tetrachloride	19.4	154	.5576	989.7
Benzene.....	132.8	78	.3045	1002
Chloro benzene	72.98	112.5	.3654	980.6

of the values of M for each liquid considered in Table X. A rearrangement of equation (11) yields

$$\frac{Mm^{\frac{7}{3}}\rho_c^{\frac{2}{3}}}{(\Sigma c_a)^2} = \frac{W}{5.57 \times 10^{-1}},$$

where everything is constant. The values of the left-hand side are calculated in Table XI. with the aid of the values of c_a obtained from Table VIII. The resulting value is very nearly the same for the five liquids, as should be the case since W approximately is independent of the nature of the liquid. If we take the mean value of the constant as 1004.5 the equation for the internal latent heat becomes

$$L = \frac{\rho^2(\Sigma c_a)^2}{m^{\frac{7}{3}}\rho_c^{\frac{2}{3}}} 1004.5.$$

This equation may be used to find the approximate internal latent heat of evaporation of a liquid.

Some further results of interest can be deduced from some of the equations given. Thus equation (10) states

$$\frac{LTm^{\frac{1}{3}}\rho_c^{\frac{2}{3}}}{T_c} 5.57 \times 10^{-1} = E,$$

which may be written

$$\frac{L\beta m^{\frac{1}{3}}\rho_c^{\frac{2}{3}}}{\alpha^{\frac{2}{3}}} 5.57 \times 10^{-1} = E,$$

since $T = \beta T_c$ and $\rho = \rho_c \alpha$. Therefore

$$\frac{Lm^{\frac{1}{3}}\rho_c^{\frac{2}{3}}}{E} = \frac{\alpha^{\frac{2}{3}}}{\beta 5.57 \times 10^{-1}} = \text{constant}$$

for liquids at corresponding states. And since $E/\lambda = \text{constant}$ at corresponding states, it further follows that

$$\frac{Lm^{\frac{1}{3}}\rho_c^{\frac{2}{3}}}{\lambda} = \text{constant}$$

for corresponding states.

Table XII. contains the values of the expressions on the left-hand side of the two last equations for five liquids at

TABLE XII.

Name of liquid.	L_1 .	$\frac{L_1 \rho_1^{\frac{2}{3}} m_1^{\frac{1}{3}}}{E_1}$.	$\frac{L_1 \rho_1^{\frac{2}{3}} m_1^{\frac{1}{3}}}{\lambda_1}$.
Ether.....	75.44	60.6	17.4
Methyl formate	92.85	59.3	17.5
Carbon tetrachloride	39.87	56.3	16.3
Benzene.....	81.73	56.5	16.6
Chloro benzene	65.88	58.5	17.4

temperatures corresponding to $T_c/3$. The data used in the calculations are contained in part in the same table, the remainder being given in Table III. It will be seen that the value of each expression is practically independent of the nature of the liquid to which it refers, as is indicated by the above equations.

An equation may also be deduced from the foregoing equations which contains Σc_a but not E or λ . Thus from equations (2) and (10) we have

$$\frac{LTm^{\frac{1}{3}}\rho_c^{\frac{2}{3}}}{T_c} 5.57 \times 10^{-1} = \frac{\kappa' \rho^2}{m^2} (\Sigma c_a)^2.$$

Substituting for T_c and ρ_c from the equations $T = \beta T_c$, $\rho = \alpha \rho_c$, and rearranging the equation we have

$$\frac{L_1^{\frac{1}{2}} m^{\frac{7}{6}}}{\rho_1^{\frac{2}{3}}} = \left(\frac{\kappa' \alpha^{\frac{2}{3}}}{\beta 5 \cdot 57 \times 10^{-1}} \right)^{\frac{1}{2}} \Sigma c_a = B \Sigma c_a, \quad \dots (12)$$

where B is a function which has the same value for different liquids at corresponding states. This equation is more useful in finding values of Σc_a than equation (2) or (4), since it involves data which are available for a larger number of liquids.

The agreement of this equation with the facts is shown in Table XIII. The fifth column of the table contains the value

TABLE XIII.

Name of liquid.	m .	L_1 .	ρ_1 .	$\frac{L_1^{\frac{1}{2}} m^{\frac{7}{6}}}{\rho_1^{\frac{2}{3}}}$.	$B \Sigma c_a$.	$B c_a$.
Ether $C_4H_{10}O$	74	75.44	.6907	1686	...	H = 45.4
Methyl formate $C_2H_4O_2$	60	92.85	.9283	1202	...	C = 240.6
Carbon tetrachloride CCl_4 ...	142	39.87	1.4385	1766	...	O = 269.6
Benzene C_6H_6	78	81.73	.7826	1716	...	F = 261.4
Fluor-benzene C_6H_5Fl	96.09	69.71	.9233	1932	...	Cl = 381.4
Bromo-benzene C_6H_5Br	157	48.84	1.2792	2154	...	Br = 483.4
Iodo-benzene C_6H_5I	203.9	40.79	1.5347	2372	...	Sn = 666.4
Stannic chloride $SnCl_4$	260.8	27.11	1.9597	2192	...	I = 703.4
Di-isobutyl C_8H_{18}	114	66.05	.6333	2762	2742	
Chloro-benzene C_6H_5Cl	112.5	65.88	.9611	2060	2052	
Pentane C_5H_{12}	72	75.47	.6057	1784	1748	c_a .
Heptane C_7H_{16}	100	71.40	.6247	2494	2170	
Octane C_8H_{18}	114	66.05	.6299	2770	2742	H = 1
Hexane C_6H_{14}	86	72.61	.6167	1941	1988	C = 5.30
Acetic acid $C_2H_4O_2$	60	86.08	.9315	1155	1202	O = 5.94
Iso-pentane C_5H_{12}	72.1	71.09	.6048	1734	1748	F = 5.76
Hexamethylene C_6H_{14}	84	76.25	.7047	2128	2079	Cl = 8.40
Di-isopropyl C_6H_{14}	76	69.39	.6237	1782	1625	Br = 10.65
Propyl formate $C_4H_8O_2$	88	78.35	.8304	1860	1865	Sn = 14.68
Ethyl acetate $C_4H_8O_2$	88	78.61	.8305	1862	1865	I = 15.49
Methyl propionate $C_4H_8O_2$...	88	79.72	.8408	1861	1865	
Propyl acetate $C_5H_{10}O_2$	102	74.30	.8046	2197	2196	
Ethyl propionate $C_5H_{10}O_2$...	102	74.38	.8071	2193	2196	
Methyl butyrate $C_5H_{10}O_2$...	102	71.07	.8147	2130	2196	
Methyl isobutyrate $C_5H_{10}O_2$...	102	69.74	.8102	2117	2196	
Methyl acetate $C_3H_6O_2$	74	89.08	.8741	1565	1533	
Ethyl formate $C_3H_6O_2$	74	86.93	.8658	1556	1533	
Water H_2O	18	455.4	.9173	660	360	
Methyl alcohol CH_3O	32	244.9	.7470	1082	692	
Propyl alcohol C_3H_8O	60	159.5	.7475	1821	1355	

of the expression on the right-hand side of the equation calculated from data given in the preceding columns of the table. The values for the first eight liquids in this column

furnished the data from which the value of Bc_a for each of the atoms H, C, O, F, Cl, Br, Sn, I, was calculated. The upper half of the seventh column of the table contains the values of Bc_a , and the lower half the values of c_a . The values of Σc_a for the remaining liquids were calculated by means of the values of Bc_a , and are given in the sixth column of the table. They agree very well with the values contained in the fifth column which have been calculated from experimental data, except in the case of water, methyl alcohol, and propyl alcohol, the values for these liquids showing a considerable disagreement between calculation and experiment.

The reason that these liquids do not fit in with the others is probably due to polymerization of their molecules. The existence of such an effect would modify the various quantities in equation (12) to a certain, though at present unknown, extent. The evidence on the polymerization of liquids founded on the deviations from the laws of osmotic pressure, and of surface-tension, &c., shows that each of the three liquids in question, besides a few others, is polymerized to a certain degree which depends on the nature of the liquid and its temperature. The deviations from equation (12) thus seem to indicate the same thing; and another method is thus furnished for testing whether a liquid is polymerized or not.

Equation (12) may be transformed into one involving T_1 instead of L_1 . The internal heat of evaporation of a liquid is given by the well-known thermodynamical relation

$$L_1 + p(v_1 - v_2) = (v_1 - v_2)T_1 \frac{dp}{dT_1},$$

where v_1, v_2 , denote the volume of one gram of vapour and liquid respectively at the pressure p and temperature T . Consider the temperature taken so low that the vapour obeys approximately Boyle's law. The equation then becomes

$$L_1 + \frac{RT_1}{m} = \frac{RT_1^2}{m} \frac{dp}{dT_1},$$

which may be written

$$\frac{L_1 m}{T_1} = -R + \frac{T_1}{p} \frac{dp}{dT_1}.$$

Substituting for p and T on the right-hand side of this equation from the equations $T = \beta T_c$, $p = \gamma p_c$, we obtain

$$\frac{L_1 m}{T_1} = -R + \frac{\beta}{\gamma} \frac{d\gamma}{d\beta} = u = \text{constant} \quad . \quad . \quad (13)$$

for liquids at corresponding states. (Modified Trouton's law.)

With the help of equation (13) equation (12) can be written

$$\frac{T_1^{\frac{1}{2}} m^{\frac{2}{3}}}{\rho^{\frac{2}{3}}} = H \Sigma c_a, \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where H is a function which has the same value for different liquids at corresponding states.

This equation has been tested for the liquids contained in Table XIII., the result being exhibited by Table XIV. The

TABLE XIV.

Name of liquid.	$\frac{2}{3}T_c$.	$\frac{T_1^{\frac{1}{2}} m^{\frac{2}{3}}}{\rho_1^{\frac{2}{3}}}$.	$H \Sigma c_a$.	$H c_a$.
Ether.....	311.7	398.3	...	H= 9.05
Methyl formate	324.7	290.2	...	C= 60.3
Carbon tetrachloride	371.0	434.2	...	O= 66.6
Benzene.....	374.3	415.9	...	F=110.7
Fluor-benzene	373	427.3	...	Cl= 93.47
Bromo-benzene	446.7	522.6	...	Br=115.6
Iodo-benzene	481	570.9	...	Sn=143
Stannic chloride	394.5	517.6	...	I=163.9
Di-isobutyl	362.5	607.3	645.7	
Chloro-benzene.....	422	491.5	500.5	
Pentane	313.5	428.4	410	
Heptane	359.9	559.7	506.6	
Octane	379.5	623.6	645.3	
Hexane	338.5	495.1	488.5	
Acetic acid	396.4	320.0	290	
Iso-pentane	307.2	425.5	401.1	
Hexamethylene	368.7	465.4	470.4	
Di-isopropyl.....	333.6	448.8	398	
Propyl formate	355.9	422.4	446.8	
Ethyl acetate	348.3	417.9	446.8	
Methyl propionate	352.5	416.6	446.8	
Propyl acetate	366.1	482.8	525.2	
Ethyl propionate	363.6	480.1	525.2	
Methyl butyrate	367.3	479.6	525.2	
Methyl isobutyrate	360.3	476.7	525.2	
Methyl acetate	337.3	354.1	368.4	
Ethyl formate	335.5	355.3	368.4	
Water	424.9	150	64.7	
Methyl alcohol.....	342	226.4	163	
Propyl alcohol	357.8	352.0	319.9	

data used in the calculations involved corresponded to $\frac{2}{3}T_c$. The vapour of each of the liquids at that temperature very approximately obeys Boyle's law. The table will be readily understood without any further explanation, as it resembles

in outline Table XIII., of which it may be said to be a continuation. The values of Σc_a calculated by means of the values of Hc_a , omitting those referring to water, methyl alcohol, and propyl alcohol, agree approximately with the values contained in the third column which were derived directly from experimental data. The agreement is, however, not so good as that exhibited in Table XIII. From the way equation (14) was obtained it is obvious that this must be due to deviations from the relation expressed by equation (13).

Cambridge, June 24, 1909.

LVII. *The Principle of Relativity, and Non-Newtonian Mechanics.* By GILBERT N. LEWIS and RICHARD C. TOLMAN*.

UNTIL a few years ago every known fact about light, electricity, and magnetism was in agreement with the theory of a stationary medium or æther, pervading all space, but offering no resistance to the motion of ponderable matter. This theory of a stagnant æther led to the belief that the absolute velocity of the earth through this medium could be determined by optical and electrical measurements. Thus it was predicted that the time required for a beam of light to pass over a given distance, from a fixed point to a mirror and back, should be different in a path lying in the direction of the earth's motion and in a path lying at right angles to this line of motion. This prediction was tested in the crucial experiment of Michelson and Morley †, who found, in spite of the extreme precision of their method, not the slightest difference in the different paths.

It was also predicted from the æther theory that a charged condenser suspended by a wire would be subject to a torsional effect due to the earth's motion. But the absence of this effect was proved experimentally by Trouton and Noble ‡.

The skill with which these experiments were designed and executed permits no serious doubt as to the accuracy of their results, and we are therefore forced to adopt certain new views of far-reaching importance.

It is true that the results of Michelson and Morley might be simply explained by assuming that the velocity of light depends upon the velocity of its source. Perhaps this assumption has formerly been dismissed without sufficient

* Communicated by the Authors.

† Amer. Jour. Sci. xxxiv. p. 333 (1887).

‡ Phil. Trans. Roy. Soc. (A) ccii. p. 165 (1904).

reason, but recent experimental evidence, to which we shall revert, seems to prove it untenable.

This possibility being excluded, the only satisfactory explanation of the Michelson-Morley experiment which has been offered is due to Lorentz*, who assumed that all bodies in motion are shortened in the line of their motion by an amount which is a simple function of the velocity. This shortening would produce a compensation just sufficient to offset the predicted positive effect in the Michelson-Morley experiment, and would also account for the result obtained by Trouton and Noble. It would not, however, prevent the determination of absolute motion by other analogous experiments which have not yet been tried.

Einstein† has gone one step farther. Because of the experiments that we have cited, and because of the failure of every other attempt that has ever been made to determine absolute velocity through space, he concludes that further similar attempts will also fail. In fact he states as a law of nature that absolute uniform translatory motion can be neither measured nor detected.

The second fundamental generalization made by Einstein he calls "the law of the constancy of light velocity." It states that the velocity of light in free space appears the same to all observers, regardless of the motion of the source of light or of the observer.

These two laws taken together constitute *the principle of relativity*. They generalize a number of experimental facts, and are inconsistent with none. In so far as these generalizations go beyond existing facts they require further verification. To such verification, however, we may look forward with reasonable confidence, for Einstein has deduced from the principle of relativity, together with the electromagnetic theory, a number of striking consequences which are remarkably self-consistent. Moreover the system of mechanics which he obtains is identical with the non-Newtonian mechanics developed from entirely different premises by one of the present authors‡. Finally, one of the most important equations of this non-Newtonian mechanics has within the past year been quantitatively verified by the experiments of

* *Abhandlungen über theoretische Physik*, Leipzig, 1907, p. 443.

† An excellent summary of the conclusions drawn from the principle of relativity, by Einstein, Planck, and others is given by Einstein in the *Jahrbuch der Radioaktivität*, iv. p. 411 (1907). An interesting treatment of certain phases of this problem is given by Bumstead, *Amer. Jour. Sci.* xxvi. p. 493 (1908).

‡ Lewis, *Phil. Mag.* xvi. p. 705 (1908).

Bucherer* on the mass of a β particle, to which we shall refer later.

Therefore, in as far as present knowledge goes, we may consider the principle of relativity established on a pretty firm basis of experimental fact. Accepting this principle, we shall accept the consequences to which it leads, however extraordinary they may be, provided that they are not inconsistent with one another, nor with known experimental facts.

The consequences which one of us has obtained from a simple assumption as to the *mass of a beam of light*, and the fundamental *conservation laws* of mass, energy, and momentum, Einstein has derived from the *principle of relativity* and the *electromagnetic theory*. We propose in this paper to show that these consequences may also be obtained merely from the *conservation laws* and the *principle of relativity*, without any reference to electromagnetics.

In dealing with such fundamental questions as we meet here it seems especially desirable to avoid as far as possible all technicalities. We have endeavoured to find for each of the following theorems the simplest and most obvious proof, and have used no mathematics beyond the elements of algebra and geometry.

The Units of Space and Time.

The following development will be based solely upon the conservation laws, and the two postulates of the principle of relativity.

The first of these postulates is that there can be no method of detecting absolute translatory motion through space, or through any kind of æther which may be assumed to pervade space. The only motion which has physical significance is the motion of one system relative to another. Hence two similar bodies having relative motion in parallel paths form a perfectly symmetrical arrangement. If we are justified in considering the first at rest and the second in motion, we are equally justified in considering the second at rest and the first in motion.

The second postulate is that the velocity of light as measured by any observer is independent of relative motion between the observer and the source of light†. This idea, that the velocity of light will seem the same to two different observers, even though one may be moving towards and the

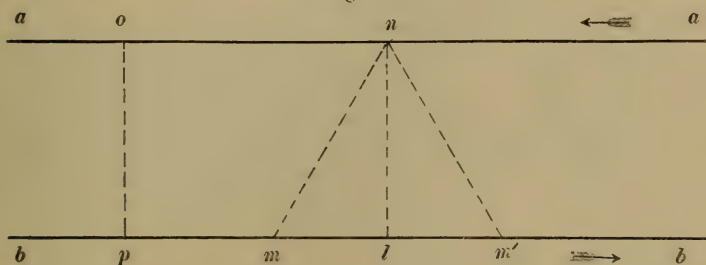
* *Ber. Phys. Ges.* vi. p. 688 (1908); *Ann. Physik*, xxviii. p. 513 (1909).

† We will imagine that the observer measures the velocity of light by means of two clocks placed at the ends of a metre stick which is situated lengthwise in the path of the light.

other away from the source of light, constitutes the really remarkable feature of the principle of relativity, and forces us to the strange conclusions which we are about to deduce.

Let us consider two systems, moving past one another, with a constant relative velocity, provided with plane mirrors aa and bb parallel to one another and to the line of motion (fig. 1). An observer A on the first system sends a beam of light across to the opposite mirror, which is reflected back to the starting-point. He measures the time taken by the light in transit.

Fig. 1.



A, assuming that his system is at rest (and the other in motion), considers that the light passes over the path opo , but he believes that if a similar experiment is conducted by an observer B in the moving system, the light must pass over the longer path mnm' , in order to return to the starting-point. For the point m moves to the position m' while the light is passing; he therefore predicts that the time required for the return of the reflected beam will be longer than in his own experiment. A, however, having established communication with B, learns that the time measured is the same as in his own experiment*.

The only explanation which A can offer for this surprising state of affairs is that the clock used by B for his measurement does not keep time with his own, but runs at a rate which is to the rate of his own clock, as the lengths of the paths, opo to mnm' .

B, however, is equally justified in considering his system at rest, and A's in motion, and by identical reasoning has come to the conclusion that A's clock is not keeping time.

* This is evidently required by the principle of relativity, for contrary to A's supposition the two systems are in fact entirely symmetrical. Any difference in the observations of A and B would be due to a difference in the absolute velocity of the two systems, and would thus offer a means of determining absolute velocity.

Thus to each observer it seems that the other's clock is running too slowly.

This divergence of opinion evidently depends not so much on the fact that the two systems are in relative motion, but on the fact that each observer arbitrarily assumes that his own system is at rest. If, however, they both decide to call A's system at rest, then both will agree that in the two experiments the light passes over the paths opo and $mn m'$ respectively, and that B's clock runs more slowly than A's. In general, whatever point may be arbitrarily chosen as a point of rest, it will be concluded that any clock in motion relative to this point runs too slowly.

Consider fig. 1 again, assuming system a at rest. We have shown that it is necessary to assume that B's clock runs more slowly than A's in the ratio of the lengths of the path opo to the path $mn m'$; in other words, the second of B's clock is longer than the second of A's, in the ratio $mn m'$ to opo . This ratio between the two paths will evidently depend on the relative velocity of the two systems, v , and on the velocity of light, c .

Obviously from the figure,

$$(op)^2 = (ln)^2 = (mn)^2 - (ml)^2.$$

Dividing by $(mn)^2$,

$$\frac{(op)^2}{(mn)^2} = 1 - \frac{(ml)^2}{(mn)^2}.$$

But the distance ml is to the distance mn as v is to c .

Hence

$$\frac{mn}{op} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Denoting the important ratio $\frac{v}{c}$ by the letter β , we see that in general a second measured by a moving clock bears to a second measured by a stationary clock the ratio

$$\frac{1}{\sqrt{1 - \beta^2}}.$$

Whatever assumption the observers A and B may make as to their motion, it is obvious that their measurements of length, at least in a direction perpendicular to their line of relative motion, will lead to no disagreement. For evidently, if each observer with a measuring-rod determines the distance from his system to the other, the two determinations must

agree. Otherwise the condition of symmetry required by the principle of relativity would not be fulfilled.

But let us now consider distances *parallel* to the line of relative motion.

Fig. 2.



A system (fig. 2) has a source of light at m and a reflecting mirror at n . If we consider the whole system to be at absolute rest, it is evident that a light-signal sent from m to the mirror, and reflected back, passes over the path mn . If, however, the entire system is considered to be in absolute motion with a velocity v , the light must pass over a different path $mn'm'$ where nn' is the distance through which the mirror moves before the light reaches it, and mm' is the distance traversed by the source before the light returns to it.

Obviously then,

$$\frac{nn'}{mn} = \frac{v}{c}, \quad \text{and}$$

$$\frac{mm'}{mn} = \frac{v}{c}.$$

Also from the figure,

$$mn' = mn + nn',$$

$$mn'm' = mn + 2nn' - mm'.$$

Combining we have,

$$\frac{mn'm'}{mn} = \frac{1}{1 - \frac{v^2}{c^2}} = \frac{1}{1 - \beta^2}.$$

Hence if we call the system in motion, instead of at rest, the calculated path of the light is greater in the ratio $\frac{1}{1 - \beta^2}$.

Now the velocity of light must seem the same to the observer, whether he is at rest or in motion. His measurements of velocity depend upon his units of length and time. We have already seen that a second on a moving clock is

lengthened in the ratio $\frac{1}{\sqrt{1 - \beta^2}}$, and therefore if the path

of the beam of light were also greater in this same ratio, we should expect that the moving observer would find no discrepancy in his determination of the velocity of light. From

the point of view of a person considered at rest, however, we have just seen that the path is increased by the larger ratio $\frac{1}{1-\beta^2}$. In order to account for this larger difference, we must assume that the unit of length in the moving system has been *shortened* in the ratio $\frac{\sqrt{1-\beta^2}}{1}$.

We thus see that a metre-stick, which, when held perpendicular to its line of motion, has the same length as a metre-stick at rest, will be shortened when turned parallel to the line of motion in the ratio $\frac{\sqrt{1-\beta^2}}{1}$, and indeed any moving body must be shortened in the direction of its motion in the same ratio*.

Let us emphasize once more, that these changes in the units of time and length, as well as the changes in the units of mass, force, and energy which we are about to discuss, possess in a certain sense a purely factitious significance;

* Certain of Einstein's other deductions from the principle of relativity will not be needed in the development of this paper, but may be directly obtained by the methods here employed. For example, the principle of relativity leads to certain curious conclusions as to the comparative readings of clocks in a system assumed to be in motion. Consider two systems in relative motion. An observer on system *a* places two carefully compared clocks, unit distance apart, in the line of motion, and has the time on each clock read when a given point on the other system passes it. An observer on system *b* performs a similar experiment. The difference between the readings of the two clocks in one system must be the same as the difference in the other system, for by the principle of relativity, the relative velocity *v* of the systems must appear the same to an observer in either. However, the observer A, considering himself at rest, and familiar with the change in the units of length and time in the moving system which we have already deduced, expects that the velocity determined by B will be greater than that which he himself observes in the ratio $\frac{1}{1-\beta^2}$, since he has concluded that B's unit of time is longer, and his unit of length in this direction is shorter, each by a factor involving $\sqrt{1-\beta^2}$. The only possible way in which A can explain this discrepancy is to assume that the clocks which B claims to have set together are not so in reality. In other words he has to conclude that clocks which in a moving system appear to be set together really read differently at any instant (in stationary time), and that a given clock is "slower" than one immediately to the rear of it by an amount proportional to the distance. From what has preceded it can be readily shown that if in a moving system two clocks are situated, one in front of the other by a distance *l*, in units of this system, the difference in setting will be $\frac{lv}{c^2}$. From this point Einstein's equations concerning the addition of velocities also follow directly.

although, as we shall show, this is equally true of other universally accepted physical conceptions. We are only justified in speaking of a body in motion when we have in mind some definite, though arbitrarily chosen, point as a point of rest. The distortion of a moving body is not a physical change in the body itself, but is a scientific fiction.

When Lorentz first advanced the idea that an electron or in fact any moving body is shortened in the line of its motion, he pictured a real distortion of the body in consequence of a real motion through a stationary æther, and his theory has aroused considerable discussion as to the nature of the forces which would be necessary to produce such a deformation. The point of view first advanced by Einstein, which we have here adopted, is radically different. Absolute motion has no significance. Imagine an electron and a number of observers moving in different directions with respect to it. To each observer, naïvely considering himself to be at rest, the electron will appear shortened in a different direction and by a different amount; but the physical condition of the electron obviously does not depend upon the state of mind of the observers.

Although these changes in the units of space and time appear in a certain sense psychological, we adopt them rather than abandon completely the fundamental conceptions of space, time, and velocity, upon which the science of physics now rests. At present there appears no other alternative.

Non-Newtonian Mechanics.

Having obtained these relations for the units of space and time, we may turn to some of the other important quantities used in mechanics.

Let us again consider two systems, a and b , in relative motion with the velocity v . An experimenter A on the first system constructs a ball of some rigid elastic material, with a volume of one cubic centimetre, and sets it in motion, with a velocity of one centimetre per second, towards the system b (in a direction perpendicular to the line of relative motion of the two systems). On the other system, an experimenter b constructs of the same material a similar ball with a volume of one cubic centimetre in his units, and imparts to it, also in his units, a velocity of one centimetre per second towards a . The experiment is so planned that the balls will collide and rebound over their original paths. Since the two systems are entirely symmetrical, it is evident by the principle of relativity, that the (algebraic) change in velocity of the first

ball, as measured by A, is the same as the change in velocity of the other ball, as measured by B. This being the case, the observer A, considering himself at rest, concludes that the real change in velocity of the ball *b* is different from that of his own, for he remembers that while the unit of length is the same in this transverse direction in both systems, the unit of time is longer in the moving system.

Velocity is measured in centimetres per second, and since the second is longer in the moving system, while the centimetre in the direction which we are considering is the same in both systems, the observer A, always using the units of his own system, concludes that the change in velocity of the ball

b is smaller in the ratio $\frac{\sqrt{1-\beta^2}}{1}$ than the change in velocity

of the ball *a*. The change in velocity of each ball multiplied by its mass gives its change in momentum. Now, from the law of conservation of momentum, A assumes that each ball experiences the same change in momentum, and therefore since he has already decided that the ball *b* has experi-

enced a smaller change of velocity in the ratio $\frac{\sqrt{1-\beta^2}}{1}$, he

must conclude that the mass of the ball in system *b* is greater than that of his own in the ratio $\frac{1}{\sqrt{1-\beta^2}}$. In general,

therefore, we must assume that the mass of a body increases with its velocity. We must bear in mind, however, as in all other cases, that the motion is determined with respect to some point *arbitrarily* chosen as a point of rest.

If *m* is the mass of a body in motion and *m*₀ its mass at rest, we have *

$$\frac{m}{m_0} = \frac{1}{\sqrt{1-\beta^2}} \cdot \cdot \cdot \cdot \cdot \cdot (1)$$

The only opportunity of testing experimentally the change of a body's mass with its velocity has been afforded by the experiments on the mass of a moving electron or β particle. The actual measurements were indeed not of the mass of the electron, but of the ratio of charge to mass $\left(\frac{e}{m}\right)$.

* This equation and others developed in this section are identical with those obtained through an entirely different course of reasoning by Lewis (Phil. Mag. xvi. p. 705, 1908). The equations were there obtained for systems in motion with respect to a point at absolute rest. We shall show here, however, that they are true, whatever arbitrary point is selected as a point of rest.

It has, however, been universally considered that the charge e is constant. In other words, that the force acting upon the electron in a uniform electrostatic field is independent of its velocity relative to the field. Hence the observed change in $\frac{e}{m}$ is attributed solely to the change in mass. It might be well to subject this view to a more careful analysis than has hitherto been done. At present, however, we will adopt it without further scrutiny.

The original experiments of Kaufmann* showed only a qualitative agreement with equation (1). Recently, however, Bucherer† by a method of exceptional ingenuity, has made further determinations of the mass of electrons moving with varying velocities, and his results are in remarkable accord with this equation obtained from the principle of relativity.

This very satisfactory corroboration of the fundamental equation of non-Newtonian mechanics, must in future be regarded as a very important part of the experimental material which justifies the principle of relativity. By a slight extrapolation we may find with accuracy from the results of Bucherer that limiting velocity at which the mass becomes infinite, in other words, a numerical value of c which in no way depends upon the properties of light. Indeed merely from the first postulate of relativity and these experiments of Bucherer we may deduce the second postulate and all the further conclusions obtained in this paper. This fact can hardly be emphasized too strongly.

Leaving now the subject of mass, let us consider whether the unit of force depends upon our choice of a point of rest. An observer in a given system allows such a force to act

upon unit mass as to give it an acceleration of one $\frac{\text{cm.}}{\text{sec.}^2}$, and

calls this force the dyne. If now we assume that the system is in motion, with a velocity v , in a direction perpendicular to the line of application of the force, we conclude that the acceleration is really less than unity, since in a moving

system the second is longer in the ratio $\frac{1}{\sqrt{1-\beta^2}}$ and the

centimetre in this transverse direction is the same as at rest. On the other hand, the mass is increased owing to the motion

of the system by the factor $\frac{1}{\sqrt{1-\beta^2}}$. Since the time enters to the second power, the product of mass and acceleration

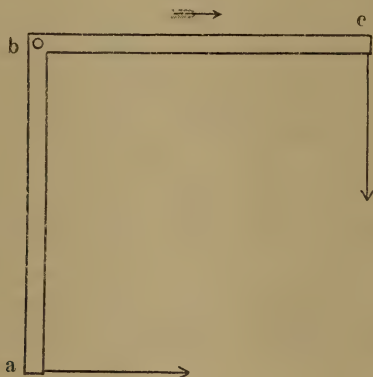
* See Lewis, *loc. cit.*

† Bucherer, *loc. cit.*

is smaller by the ratio $\frac{\sqrt{1-\beta^2}}{1}$ than it would be if the system were at rest. And we conclude, therefore, that the unit of force or the dyne in a direction transverse to the line of motion is smaller in a moving system than in one at rest by this same ratio.

In order now to obtain a value for the force in a longitudinal direction in the moving system, let us consider (fig. 3) a rigid lever abc , whose arms are equal and perpendicular, and equal forces applied at a and c in directions parallel to bc and ba . The system is thus in equilibrium.

Fig. 3.



Now let us assume that the whole system is in motion with velocity v in the direction bc . Obviously, merely by making such an assumption we cannot cause the lever to turn, nevertheless we must now regard the length bc as shortened in

the ratio $\frac{\sqrt{1-\beta^2}}{1}$, while ab has the same length as at rest.

We must therefore conclude that to maintain equilibrium the force at a must be less than the force at c in the same ratio.

We thus see that in a moving system unit force in the longitudinal direction is smaller than unit transverse force in the

ratio $\frac{\sqrt{1-\beta^2}}{1}$, and therefore, by the preceding paragraph, smaller than unit force at rest in the ratio $\frac{1-\beta^2}{1}$. It is

interesting to point out, as Bumstead* has already done, that the repulsion between two like electrons, as calculated from

* Bumstead, *loc. cit.*

the electromagnetic theory, is diminished in the ratio $\frac{\sqrt{1-\beta^2}}{1}$ if they are moving perpendicular to the line joining them; and in the ratio $\frac{1-\beta^2}{1}$ if moving parallel to the line joining them.

From the standpoint of the principle of relativity, one of the most interesting quantities in mechanics is the so-called kinetic energy, which is the increase in energy attributed to a body when it is set in motion with respect to an arbitrarily chosen point of rest. Knowing the change of the mass with velocity as given by equation (1), the general equation for kinetic energy *, E' , may readily be shown to be,

$$E' = m_0 c^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right). \quad . \quad . \quad . \quad (2)$$

From equations 1 and 2 we may derive one of the most interesting consequences of the principle of relativity. If E is the total energy (including internal energy) of a body in motion, and E_0 is its energy at rest, the kinetic energy E' is equal to $E - E_0$ and equation (2) may be written,

$$E - E_0 = m_0 c^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right). \quad . \quad . \quad . \quad (3)$$

Moreover, we may write equation (1) in the form

$$m - m_0 = m_0 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right), \quad . \quad . \quad . \quad (4)$$

and dividing (3) by (4),

$$\frac{E - E_0}{m - m_0} = c^2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In other words when a body is in motion its energy and mass are both increased, and the increase in energy is equal to the increase in mass multiplied by the square of the velocity of light. From the conservation laws we know that when a body is set in motion and thus acquires mass and energy, these must come from the environment. So also when a moving body is brought to rest it must give up mass as well as energy to the environment. The mass thus acquired by the environment is independent of the particular form

* Consider a body moving with the velocity v subjected to a force f in the line of its motion. Its momentum M and its kinetic energy E' will be changed by the amounts $dM = f dt$, $dE' = f dl = f v dt$. Hence $dE' = v dM$ or substituting mv for M , $dE' = m v dv + v^2 dm$. Eliminating m between this equation and equation (1), and integrating, gives at once the above equation (2).

which the energy may assume and we are thus forced to the important conclusion that *when a system acquires energy in any form it acquires mass in proportion*, the ratio of the energy to the mass being equal to the square of the velocity of light. We might go further and assume that if a system should lose all its energy it would lose all its mass. If we admit this plausible although unproved assumption, then we may regard the mass of every body as a measure of its total energy according to the equation,

$$m = \frac{E}{c^2} \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \quad (6)$$

For a body at rest

$$m_0 = \frac{E_0}{c^2}.$$

Combining this equation with (3) gives

$$\frac{E}{E_0} = \frac{1}{\sqrt{1-\beta^2}}.$$

We thus see that energy changes with the velocity in the same way that mass does, and that the so-called kinetic energy is a "second-order effect" of the same character as the change of length and the change of mass. The only reason that this effect is easily measured, and has become a familiar conception in mechanics, while the others are obtainable only by the most precise measurements, is that we are in the habit of measuring quantities of energy which are extremely minute in comparison with the total energy of the systems investigated.

Conclusion.

We have shown how observers stationed on systems in motion relative to one another have been able to preserve their fundamental principles of mechanics only by adopting certain novel conclusions. These conclusions are self-consistent ; in the one case where they have been tested they are in accord with experiment ; and they enable us to save all the fundamental physical concepts which have been found useful in the past. We have, however, considered primarily only systems which are initially in uniform relative motion. Whether our conclusions can be retained when we consider processes in which the relative motion is being established, in other words, processes in which acceleration takes place, it is not our present purpose to determine.

The ideas here presented appear somewhat artificial in character, and we cannot but suspect that this is due to the

arbitrary way in which we have assumed this point or that point to be at rest, while at the same time we have asserted that a condition of rest in the absolute sense possesses no significance.

If our ideas possess a certain degree of artificiality, this is also true of others which have long since been adopted into mechanics. The apparent change in rate of a moving clock, and the apparent change in length and mass of a moving body, are completely analogous to that apparent change in energy of a body in motion which we have long been accustomed to call its kinetic energy. We may with equal reason speak of the kinetic mass found by Kaufmann and Bucherer, or the kinetic length assumed by Lorentz. We say that the heat evolved when a moving body is brought to rest comes from the kinetic energy which it possessed. We thus preserve the law of conservation of energy. It is in order to maintain such fundamental conservation laws, and to reconcile them with the Principle of Relativity, which rests on the experiments of Michelson and Morley and of Bucherer, that we have adopted the principles of non-Newtonian Mechanics.

These principles, bizarre as they may appear, offer the only method of preserving the science of mechanics substantially in its present form. If later, when more complex systems are considered, and especially when we deal with acceleration, these views prove untenable, it will then be necessary to revolutionize the whole of mechanics.

Research Laboratory of Physical Chemistry,
Mass. Inst. of Technology, Boston,
May 11th, 1909.

V91

LVIII. *On the Moving Force of Terrestrial and Celestial Bodies in Relation to the Attraction of Gravitation.* By HENRY WILDE, D.Sc., D.C.L., F.R.S.*

1. **I**N the course of a lecture which I delivered before the Society in 1902, "On the Evolution of the Mental Faculties in relation to some Fundamental Principles of Motion," prominence was given to the historic controversy respecting the measure of moving force of terrestrial bodies which has exercised the minds of distinguished men of science and learning for more than two centuries.

* Communicated by the Author. Reprinted from the *Memoirs and Proceedings of the Manchester Literary and Philosophical Society*, vol. liii. pt. ii. (1909).

2. The proposition was enunciated by Descartes in his 'Principia'*, "That when a part of matter is moved with double the quickness of another, and that other is twice the size of the former, there is just precisely as much motion, but no more, in the less body as in the greater." Forty years later Newton adopted in his 'Principia' Descartes' definition of the quantity of motion in a moving body in substantially the same terms as follows:—"The quantity of motion is the measure of the same arising from the velocity and quantity of matter conjointly. The motion of the whole is the sum of the motion of all of its parts; and therefore in a body double in quantity with equal velocity the motion is double; with twice the velocity it is quadruple." To make this definition more explicit, Newton states under his second law, "if any force generates a motion, a double force will generate double the motion, a triple force triple the motion, *whether that force be impressed altogether and at once, or gradually and successively.*"

3. Although Galilei had long before demonstrated that the spaces described by heavy bodies from the beginning of their descent are as the squares of the times and also of the velocities acquired in falling through those spaces, yet the significance of this law in relation to the moving force of bodies was entirely overlooked until Leibnitz made the announcement that the force of a body in motion, by the free action of gravity, is as the square of the velocity. To this measure of moving force Leibnitz applied the term, *vis viva*, or living force.

4. The controversy which has since gathered round the question of the measure of moving force, and still remains unsettled, forms a remarkable chapter in the history of the physical sciences. As might have been anticipated *à priori*, philosophers, mathematicians, metaphysicians, and men of letters, unskilled in experimental methods of interrogating nature, adopted the Cartesian measure of moving force. Of these may be mentioned, Maclaurin, Hutton, and Young; Locke, Kant, Schopenhauer, Voltaire, and other writers of more or less note up to the present epoch. Happily for the progress of science a number of natural philosophers, among whom Smeaton, Wollaston, Ewart, Dalton, Joule, and Fairbairn stand pre-eminent, have proved conclusively by various methods, that the true measure of the moving force of a body under the free action of gravity is as the square of the

* *Principia Philosophiæ*, Pars. 2, § XXXV., 1643.

velocity *. Nevertheless, modern scholasticism has not yet pronounced decidedly in favour of the law, and the traditional error of Descartes and Newton still survives in manuals of elementary and advanced science, under the name of momentum, in contradiction of Newton's second law which expressly excludes the element of time in the measure of the quantity of motion in a body.

5. In the preface to his 'Principia' Newton set forth with singular lucidity and ingenuousness, the dependence of the mathematical principles of natural philosophy upon experimental mechanics in their application to the motions of celestial bodies. It will therefore be obvious that the question, whether the quantity of motion in a planetary system is simply as the velocity, or as the square of the velocity, is one of fundamental importance. So far as I know, no attempt has yet been made to deal with this problem, and no explication of astronomical science can be considered complete so long as it remains unsolved.

6. In order to demonstrate that the moving force by which the moon and other celestial bodies are maintained in their orbits is as the square of the velocity, it is postulated as general knowledge in physical astronomy :—

- (a) That the equatorial circumference of the earth is 24,900 miles.
- (b) That the versed sine of five miles (or more exactly, 4.936 miles) of the earth's circumference, is 193 inches = 16 feet 1 inch.
- (c) That a body at rest near the earth's surface falls perpendicularly through the versed sine of 4.936 miles of arc of the earth's circumference = 16 feet 1 inch during one second of time.
- (d) That as versed sines are as the squares of their arcs, and the accelerative force of gravity increases in the same proportion, a body projected horizontally near the earth's surface with a velocity 4.936 miles (26.062 feet) per second, would revolve round the earth continually without touching it.

* These results have been abundantly confirmed by my experiments with the gyroscope described in the lecture referred to, wherein it was shown (1) that four times the weight falling from the same height were required to generate a double velocity of the revolving disk; (2) that one unit of weight falling through four times the height also generates a double velocity of the disk; (3) that the moving force required to generate a double velocity of the disk is independent of the time of its application and is as the square of the velocity.—*Manchester Memoirs*, vol. 46, 1902.

(e) That $\frac{24900 \text{ miles}}{4.936 \text{ miles}} = \frac{5044^s}{60^s} = 84.07 \text{ minutes} = \text{the time}$
 of revolution of a body round the earth with an
 angular velocity of 26,062 feet per second (d).

7. The mean distance of the moon according to modern astronomy, is 60.28 semi-diameters of the earth, and the time of its revolution in respect to the fixed stars, 27 days, 7 hours, 43 minutes = 39343 minutes. Now as the times and velocities of rotation are reciprocally equivalents of each other, we have $\frac{39343^m}{84^m.07} = 468$ as the ratio of the orbital times or velocities of a body revolving round the circumference of the earth and the moon's orbit respectively. Therefore $468 \times 84.07 = 39344^m = 27 \text{ days, } 7 \text{ hours, } 44 \text{ minutes,}$ for the time of the moon's orbital revolution.

8. Again, from the square of 468, as a velocity, divided by the moon's distance in semi-diameters of the earth we have $\frac{468^2}{60.28} = 3634$, or reciprocally $\frac{1}{3634}$ the total moving force of a body revolving round the earth's circumference and the moon's orbit respectively, the numbers being the same as those deduced from the law of attraction of gravitation for the same distance.

9. As the radius of the moon's orbit is 60.28 semi-diameters of the earth, we have with the earth's radius in miles, $3964 \times 2 \times 60.28 \times 3.1416 = 1501370$ miles for the circumference of the moon's orbit. Dividing this by the time of revolution in minutes we have $\frac{1501370 \text{ miles}}{39344 \text{ miles}} = 38.160$ miles per minute = 3358 feet per second for the moon's orbital velocity.

10. That forces of any kind radiating from every point of a body in free space are inversely proportional to the square of the distance is a proposition requiring but little demonstration seeing that it flows directly from the geometry of space. It will be sufficient if I mention in this connexion the intensity of light which increases and diminishes in this same ratio, as is commonly demonstrated by photometers. Experimental mechanics were not sufficiently advanced in Newton's time to enable him to determine the ratios of the attractive and moving forces of celestial bodies, as there was apparently no physical connexion between such bodies, and it was chiefly from observation and his geometry that the law of attraction of gravitation as the inverse square of the distance was established.

11. The application of the law to the moon's orbit is the subject of the fourth proposition of the third book of the 'Principia,' and is of so much importance in relation to my paper that I have thought it well to give an abstract of the text from Motte's classical translation. The linear quantities in the original are expressed in Paris feet, but are here given in English measure in accordance with modern usage:—

“Let us assume the mean distance of the moon to be 60 semi-diameters of the earth, and if we imagine the moon deprived of all motion to be let go, so as to descend towards the earth with the impulse of all that force by which it is retained in its orbit, it will, in the space of one minute of time, describe in its fall 16 feet 1 inch. For the versed sine of that arc which the moon, in the space of one minute of time, would by its mean motion describe at the distance of 60 semi-diameters of the earth is nearly 16 feet 1 inch. Wherefore since that force in approaching to the earth increases in the reciprocal duplicate proportion of the distance, and, upon that account, at the surface of the earth 60×60 times greater than at the moon, a body in our regions falling with that force ought, in the space of one minute of time, to describe $60 \times 60 \times 16$ feet 1 inch. And with this very force we actually find that bodies here upon earth do really descend. And therefore the force by which the moon is retained in its orbit becomes, at the very surface of the earth, equal to the force of gravity which we observe in bodies there. And therefore the force by which the moon is retained in its orbit is that very same force which we commonly call gravity.”

12. I have already demonstrated that the moon's orbital velocity is 38·160 miles per minute, and that a body would describe an arc of 4·936 miles of the earth's circumference in one second, during which time it would fall through the versed sine of that arc = 193 inches. Now as the versed sines of arcs of the same length are as the radii inversely from the centre, we have $\frac{193 \text{ inches}}{60 \cdot 28} = 3 \cdot 21$ inches as the versed sine of 4·936 miles of the moon's orbit. Again, as versed sines are as the squares of their arcs, we have $\frac{38 \cdot 160 \text{ miles}}{4 \cdot 936 \text{ miles}} 7 \cdot 73^2 \times 3 \cdot 21$, or nearly 16 feet 1 inch, as the versed sine of 38·160 miles through which the moon falls towards the earth during one minute of time in accordance with Newton's demonstration.

13. The assumption by Newton of the whole number 60 as the distance of the moon in semi-diameters of the earth in combination with the same even number in seconds and minutes of time greatly facilitated the demonstration of the

law of the attraction of gravitation, the truth of which could at once be perceived by a simple mental calculation. Thus $60 \times 60 = 3600$ is the attractive force at the earth's surface, and reciprocally $\frac{1}{3600}$ at the distance of the moon's orbit.

Now this same demonstration is equally applicable to the quantity of moving force of terrestrial bodies as well as at the distance of the moon's orbit. For as the moving force is as the square of the velocity, a body falling during one minute through a distance of 16 feet 1 inch at the moon's orbit, that is to say, with a velocity 60 times less than at the earth's surface, its moving force would be $60 \times 60 = 3600$ times less than that of a body falling the same distance during one second of time on the terrestrial globe. Therefore the moving force and the attraction of gravitation are alike inversely proportional to the square of the distance, and are correlated equally in amount to maintain and retain the moon in its orbit during the course of its revolution round the earth.

14. The quantitative relations of the moving and attractive forces of the planetary bodies are set forth in the following new calculus of elements, wherein Mercury is taken as unity instead of the Earth, as in ordinary astronomical tables.

15. The masses of the bodies are taken as equal and require to be multiplied by the specific mass of each planet for the total force.

16. As the moving and attractive forces are correlatively equal they are expressed by the same numbers and their reciprocals, Mercury = 1.000, and Neptune = $\frac{1}{6020} = 0.00016$, as in the similar calculus of moving and attractive forces of the moon during its orbital revolution.

17. The planetary distances in radii of that of Mercury may be taken as astronomically correct, as they are derived from the periodic times; but the numerical expression of them in miles is necessarily only a near approximation, from the fact that the value of the unit of distance has not yet been exactly determined. Accepting, provisionally, the unit distance of Mercury to be 35,860,000 miles, we have for the mean distance of the Earth from the Sun $2.583 \times 35,860,000 = 92,626,380$ miles, and of Neptune $77.59 \times 35,860,000 = 2,782,377,400$ miles. In like manner the intermediate planetary distances in miles may also be determined.

NEW TABLE OF ELEMENTS OF THE LUNAR AND PLANETARY ORBITS.

	Distances in Radii.		Periodic Times. Days.	Moving and Attractive Forces.	
				Radii ² .	Reciprocals.
Mercury.....	1.000 ³ = ...	$\sqrt{1.000}$ =	1.000 × 87.97 =	1.000 =	1.00000
Venus.....	1.868 ³ = ...	$\sqrt{6.522}$ =	2.554 × 87.97 =	3.489	0.28661
Earth.....	2.583 ³ = ...	$\sqrt{17.239}$ =	4.152 × 87.97 =	6.671	0.14990
Mars	3.936 ³ = ...	$\sqrt{60.980}$ =	7.809 × 87.97 =	15.492	0.06455
Ceres	7.148 ³ = ...	$\sqrt{365.200}$ =	19.110 × 87.97 =	51.094	0.01957
Jupiter	13.437 ³ = ...	$\sqrt{2426.000}$ =	49.250 × 87.97 =	180.633	0.00553
Saturn	24.640 ³ = ..	$\sqrt{14957.000}$ =	122.300 × 87.97 =	607.129	0.00165
Uranus	49.550 ³ = ..	$\sqrt{121661.000}$ =	348.840 × 87.97 =	2455.202	0.00041
Neptune ..	77.590 ³ = ..	$\sqrt{467142.000}$ =	633.494 × 87.87 =	6020.208	0.00016
Moon	60.28 ³ = ..	$\sqrt{219038.000}$ =	468.000 × 84.07 =	3634.000	0.00027

LIX. *The Ultra-violet Absorption, Fluorescence, and Magnetic Rotation of Sodium Vapour.* By R. W. WOOD, Professor of Experimental Physics, Johns Hopkins University*.

[Plates XV. & XVI.]

THE present paper deals with the optical properties of sodium vapour in the ultra-violet, and forms a continuation of an extended series of papers already published regarding this remarkable metallic vapour.

For a proper understanding of its behaviour in the ultra-violet, a brief review of its properties in the visible region will be necessary.

The absorption is of two types. A Balmer series of double lines, beginning with the D lines and extending into the remote ultra-violet. Of these but seven members were known previous to the publication of my last paper on the subject in this journal†. At that time I had succeeded in raising the number to about thirty, since which time, by employing a larger quartz spectrograph, I have raised the number to forty-eight. The wave-lengths, as measured from the new plates, are given in the following table, and full details of the work will be found in the *Astro-physical Journal* for March 1909. This forms the most complete Balmer series thus far observed, only 12 lines appearing in terrestrial hydrogen, and 30 in solar.

Wave-lengths of Lines in Balmer Series of Sodium Vapour.

<i>n.</i>	λ .		<i>n.</i>	λ .		<i>n.</i>	λ .
3	5896.16	} Previous observations	18	2433.85		35	2417.38
	90.19		19	31.43		36	17.10
4	3303.07		20	29.42		37	16.80
	02.47		21	27.72		38	16.56
5	2852.91		22	26.28		39	16.33
6	2680.46		23	25.00		40	16.11
7	2594.05		24	23.88		41	15.89
8	43.82		25	22.90		42	15.70
9	12.15		26	22.04		43	15.52
10	2490.70		27	21.29		44	15.37
11	75.60		28	20.60		45	15.21
12	64.53		29	20.02		46	15.06
13	56.02		30	19.50		47	14.94
14	49.46		31	19.00		48	14.78
15	44.24		32	18.44		49	14.64
16	40.06		33	18.09		50	14.50
17	36.70		34	17.71			

* Communicated by the Author.

† "An Extension of the Principal Series of Sodium," *Phil. Mag.* Dec. 1908.

As will be seen from the table, the last 22 lines, *i. e.* nearly one-half of the whole series thus far observed, fall in a region of the spectrum not wider than the distance between the D lines!

Kayser gives on page 521 the constants of the Balmer formula for the sodium series, computed from the seven lines known at the time.

These seven lines are well represented by the expression

$$10^8\lambda^{-1} = 41496.34 - 127040 n^{-2} - 843841 n^{-4}.$$

With the complete series now at our disposal it will be necessary to redetermine the constants.

I have calculated the wave-lengths of the 32nd and the 50th line from the formula, finding 2417.5 instead of 2418.44 (observed), and 2415.2 instead of 2414.5; that is, the calculated values are separated from the observed by a distance equal to the distance between four of the lines at this point of the spectrum.

Photographs of the series are reproduced on Pl. XVI. Fig. 1 is an enlargement showing the lines from $n=12$ to $n=40$. The last ten members are too close together to appear. Fig. 2 is reproduced natural size, and shows the series from $n=8$. The source of light was the cadmium spark, and the iron comparison spectrum was taken on the same plate, the image of the arc being projected on the slit just below the point illuminated by the light of the spark after it had traversed the sodium vapour.

It is extremely interesting to note that a general absorption begins at the head of the series, which extends down to the end of the spectrum. In other words, the vapour is much more transparent to the light between the lines of the series than in the region below the head. This is in accord with the recorded observation of a faint continuous emission spectrum below the head of the series in the case of solar hydrogen. Pl. XVI. fig. 3 shows a series of spectra taken with progressively increasing vapour density, in which the complete Balmer series appears, with the exception of the first member ($n=3$), the D lines, the position of which is however indicated. As the vapour density increases the higher members of the series appear, and the lower members are drowned out by channelled spectra which accompany them.

These channelled spectra are analogous to the one which accompanies the D lines, which is the one previously studied with respect to fluorescence and magnetic rotation. They differ from it in one respect, however. The channelled spectrum in the visible region consists of two parts, as is

shown in Pl. XV. fig. 3. Spectrum *d* was made with moderate vapour density. The position of the D lines is indicated, though in the present case they have widened into a very broad band. A channelled spectrum appears in the red, orange, and yellow, and another in the green-blue, the yellow-green region being transmitted freely. The red-orange portion spreads over and below the D lines, as the vapour density increases; while the blue-green one pushes up in the other direction, the two eventually meeting at wave-length 5500, where a broad hazy band appears, which is just appearing in spectrum *b*, and is very distinct in *a*. This point appears to be the "centre of gravity" (if we may use the term) between the two channelled spectra which border the D lines. With a spectroscope of low dispersion we see merely a close double green line at the point, and the extreme violet-blue, everything else being cut off. The colour of the transmitted light is as deep as, and about the colour of, the light which gets through the densest cobalt glass.

As I have already shown, there is some direct connexion between the mechanism which produces this channelled spectrum and the one which produces the D lines, for absorption of blue-green light, carefully freed from yellow by prismatic analysis and colour screens, causes the vapour to emit yellow light of the wave-length of the D lines, in addition to the more powerful emission (line spectrum) in the blue-green region.

I find now that channelled spectra accompany the other lines of the Balmer series, but that they are by no means the counterpart of the one around the D lines. They are *not* made up of two portions, with a comparatively bright region between, but spread out in both directions, both above and below the lines which they accompany. I have found traces of them as far down as the line for which $n=8$. They can be well seen in Pl. XVI. fig. 3, spectra *d*, *e*, and *f*, while fig. 4 shows the one accompanying the first ultra-violet line ($\lambda=3300$) on a larger scale; this photograph was made with a concave grating, "*b*" with rare, "*a*" with denser vapour. The Balmer line 3303 appears to widen on the side towards the visible spectrum (see (*a*)), though this may be due to the presence of a group of heavy lines in the channelled spectrum. An enlargement of this region is reproduced on Pl. XV. fig. 2 to show the complexity of the spectrum. The spectrum resembles the one in the blue-green region in its general appearance.

The question now suggests itself:—Do these ultra-violet channelled spectra exhibit the fluorescence phenomena shown

in the visible region? This question I can answer in the affirmative for the first one (at line 3303). The fluorescence can be powerfully excited by the triplet in the zinc arc at wave-length 3300. The fluorescent light is of course invisible, which makes its detection difficult. The sodium tube was closed with a large quartz lens, and powerful fluorescence excited with a Heraeus zinc arc in quartz. The image of the green fluorescent spot was focussed upon the slit of a quartz spectrograph by means of a quartz lens. Under these conditions ultra-violet fluorescent light, if present, should appear in the spectrum. Such was found to be the case, as is shown by Pl. XV. fig. 1; the zinc arc spectrum is shown above, the fluorescent spectrum below. The banded fluorescent spectrum is strongest at some distance above the zinc lines (*i. e.* on the side of longer wave-lengths). The intervention of a glass plate between the arc and the sodium tube caused the disappearance of the ultra-violet fluorescent bands, showing that they were due to the absorption of ultra-violet and not of visible light.

An attempt was next made to see whether any mechanical connexion existed between the mechanisms which produced the D lines and the visible channelled spectra (which have been shown to be connected) and the ultra-violet lines. The zinc ultra-violet triplet was isolated by means of a quartz spectrograph and focussed on the aperture of the sodium chamber. The room was absolutely dark, and all extraneous light carefully screened off. No visible fluorescence was observed. The ultra-violet fluorescence was, however, photographed with a second quartz spectrograph. In this case one is working wholly in the dark, and the difficulties attending the adjustment of the two optical trains may be well imagined.

This experiment indicates absence of connexion between the two systems.

The converse experiment was next tried. Powerful stimulation by the entire visible spectrum failed to show any trace of the ultra-violet band in the spectrum of the fluorescent light, even though the visible region was overexposed ten fold. This again indicates absence of connexion.

It looks very much as if the different lines of the principal (Balmer) series and their accompanying channelled spectra may be considered as produced by different entities. The enormous increase in vapour density necessary to bring out the higher members of the series may perhaps be ascribed to the possible circumstance that the entities producing them are present in smaller numbers.

It appears to me that there are two hypotheses which we may make: First, that the Balmer lines and their accompanying spectra are caused by atoms which have lost one, two, three, four, &c. electrons. Secondly, that they are produced by aggregates or complexes of one, two, three, or more atoms. In either case it seems probable that the members would be present in continuously decreasing numbers.

I feel inclined to favour the first hypothesis, for the reason that the channelled spectrum accompanying the D lines appears to be the most complex, the ultra-violet channelled spectra decreasing in complexity as we pass down the spectrum. It seems probable that complexes made up of large numbers of atoms would have more available frequencies than the single atom, whereas the loss of a large number of electrons might be expected to decrease the number of different frequencies. This, however, is a point which can be handled by mathematical physicists. Of course I have not brought as high dispersion to bear upon the ultra-violet region as has been used in the visible, for this region has been photographed by Mr. Clinkscales, one of my students, in the second order spectrum of the 21 foot grating*. This is owing to the fact that it is difficult to get a sufficiently brilliant source giving a continuous spectrum below wave-length 3300. The crater of the arc can be used for the first band, but we have to rely upon the spark below this point. The crater is comparatively weak, even in the region of wave-length 3300.

Mr. Clinkscales's photographs of the visible channelled spectrum show that there are, in round numbers, about 6000 absorption-lines, as many as thirty being crowded together in a region not wider than the distance between the D lines.

Magnetic Rotation.

The study of the magneto-optical properties of the vapour in the ultra-violet is attended with great experimental difficulties. The only positive result thus far obtained is that the magnetic rotation at the first two ultra-violet members of the Balmer series is in the same direction as at the D lines. The method employed was the usual one, polarized light being passed in succession through the magnetized vapour, a Fresnel double prism of right- and left-handed quartz, and a second polarizing prism, and the image of the horizontal fringes focussed on the slit of the quartz spectrograph.

* Wood, "Resonance Spectra of Sodium Vapour," *Phil. Mag.* [6] xv. p. 581 (May 1908); *Phys. Zeit.* ix. no. 14, p. 450.

Nicol prisms cannot be used on account of the absorption of the ultra-violet by the Canada balsam. We may, however, substitute glycerine for the balsam, or employ the Foucault prism with its air-film. Both methods were used. The curvature of the fringes at the ultra-violet lines was very slight at the 3303 line, and scarcely discernible at the 2852 line, but it was in the same direction on each side of the absorption line, and in the same absolute direction as at the D lines. Nothing has as yet been accomplished with the lines in the ultra-violet channelled spectrum, and I feel very doubtful about getting results in these regions. In the visible spectrum, as I have already shown*, the rotation is positive at some of the lines and negative at others.

More complete data regarding this point will appear in a paper which will appear shortly in the *Astro-physical Journal*, in which the more recent results obtained by Dr. Hachett and myself on the fluorescence and magnetic rotation of the vapour will be described.

July 1908.

LX. *High Purity Interference Phenomena of Chlorate of Potash Crystals.* By R. W. Wood †.

[Plate XV. fig. 4.]

SOME years ago I endeavoured to prepare large crystal laminae of chlorate of potash for the purpose of isolating spectrum lines by reflexion for monochromatic illumination of fluorescent vapours. The experiments resulted in only moderate success; but I obtained a few crystals which I believe showed narrower reflexion maxima than any that have been previously described.

One flake, measuring about 6 mm. on a side, exhibited total reflexion at normal incidence of a region of the spectrum only 10 or 12 Ångström units in width, that is, only double the distance between the D lines. The spectrum of the transmitted light exhibited a very black band at the same point, and of the same width. This band was photographed, after having been brought into the vicinity of the D lines by suitable inclination of the plate, and the D lines themselves impressed on the plate by holding a sodium flame in front of the slit for a few seconds. The photograph is reproduced on Plate XV. fig. 4, spectrum "c." The position of the D lines

* "On the Existence of Positive Electrons in the Sodium Atom," *Phil. Mag.* Feb. 1908.

† Communicated by the Author.

I have marked on the spectrum immediately above this one. (The marks refer to the lower spectrum only.)

By increasing the angle of incidence, the band can be made to move down the spectrum, widening as it moves. When in the green it appears as in spectrum "b" and is accompanied by fainter lateral minima. The narrowness of the reflected region has been shown by Lord Rayleigh to be due to multiple twin planes, sensibly equidistant. From the width of the reflected region we can form an estimate of the number of laminae present in the crystal plate. Each lamina gives us by reflexion a virtual image of the source, these images being in line, one behind the other, at normal incidence. The action is not unlike that of a diffraction grating when the diffracted ray is at grazing emergence. If we are dealing with a first-order spectrum, 1000 lines are necessary to resolve the D lines. As I have shown in a previous paper ("Interference-phenomena of Chlorate of Potash crystals")* we have in general a number of orders present, and if we photograph those in the ultra-violet, we can determine the order of the one (or more) in the visible. (Each order is represented by a narrow region of wave-lengths selectively reflected.)

In the present case since the visible spectrum shows but a single reflected band (in the yellow) we are clearly dealing with the first order. The second order will be found at wave-length 29 or thereabouts. If our visible band at wave-length λ was a second order one we should have a third order band at wave-length $\frac{2}{3}\lambda$ or 4000 which would be easily visible.

If now we compare the width of the band in the photograph with the distance between the D lines, it is clear that the crystal plate is very nearly able to separate or resolve the D lines. In other words, if we incline the plate a little less, causing the band to move up the spectrum, it will reflect D_2 before it reflects D_1 . It was, however, not quite able to do this, but would easily separate lines of twice the separation of the D's. From this we may infer that the number of twin planes in the crystal is somewhere between 500 and 1000 say roughly 700. If we are dealing with a first-order spectrum the path difference between rays reflected from two adjacent twin-planes must be equal to the wave-length of the light in the crystal. Assuming no phase-change this makes the thickness of each lamina about 0.0002 mm., and multiplying this by 700 gives us 0.14 mm. as the thickness of the crystal plate, which was not very far from the truth.

* Phil. Mag. [6] xii. p. 67.

This particular crystal plate was selected from a very large number. I have given directions for preparing the plate in a former paper *. Possibly by taking a number of selected plates and immersing them in a solution which is cooling off and throwing down crystals, we may be able to build up plates with even larger numbers of twin planes. I have not yet tried this method. If somewhat larger plates could be made they would prove superior to the spectroscope for the isolation of spectral lines for monochromatic illumination, for a much wider source of light could be used.

It is interesting to compare the crystal described in this note with the best Lippmann photographs, which show colour resulting from the same type of interference.

Mr. H. E. Ives has made the best Lippmann photographs with monochromatic light that I know of. The spectrum of the band reflected from his best plate is shown in Pl. XV. fig. 4, "a," on the same scale of wave-lengths, *i. e.* taken with the same spectroscope. It has a width about three times as great as the band reflected from the crystal. If I remember rightly, Mr. Ives sectioned a similar film and counted about 250 silver laminae, built up by the stationary waves. This is in good agreement with our estimate of the number in the chlorate crystal.

It is most remarkable that the laminae are of such constant thickness in a given plate and yet vary over such a wide range when we consider different; a plate which starts with twin planes .0002 mm. apart apparently builds up 700 laminae of the same thickness, while another plate starting with a different "grating constant" sticks to it to the end.

The process of formation of these crystals is well worth a further study. It would be very interesting to see whether a crystal once started could be induced to change its mind about its "constant" by varying the concentration or temperature of the mother liquor in which it was forming.

East Hampton, Long Island.
July 1909.

* "Interference-colours of Chlorate of Potash Crystals," *Phil. Mag.* [6] xii. p. 67.

LXI. *On Velocity of Molecular and Chemical Reactions in Heterogeneous Systems.* By Dr. MEYER WILDERMAN, Ph.D., B.Sc. (Oxon.)*.

PART I.

[Plates XVII. & XVIII.]

I. *On Velocity of Molecular Reactions in Heterogeneous Systems and the so-called Diffusion Theory.*

§1. **I**N the report of the British Association, 1896, *Zeitschrift für physikalische Chemie*, 1899, xxx. pp. 348–368, and *Phil. Mag.* July 1901, p. 50, and some other publications, I showed that the velocity of all molecular reactions between two parts of the heterogeneous system, such as transformation of water into ice, ice into water, separation of salt from an oversaturated solution, solution of salt, evaporation of liquids, condensation of vapour, solidification of solutions to solids or to solid solutions, &c., all follow the same law :

$$\frac{dt}{d\tau} = C (t_o - t)(t - t_{ov}) = c \Sigma \tau (t_o - t),$$

i. e. the velocity of reaction is at the time τ directly proportional to the remoteness of the system at the time τ from the point of equilibrium, $t_o - t$, and to the surface of contact of the reacting parts of the heterogeneous system, $t - t_o^*$, which is proportional to $\Sigma \tau$. Further improvements in the methods employed and the photographing of the reactions showed, however, the necessity of introduction of the constant K , which I called the “instability constant,” i. e.

$$\frac{dt}{d\tau} = C (t_o - t)(t - t_{ov} + K)$$

(see the above papers in the *Zeitsch. für physikalische Chemie* and *Phil. Mag.*)

$$= K (t_o - t) \text{ for constant surface.}$$

The above equation was therefore given by me in a most general way, including the solution and separation of salts, as a particular case only, in a series of publications before Noyes and Whitney (*Zeitsch. phys. Chem.* 1897, xxiii. p. 689), before L. Bruner and Tolloczko (*Zeitsch. phys. Chem.* 1900, xxxv. p. 283), and long before Nernst and E. Brunner (*Zeitsch. phys. Chem.* 1904, xlvii. p. 52), who all use it for the particular case of solutions of salts only. The equation

* Communicated by the Author.

used by Noyes and Whitney is $\frac{dt}{d\tau} = C(t_0 - t)$ (they take the surface to remain constant); Bruner and Tolloczko use my equation $C\Sigma\tau(t_0 - t)$; the equation used by Nernst and Brunner is $\frac{dt}{d\tau} = \frac{D}{\delta}\Sigma(t_0 - t)$, where $\frac{D}{\delta}$ are also constants, *i. e.* the equation of the above authors is identical with the first (less accurate one) of mine. As my papers were published also in the very same *Zeitsch. für. phys. Chem.* in which these and other authors * later published their papers, I must clearly uphold here my priority rights, since all these authors do is to give their own interpretation to my constant. All the more so, since Nernst and E. Brunner, L. Bruner and Tolloczko, give us in an arbitrary manner (with their methods, see §2) for a part of the curve what was established by me before, on careful experiments, for the whole curve; my reactions were photographed and carried out by methods of infinitely greater accuracy and reliability than theirs.

In my general equation the assumption was made that the volume remains the same during the whole time of the reaction. Mr. Drucker also investigated the effect of the volume upon the speed of solution of solids, and found that the speed is indirectly proportional to the volume of the liquid. I wish to add that, from the standpoint of my general equation, this is, at a constant surface of the solid, a self-understood, unavoidable necessity. Because, as the reaction goes on *at the surface of the solid in contact with the liquid*, the amount of substance going into solution is, at a constant surface, at the time τ only dependent upon the concentration of the solution, and this amount is according to our conception subsequently distributed by the stirrer, through the *whole volume* of the liquid. If the volume is, for example, twice as large, the increase in its concentration must be twice as slow, *i. e.* the speed of solution, *as measured from the concentrations of the solution*, will be twice as small. This naturally applies to *all molecular reactions*, of which solution of solids forms only one particular case. But I wish clearly to point out, that *this dependence upon the volume is only an apparent one*, for the reason that we are able only to measure the concentrations of the solution. The *true reaction* takes place only *at the surface of the solid*, and here the phenomenon of solution is *independent of the volume* of the rest of the solution. Only the surface of contact and the concentration of the solution in contact with the solid are the working factors and come into consideration here.

* See Karl Drucker (*Zeitsch. für phys. Chem.* 1901, p. 173, and p. 693), who wrongly attributes my equation to Noyes & Whitney.

§2. *On the Method of Investigation.*

- (a) *On the artificial production of solid blocks suitable for investigation in this region. Also on the solids used by Nernst and E. Brunner and others to prove the Diffusion Theory.*

The speed of reaction is directly proportional to the surface of the solid, and it is quite evident that no reliable research at a constant surface is possible, and still less are comparative quantitative experiments possible, so long as we cannot procure a solid, with a sufficiently large surface, which is quite uniform throughout its whole thickness, which can be reproduced every time we want it, which is chemically pure, which is suitable for quick and reliable measurements of concentration, and which represents a reaction free from subsidiary reactions and from other complications. To get all these requirements realized at once is by no means easy. Metals seem to answer the requirements of surface, but, except Pt, Ag, and the liquid mercury, they all cannot be got chemically pure, and insurmountable difficulties are created for correct measurements of the speed of reaction by the interference of local action due to small variations in the impurities. Again, some of the minerals, such as marbles, gypsums, and some other few substances, present the best material for getting suitable blocks for an investigation, but these minerals are seldom chemically pure, few of them are uniformly the same everywhere, and still fewer of them are substances which are suitable for a rapid and reliable titration. The solid substances, which are most suitable and desirable for research, are for this reason those which are artificially prepared in the laboratory, but the difficulty is that these substances are almost without exception obtainable either in small crystals or powders, &c., while we require for the investigation blocks with large surfaces. How Nernst and Brunner try to overcome the difficulties here is best to be seen from the *two principal substances of their investigation*, upon which they tried to build up experimentally their diffusion theory, namely, benzoic acid and "magnesium hydroxide."

About the first we read that, to create a block, they "melted the benzoic acid in a test-tube, poured it into a porcelain cover of a crucible, 5 cm. diameter. On cooling the same separated from the cover, and they fixed it in again with shellac; the surface was then rubbed down with a knife and sandpaper. The surface still clearly retained its crystalline structure. This cake still contained scratches at the rim, and they filled these now up with paraffin." With

this method we always get air-tubes and spaces enclosed in the acid, and this cannot be avoided even by removing the air from the liquid acid in vacuum or by very slow cooling of the acid in a double-walled air-bath. The substance does not crystallise as one uniform mass. Photo (1) (Pl. XVII.) gives the surface of the block of benzoic acid as soon as the top thin film is removed during the reaction. There is no need for me to add that such a block cannot possibly be suitable for any comparative quantitative measurements. This process of melting can naturally find only a limited application, cannot e. g. be applied to inorganic substances requisite for the research. From among the last Nernst and Brunner chose for their research "magnesium hydroxide." We are told that they "took advantage of the property of calcined MgO mixed with water, to dry to a thick mass and to become hard." As a matter of fact, ordinary analysis shows that the $\text{Mg}(\text{OH})_2$ always contains, under their conditions of work, very much carbonate, and we find that $\text{Mg}(\text{OH})_2$ cannot stick together without it. I find that if we mix MgO with water and let it dry in an ebonite lid with vertical rims over CaCl_2 in vacuum, so as to prevent absorption of CO_2 by the MgO or $\text{Mg}(\text{OH})_2$, we get the block as in photo (2); if we dry pure $\text{Mg}(\text{OH})_2$ mixed with H_2O over CaCl_2 in vacuum, we get a block as in photo (3) (in which one part of the surface has been polished). Both of them show a great contraction on drying, contain numerous air-holes, before and after their surfaces are cut away with a razor, contain big cracks across, and *fall to pieces as soon as they come into contact with water*, as is to be seen from photo (4), in which the breaking up of $\text{Mg}(\text{OH})_2$ under water is shown. If we mix MgO with 10 per cent. gum-arabicum, fill with it the ebonite lid and dry over CaCl_2 , we get a disk, which sticks together as in photo (5), which at the same time shows the enormous contraction of the mixture; but as soon as we bring this disk in contact with water it breaks up to pieces as is seen from photo (6). *There is no possible way of getting a block of pure $\text{Mg}(\text{OH})_2$ which could be brought into contact with water without falling at once to pieces!* I lost quite a considerable time in my attempts to produce such a block, but entirely failed, though I was with my method quite successful in a great number of other cases, namely: I compressed the mixture of MgO and H_2O directly after they were mixed, and the mixture MgO and H_2O after standing in a closed up vessel for about two weeks; then I compressed dry $\text{Mg}(\text{OH})_2$, as well as $\text{Mg}(\text{OH})_2$ mixed with water in moulds, under a pressure of several thousand atmospheres; and though I succeeded with a conical piston in

this way in getting beautiful uniform blocks free from air-holes and cracks and the blocks seemed quite excellent, yet when the same were brought into contact with H_2O they immediately fell to pieces. The solids on coming into contact with H_2O expand again to such a degree that they fall in no time to pieces. On the contrary, $MgCO_3$ shows no such property, but gives excellent blocks which stand water very well. Photo 7 gives a block of $MgCO_3$ compressed under 2500 atmospheres, in which the impression of the piston in the centre is still to be seen, and then allowed to remain under water. Photo 8 shows a block of $MgCO_3$, which was compressed in an ebonite lid in a special arrangement by means of a screw, after it was first mixed with water to a paste, and then allowed to remain under water. Photo 9 shows $MgCO_3$ as got in an ebonite lid after the same was mixed with water to a paste and dried, and then the lid and the dry disk were allowed to remain for a long time under a dilute HCl solution; for they show at the same time that water does not break up the same. Nernst and Brunner give us, on p. 79 of their paper, a series of apparent reasons why their theory does not require to hold good in case of Oxides, Carbonates, &c. which do not agree with their theory, and why this one substance, $Mg(OH)_2$, is to be of all substances *the* substance to be taken for the test of their theory, and we are invited to accept their theory on the basis of " $Mg(OH)_2$ " alone. Now we find that Nernst and Brunner's so-called $Mg(OH)_2$ is nothing but a *mixture* of oxide, hydroxide, and carbonate of magnesia. Not contented with this, since their "magnesium hydroxide" blocks do not hold sufficiently well, they mix it with gum arabicum, which, as known, is the potassium and calcium salt of "Arabinsäure" or "Gumisäure," and so their mixture contains besides also the Magnesium salt of "Arabinsäure." As the conditions of preparation of their blocks were never identical in respect of time, quantity of gum arabicum used, &c., the composition of their mixture was never the same. And with such mixtures Nernst and Brunner give us their quantitative experiments upon speed of reaction, their comparative experiments, their principal experimental proofs of the diffusion theory!

The method I use for getting from a substance chemically pure and suitable for the research, blocks of any desired thickness, of the same uniformity through the whole of its mass, free from air and cracks, and enabling us to use the same blocks for a series of experiments for comparative experiments and to reproduce the same, consists in their *artificial preparation under the very high pressure of several*

thousand atmospheres. The substance is brought into a steel ring of an internal diameter = 51 mm., external diameter = 127 mm., placed in the steel base (Pl. XVIII.), and is compressed here under several thousand atmospheres with the piston, which is 51 mm. diameter and 12 cm. long. The diameter of the piston is 51 mm., or very nearly 2 inches. The diameter of the ram of the hydraulic press is 10 inches, the ram is compressed to 75 or 100 atmospheres on the gauge, thus producing a pressure on the piston and the substance of $75 \times 25 = 1875$, or $100 \times 25 = 2500$ atmospheres per square inch. The maximum compression of the solid usually takes place much below these pressures, already at about 1000 to 1400 atmospheres, and often below this. The substances are left under the above pressure usually for 16 to 24 hours, when the pressure is generally released; the base is then removed from the ring containing the compressed block of the substance, and the latter is pushed out from the ring (the pressure usually required is 70 to 350 atmospheres per square inch), with the piston, into a steel ring of a larger diameter placed below the ring, it being arranged that the disk should fall on a piece of soft felt below. The ring, base, and piston must be perfectly polished so as to get a perfect uninjured block. The ring itself was a little modified, made internally a little conical (a fraction of a mm. suffices) from top to bottom, in order that the block of substance shall leave the ring at once, as soon as it is moved by the piston, instead of its requiring to be pressed through the ring through its whole thickness. In such a way we get beautiful disks, with perfect surfaces, free from any air-holes, cracks, &c., requiring no admixtures in order to make the substance stick well together, with perfect uniformity through their whole thickness; whether the substance is crystalline or amorphous, an organic or inorganic solid, a compound or an element (S, C, metal powders, &c.), it is crushed to a uniform body; one disk 1.5 to 2 cm. thick is quite enough for carrying out a long series of comparative experiments, and we are able to *polish* and to rectify over and over again the surface of the same block for the same experiment, because it must be a strict rule to allow the solid to corrode during solution only as little as possible and to repolish the same as often as is practicable. We can keep the dissolving surface of the solid always at nearly the same height in the liquid, by fixing the blocks to ebonite disks of different thicknesses. Photo 10 (Pl. XVII.) shows a disk of benzoic acid before treatment; photo 10 A shows the block of benzoic acid after solution in water

during two hours. The surface is still excellent. I find, however, that while some of them (such as benzoic acid, most organic substances, MgCO_3 , &c.) remain on placing the disk into water (under stirring) in perfect condition; others, such as $\text{Mg}(\text{OH})_2$, $\text{Fe}_2(\text{OH})_6$, & ., break up mechanically at once, since when the disks are brought into contact with water they suffer too great an expansion. All the same, the above method now opens the way for a true and reliable investigation in this region, putting at our disposal an exceedingly great number of chemically pure substances, suitable and desirable for this research, while before we practically had none, and we were dependent upon minerals, which are seldom perfectly pure. Care was taken to use only the purest chemicals for this research, and the firm of Merck prepared the same for me with special care and under special instructions; and where minerals had for special reasons to be used, only the best and purest specimens procurable were employed.

(b) *On the Arrangement of Constant Speeds of Stirrer.*

If we have a super-cooled or a super-saturated solution, so that very fine particles of ice or salt separate from the whole of the liquid, and through the whole of the liquid, then, by the use of an effective stirrer creating currents in all directions, a uniform state of the whole system during the reaction can be obtained, as can be seen from the photographic curves in my paper (*Zeitschr. für phys. Chemie*, 1899) giving the speed of these reactions. When, however, we have a solid piece, salt or ice in a liquid mass, so that the contact between the solid and the liquid is only local, and reduced to a comparatively very small surface of contact, it is extremely difficult to get the liquid uniform through the whole mass, as already mentioned in the above paper. Nernst and Brunner use an air-motor, which is known to be inconstant, and the variations of its speed were, according to Nernst and Brunner, as much as 10, 15, sometimes even 30 per cent. The direct experiments with pieces of rubber, of the same specific gravity as water, showed me, that if we keep the solid fixed at the bottom and move the liquid by the stirrer, the number of revolutions which the liquid makes past the solid is not directly proportional to the number of revolutions of the stirrer, but is considerably smaller. The liquid is also never equally moved in all its layers, as is to be seen from the fact, that a cone is formed in it with the apex (smallest speed) on the stem of the stirrer, and the base (biggest speed) above it, which is the higher the quicker the

stirring. For this reason I preferred to fix the solid disk, as Drucker did, to the stirrer itself, so as to make the number of revolutions of the same equal to the number of revolutions of the stirrer, and I placed the investigated disk, always of nearly the same thickness, between the two stirring plates of the double stirrer (see Pl. XVIII. fig. 1), so that, besides moving the block, the liquid was stirred above and below it. The double stirrer is made of ebonite, which is affected neither by acids nor by alkalis. The two stirrers are connected with ebonite rods forming a cage for the block of substance to be investigated. The last is covered with paraffin of a high melting-point at all surfaces except the top, a rubber ring is put on it and prevents it from being damaged after it is fixed between the rods. Though apparently little work had to be done, I found that to get constant speeds I had to use a powerful electromotor ($\frac{1}{8}$ H.P.) to prevent its heating up, and that constant voltage as well as a regulation of the speed had to be provided for. Both the field and armature were supplied with a series of resistances and with continuous spiral resistance when required. In all experiments I usually employed 200 volts thrown upon the common terminals connecting the field and armature. For the variation of speed within very great limits, I used in the first instance different sets of wheels of different diameter, which could be changed as desired, and the wheels were driven by a flexible steel spiral which could be applied to all of them by means of the arrangements seen in fig. 2 (Pl. XVIII.). To get the variations of speed in still greater limits the volts thrown upon the field and armature were different, use being made of accumulators. I was thus able to change *the speed of the stirrer* from about 1 revolution per minute to about 500 and to keep the speed practically constant. The best results were obtained for speeds of about 100 to 150 revolutions per minute, when the variation after hours did not amount to 1 revolution per minute, and which I adopted for my investigations. As the effective stirring depends more upon the construction of the part of the stirrer in the liquid than upon the number of revolutions, I reduced the last by improvements in the first, and this enabled me to get with an ordinary one-fifth second stop-watch, easy, reliable counting of the number of revolutions of the stirrer, which usually were kept at about 130 per minute. The speed-indicator was used only for speeds above 250 revolutions per minute. I found that no permanent speed could be got for longer periods when ordinary cords are used, because as these stretch under work the speed changes also. A thin steel wire was for this reason

wound to a spiral on a lathe supplied with small hooks and used instead of cords. This has a wonderful flexibility, gives an excellent grip, and allows good adjustment, if required, by cutting off a few rings or by forcible stretching of a few of them, and gives always the same constant speed. By an arrangement, as seen at C of the photograph (fig. 2), the same spiral can be used for the different pulleys.

(c) *Further details of the Method* (see fig. 1, Pl. XVIII.).

The beaker with 1500 c.c. of the liquid was kept in a bath of a constant temperature, a 1/10 degree thermometer having been immersed in the liquid, with its bulb close to the solid block; the effect of stirring upon the temperature of the liquid seemed to be but small, but had to be adjusted sometimes by the temperature of the outer bath, which was continuously stirred by the second stirrer driven by a pulley from the first. In case of substances of very small solubility, such as the gypsums, which do not admit sensitive titration, especially since very dilute solutions had to be investigated here, the method of electric conductivity after Kohlrausch had to be used. Two platinum electrodes (see fig. 1) were fixed in the ebonite lid of the beaker, for making these measurements, the electrodes coming into the middle between the two stirrers. A series of known concentrations of the same $\text{CaSO}_4 + 2\text{H}_2\text{O}$, which later on had to be investigated as a solid disk, was prepared, and small intervals of concentrations were investigated with a series of resistances, and so a scale was got for them on the bridge, for the same temperature and speed of stirrer, which subsequently was used when the solution of the disk was investigated. The water used in these experiments was the same. The volume of the solution remained for the whole series the same, namely 1500 c.c.

In case of other substances, such as benzoic acid, or of mixtures, the speed of reaction was measured by taking out samples and titrating the same. The samples were drawn off with a 10 c.c. pipette through the glass tube fixed in the ebonite lid of the beaker, always from the same place between the two stirrers; after the pipette was first twice filled with the solution, and the solution let back into the beaker, the final filling of the pipette for titration was carried through within the last 5 to 10 seconds of the minute.

Very great and quite exceptional precautions had to be taken with the titrations of this acid. I titrated the benzoic acid with a .088 normal baryta, back titrating the same with a .022 normal benzoic acid. The baryta was added from a

thin well-running tube, 1 mm. of which indicated $\cdot 01$ c.c., and I read the tube with a lens to about $\cdot 2$ mm. or to $\cdot 002$ c.c., and the excess of baryta had to be titrated with the more dilute $\cdot 022$ normal benzoic acid so that the last drop should not affect the result much, and this operation had to be repeated twice to further reduce the reading-error. As 50 c.c. of ordinary distilled water contain already an amount of CO_2 corresponding to $0\cdot 3$ – $0\cdot 4$ c.c. of a $\cdot 022$ normal solution, and at least 50 c.c. of water are required for washing out the sample bottles and for titration, the CO_2 of the water alone is enough to cause an error of 10, 20, or 30 per cent. in the total difference measured. Therefore the water in the wash-bottle had to be freed from CO_2 by boiling and then protected by a tube of soda-lime to prevent its contamination with CO_2 from the air or through our blowing, the total amount of wash-water used for each titration had to be kept the same in each experiment, and the water itself had to be titrated and the samples freed from CO_2 before titration by boiling. As rapid stirring mixes energetically the solution with the air containing CO_2 , and the experiment lasts for several hours, the water in the beaker had to be protected against contamination with CO_2 , as shown in fig. 1 (Pl. XVIII.). The glass cylinder was closed up with a thick ebonite piece and sealed up with Crooks' cement between glass and ebonite. In the ebonite piece a circle was eccentrically cut out to allow the blades of the stirrer mixing the solution to pass through it into the beaker. A deep circular groove 17 mm. wide was cut out in the remaining ring, concentric with the cut-out circle; to the stem of the stirrer a circular lid with a deep projecting circular rib 7 mm. thick, cut out of one piece of ebonite, was fixed, and the stirrer was fixed so that the circular rib entered deep into the groove, leaving 5 mm. free space all round without touching the circular ring at the bottom; this groove was filled with distilled water sufficient to form a secure seal with the rotating circular rib of the lid of the stirrer, and the stirrer was thus always kept in the same position in the beaker. None of these very necessary precautions in the titration were taken by Nernst and Brunner, Bruner and Tolloczko, and others.

All the above precautions in the preparation of the blocks, in the arrangement of constant speeds, in the titration &c., I had to take before I was able to get the constants, as I give them in this paper, which are considerably worse than those supplied by Nernst and Brunner and others. It is true, Nernst and Brunner tell us, that "the greater variations of A were omitted if they differed much from the rest," but

this is a very strange thing to do, when we are asked to take their results as quantitative measurements and to give them all the credence we have to give the quantitative results, instead of leaving it to the reader to see for himself what value those A's really possess. Especially is this regrettable when we are invited to accept a theory on the basis of conclusions which are mostly based upon differences in the values of A, which, it is quite evident, are often smaller than the experimental error of the method itself.

§ 3. *The Theory of Molecular Reaction, including the Solution of Solids in a Solvent. Also on the Diffusion Theory.*

In the first place let me state again, in somewhat greater detail, the content of my equation as far as it concerns the solution of salts, before I pass to the consideration of its content according to the diffusion theory. I assume that every substance has a solution pressure, which causes the solid to pass into the solvent, whenever brought into contact with the same; that this goes on until the solution becomes saturated, so that the solution pressure of the solid is kept in equilibrium by the counter osmotic pressure of the saturated solution, which (leaving out of consideration galvanic cells where we have to deal with electrostatic forces) at equilibrium are equal; that in case of an unsaturated solution, before equilibrium is reached, the solution pressure of the solid is greater than the counter osmotic pressure of the solution, and the substance of the solid is still driven into the solution by a force, which is directly proportional to the difference of the existing solution pressure and the counter osmotic pressure, or to the difference of the osmotic pressure of the solution at equilibrium (which is equal to the solution pressure of the solid) and the osmotic pressure of the solution at the time τ , or also directly proportional to the difference of the concentration of the solution at equilibrium and its concentration at the time τ , since the osmotic pressure is directly proportional to the concentration. Similarly, in case of an over-saturated solution, the solution pressure of the solid is smaller than the counter osmotic pressure of the solution, and the substance in solution is driven into the solid by a force, which is directly proportional to the difference of the existing osmotic pressure of the solution and the solution pressure of the solid (or osmotic pressure of the solution at equilibrium). The speed with which the solid passes into the solution or from the solution into the solid, I therefore take to be directly proportional to the driving force, and the number of molecules passing from the solid

into the solution, and *vice versa*, is evidently the greater, the larger the surface from which they pass into the solution, or the larger the surface of contact of the solid and liquid, because the reaction goes on all along at every point of the solid in contact with the solution. Hence my equation

$$\frac{dt}{d\tau} = C\Sigma\tau(t_0 - t).$$

These extremely simple and clear conceptions about the cause of the shape of my *general* equation, Noyes and Whitney, Bruner and Tolloczko, Nernst and Brunner tried to replace, singling out a special case of solution of solids from all the rest of the reactions, by other conceptions. Let us now see what these conceptions are, how they simplify matters purely theoretically, and how far they are confirmed by true experiments. Noyes and Whitney, dealing with solubility of salts, write :—" We can imagine that the solid substance is always surrounded with an infinitely thin layer of a saturated solution, and that the process consists in a diffusion from this layer to the rest of the solution, which is kept by stirring homogeneous. Then according to the known laws of diffusion the speed of reaction will be directly proportional to the differences of concentration of the saturated solution

and that of the solution, i. e., $\frac{dx}{d\tau} = A'(C' - C).$ " This view was adopted first by L. Bruner and Tolloczko, who went a step further, and assumed that the velocity constant is nothing else but the *diffusion constant*, and later on they introduce also the idea of a finite layer δ &c. The views of these investigators were later on fully adopted and followed by Nernst and Brunner, who instead of D use Bruner and

Tolloczko's $\frac{D}{\delta}$, and make a step further still, by trying to explain the equations of Boguski, Spring, Haber, and Lorenz, giving speed of chemical reactions in heterogeneous systems, as diffusion equations also. With the last we shall deal in detail later on. In this first publication I shall restrict myself principally to molecular reactions only.

Returning to the work of Bruner and Tolloczko, as $\frac{D}{\delta}$ had to be made equal to the velocity constant, they soon got the result that if the diffusion constants were not to upset the theory, δ or the layer of Noyes and Whitney cannot be taken to be "infinitely thin," but must have a measurable thickness. In this, as well as in my identical equation,

every letter denotes the same thing, but C' is in my conception the concentration of saturation which the system is striving to reach, owing to *the solution in contact with the solid being unsaturated*, $C' - C$ being directly proportional to the force driving it to the state of equilibrium, while Noyes and Whitney assume that the concentration of saturation is *always* present near the solid salt, while the concentration of the rest of the solution is entirely another one. To make, however, this assumption of constant saturation at a surface possible, Bruner and Tolloczko had further to introduce the conception of infinite speed of reaction at the surface of the solid, while following them Nernst *further introduced also the idea of "infinitely great forces acting at the surface,"* since without such assumptions the above explanation cannot, evidently, be upheld; and then to suit the idea of the δ a second hypothesis had to be made, by Tolloczko and Bruner, also adopted by Nernst and Brunner, that *the solution cannot be mixed up to the solid itself, but the mixing stops at a distance from the solid*. Since without this mysterious property we get from $\frac{D}{\delta} = K$, that if δ were according to

Noyes and Whitney infinitely small, D , or the diffusion constants as calculated from this equation, would only prove the *falsity* of the theory. This is how the original hypothesis of Noyes and Whitney, which seemed to be very simple, grew up into a conglomerate of assumptions. I shall now test, step by step, what this theory and all its assumptions are really worth.

(a) Let us now first see, how Noyes and Brunner support their hypothesis, following in all essential features the ideas given by Bruner and Tolloczko. About the first Noyes writes: "At the surface of contact of two phases (or parts) of the heterogeneous system equilibrium is extremely quickly established." "This," in Nernst's opinion, "follows already *a priori* theoretically as a necessity, because otherwise noticeable difference of the chemical potential would exist at the separating surface, *i. e.*, at infinitely near points, which evidently would lead to infinitely great forces and velocities of reaction. This, however, means nothing else than that in the immediate vicinity of the two surfaces the equilibrium is established with infinite speed. If we assume what is more nearly true, that the surface of separation is not a mathematically sharp plane, we have still to deal with dimensions of the order of the spheres of action of molecular forces, and if we can no longer speak of infinitely great velocities of reaction, still they must be of a very great order, which practically

amounts to the same." This says in so many words what Bruner and Tolloczko told us before (*Zeitsch. für anorg. Chemie*, 1901, p. 328) that "the saturated solution at the surface of the solid is formed instantaneously." Now, this fundamental assumption of Nernst is quite wrong, as can be seen from the following: when we mix two liquids, say in equal parts, and a reaction goes on between them, we have, in the first instance, a certain difference of chemical potentials acting between the molecules in the system, since otherwise no reaction would go on; secondly, the acting molecules can only be removed from one another by double the distance of the spheres of the molecules themselves. According to Nernst all such reactions, such as solution of anilin in acetic acid, &c. ought to go on with an infinitely great speed in the homogeneous system.

Now, a very great number of reactions in *organic* chemistry are carried out by mixing two liquids together in the above manner, and all organic reactions, with few exceptions, belong to the *slow, often to the extremely slow*, not to the instantaneous, reactions. On the other hand, many reactions, *even in very dilute solutions*, go on almost instantaneously, though the reacting molecules in the solution are very considerably removed from one another. This is so well known to every chemist, that I need not dwell upon it any longer to show the utter futility of the above assumption. Nernst's assumption that "infinitely great forces" are acting between different parts of the heterogeneous system, when it is not in equilibrium, is therefore absolutely unsupported by daily experience. Here we have to deal with ordinary solution of solid salts, where no electrostatic forces come into play, and consequently where the osmotic pressure of the solution at equilibrium equals the total solution pressure of the solid. We shall see later on that the forces coming into consideration here for the different kinds of reaction are not "infinitely great," but of measurable, and often of very small values indeed. So far I much prefer the less detailed views of Bruner and Tolloczko, who speak of "instantaneous reactions," without binding themselves to "infinitely great forces."

(b) I now turn my attention to Nernst and Bruner's support of the hypothesis that at the surface of the solid a saturated solution is formed, which is only capable of passing into the region of the action of the stirrer by diffusion. My question is, where is the proof for the same? What is there to prevent the liquids from passing to the very solid? In my opinion neither their deduction of the equation on p. 63, nor

their own experiments on p. 62, giving the variation of the constant A with temperature—the only experiments which ought to have brought some support to their assumptions—tend to support their views, but on the contrary disprove the same. The equations (3) and (4) of Nernst and Brunner are not deduced according to proved conceptions, but are nothing else than a graphical adaptation to the above mentioned conceptions, which became unavoidable, to make them artificially fit into a known equation. But as Nernst and Brunner admit, that it is impossible to assume that there should be on the one hand a layer of complete rest, where the equalization takes place only by diffusion, and on the other hand a layer outside the same, where equalization takes place only by convection caused by the stirring,—it follows from this that even if Bruner and Tolloczko's, Nernst and Brunner's assumption of two layers be made, a gradual transformation of the second layer into the first under a gradual diminution of convection must take place; the straight line AB is therefore arbitrary, and must be replaced by a curve, of an unknown equation, in all probability by a logarithmic curve, and the value D is not CB, but a value which may be considerably greater; and the whole deduction of equation 4, that is of my equation, from the diffusion conceptions is arbitrary: the strict existence of my equation is, in my opinion, for this reason, if anything, a proof that their conceptions are wrong, not that they are right. Moreover, for all we know, there is no reason whatever to assume, that even if a saturated solution be formed at the surface of the solid, this should not form only an infinitely thin film, as originally assumed by Noyes and Whitney, that this film should not be of the size of the sphere of a molecule, that is of the dimensions 10^{-15} mm. instead of Nernst's 0.02 mm., and that the liquid from outside this film should not enter it, *i. e.* should not reach the very surface of the solid itself diluting the same. Indeed, in case of solidification of water in a U-tube, *i. e. even without stirring*, the water reaches the ice or the ice the water to a proximity of about 10^{-15} mm.—why should this not be the case with the liquid in contact with the solid especially when it is stirred? The very value calculated by Nernst and Brunner, by the use of diffusion constants, for “ δ ” as being equal to 0.02 mm., which seems to them “as very probable,” seems to me on the contrary *very improbable*, in view of the results obtained for solidification of water. With these much more natural assumptions for δ , &c., their deduction of equation (4) would again cease to hold good, and *the actual values of the diffusion*

constants would present, in my opinion, a striking proof that the application of the above diffusion conceptions here is entirely out of place.

(c) Not only is there no proof for their values of δ , but the assumption of the existence of the layer at all is extremely unnatural, and is evidently wrong. My equation holds good also for evaporation of solids and liquids, where the diffusion of the gas into the whole gaseous space is instantaneous, and where there is nothing to keep the gas saturated at the surface of the solid, and in the rest of the space unsaturated, where there is nothing to prevent the moving molecules from reaching the very solid on its way. If this is so for the gaseous space, where is the need and the force of these most unnatural assumptions as regards the solution?

Note also, that the laws of Boyle and Gay Lussac and the whole kinetic theory were deduced without the assumption of the existence of such a layer and presuppose that the gas can reach the very solid. Note, that van't Hoff's laws of Boyle and Gay Lussac in solutions presuppose that the substance in solution behaves in a similar manner to the substance in the gaseous space. All this is, in my opinion, a solid proof against the above fundamental assumptions of the diffusion theory.

(d) I now pass to an important experimental test of the diffusion theory, and this is the influence of temperature upon the speed of reaction. If the velocity of reaction is, as we are told, regulated by diffusion, then A must vary with temperature as the diffusion constant F does. This, in my opinion, would be a serious proof for the diffusion theory if it proved to hold good, since it would have represented a quantitative test of the theory instead of remaining a mere assertion with no proof whatever. Now Nernst and Brunner investigated the speed of solution of benzoic acid at different temperatures. They find A at $20^\circ = 2.30$, at $30^\circ = 1.5 \times 2.30$, i. e. an increase of 5 per cent. per degree, while it ought to have been only 2.5 per cent. according to the variation of the diffusion constant. One would say, this is a most decisive proof against their theory; but there is always, according to Nernst and Brunner, a way out of the difficulty. It is enough for this to assume that the adhesion and with it the thickness of the layer δ , become smaller with temperature. The sole effect of the introduction of this layer δ is to prove something without proof. To this δ all sorts of properties are now attributed, without, however, the least evidence being produced. The diffusion-constant F is a clear, known physical

constant and can be tested in an independent manner, while δ presents no such embarrassing feature, as it *cannot be tested in any independent way*. It will be shown later on that the apparent proof of " δ " consisting in *the would-be fact, according to Nernst and Brunner, that the speed of solution of magnesium hydroxide, magnesium metal in benzoic acid, is the same as the speed of solution of benzoic acid itself, does not exist and is based only upon quite arbitrary experiments*. About this δ we read:—"This δ should depend only upon speed of stirring and temperature and not upon the nature of the substance in solution;" but a little further again "this, however, is not the case, because this layer has a resting surface on one side and moving on the other, and the effect of the last becomes the greater the more we are removed in this layer from the resting surface," "nor is it independent of the value F itself" (*i. e.* it depends upon the nature of the substance), "nor is it the same at all places of the surface of the solid," and lastly, "whether δ depends only upon temperature and speed of stirring it is difficult to say in advance." In short we know absolutely *nothing about this very δ , about which we ought to know everything and upon the value and existence of which the whole theory depends*.

It is quite evident that until some independent way be used to determine the value δ in each case, so as to show that my velocity-constant A is, in each case, really composed of the two known values F and δ , *not an atom of evidence can be brought for the existence of such a relationship*. All Nernst and Brunner do is to divide F by my constant A, so as to get the value which δ *ought* to have, in order that $\frac{F}{\delta}$ *should* be equal to A which is found in the experiment, as required by their theory, while what *we wish to know, require to know, is, does δ really possess such a value, and does it exist at all?*

I now give here a very careful investigation of the *influence of temperature upon the velocity-constant*, carried through with the same benzoic acid which Nernst and Brunner investigated, and for a series of temperatures between 1° C. and 60° C.

In the following table—

b' gives the number of cc. Baryta requisite for neutralization of 10 cc. of the benzoic acid in solution.

C_1 gives the concentration of benzoic acid in solution at the time τ , if the concentration of the benzoic acid solution saturated at $17^{\circ}\cdot 5$ C. is taken as unit and 10 cc. are put equal to 10.

v is the volume of the solution at the time τ .

C is the concentration of the benzoic acid at the given temperature, expressed in unit concentrations of the benzoic acid saturated at $17^{\circ}\cdot 5$ C., which we denote with letter (a) .

A is $\frac{v}{d\tau} \ln \frac{C-C_1}{C-C_2}$ or the *velocity constant* of the reaction of solution of benzoic acid.

TABLE I.

Influence of Temperature upon the Velocity Constant of Molecular Reactions at different Temperatures. (Speed of solution of benzoic acid in water at 60° C., 40° C., 31° C., $17^{\circ}\cdot 5$ C., $1^{\circ}\cdot 5$ C.)

No.	b' .	C_1 .	v .	$C-C_1$.	$d\tau$.	A .	
<i>Speed of solution of benzoic acid in water at 60° C.</i>							
1 ...	0.843	3.338		41.862			14th May, 1907.
2 ...	1.449	5.738	1490	39.462	10	8.797	$T=333^{\circ}$.
3 ...	2.058	8.150	1480	37.050	10	9.327	C at equilibrium at 60° C.
4 ...	2.711	10.73	1470	34.47	10	10.610	$=1.2132$ gr. in 100 cc. of
5 ...	3.259	12.90	1460	32.30	10	9.487	water; <i>i. e.</i> 10 cc. of the
6 ...	3.798	15.04	1450	30.16	10	9.947	saturated solution $=45.2$ cc.
7 ...	4.325	17.13	1440	28.07	10	10.340	of (a) .
Average ...							$n=130$ rev. per minute.
							Corr. for water $=0.05$ cc.
							Pipette $=10$ cc.
<i>Speed of solution of benzoic acid in water at 40° C.</i>							
1 ...	0.428	1.695		18.985			21st May, 1907.
2 ...	1.077	4.265	1470	16.415	42	5.089	$T=313^{\circ}$.
3 ..	1.425	5.642	1450	15.038	22	5.775	C at equilibrium at 40° C.
4 ...	1.839	7.282	1440	13.398	30	5.543	$=0.5551$ gr. in 100 cc. of
5 ...	2.244	8.886	1430	11.794	31	5.883	water; <i>i. e.</i> 10 cc. of the
6 ...	2.588	10.250	1420	10.430	30	5.819	sat. sol. $=20.68$ cc. of (a) .
Average ...							$n=130$ rev. per minute.
							Corr. for water $=0.03$ cc.
							Pipette $=10$ cc.
Average ...							
5.756							

No.	b'	C_1	v	$C - C_1$	$d\tau$	A	
<i>Speed of solution of benzoic acid in water at 1°·5 C.</i>							
1 ...	·011	·0436	1490	6·2494	23·75	1·326	9th May, 1907. $T = 274^{\circ}\cdot 5$. C at equilibrium at $1^{\circ}\cdot 5\text{ C.}$ $= 0\cdot 1689$ gr. in 100 cc. of water; <i>i. e.</i> 10 cc. of the sat. sol. $= 6\cdot 293$ cc. of (a). $n = 129$ rev. per minute. Correction for water = $0\cdot 045$ cc. Volume of pipette = 10 cc.
2 ...	·044	·1742	1480	6·1188	19·25	1·558	
3 ...	·075	·2970	1470	5·996	30·0	1·881	
4 ..	·132	·5228	1460	5·7702	50·0	1·627	
5 ...	·211	·8356	1450	5·4574	58·0	1·377	
6 ...	·285	1·128	1440	5·1650	60·0	2·218	
7 ...	·398	1·584	1430	4·709	45·0	1·225	
8 ...	·445	1·762	1430	4·531	40·0	1·487	
9 ...	·492	1·948		4·345			
Average ...						<u>1·587</u>	

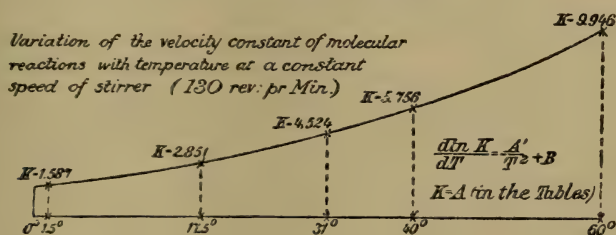


Diagram 1

Influence of Temperature upon the Velocity Constant A of Molecular Reactions.

We find

$$\frac{d \ln K}{dt} = \frac{A'}{T^2} + B; \text{ if } B=0, \frac{d \ln K}{dT} = \frac{A'}{T^2}$$

and

$$\ln K = -\frac{A'}{T} + \text{const.} \quad \ln K_{T_0} - \ln K_{T_1} = A' \frac{T_0 - T_1}{T_0 T_1}$$

$t =$	1°·5	17°·5	31°	40°	60°
$T =$	274·5	290·5	304	313	333
$A =$	1·587	2·851	4·524	5·756	9·946

It follows from this :

$$\ln \frac{2\cdot 851}{1\cdot 587} = \frac{A' \times 16}{274\cdot 5 \times 290\cdot 5} \text{ and } A' = 2920 ;$$

$$\ln \frac{4\cdot 524}{2\cdot 851} = \frac{A' \times 13\cdot 5}{290\cdot 5 \times 304} \text{ and } A' = 3020 ;$$

$$\ln \frac{5\cdot 756}{4\cdot 524} = \frac{A' \times 9}{304 \times 313} \text{ and } A' = 2550 ;$$

$$\ln \frac{9\cdot 946}{5\cdot 756} = \frac{A' \times 20}{333 \times 313} \text{ and } A' = 2850 .$$

Thus I find that the law regulating the variation of the velocity-constant with temperature is for molecular reactions similar to the law of van't Hoff for homogeneous reactions; it has the same thermodynamic explanation, i. e. is in connexion with the maximum work performed during the reaction in a reversible cycle. I will show in my publication on chemical reaction in heterogeneous systems that the same equation holds good in case of reaction of acids &c. upon little soluble solids and confirm it also upon data taken from the investigations of other reliable scientists.

The above is a very strong and reliable proof against the diffusion conceptions of Noyes and Whitney, Bruner and Tolloczko, Nernst and Brunner. As to the percentage increase of A, this gradually drops from 5.1 per cent. per degree between $1^{\circ}5$ C. and $17^{\circ}5$ C. to 3.64 per cent. between 40° C. and 60° C., instead of being 2.5 per cent., as required by the diffusion theory; and this increase of A drops with rise of temperature instead of rising.

(e) My further question is, what do we gain by the Bruner-Tolloczko's, Nernst and Brunner's hypothesis? Nothing either in clearness or in generality of conception. In the first instance, what has become of the speed with which the solid passes into the thin film of the solution, the heterogeneous reaction proper? and which law regulates this reaction? What do we know of the "infinite forces at play," mentioned by Nernst? Are we to be satisfied by mere assertions without proof, or are we going to know anything about this speed and about the forces at play and the mode of their action? In the theory of galvanic cells we are, at any rate, consistent, we calculate and measure the solution-pressures, whether they be only an integration-constant or not. We get solution-pressures of 10^{15} or 10^{-15} atmospheres, to which many a serious scientist objected, but we, from our point of view, adhere to them. These forces are great enough and small enough to show us that if Nernst and Brunner's theory of the process of the ordinary solution of solids at their surface be true, this theory, as given to us at present, still teaches us nothing about the heterogeneous reactions proper even in the sphere of pure abstraction. I, for one, would object to this method of treating the subject of molecular reactions as a clear and lucid conception of a phenomenon which we wish to know, even if pure abstractions were all that interested us. Our conceptions, instead of becoming simpler, became complicated, and we are thrown face to face with new unknown problems which are very difficult and inaccessible to test, while the content and the conceptions contained in the equation,

as given by myself, are very clear and simple, are almost axiomatic in their character.

I return to the argument I have already given in the Rep. Brit. Assoc. Liverpool, 1896, in connexion with Co. My equation is shown to hold for *all* molecular reactions, including such reactions as the transformation of ice into water, water into ice, &c., where concentration does not come into consideration. *It is evident that a law of such a generality cannot find an explanation in diffusion*, which can have only a limited application. This is evident *proof* that there is no necessity whatever in diffusion conceptions to explain the meaning of my equation, and that to give the correct natural interpretation to its meaning, we must take as a basis the *general* features underlying *all* of them, as I did in my papers and here, and not follow the particular line of diffusion.

The *analogy* between the equation for velocity of solution of salt with the equation for diffusion evidently seemed to Noyes and Whitney, Bruner and Tolloczko, Nernst and Brunner peculiar and led them to try to *identify* them and to adapt the conceptions accordingly.

This coincidence may appeal to some as remarkable, but is of no value *so long as the physical constants do not prove this identity*, because buoyancy is also regulated by a law of a similar nature, so is the law of Newton for cooling &c. Why not *identify* velocity of molecular reaction with the law of Newton? With many assumptions, plenty of hypothesis, and no experimental proof, a much more fascinating theory could be developed here. But, as already explained in my paper (Phil. Mag. July 1901) in great detail, there is no more and no less analogy between *all* these phenomena (speed of molecular reaction, diffusion, buoyancy, conduction of heat), they *all* have in common, that, whenever a kind of energy (chemical, thermal, &c.) is removed from the state of equilibrium, the speed with which it strives to attain the same is directly proportional to its remoteness at the time τ from its point of equilibrium. This, nevertheless, does *not* make all these phenomena *identical*.

(*f'*) Let us now assume for a moment that the speed of solution is regulated by

$$\frac{dt}{dy} = \frac{F}{\delta} \Sigma (C-c) \quad \text{or} \quad \frac{F'}{\delta} (C-c), \quad \text{for } \Sigma=1.$$

Now the velocity of evaporation also follows my equation, and the well established thermodynamic analogy between solution and evaporation makes this also self-evident. In case of gases the diffusion constant D is about 10,000 times greater

than in water ; δ , if it existed at all, is evidently much smaller than in solution (we look away from the fact that it is inconceivable that a layer of gas should be formed near the solid the molecules of which cannot move into the rest of the gaseous space, or that the gas from the total volume should not be capable to reach the solid). In view of the above equation for solution, the velocity of evaporation ought to go on with an infinitely greater speed than velocity of solution. We find, however, on the contrary, that the speed of evaporation of liquids and solids belongs in comparison to the slow reactions, as is to be seen already from the fact how difficult it is to reach the maximum of vapour-pressure in all the researches.

This is a further proof against the diffusion conceptions.

(g) I shall next deal with the variation of the constant A with the number of revolutions of the stirrer. There is no reason why the ordinary velocity-constant should not change with the speed of the stirrer, considering that it does so enormously change with the solvent, with the size and dimensions of the apparatus and other mechanical conditions, but whatever its cause, I will show, however, that this variation of the velocity-constant with the number of revolutions proves, if anything, the *improbability* of the diffusion conceptions of Nernst and Brunner. Formerly Tolloczko and Bruner found by the very same method which Nernst and Brunner use, that the speed of solution of the same benzoic acid is directly proportional to the speed of stirring—a rate of variation which does not seem probable for the layer δ . Nernst and Brunner seem to have got at first the same results, but they tell us that this was due to some cracks at the rims of the acid, and that as soon as these cracks were filled up with paraffin the value of A changed as the two-thirds power of the speed of the stirrer. It will, however, be seen from the details of the method used by Nernst and Brunner (dealt with in § 2) and from the photographs of the benzoic acid which I obtained by their method, that their benzoic acid was never free from holes, that the speed of their motor changed too considerably for accurate measurements to be carried out in this region, and that even their titrations were not arranged accurately enough for the very dilute solutions they dealt with. With such a method they nevertheless changed the variations of speed (see pp. 60–61) of their stirrer only from 144 to 202 and from 135 to 180, *i. e.* only by 25 to 30 per cent., which is not enough to distinguish between the results calculated after one equation or the other, even when the

results are obtained with very much better methods. On page 61 they give us

$$n=200 \text{ rev. and } A=2.63; \quad n=177 \text{ rev. and } A=2.76,$$

i. e. a diminution of A with the increase of the speed, showing the lack of sensitiveness of their method. On page 72 they give us the variation of A for their so-called "Magnesium hydroxide" in benzoic acid:—

$$n=135; \quad A=1.35, 1.88, 1.76=\text{aver. } 1.66.$$

$$n_1=182; \quad A_1=2.14, 1.71, 1.90, 1.87, 1.89=\text{aver. } 1.92.$$

$$n_2=178; \quad A_2=2.14, 2.08=\text{aver. } 2.11.$$

If we take the velocity constant to be directly proportional to the speed of the stirrer, we get—

$$\frac{n_1}{n} = \frac{182}{135} = 1.348 \quad \text{and} \quad \frac{A_1}{A} = \frac{192}{166} = 1.157,$$

i. e., a difference of 14.2 per cent.

$$\frac{n_2}{n} = \frac{178}{135} = 1.318 \quad \text{and} \quad \frac{A_2}{A} = \frac{211}{166} = 1.272,$$

i. e., a difference of 3.5 per cent.

If we take, with Nernst and Brunner, the velocity constant to be directly proportional to the two-third power of the speed of the stirrer we get:

$$\frac{\lg 192 - \lg 166}{\lg 112 - \lg 135} = .487 \text{ instead of } .667,$$

i. e., a difference of -26.98 per cent.; and

$$\frac{\lg 211 - \lg 166}{\lg 178 - \lg 135} = .8676 \text{ instead of } .667,$$

i. e., a difference of $+30.08$ per cent.

Thus Nernst and Brunner's data, if anything, prove more direct proportionality and not the two-third power of n , and in reality they can neither be used for the proof of one nor for the proof of the other, especially considering the fact that besides general great experimental errors of their method their "Magnesium hydroxide" proves also to have been a mixture of MgO , $\text{Mg}(\text{OH})_2$, MgCO_3 , &c., &c.

I now give here a careful investigation of this point carried out with speeds from about 2 to 400 revolutions per minute and more. (See Table II. and diagram 2.)

TABLE II.—*Variation of the Velocity Constant with the number of revolutions of the stirrer per minute.*

$$\frac{v}{d\tau} \log \frac{O-C_1}{C-C_2} = K_1; \quad K_1 \times 2.3026 = A_1.$$

No.	Benzoic acid, Molecule normal <i>a.</i>	C ₂ . Benzoic acid in c. c. of the saturated solution of benzoic acid Elect. conduct. by Titration		C - C ₁ Electric conduct.	C - C ₁ . By titration.	<i>v.</i>	<i>dτ</i> in minutes.	Electric conductivity. K ₁ × 2.3026 = A ₁ .	Titration. K' × 2.3026 = A'.		
<i>Diffusion of benzoic acid in water, when not stirred.</i>											
1'...	·000896	·4072	·4376	9.5928	9.5624	1470	1060	·09554	·10130	2nd May, 1907.	
2'...	·002302	1.046	1.1110	8.954	8.8890						
1''.	·003560	1.618	1.653	8.382	8.347	1430	2500	·07182	·07392	Temp. 16°·0–16°·5 C.	
2''.	·005737	2.607	2.665	7.393	7.335						
1'''.	·006533	2.968	3.015	7.032	6.985	1410	1008	·05584	·06316		
2'''.	·007137	3.243	3.322	6.757	6.678						
<i>i. e.</i> , the velocity constant of diffusion seems to become smaller, the greater the concentration of the solution.											
<i>Speed of solution of benzoic acid in water at 2.42 rev. per minute.</i>											
1 ...	·005737	2.607	2.665	7.393	7.335	1420	367	·1937	·1892	30th April, 1907.	
2 ...	·003533	2.968	3.015	7.032	6.985						
<i>Speed of solution of benzoic acid in water at 2.86 rev. per minute.</i>											
1 ...	·000108	0.0491	0.0198	9.9509	9.9802	1490	115	·2276	·1969	Temp. 17°·6 C.	
2 ...	·000489	0.2222	0.1703	9.7778	9.8297						
3 ...	·000896	0.4072	0.4376	9.5928	9.5624	1480	152	·1861	·2685		
Average...								·2068	·2326		
<i>Speed of solution of benzoic acid in water at 3.00 rev. per minute.</i>											
1 ...	·002451	1.114	1.111	8.886	8.8890	1460	113.5	·2811	·3267?		
2 ...	·002873	1.306	1.336	8.694	8.664						
3 ...	·003345	1.520	...	8.480	...	1450	152	·2376	·2271		
4 ...	·003560	1.618	1.653	8.382	8.347	1440	76	·2203	...		
Average...								·2464	·2588		

TABLE II. (continued).

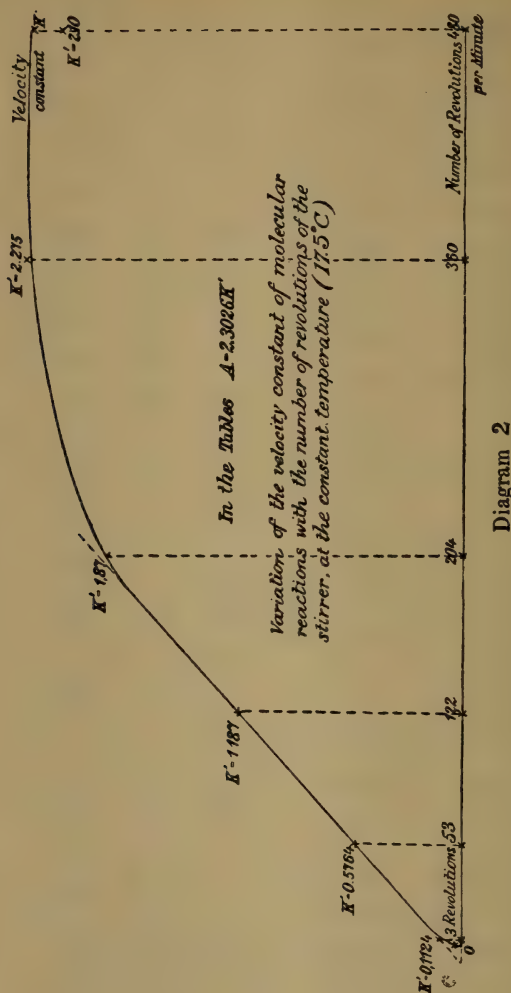
No.	Benzoic acid. Mol. norm. α .	C_1 . Elect. cond.	C' . Titra- tion.	$C-C_1$. Elect. cond.	$C-C'$. Titra- tion.	v .	$d\tau$.	$K_1 \times 2.3026$ $=A_1$.	$K' \times 2.3026$ $=A'$.	
<i>Speed of solution of benzoic acid in water at 53 rev. per minute.</i>										
1...	·000740	·3364	·4222	9·6636	9·5778	1470	19·8	1·383	1·443	21-22 May, 1907.
2...	·001085	·4931	·6075	9·5069	9·3925	1460	26·5	1·326	1·465	
3...	·001582	·7190	·8530	9·2810	9·1460	1450	28·5	1·279	1·502	Temp. 17°·75 C.
4...	·002089	·9496	1·120	9·0504	8·880	1440	29·75	·909 ??	1·041	
5...	·002573	1·1170	1·309	8·8820	8·691	1430	31·75	1·187	1·385	
6...	·003068	1·3950	1·572	8·6050	8·428	1420	53·0	1·464	1·047	
7...	·003983	1·8100	1·895	8·1900	8·105	1410	64·25	1·294	1·248	
8...	·005015	2·2790	2·343	7·7210	7·657	1400	53·25	1·212	1·538	
9...	·005780	2·6270	2·778	7·3730	7·222	1390	50·0	1·332	1·277	
10...	·006538	2·9720	3·102	7·0280	6·893					
Average...								1·266	1·327	
<i>Speed of solution of benzoic acid in water at 122 rev. per minute (17°·5 C.).</i>										
1...	·000484	·220	·2154	9·7800	9·7846	1480	6·0	2·397	3·168	15-20 March 1907.
2...	·000692	·3145	·3404	9·6855	9·6596	1465	25·5	2·675	2·781	
3...	·001665	·7568	·7971	9·2432	9·2029	1445	46·5	2·452	2·309	Temp. 17°·8 C.
4...	·003204	1·456	1·475	8·544	8·525	1430	39·75	2·381	2·567	
5...	·004407	2·003	2·062	7·997	7·938	1420	31·5	2·303	2·254	
6...	·005282	2·401	2·449	7·599	7·551	1410	23·5	2·212	3·324	
7...	·005887	2·676	2·856	7·324	7·144					

TABLE II. (*continued*).

No.	Benzoic acid. Mol. norm. a.	C ₁ . Elect. cond.	C'. Titra- tion.	C-C ₁ . Elect. cond.	C-C'. Titra- tion.	v.	dτ.	K ₁ × 2·3026 = A ₁ .	K' × 2·3026 = A'.	
8...	·005556	2·525	2·555	7·475	7·445	1490	18·5	2·895	2·482	Temp. 17°·4 C.
9...	·006137	2·789	2·770	7·211	7·230	1475	66·5	2·224	2·503	
10...	·007655	3·479	3·55	6·521	6·450	1460	30·5	2·703	3·088	
11...	·008440	3·837	3·953	6·163	6·047	1445	59·0	2·443	2·395	
12...	·009729	4·422	4·516	5·578	5·484	1430	24·5	2·099	2·191	
13...	·010163	4·619	4·718	5·381	5·282					
14...	...	...	4·645	...	5·355	1490	36·5	...	2·793	Temp. 17°·7 C.
15...	...	...	4·999	...	5·001	1480	38·0	...	2·351	
16...	...	...	5·292	...	4·708					
17...	...	...	6·679	...	3·321	1485	162	...	3·387	Temp. 17°·4 C.
18...	...	...	7·705	...	2·295	1465	157	...	2·533	
19...	...	..	8·252	...	1·748					
Average...								2·413	2·676	
<i>Speed of solution of benzoic acid in water at 204 rev. per minute.</i>										
1...	·001022	·4645	...	9·5355	...	1480	10·0	4·188	...	19-22 April 1907.
2...	·001607	·7304	...	9·2696	...	1470	15·0	4·598	...	
3...	·002532	1·151	1·045	8·8490	8·955	1460	15·4	4·914	4·911	Temp. 18° C.
4...	·003515	1·598	1·497	8·4020	8·503	1450	14·8	4·46	3·495	
5...	·004338	1·972	1·795	8·0280	8·205	1440	22·5	4·246	4·918	
6...	·005471	2·487	2·402	7·5130	7·598	1430	20·7	5·163	3·949	
7...	·006664	3·028	2·824	6·9720	7·176	1420	21·8	4·02	4·393	
8...	·007581	3·445	3·292	6·5550	6·708	1410	22·0	3·857	4·175	
9...	·008425	3·828	3·715	6·1720	6·285					
Average...								4·426	4·306	

No.	Benzoic acid. Mol. n. a.	C ₁ . Elect. cond.	C'. Titra- tion.	C-C ₁ . Elect. cond.	C-C'. Titra- tion.	v.	dτ.	K ₁ × 2·3026 = A ₁ .	K' × 2·3026 = A'.	
Speed of solution of benzoic acid in water at 365 rev. per minute.										
1...	·000605	0·2749	...	9·7251	...	1490	4	6·132	...	3 May 1907. Temp. 17°·5 C.
2...	·000953	0·4332	...	9·5668	...	"	4	5·285	...	
3...	·001251	0·5686	...	9·4314	...	"	10	5·702	...	
4...	·002030	0·9227	...	9·0773	...	"	2	4·992	...	
5...	·002163	0·9832	0·9465	9·0168	9·0535	1480	3	4·987	4·656	
6...	·002362	1·074	...	8·926	...	"	4	6·218		
7...	·002675	1·216	...	8·784	...	"	5	5·174		
8...	·003032	1·378	...	8·622	...	"	2·5	5·112	5·757	
9...	·003194	1·452	1·323	8·548	8·677	1470	10·5	5·188		
10...	·003879	1·763	...	8·237	...	"	4·5	5·317		
11...	·004172	1·896	1·818	8·104	8·182	1460	15·5	4·743	5·135	
12...	·005048	2·294	...	7·706	...	"	9	4·721		
13...	·005534	2·515	...	7·485	...	1450	24	5·216		
14...	·006896	3·134	3·105	6·866	6·895	1440	32	4·969	5·345	
15...	·008476	3·852	3·877	6·148	6·123					
Average...								5·268	5·222	

<i>Speed of solution of benzoic acid in water at 470 rev. per minute.</i>										
1...	·000597	·2713	...	9·7287	...	1500	2	4·559	...	7 May 1908. Temp. 18° C.
2...	·000727	·3304	...	9·6696	...	"	3	4·835	...	
3...	·000932	·4235	...	9·5765	...	"	2	4·26	...	
4...	·001051	·4776	...	9·5224	...	"	4	5·043	...	
5...	·001330	·6045	...	9·3955	...	"	3	4·444	...	
6...	·001513	·6875	...	9·3125	...	"	7	5·158	...	
7...	·001999	·9085	...	9·0915	...	"	12	4·951	...	
8...	·002777	1·262	...	8·738	...	"	1·5	5·389	...	
9...	·002879	1·309	...	8·691	...	"	8·5	4·974	...	
10...	·003413	1·551	...	8·449	...	"	4	5·504	...	
11...	·003683	1·674	...	8·326	...	"	15	4·145	...	
12...	·004428	2·013	...	7·987	...					
Average...								4·835	...	



The results found here are therefore the following:—

1. When the liquid is not stirred at all, the only way for the substance, passing into the solution from the surface of the solid, to pass into the rest of the solution is naturally, as known long ago, by diffusion. The amount passing in this manner is, however, in comparison with what passes at the maximum revolutions or even at any ordinary speeds, only

a small fraction of a per cent. As the assumption that the stirring should produce such a variation in the diffusion-layer is not at all probable, this certainly is, in my opinion, a proof against the diffusion theory.

2. Up to about 3 revolutions of the stirrer, the increase of A is greater than in direct proportion to the number of revolutions, as the same follows from the curve between 3 and 180 revolutions; the preponderating effect of stirring over that of diffusion is seen even at these slow speeds; the speed of solution is here due to stirring and to diffusion, and the last cannot yet be neglected.

3. From about 3 revolutions to about 180 revolutions the value of A is directly proportional to the number of revolutions of the stirrer (into this region fall the investigations of Bruner and Tolloczko under their experimental conditions, who found the same result). Thus from about 3 revolutions already the value of transportation due to diffusion no longer interferes with the value of speed of solution, as effected by stirring only.

4. From about 180 to about 400 revolutions, the increase in the value of A first gradually becomes smaller, until it becomes independent of the number of revolutions of the stirrer, *i. e.* the maximum value for A has been reached. Into this region fall the experiments of Lorenz and Siegrist under their experimental conditions.

It is, however, evident that the more soluble a substance is, the less will be the number of revolutions requisite to reach a constant maximum value of A' .

Nernst and Brunner's idea that diffusion is the sole factor in transportation of benzoic acid from the surface of the solid to the rest of the solution is, in my opinion, in no way supported by the character of the above results; quite the contrary. One or two revolutions of the stirrer already transports considerably more than the whole process of diffusion can possibly do alone without stirring; and I fail to see any reasonable support, any reliable proof, that diffusion alone should be operative and that as such it should be (owing to variation of δ) *accelerated* by stirring to such an enormous extent *instead of being more or less destroyed* by the same, owing to the *destruction* of the drop in concentration near the surface of the solid, which is naturally formed when the liquid is not stirred at all. Indeed, as already mentioned, there is nothing to prevent the stirrer from mixing the liquid up to the very solid, and if the liquid can reach the solid during

solidification to a proximity of about 10^{-15} mm., when the liquid is not stirred, we have no reason whatever to assume that the liquid does not reach the solid to the same proximity when we stir the same. For the diffusion conceptions to find any application at all it is necessary *to prove*, not that a saturated solution can be formed at the surface of the solid (when not stirred), but that the solution *is incapable of entering* the layer near the solid when it is moved into it by the stirrer, diluting the same. The assumption that a *saturated* solution is formed at the solid which passes into the rest of the solution by diffusion, overlooks also the fact that *the solid in contact with its saturated solution ought to be in equilibrium with it, ought not to be able to dissolve any more*, and that a reaction of solution of the solid cannot possibly go on, unless the layer in the immediate contact with the solid is *unsaturated*. In this notion we are confirmed by the behaviour of a solid in contact with its gas, and this forms at present an axiomatic notion, which existed for ever so long in physics and chemistry, which led to many useful and fruitful results which cannot be given up for the sake of peculiar diffusion conceptions, even if they should seem capable of explaining some particular case.

(h) I will next consider the "important argument" of Nernst and Brunner "for the conception of the diffusion process." They tell us:—"The true size of the surface is of no importance; when the originally smooth surface of the solid becomes corroded, the velocity constant A does not change, in spite of the great increase of the surface. Only the quadratic dimensions or contour of the surface and not the microscopic properties of the same are of importance; the last should be of importance, if the reaction takes place on the surface itself." Assuming, however, that this is true, which it is not *, it is not to be seen, how this is to be taken as a proof for the diffusion conceptions of Nernst and Brunner? If the diffusion layer δ were, in comparison with the depth of corrosions, very thick, so that the depths of the corrosions could be neglected, such a conclusion would be admissible. But when the diffusion layer δ is even according to Nernst and Brunner only 0.02 mm. thick, while the corrosions according to them can be as deep as 1 mm. (without affecting the constant), it is quite evident that the diffusion layer or surface will be as large as the corrosive surface of the

* In my next publication I shall furnish further experimental proof that reaction *does take place at the surface* of the solid even in case of substances of no small solubility.

solid itself, and the increase of the diffusion surface will be the same as that of the surface of the solid during corrosion, *i. e.*, we gain nothing whatever here by the assumption of a diffusion layer.

The true position of the facts here is, in my opinion, the following:—

Surfaces which appear to the naked eye smooth are in reality, on inspection with a lens, corroded surfaces, and such surfaces are not always essentially smaller than the surface of the same solid when its corruptions are perceivable; influences of an opposite kind may often counteract one another: on the other hand, I must assert that *very often* we find an increase of the velocity constant with the increase of corrosion, when this is still very far below the values mentioned by Nernst and Brunner, and I find that repeated polishing of the surfaces is in this region imperative.

PART II.

(i.) I now pass to the most important and most direct test of the diffusion theory. This is given to us by the investigation:—(1) of the *speed of solution of different modifications of the same substance in the same solvent*; (2) of the *speed of solution of the same crystal, cut in different directions to its axis, in the same solvent*. Because if the velocity, with which a solid dissolves, depends, as we are told, upon the speed of diffusion of its saturated solution from the solid into the rest of the solution, and if the saturated solution is formed instantaneously at the solid, then all the $\text{CaSO}_4 + 2\text{H}_2\text{O}$ of the different Gypsums giving the same saturated solution* must dissolve with the same speed, and the same crystal Selenite must dissolve in different directions of its axis with the same speed. *If this is not the case, this is the most decisive and direct proof one can possibly imagine, that these fundamental assumptions of the diffusion theory do not hold good, and the whole diffusion theory tumbles to the ground.* Bruner and Tolloczko found (*Zeitsch. für anorg. Chemie*, 1901, p. 325) for Alabaster, the velocity constant $A=0.00230$ ($V=2000$ cc., $f=22.91$ cm.², number of revolutions 400) and for Marienglass or Selenite $A=0.00079$ ($V=2400$ cc., $f=20.9$ cm.², number of revolutions 400), *i. e.* A Alabaster : A Marienglass = 3.44 : 1. In *Zeitsch. für anorg. Chemie*, 1903, pp. 29, 30, they try to get over this difficulty. They

* See the investigations of Kohlrausch and Rose and others.

give us $A \text{ Alabaster} : A \text{ Marienglass} = 2.5 : 1$, but they inform us that the surface of Marienglass is smooth, the surface of Alabaster not, and when we polish Alabaster to a smooth surface the constant A drops to half its value. The impression is thus created, that the constants of both are really the same. Note in the first instance, that this explanation is now brought forward by Bruner and Tolloczko, who were the first to find: "The important argument for the diffusion theory consisting in the fact that corrosion as deep as 1 mm. has no effect on the velocity constant;" that they no longer accept this statement in case of Alabaster, which very easily gives perfect surfaces, much better than marble, while they do not take into account the corrosion of all other substances used by them, which are not able to give such smooth surfaces as Alabaster, but for which they nevertheless furnish us with perfect and impossible constants, under conditions when the substances must have corroded during the reaction very considerably! With "Plâtre" Bruner and Tolloczko found A to be twice as big as that of Alabaster, but according to them this is only due to the fact that some substance is coming off from the solid. As the method of Bruner and Tolloczko seemed to me quite unsuitable for a reliable investigation here, this matter had to be investigated again. The solubility of the Gypsums is only about 2 gr. per litre, and some of the solutions and differences investigated by them contained no more than .003 gr. in the 25 cc. which they investigated. Such amounts of gypsums Bruner and Tolloczko wish to measure quantitatively either by evaporation or by the more complicated and unreliable method of precipitation with oxalic acid (collecting, washing the same), the decomposition with SO_4H_2 and titration with permanganate, and yet they supply us by such methods with constants differing in a series only by about 2 per cent. ! To remove, however, all possible doubt here about the reason for the difference found between Marienglass and Alabaster and to investigate the same in very dilute solutions, when the surface had no time yet to corrode except but very little, I employed the method of electric conductivity as described in paragraph 2, which enables us to make very much more accurate and instantaneous measurements than it is possible to expect from the methods of Bruner and Tolloczko, and very great care was taken to polish the surfaces of the gypsums as perfectly as possible.

Table III. A gives the speed of solution of the following Gypsums $\text{CaSO}_4 + 2\text{H}_2\text{O}$:—Selenite or Marienglass from

Italy, Alabaster from Leicester, fibrous Gypsum from Nottingham, all perfect specimens, perfectly polished and washed and dried up well and quickly; their surface is almost the same, about 4 sq. in., all were investigated at the same temperature of very near $17^{\circ}5$ C., with the same speed of stirrer (130 to 132 revolutions per minute) and the same volume (1500 cc.) which remained constant during the whole time of the reaction. The surface of the selenite exposed to solution was the flat large side of the twin-crystal, the surface of the fibrous gypsum was cut along the fibre, quite parallel to the same, while the surface of alabaster could be taken in any direction. Photos 11, 12, 13 (Pl. XVII.) show Selenite, Alabaster, fibrous Gypsum, their structure and their surfaces *after* solution. It will be seen from the same that their surfaces were not allowed to corrode. The selenite is transparent like glass, and what we see on the photo is the paraffin below the bottom surface. On the contrary, in case of alabaster and fibrous gypsum, the photos show their structure, not corrosions, which are extremely small. The velocity constant of Selenite is from .48 to about .65, of the fibrous Gypsum is about 2.0, of the Alabaster from 3.1 to about 4.0, the average ratio of the constants of Selenite : fibrous Gypsum : Alabaster at the same surface is about 1:3.5:5.4. Thus Marienglass and Alabaster give, not as Bruner and Tolloczko lead us to think almost equal constants, but very different constants indeed, and all the three above specimens of the same Gypsum give undoubtedly different constants, whether we take extremely dilute solutions given by the first three dilutions in the above tables, where the ratio is 1:4:6, or the more concentrated ones. Therefore, as the solubility of all modifications is the same, it is sufficient that the same monoclinic crystal should configure itself once into a twin-crystal, another time into a fibrous structure, another time into a granular crystalline form; and this physical modification in the configuration of the very same components is already alone sufficient to cause the solid to acquire some new physical properties, of which the speed of solution is one. All the above three specimens are as we find them in nature, their velocity constants undoubtedly increase as the reaction goes on, and this may be due either to corrosion or also to some other influences. It is not likely to be due to the presence of some more easily soluble substances as impurities, because the velocity constants ought in such a case to decrease with time, instead of increasing.

TABLE III A.

Molecule normal. C_1 .	$d\tau$ in minutes.	$O-C_1$.	$\frac{v}{d\tau} \log \frac{O-C_1}{O-C_2} \times 2.3026 = K$.
A. <i>Selenite or Marienglass</i> (from Italy). 18° C.; 131 rev. per minute; $v=1500$ cc.; $\Sigma=26$ cm. ² 14-15th May, 1908.			
·0001600	25	·0143000	·4864
·0002754	20	·0141846	·5026
·0003701	20	·0140899	·5571
·0004743	10	·0139857	·6638
·0005360	10	·0139240	·5700
·0005890	10	·0138710	·5764
·0006420	10	·0138180	·6080
·0006980	10	·0137620	·6610
·0007585	15	·0137015	·6683
·0008499	15	·0136101	·6578
·0009392	15	·0135208	·6976
·0010328	15	·0134272	·6398
·0011182	15	·0133418	·6365
·0012026	15	·0132574	·6658
·0012910		·0131690	
		Average	·6137
Polished block again.			
·001291	15	·0131690	·6407
·001375	15	·0130852	·6002
·001453	30	·0130070	·6050
·0016096	15	·0128504	·7054
·001700		·012760	
		Average	·6378
Excellent surface, transparent like glass. For $\Sigma = 20.4$ cm. ² $K' = .49$ (average) ·40 (in the three greatest dilutions).			
Molecule normal. C_1 .	$d\tau$ in minutes.	$O-C_1$.	$\frac{v}{d\tau} \log \frac{O-C_1}{O-C_2} \times 2.3026 = K$.
B. <i>Fibrous Gypsum</i> (from Nottingham). 17°·55 C.; 131 rev. per minute; $v=1500$ cc.; $\Sigma=25.5$ cm. ² 20th May, 1908.			
·0001730	5	·0142870	2.065
·0002720	5	·0141880	1.925
·0003629	6	·0140971	1.987
·0004743	6	·0139857	1.823
·0005758	6	·0138842	1.883
·0006802	6	·0137798	1.811
·0007797	6	·0136805	1.866
·0008815	23	·0135785	1.840
·0012586	19	·0132014	1.860
·0015750	19	·0128850	2.176
·0019255	15	·0125345	1.848
·0021550	15	·0123050	1.902
·0023860	15	·0120730	1.807
·0026030	15	·0118570	2.017
·0028300	15	·0116300	2.037
·0030740		·0113860	
		Average	1.923
Polished block again.			
·0032010	16	·0112580	1.960
·0034340	15	·0110250	1.738
·0036250	16	·0108350	1.845
·0038360	23	·0106240	2.285
·0042020	18	·0102580	2.031
·0044490	19	·0100110	1.937
·0046920	44	·0097680	2.094
·0052740	15	·0091860	2.247
·0054780	17	·0089820	2.328
·0057120	13	·0087480	2.330
·0058870		·0085730	
		Average	2.080
Excellent surface. For $\Sigma = 20.4$ cm. ² $K' = 1.7$.			
Molecule normal. C_1 .	$d\tau$ in minutes.	$O-C_1$.	$\frac{v}{d\tau} \log \frac{O-C_1}{O-C_2} \times 2.3026 = K$.
C. <i>Alabaster</i> (from Leicester). 17°·8 C.; 131 rev. per minute; $v=1500$ cc.; $\Sigma=27.56$ cm. ² 16th May, 1908.			
·000093	3	·0143670	3.143
·0001830	3	·0142770	3.413
·0002802	3	·0141798	3.287
·0003730	3	·0140870	3.333
·0004665	3	·0139935	3.580
·0005662	3	·0138938	3.574
·0006654	3	·0137946	3.450
·0007603	3	·0136997	3.640
·0008597	3	·0136003	3.430
·0009524	6	·0135076	3.547
·0011428	3	·0133172	3.357
·0012320	3	·0132280	3.573
·0013292	3	·0131308	3.763
·0014278	6	·0130322	3.513
·0016096	8	·0128504	3.597
·0018540	7	·0126060	3.602
·0020640	19	·0123960	3.742
·0026380	10	·0118220	3.778
·0029320	30	·0115280	3.820
·0037800		·0106800	
		Average	3.534
Polished block again.			
·0051860	12	·0092740	3.180
·0054190	15	·0090410	3.625
·0057410	15	·0087190	3.750
·0060620	46	·0083980	3.540
·0069260	30	·0075340	3.800
·0074070	29	·0070580	3.779
·0079040	31	·0065560	3.457
·0083560	30	·0061040	3.695
·0087910	30	·0056690	3.594
·0091830	30	·0052770	3.992
·0095880		·0048720	
		Average	3.56
Excellent surface. For $\Sigma = 20.4$ cm. ² $K' = 2.6$ (average) = 2.4 (in the three greatest dilutions)			

TABLE III B.

A. Selenite or Marienglass (the large, flat surface of the twin crystal). 18° C.; 131 rev. per min.; $v=1500$ cc.; $\Sigma=26$ cm. ²				B'. Selenite or Marienglass (surfaces cut vertically to the surface of A' and cemented with paraffin). 17°·3 C.; 130 rev. per min.; $v=1500$ cc.; $\Sigma=24$ cm. ² 25th May, 1908.				C'. Selenite or Marienglass (powdered and compressed to a disk under 2500 atm. during 16 hours). 17°·4 C.; 131 rev. per min.; $v=1500$ cc.; $\Sigma=20\cdot4$ cm. ²					
Molecule normal. O_1 .	$d\tau$ in minutes.	$O-O_1$.	$\frac{v}{d\tau} \log \frac{O-O_1}{O-O_2} \times 2\cdot3026 = K$.	Molecule normal. O_1 .	$d\tau$ in minutes.	$O-O_1$.	$\frac{v}{d\tau} \log \frac{O-O_1}{O-O_2} \times 2\cdot3026 = K$.	Molecule normal. O_1 .	$d\tau$ in minutes.	$O-O_1$.	$\frac{v}{d\tau} \log \frac{O-O_1}{O-O_2} \times 2\cdot3026 = K$.		
001600		·0143000		·0001392		·0143208		·0000680		·0143920			
002754	25	·0141846	·4864	·0001722	1	·0142878	3·457	·0000930	1	·0143670	2·590		
003701	20	·0140899	·5026	·0002019	1	·0142581	3·018	·0001452	2	·0143148	2·740		
004743	20	·0139857	·5571	·0002308	1	·0142292	2·970	·0002067	2	·0142533	3·240		
005360	10	·0139240	·6638 ?	·0002634	1	·0141966	3·595	·0002611	2	·0141989	2·858		
005890	10	·0138710	·5700	·0002929	1	·0141671	3·035	·0003130	2	·0141470	2·742		
006420	10	·0138180	·5767	·0003555	2	·0141045	3·376	·0003674	2	·0140926	2·902		
006980	10	·0137620	·6080	·0005470	6	·0139130	3·415	·0004220	2	·0140380	2·900		
007585	10	·0137015	·6610	·0006472	3	·0138762	3·620	·0004780	2	·0139820	3·004		
008499	15	·0136101	·6683	·0008964	7	·0138128	3·895	·0005864	4	·0138736	2·920		
009392	15	·0135208	·6578	Excellent surface. For $\Sigma=20\cdot4$ cm. ² $K'=2\cdot87$.		Average	3·376	·0006367	2	·0138233	2·728		
0110328	15	·0134272	·6976			·0008514	8	·0136086	2·931				
011182	15	·0133418	·6398			·0011171	10	·0133429	2·955				
012026	15	·0132574	·6365			·0015082	15	·0129518	2·977				
012910	15	·0131690	·6658			·0020454	21	·0129518	3·016				
Average			·6137			·0020454	12	·0124146	2·950				
Block polished again.						·0023390	11	·0121250	2·973				
						·0025910	23	·0118690	2·990				
						·0031230		·0113370					
						Average		2·907					
						Very good surface. For $\Sigma=20\cdot4$ cm. ² $K'=2\cdot9$.							
01291					·0131690								
01375				15	·0130852	·6407							
01453				15	·0130070	·6002							
016093				30	·0128504	·6050							
01700				15	·0127600	·7054							
				Average		·6378							
Excellent surface. or $\Sigma=20\cdot4$ cm. ² $K'=49$ (average). ·40 (in the three greatest dilutions).													

TABLE III C.

Molecule normal. O_1 . dt in minutes. $O - O_1$. $v \log \frac{O - O_1}{O - O_2}$ $\times 2.3026 = K$.				Molecule normal. O_1 . dt in minutes. $O - O_1$. $v \log \frac{O - O_1}{O - O_2}$ $\times 2.3026 = K$.			
A". $\text{CaSO}_4 + 2\text{H}_2\text{O}$ precipitated; with excess of water dried in open air; compressed under 2500 atm. for 16 hrs. 18°C .; 131 rev. per min.; $v = 1500 \text{ cc}$.; $\Sigma = 20.4 \text{ cm}^2$ 29th May, 1908.				B". Anhydride CaSO_4 , Nova Sco- tia: cut to a disk of 50 mm. diameter; $17^\circ.5 \text{ C}$.; 132 rev. per min.; $v = 1500 \text{ cc}$.; $\Sigma =$ 19.63. 10th June, 1908.			
C". CaSO_4 Anhydride, pre- cipitated. Got by heating at 310°C . for 4 to 5 hours, compressed to a disk at 2500 atm.; $\Sigma = 20.4 \text{ cm}^2$				From the top surface powder comes off, <i>i.e.</i> can- not be investigated. It is well known that while CaSO_4 prepared from $\text{CaSO}_4 + 2\text{H}_2\text{O}$ at 80°C . in air current absorbs water rapidly forming $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ and dissolving in water, the CaSO_4 prepared from $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$ by heating at about 300°C . (as in our case) absorbs water and dissolves in water very slowly. Thus, again, the speed of solution of the same CaSO_4 -Anhydride, differ- ently prepared, is <i>different</i> , the speed of combination of the CaSO_4 's with water, which is also a heteroge- neous reaction, is different, and the speed of reaction is <i>dependent</i> upon the phy- sical state of the solid, <i>not</i> independent of the same.			
•000067		•0143930		•000050		•0144100	
	1		2.797		8		.5698
•000094		•0143660		•000094		•0143660	
	1		3.142		8		.6173
•000124		•0143360		•000141		•0143190	
	1		3.039		4		.6302
•000153		•0143070		•000165		•0142950	
	1		3.384		6		.6792
•000185		•0142750		•0002040		•0142560	
	1		2.837		7		.6140
•000212		•0142480		•0002444		•0142155	
	2		3.004		8		.5820
•000269		•0141910		•0002889		•0141711	
	3		3.056		10		.5971
•000355		•0141045		•0003452		•0141148	
	3		3.073		15		.6536
•000442		•0140178		•0004372		•0140228	
	6		3.063		6		.6636
•000613		•0138471		•0004743		•0139857	
	6		3.029		?		?
•000780		•0136803		•0005060		•0139540	
	11		2.986		20		.6754
•001076		•0133839		•0006314		•0138286	
	45		3.166		15		.6557
•002289		•0121710		•0007217		•0137383	
	43		3.087		18		.6782
•003330		•0111300		•0008332		•0136268	
	27		3.313		17		.6886
•003965		•0104950		•0009392		•0135208	
	30		2.908		20		.6976
•004558		•0099020		•0010642		•0133958	
	10		3.090			Average	.643
•004760		•0097000		Excellent surface; polishes better than marble. For $\Sigma = 20.4 \text{ cm}^2$ $K' = .67$.			
	10		3.156				
•004962		•0094980		Excellent surface. For $\Sigma = 20.4 \text{ cm}^2$ $K' = 3.054$.			
	10		2.852				
•005141		•0093190					
		Average	3.054				

In Table III. B the same selenite is investigated in two different directions in the crystal: the first one is the large flat surface of the twin-crystal already mentioned before (A Selenite, photo 11); the second is a surface cut vertically to the surface A, as in the annexed figure. I cut the selenite crystal, as in the figure, into several strips and connected the vertical surfaces to one surface, the cut-off strips separating into several thin plates, and I made up a block connecting the same by means of paraffin (see photo 14). The surface of the



crystal without paraffin was 49 mm. in length and 49 mm. in width; through the paraffin it became 49 mm. in length and 52 mm. in width. As in case of all other gypsums, all sides were covered with paraffin, except one which was exposed to the action of the solution. The surface was perfectly polished, as is to be seen also from photo 14; the velocity constant obtained for this vertical surface is about 3.376, *i.e.* about 6 times as large as the velocity constant of solution of the other surface of the same crystal (which is .6378), a difference which is truly enormous, and cannot possibly be attributed either to the difference in the surface or to the method employed. This experiment proves beyond any possible doubt that the same crystal dissolves at different rates in different directions to the axis of the crystal, and is at the same time an absolute proof that the fundamental assumptions of the diffusion theory mentioned above are thoroughly wrong and belong only to the world of pure imagination. This experiment also explains the reason, why the different modifications of gypsum give us different velocity constants. In the fibrous gypsum all the crystals of which the gypsum is formed configurate in one direction and not in the other; in the selenite the crystals configurate in all directions of the twin-crystal; while in alabaster, precipitated $\text{CaSO}_4 + 2\text{H}_2\text{O}$, or in powdered selenite they configurate in no direction. Owing to this in the selenite twin-crystal in the large side of the same, always the same surfaces of all the crystals (which are least easily dissolving) are exposed to solution, in the fibrous gypsum the surfaces of the crystals lying in two directions are exposed to solution, but not the surfaces in the third direction, which combine to form the fibre, while in the alabaster the surfaces of the crystal in all three directions are exposed to solution in equal or nearly equal shares. So we find for $\Sigma = 20.4 \text{ cm.}^2$ the velocity constant of alabaster is about 2.6; the velocity constant of selenite in powder and compressed to a disk is for the same surface = 2.9, while the velocity constant of $\text{CaSO}_4 \cdot 2\text{H}_2\text{O}$

precipitated and compressed to a block is for the same surface = 3.0, *i. e.* is only a little different (photograph 15 shows block of powdered selenite *after* solution; the corrosion is visible); which seems to prove that when the crystal has no special configuration and all its surfaces are equally exposed to solution, the speed of solution is practically the same.

It should be noted at the same time that the powdered and compressed selenite (see Selenite) and the fine precipitated and compressed $\text{CaSO}_4 + 2\text{H}_2\text{O}$ seem to give a very good constant from beginning to the end, without any essential increase in the same.

(ii.) $\text{CaSO}_4 + 2\text{H}_2\text{O}$ and CaSO_4 Anhydride.—If a substance forms a saturated solution at the surface of the solid with infinitely great speed, and if it passes from the surface of the solid to the rest of the solution only by diffusion, then $\text{CaSO}_4 + 2\text{H}_2\text{O}$ must dissolve with greater speed than anhydride CaSO_4 . Because the CaSO_4 must either first form $\text{CaSO}_4 + 2\text{H}_2\text{O}$ in the solution at the surface of the solid, before it diffuses into the rest of the solution in which we know it only as $\text{CaSO}_4 + 2\text{H}_2\text{O}$, not as CaSO_4 , or $\text{CaSO}_4 + 2\text{H}_2\text{O}$ must be formed first on the solid CaSO_4 itself, as a thin solid layer, under the action of water, and then as such pass into the solution. Whether this reaction of transformation of CaSO_4 into $\text{CaSO}_4 + 2\text{H}_2\text{O}$ be quick or slow, whether it takes place at the solid or in solution, the passing of CaSO_4 into the whole of the solution by diffusion can only be longer, but not shorter than in the case of $\text{CaSO}_4 + 2\text{H}_2\text{O}$, if the fundamental assumptions of the diffusion theory be true, and a reverse phenomenon should not be possible. Table III. c gives the speed of solution of anhydride CaSO_4 , Nova Scotia (photo 16 gives Nova Scotia *after* solution), which was turned to a disk of 50 mm. diameter. The velocity constant of the same is about .64 for its surface; on the other hand, we found the Selenite $\text{CaSO}_4 + 2\text{H}_2\text{O}$ gave for the same surface $\Sigma = 20.4 \text{ cm.}^2$, and under all other identical conditions of temperature, stirring, volume of solution, &c., a constant of about .49; an inspection of the two blocks Selenite and Nova Scotia after they were dissolving in solution, also show a greater corrosion in the last, which is indicative of a greater speed of solution, than in the case of Selenite.

Conclusion.

I have thus proved, both theoretically and experimentally, that the explanation of my equation for molecular reactions in case of solution of salts, as a diffusion equation, and all

the fundamental assumptions connected with the diffusion conceptions of Nernst, Brunner, and others, have no theoretical and no experimental support whatever. I have traversed in this first publication almost all the experimental material of Bruner and Tolloczko, Nernst and Brunner, so far as it deals with molecular reactions in solution. I have shown step by step, that all their experimental proofs here are nothing but arbitrary experiments, that as soon as they are replaced by accurate reliable results the very same substances furnish us with a solid proof against the diffusion theory. In my next publication I will deal with the speed of *chemical* reaction in heterogeneous systems, and I shall have then a still wider opportunity to show, traversing again the whole of the experimental material of Nernst and Brunner, that from the beginning to the end there is not one experiment which is not arbitrary, and which, when replaced by correct results, does not most decidedly refute the so-called diffusion theory with all its numerous and unnatural assumptions. It is a theory, which is neither theoretically sound, nor is it supported in the least degree by direct experiment.

Davy-Faraday Laboratory
of the Royal Institution, London,
January 1909.

V711

LXII. *On the Radioactivity of certain Lavas.*

By J. JOLY, F.R.S.*

SOME months ago, when determining the radium content of lavas from various parts of the world, I noticed with surprise the relatively large amount of radium contained in one from Vesuvius. This lava was from the eruption of 1855. The reading was so high that before accepting it I re-determined it upon a second chip taken from the interior of the same hand specimen. I obtained identically the same value.

Returning to the subject at a more recent date I examined the series of dated Vesuvian lavas in the Museum of Geology in Trinity College, and found quantities of radium in every case supporting my original determination. But the fact seemed brought out that the earlier lavas of the last three hundred years possessed a lower radium content than the more recently ejected rocks. At this stage Prof. Johnston-Lavis was so kind as to send me some samples of the lava-flow of April 1906. Subsequently he also supplied me with one of the oldest lavas: what he has classified as Dyke No. 1 of the ancient volcano of Monte Somma. This last rock is—

* Communicated by the Author.

in common with all the Vesuvian lavas—rich in leucite, and is, apparently, a leucite-tephrite.

An examination of these rocks afforded results in harmony with the indications of those already obtained. The recent lava was considerably richer in radium than any of the others and the ancient dyke-rock was much the poorest in radium of any yet examined; in fact, it was rather below the normal of igneous rocks.

Among prior observations upon the lavas of Vesuvius, I am acquainted with only one, that made by the Hon. R. J. Strutt, effected in the same manner; that is, by solution of the material and boiling off the emanation*. This observation revealed no abnormality. After what has been said, however, I do not think any contradiction with the present results need be supposed to exist. Thus the oldest leucite-tephrite I examined resembled many feebly radioactive lavas, and it is probable that among the earlier lavas similar rocks abound. Experiments have also been made upon the lavas of 1906 by Nasini and Levi (*Atti R. Acc. Lincei*, ser. 5, Rendic. xv. sem. 2, p. 391, 1906) and on the lava of 1904 by Tommasina (*Phys. Zeit.* vi. 1905, p. 707). In both cases the investigation was carried out by the dispersion-method; the powdered rock being introduced into the electroscope and the increased current observed. With regard to the lava of 1906 Nasini and Levi found the greatest effect from the lapilli and dust; obtaining (under certain conditions) a saturation current of 33.4×10^{-15} amp. On the other hand, they concluded that the solidified lavas and scoria were inactive. They examined the ejecta of some other outflows. The lapilli and dust of 1872 gave 19.2×10^{-15} amp. under the same conditions, and the lava 13×10^{-15} amp. For ejecta of various years the effects upon the electroscope, to the same order of magnitude as the above results, were:—

1631.....	17.0
1769.....	19.7
1858.....	14.7
1868.....	23.2
1895-99	4.9

The authors conclude that a lava inactive immediately after solidification may slowly acquire radioactivity.

Tommasina found, respecting the lava of 1904, that while obsidian from Lipari showed no determinable effect upon the electroscope the lava of Vesuvius increased the rate of fall of potential from a normal 10 volts per hour to 20 volts, and again the rate of loss of potential might even be increased rather more than three-fold.

* Proc. Roy. Soc. A. vol. lxxvii. p. 472.

The method pursued in these experiments, depending upon that variable property, the "emanating power," is liable to give results lesser or greater according to the ultimate compactness or porosity of the material under examination. This renders its indications uncertain. These observations seem, however, to point to an abnormally high radioactivity and there are some indications of the rise in radioactivity of the more recent lavas. On the point of the lavas gaining in radioactivity after solidification, if this gain is supposed to be rapid, my experiments are not relevant; as in every case the materials which I examined were some years old. But if the gain is supposed to take place over centuries my results do not support such a view. In fact, we would expect to find the more anciently erupted lavas the most radioactive, whereas it is the other way according to my experiments.

In the course of my work on the Vesuvian rocks additional security against error was obtained by systematically interpolating experiments on materials from other localities. These were in all respects prepared under like conditions and examined in the same way. Of the lavas listed below the following were so interpolated:—Etna, Pantellaria, Stromboli, Chimborazo, Krakatao ash. In addition to these, specimens of Vesuvian biotite and leucite were examined; and (for other work) Moffat Shale (3.1) and Keuper Sandstone (4.0). It will be seen further on that some of these are abnormal only in the sense of being low. The experiments were in some cases repeated upon fresh material, and in every case interior chips were taken from the specimens. The usual precautions as to purity of chemicals, etc. were observed. The standard of comparison is uraninite of Joachimsthal containing, according to a recent analysis made here, 65 per cent. of uranium.

More recently I have determined the thorium content of most of the materials dealt with. In some cases this determination was carried out upon the same solutions used for measuring the radium; in some a fresh solution was prepared. In the course of these experiments, which were carried out by a method recently described by me (*Phil. Mag.* May 1909), a test of the reliability of the method was made which may with advantage be referred to.

The amount of thorium in the Krakatao pumice listed in the table below was found to be 5.2×10^{-5} gram per gram. To 10 grams of the powdered pumice I added 1.2 milligram of a specimen of thorianite kindly given to me by Professor Dunstan (*Phil. Mag.* July 1909, p. 140). The whole was carefully mixed with the fusion mixture of alkaline carbonates (25 grams) and fused in the usual manner. The melt was leached with distilled water and filtered cold. The residue,

after being brought into solution by 50 cc. of HCl and diluted with water, was tested for thorium. The filtrate was separately tested. The total quantity of thorium found was 139.5×10^{-5} gram. Of this the alkaline solution held just one-twelfth part. Deducting the natural thorium— 52×10^{-5} gram—a balance of 87.5×10^{-5} gram remains. But 1.2 milligram of thorianite should contain 83.7×10^{-5} gram of thorium, according to Dunstan's analysis. The discrepancy is within the limits of error attending the weighing out of the thorianite. A second experiment of the same character on the same pumice was made, in which 1 milligram of thorianite was taken. The filtration was effected while the solution was hot. The thorium found was 117.5×10^{-5} gram, all of which was in the acid solution. Deducting, as before, the natural thorium, a balance of 65.5×10^{-5} gram remains to account for the inserted thorium—which was, on Dunstan's analysis, 69.8×10^{-5} gram. The discrepancy is again within the errors of the weighing.

These experiments were made with the double object of testing the method of determination and of finding whether ThX was removed in the filtrate; which from the well known experiment of Rutherford and Soddy might, perhaps, have been expected. In a few cases out of many I have found some radioactivity in the alkaline solution, and I have not generally been able to connect its appearance with the temperature at which filtration is effected. The radioactivity observed occasionally in the alkaline solution seems generally to diminish in a few days but never quite to disappear. A little both of thorium and of thorium X are possibly separated in such cases. The testing of both solutions is, therefore, desirable, especially if freshly made up solutions are being dealt with. In dealing with old solutions the error, if this is not done, will generally be small. A good plan is to mix both solutions together after the residue has been brought into solution. The alkaline solution should, in this case, be first acidified. In certain basic rock-solutions this procedure is, however, not admissible; a precipitate forming.

In the thorium experiments carried out in connexion with the lavas, the velocity of air-current from the flask to the electroscope was about two cubic centimetres per second. With this current the sensibility of the electroscope was 1.55×10^{-5} gram thorium per scale-division per hour. This velocity of air-draft is sufficiently slow to secure the results against error from the passage of the short-lived emanation of actinium.

The table here given embodies the experiments on lavas*. It first deals with materials from the Vesuvian focus of eruption. Lavas ejected from the neighbouring petrographical provinces are given next. A few results from widely distributed volcanoes are added for comparison. This last list is, I am aware, too short, the subject being deserving of fuller treatment. Finally, some separated rock minerals from Vesuvius are dealt with. It may at once be pointed out that there is evidently no concentration of radium in these minerals. The radium is apparently distributed throughout the magma. In the table a query after a determination denotes the presence of some precipitate in the acid solution.

	Radium $\times 10^{-12}$	Thorium $\times 10^{-5}$
Monte Somma; very old dyke	2.8	2.2
Vesuvius, 1631; Gravina	7.8	1.7
„ 1794; Torre del Greco	9.8	0.6
„ 1832	13.0	2.3
„ 1855; Le Novelle; (12.5), (12.5)	12.5	2.5
„ 1868; (11.9), (13.3)	12.6	4.1
„ 1895-99; Colle Umberto; slow out-flow; (15.3), (14.0)	14.6	2.1
„ 1906; Torre Annunziata; interior of lava stream; (17.9), (14.2) (?)	16.0	2.6
„ 1906; Glassy lava; cooled in cistern	13.8	...
„ 1906; Flotation bomb, Boscotrecase	10.7 (?)	...
„ 1906; Ejected old lava block with tachylite filling	9.2	...
Campi Phlegraei, Monte Olibano; trachyte; (7.8). (5.8)	6.8	4.2
Ischia; olivine trachyte; (4.3), (7.8)	6.0	2.4
Lipari, Rocchi Rossi; lava; (5.8), (4.6)	5.2	4.5
„ Val di Lachi; augite andesite	1.6	0.5
Stromboli, scoria; (3.9), (3.2)	3.5	1.5
Vulcano; bomb; (5.3), (4.6)	5.0	4.6
Etna, Val del Bove; 1855; lava	6.0	1.2
Pantellaria, La Mantua; lava	3.1	2.2
Kilauea, crater, 1875; lava	3.9	1.4
Ascension Island; obsidian	1.6	2.5
Krakatao, ash; collected about 17 miles N.E. of volcano: 1883	0.6	...
„ pumice; 1883	4.5	5.2
Chimborazo, near Quito; pumice	3.2	2.4
St. Helena; lava	2.3 (?)	0.6
Martinique; bomb, much weathered	1.3	...
Iceland; lava	1.5 (?)	...
Vesuvius; cube of halite with purple streaks	0.0	...
„ leucite	1.0	0.6
„ biotite	2.6	...

* Some of the results have appeared in "Radioactivity and Geology," p. 43. As therein given they are unfortunately affected by an error which arose in calculating the constant of a recently constructed electroscope.

It will be noticed that in every case the readings of Vesuvian lavas from 1631 to the present day are remarkably high; up to three times the normal for igneous rocks and, indeed, even higher. The thorium-content, although large in comparison with what generally prevails in the rocks of the St. Gothard series (Phil. Mag. July 1909) are not conspicuously higher in the Vesuvian rocks than in the rocks from other volcanoes. The highest reading was obtained for the Krakatao pumice.

The Vesuvian lavas appear to show a progressive increase of radioactivity according as they are of more recent eruption. The old leucitic rock of Monte Somma is of special interest. According to this one sample of the earlier rocks the radioactivity was at first actually less than normal. The culminating richness in radium is attained in a sample from the interior of the lava-flow of April 1906. (The "ejected old lava block" in the list is, of course, not truly of this date.) Unless we conclude that the radium is transforming within these lavas after their ejection at a rate much greater than would be expected from the period inferred on experimental grounds, and that the loss is not being made good, we must assume that as time progresses, this volcano is tapping materials richer and richer in radium. There is no improbability in this explanation save what is involved in the rather contradictory fact that the lavas of Vesuvius have shown a remarkable constancy in chemical composition from the earliest times. This is noticed by many authorities, but specially by Rosenbusch ('Elemente der Gesteinslehre,' p. 344), who remarks that notwithstanding great variations in texture, colour, and the nature of the phenocrysts, their chemical constitution has been extraordinarily constant from the oldest pre-historic eruptions of Monte Somma to the most recent eruptions of Vesuvius. It is therefore surprising to find the radium (or uranium) content varying not less than five-fold.

The lavas from the surrounding volcanoes emanate from distinct petrographical provinces and show the most diverse chemical compositions (Bonney, 'Volcanoes,' p. 207). Their failure to show the high radioactivity of the Vesuvian lavas is, therefore, not remarkable. On the whole, the Lipari and Etna lavas stand high among lavas from other parts of the world. But an extension of the number of determinations may reveal many cases of radioactivity quite as high. And, of course, the same remark may be made in reference to the Vesuvian results.

The possibility of a connexion between radioactivity and

vulcanicity is so natural an association of ideas that even before the general prevalence of radium in rocks had been demonstrated by Strutt, a suggestion to that effect had been made. Major Dutton (*Journal of Geology*, xiv. May-June 1906, p. 259) in 1906 put forward the view that radioactive materials might give rise to fusion temperatures at points within 3 or 4 miles of the surface: indeed, his appeal to radioactive energy was mainly based on the necessity which, according to Major Dutton's views, exists of locating volcanic foci at these short distances beneath the surface of the Earth. The repetitive character of the phenomena was also regarded as more readily accounted for on the theory of the accumulation of radioactive energy than in any other manner. Major Dutton's ideas were criticised by G. D. Lauderback (*ibid.* p. 748) more especially as to the necessity of assuming a position so superficial for volcanic foci and of appealing to radium to explain the repetitive phenomena. Moreover, it was pointed out by more than one critic at the time that any theory involving haphazard pockets of radioactive materials was not in accord with the fact that the location of volcanoes was by no means haphazard, but was determined according to certain great surface features of the Globe; the geosynclines, wherein sediments collect and crust-flexure occurs.

When recently dealing with the latter subject—the very probable connexion between radioactive heating and the instability of the sediment-laden part of the crust (*Address Sect. C. Brit. Assoc.* 1908), it appeared to me that a connexion between vulcanicity and radioactivity was, in point of fact, involved in the conditions determining the rise of the geotherms under a superficial covering of some kilometres of radioactive sediments. While we are not entitled to assume upon our present knowledge the existence of radioactive magmas in pockets close to the surface and possessed of sufficient radioactivity to establish a local volcanic focus, some miles down high temperatures might be ascribed to the great sedimentary accumulations which have been sufficiently long *in situ* for the accumulation of radioactive heat. The crust buried beneath the covering of sediment is, in fact, heated both from above and from beneath, and the quantities involved, as regards depths of sedimentary deposit and average radioactivity of such materials, are such as to justify the view that in the geosynclines vulcanicity would be specially favoured. If this hypothesis is correct, not only the flexure and distortion affecting these areas, but the chains of volcanoes breaking out along the great mountain ranges, may be logically

regarded as primarily traceable to the effects of radioactive energy.

As defining a general connexion between radioactivity and vulcanicity, this view appears to be rather strengthened than otherwise by recent advances in our knowledge of the tidal stability of the earth. For if, as Professor Love concludes (*Nature*, April 29, 1909), we may not postulate the general extension of a fluid substratum, we are evidently driven to connect vulcanicity in the geosynclines with some local source of heat. It may here be observed that there does not seem to be difficulty in reconciling Prof. Love's conditions with the facts attending mountain elevation. Under the new restrictions we are compelled to assume that temperatures deep down cannot generally amount to those of liquefaction. However, "crust-creep" on a large scale must at the same time be possible, or the forces acting in the geosynclines become unaccountable. In the wide range of viscosity of siliceous substances, both conditions seem attainable. While generally there cannot prevail a temperature of fusion, conditions of rigidity toward tidal stress do not seem inconsistent with sufficient viscosity to permit yielding under the more prolonged, unidirectional, stress of secular cooling. And not without bearing on the connexion between radioactivity and vulcanicity in its wider aspect is the fact that the case for the shallow, superficial, character of the radioactive surface-materials of the earth seems strengthened by the limitation of temperature-rise downward. It appears that the gradients must ultimately be limited to such a gradual slope as will not overtake the rise under pressure of the melting-points of the rock-forming silicates*. Such a condition is absolutely inconsistent with the downward extension of the radium-richness of the surface-rocks.

These remarks apply generally to a probable connexion between radioactivity and volcanic outbreak; they do not generally deal with the particular case to which I have called attention:—the case of a volcano emitting abnormally radioactive lavas.

We might suppose that the circumstances connected with the case of Vesuvius were to a certain extent fortuitous; the exceptionally radioactive materials having been brought to light by a regional crustal disturbance unconnected with the existence of a local deep-seated radioactive magma. On such a view we may admit that the locally great radioactivity has perhaps contributed to the remarkable persistence

* For data see Vogt, *Tschermak's Min. u. Pet. Mit.* xxvii. p. 105.

of Vesuvius as an active volcano, and even assisted in maintaining the vulcanicity in the surrounding region. There may be other cases of the kind. We have only to assume that here and there in the deeper crust are rock-masses of higher radioactivity than generally prevails, and that while regional vulcanicity may require for its initiation and development prolonged and deep sedimentation, yet where there is coincidence with strongly radioactive magmas, specially energetic and persistent effects will arise. We are, of course, without guidance as to the downward extension of the radioactive laccolith concerned; or as to whether we have yet attained a true measure of its radium-content.

An explanation of the exceptional radioactivity of Vesuvian lavas may, however, also be sought on the hypothesis that the materials ejected from Vesuvius are sedimentary in origin. Such an origin has before now been ascribed to volcanic materials. On this view the richness in radium observed in these lavas is in line with the observed radium-richness of many slow-collecting sediments. There is, indeed, nothing in the general composition of these lavas to negative their sedimentary origin. They might represent certain calcareous clay slates, for instance*. It is remarkable that the percentage of potash much exceeds that of soda. Although certain, rather scarce igneous rocks—the shonkinites and some others—sometimes exhibit a similar excess of potash over soda, this feature is, generally speaking, characteristic of detrital remains which have undergone the impoverishment in sodium attendant upon denudation. The absolute amounts of alkalis present are, however, higher than is commonly found in sedimentary rocks. On the whole this view seems less probable than that which assumes the Vesuvian lavas to be drawn from a deep-seated laccolith.

Whichever explanation of the origin of these lavas is deemed most probable, it may fairly be asked if we are indeed entitled to assume that any considerable temperature-rise may be ascribed to their radioactivity. This question discloses the unsatisfactory state of our knowledge upon the subject. All depends of course on the extension of the radioactive materials. If they are laccolithic in origin we may reasonably suppose their downward extension considerable, and in this case, the intensification of volcanic conditions, by cumulative radioactive energy, appears highly probable. If, on the other hand, the lavas are sedimentary in origin, the vertical extension of beds so radioactive can hardly be great. In

* For analyses of the Vesuvian lavas, see Rosenbusch, *loc. cit.* p. 346.

this case the contributory thermal effects of the radioactive substances has, probably, been small, however long a time we are willing to assign to the genesis of heat.

An estimate of the intrinsic heating-effect of the radioactive lavas we have been considering is of interest. The mean radium-content of the lavas ejected since 1631 is 12.3×10^{-12} gram per gram. Assuming that the presence of one gram of elemental radium in equilibrium involves the generation of 201.6 gram-degrees per hour, the heating-effect in the lava is about 25×10^{-10} calorie per hour. The mean quantity of thorium is 2.3×10^{-5} gram per gram. Pegram and Webb's result, obtained by a direct thermal measurement applied to thorium oxide, gives the heat per gram of elemental thorium in equilibrium as 2.38×10^{-5} calorie. Hence the thorium present would produce 5.5×10^{-10} calorie per hour. The added results of these quantities of heat works out as giving a rise of temperature of about 150° C. in one million years; assuming no loss by conduction.

Those who look to radioactive heating as affording an explanation of volcanic periodicity would do well to reflect upon the slowness with which heat is supplied, even in exceptionally rich radioactive materials.

In the case of the Vesuvian lavas the abnormally large amount of radium present renders the heating-effect of the thorium relatively small. The mean thorium-content of the other lavas examined is 2.6×10^{-5} , and the mean radium-content of these rocks is 4.0×10^{-12} gram per gram. The heating-effects are respectively 6.2×10^{-10} and 8.1×10^{-10} gram-degree per hour. Here there is nearly equality in the thermal values. Blanc (*Rend. d. R. Accad. d. Lincei*, xviii. ser. 5, p. 289) has concluded that the thermal effect of thorium in rocks may amount to about double that due to the radium series, but his figures are based on a somewhat larger thorium- and smaller radium-content than has prevailed in the rocks which I have, so far, examined.

LXIII. *On the Electromotive Forces produced by Acid and Alkaline Solutions streaming through Glass Capillary Tubes.*
By ALEXANDER T. CAMERON and ERICH OETTINGER*.

THE electromotive forces existing between glass or other insulating material and aqueous solutions, and the consequences resulting therefrom when these solutions are moving over the insulating surface, as, *e. g.*, when they are

* Communicated by the Authors.

forced through glass tubes, have formed the subject of the theory of Helmholtz*. This is based on the facts that, firstly, when a conducting liquid streams through a channel of insulating material, an electromotive force is produced in the direction of the liquid stream or opposite to it, while, secondly, an electromotive force applied at the ends of the channel, or between two points in it separated by a convenient distance, produces a flow of liquid. Helmholtz assumed that there existed a difference of potential at the boundary between the insulating material of the tube and the conducting liquid, and derived an equation connecting this difference of potential $\phi_i - \phi_a$ (where ϕ_i is the potential of the liquid, ϕ_a that of the solid insulator) with the pressure P between two cross-sections of the channel, the electromotive force E existing between these two cross-sections, the specific conductivity κ of the solution, and its coefficient of internal friction η :

$$E = \frac{P}{4\pi\eta\kappa}(\phi_i - \phi_a).$$

The formula applies to tubes for which Poiseuille's law holds, provided also the potential difference between the two cross-sections is not led into an outside circuit, but only produces an electric current along the tube, and that there is no shifting of the last layer of the liquid adhering to the glass. This last condition may be expressed in the form that the external friction is infinite; for a finite value the term $\phi_i - \phi_a$ has to be replaced by

$$\phi_i - \phi_a + \xi \frac{\partial \phi}{\partial N},$$

where ξ is the constant of slipping, and N the normal to the tube wall. To this, the "electric moment of the double layer," Dorn gave the symbol μ . Lamb† has proposed a modification of the Helmholtz theory, which differs only by the assumption that the charge of the liquid is rigidly confined to the last layer. In Lamb's theory the same term takes the form $(\phi_i - \phi_a) \frac{l}{d}$, where d is the thickness of the double layer, *i. e.* a magnitude of the order 10^{-8} , while l is the ratio of internal and external friction, *i. e.* of the coefficients of friction and slip.

Helmholtz verified his theory by applying to it the experimental results of Quincke‡, who forced water through two

* Wied. *Ann.* vii. p. 337 (1879).

† Phil. Mag. [5] xxv. p. 52 (1898).

‡ Pogg. *Ann.* cvii. p. 1 (1859); cviii. p. 33 (1860).

glass tubes connected by a diaphragm of burnt clay and similar substances. Zöllner * showed that what held for a bundle of capillaries applied with equal force to a single tube, while further experimental work was carried out by Edlund †, Haga ‡, Clark §, Dorn ||, and Saxén ¶. Their results are in general agreement with each other, and, especially those of Dorn, entirely agree with the Helmholtz theory.

All these experimental investigations consider the factor $\phi_i - \phi_a$, or its expanded form μ (Lamb's $(\phi_i - \phi_a) \frac{l}{d}$), as a constant. Whether this is the case or not has remained unverified, since independent determination of this value is not possible. The figure calculated by Helmholtz and Dorn is about 4 daniell (5.1 volts when corrected for modern units), which seems a rather large value for the potential between glass and water, considering the fact that much smaller potentials, so far as we are aware, always produce development of hydrogen gas**. Much smaller values, strikingly altering with the concentration of the solution, have been derived from the electric endosmose experiments of Wiedemann ††, Quinke ‡‡, and Freund §§, so that one point of the theory still remains unsettled.

A way of access to the potential difference between glass and solutions has been opened recently by Haber and Klemensiewicz |||, who have been able to prove that the surface of ordinary glass acts perfectly as a so-called hydrogen electrode of constant hydrogen pressure. The change of potential produced with such an electrode by altering the acidity or alkalinity of the surrounding solution is well known. The difference of potential between two such electrodes, one dipping into acid the other into alkali, was made

* Pogg. *Ann.* cxlviii. p. 640 (1873).

† Wied. *Ann.* i. p. 161 (1877); ix. p. 95 (1880).

‡ *Ibid.* ii. p. 326 (1877).

§ *Ibid.* ii. p. 336 (1877).

|| *Ibid.* v. p. 20 (1878); ix. p. 513 (1880); x. p. 71 (1880).

¶ *Ibid.* xlvii. p. 46 (1892).

** See table of absolute potentials by Wilsmore and Ostwald, *Zeitschr. f. phys. Chemie*, xxxvi. p. 92 (1901). Billiter's absolute zero point (Drude's *Ann.* xi. p. 137, 1903), although different, is still near enough to Ostwald's for the validity of the above consideration.

†† Pogg. *Ann.* lxxxvii. p. 321 (1852).

‡‡ *Ibid.* cxiii. p. 513 (1861).

§§ Wied. *Ann.* vii. p. 44 (1879).

||| *Zeitschr. f. phys. Chem.* 1909.

by Ostwald a method for determining the dissociation of the water*.

The fact that glass acts as a hydrogen electrode enables us to state that at ordinary temperatures an increase of a tenth power in the concentration of the hydrogen ions produces a change in the electromotive force of 0.058 volt. A solution normal as regards hydrogen ions and one normal as regards hydroxyl ions show, as is well known, a ratio of 10^{-14} between their hydrogen-ion concentrations. Consequently, the change in the electromotive force between glass and surrounding liquid, when the latter undergoes this change from normal hydrogen to normal hydroxyl ions, will be $14 \times 0.058 = 0.812$ volt; where both solutions are one-hundredth normal, this value becomes $10 \times 0.058 = 0.580$, and with one-thousandth normal solutions, $8 \times 0.058 = 0.464$ volt, and so on.

The sign of the change is always such that pure water, arbitrarily taken as neutral against glass, in which the concentration of both hydrogen and hydroxyl ions is 10^{-7} , becomes positive by increase of hydroxyl ions, negative by increase of hydrogen ions.

In light of these facts the potential difference $\phi_i - \phi_a$, or its expanded form μ (Lamb's $(\phi_i - \phi_a) \frac{l}{d}$), does not seem quite so inaccessible as before. It is now to be expected that if the Helmholtz theory is tested with acid and alkaline solutions, different values of $\phi_i - \phi_a$ will result, and we may predict that if the term $\zeta \frac{\partial \phi}{\partial N}$ is zero or constant (or Lamb's $\frac{l}{d}$ is constant), the difference between the values will be equal to the electromotive force derived from the acidity and alkalinity of the applied solutions.

The irregularity of the values for $\phi_i - \phi_a$ found by different experimenters leads us to expect that the experimental verification is beset with some difficulties, so that perhaps no more than a qualitative agreement can be expected. Moreover, there exists a group of facts which will be discussed at the end of the paper, which make it doubtful whether or not the interference of another effect will not prevent more than qualitative agreement. In view, however, of the fact that the values of the single potentials existing at a phase boundary

* *Zeitschr. f. phys. Chemie*, xi. p. 521 (1893); *cp.* the correction by Nernst, *ibid.* xiv. p. 155 (1894), and the recent papers by Lorenz and his collaborators, *ibid.* lx. p. 422 (1907); lxvi. p. 733 (1909).

(metal, salt solution, as well as other two-phase systems) has for twenty years formed the subject of unsettled dispute, even such a qualitative result did not seem to lack importance. Moreover, the electromotive force at the boundary of two phase systems such as glass and liquid is of special interest, since Haber has shown that the laws governing this electromotive force are applicable to interesting electro-physiological phenomena.

This paper contains an account of attempts to verify the theory from this point of view, and though our results give only a qualitative confirmation, for the reasons just stated, it seems desirable to publish them, especially since considerable time may elapse before the experimental difficulties are sufficiently overcome to permit of completer verification.

Experimental Part : Apparatus.

The apparatus differs in a number of essentials from those employed by previous investigators, and as it seems easier to construct and to use than these, it may be described shortly. It consists essentially of four parts, an instrument for measuring potentials, a capillary tube containing electrodes, a stout glass vessel to which it is joined, containing the liquid under examination, and an apparatus for maintaining constant pressure. The apparatus was tested by repeating the experiments of Dorn with pure water; in this way a number of errors in it were detected which require to be avoided, and may therefore be mentioned.

In all cases a quadrant electrometer was used to measure the potential difference; in this way errors arising from polarization, which occur with galvanometers or with electrometers of greater capacity, were avoided. Both the Thomson instrument with bifilar suspension and that with a platinum hair-suspension and Dolezalek needle, were employed at different times. The capillaries used were calibrated with a shifting thread of mercury, and selected so that the variation in diameter was slight. Different forms of electrodes were tested. At first side tubes were blown on to the capillary, and completely filled with mercury, so that the mercury was in contact with the stream of liquid; contact was allowed with the electrometer by platinum wires sealed through the ends of the side tubes. The potentials derived from these electrodes were not proportional to the fall of pressure between them, *i. e.* did not agree with the Helmholtz formula; on using the same tubes without mercury worse results were

obtained; in this case the platinum electrodes in the side tubes were not in contact with flowing but with resting liquid. The results showing closest agreement with theory were obtained with electrodes consisting of thin platinum wires fused directly into the capillary tubes, the alteration in the capillary being made as small as possible. External contact was improved by surrounding the portion of tube containing the electrode with tinfoil, with copper wire as binding material; this was connected to the electrometer.

The vessel used to contain the liquid under examination was a five-litre bottle of Jena glass, closed by a two-holed rubber cork, which was held in position when under pressure by a simple clamp. A short glass tube dipping just below the cork connected the vessel with the pressure apparatus. A second wide tube, reaching to the bottom of the vessel, was joined by pressure tubing with the capillary, which was clamped in a horizontal position. At first the device employed by Haga (*loc. cit.*) was tried, the capillary tube was connected between two similar flasks, and the liquid forced backwards and forwards between them. However, with this method contradictory results were obtained, probably attributable to the "waterfall electricity" effect observed by Lenard or the similar effect observed by Elster*. It is probable that the curious results obtained by Haga† are attributable to one or other of these causes. For the same reasons it was found necessary to prevent the liquid flowing out of the capillary from impinging on any surface, before it was broken up into drops.

The pressure arrangement consisted of an ordinary pressure bomb containing compressed air (freed from carbon dioxide) up to a hundred atmospheres pressure. The adjustment was made with a Le Rossignol‡ valve, which, since it contains a barrel with the small angle of 4° , allows a very fine adjustment, and gives fairly constant pressures even with such slight outflow of gas as was required in these experiments. Between the valve and the five-litre vessel were inserted an open mercury manometer reading to one and a half atmosphere, a simple safety-valve consisting of a glass tube dipping below mercury, and an outlet tube, joined to pressure tubing, closed by a screw-clamp.

* Wied. *Ann.* vi. p. 553 (1879).

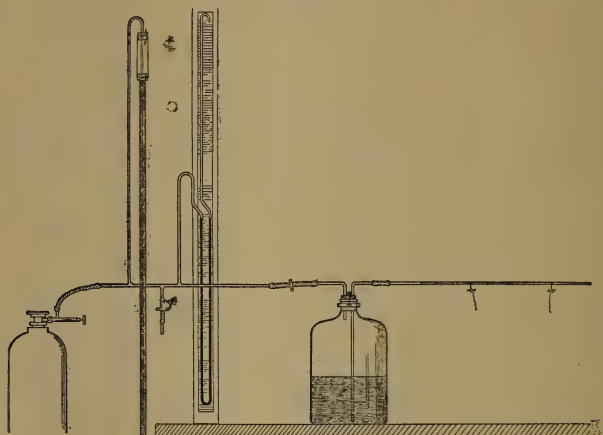
† *Loc. cit.* p. 328.

‡ *Chem. Zeit.* no. 69, 1908.

The apparatus is shown in the accompanying diagram (fig. 1).

The electrometer needle was always charged to a convenient potential by means of a Zamboni pile. One pair of quadrants was earthed, the other connected with the electrode

Fig. 1.



nearest the free end of the capillary. The other electrode was also earthed. It was found that the connexions, reversed, did not lead to such certain results; the reason was probably due to imperfect insulation of the second electrode, since connexion with the flask was allowed by the liquid. This source of error was also observed by Dorn. The sensibility of the electrometer was always determined with a normal cadmium cell.

Preliminary Experiments.

If the quantities $\phi_i - \phi_a$, η , and κ are constant, Helmholtz's formula simplifies to the form

$$E = K \cdot P,$$

where E and P are respectively the differences of potential and pressure between two electrodes, and K is a constant. Ordinary distilled water of specific conductivity between 10^{-5} and 10^{-6} was used to test the formula in this simple form, in order to control the working of the apparatus.

A thin capillary tube ab of diameter 0.65 and length 270 mm. was taken, and four electrodes α , β , γ , δ , inserted, such that $a\alpha=29$, $\alpha\beta=62$, $\beta\gamma=97$, $\gamma\delta=68$, $\delta b=14$ mm.

The tube was rinsed out with chromic acid mixture, the end *b* joined to the flask, and water forced through at different pressures.

Three difficulties prevent an exact agreement between the results obtained and the theory. The first arises from the fact that the introduction of the electrodes disturbs the flow of liquid, both through the electrodes themselves, and from the unavoidable distortion of the glass capillary at the point of insertion. Whether we neglect this disturbing influence, consider the space round the electrodes as a normal capillary, and measure the actual distance from electrode to electrode, or whether we neglect the distorted spaces themselves, assuming that there is no effect within them, and measure the distances between the beginning and end of the undistorted capillary system, we shall always make a certain error. It was thought that the error might be less if the second assumption were followed.

The second difficulty arises from the fact that, according to the results found formerly by Dorn and Clark, the effects obtained are dependent on time. Clark found that when a tube was rinsed with concentrated sulphuric acid and then distilled water, and then water forced through it continuously for many hours, a decrease in the electromotive force was obtained invariably, which amounted in some cases to as much as 25 per cent. On recleaning the tube the original value was regained. Dorn's results were not so regular. He obtained both decreases and increases of voltage with lapse of time; these were connected apparently with the method used to cleanse the tube.

The last point of uncertainty comes into the results from the calculation of the actual pressures existing between the electrodes. We know the total pressure applied. This is not entirely used up in the tube. With pressures of half an atmosphere or more the liquid does not escape in drops from the tube, but forms a ray possessing kinetic energy corresponding to a certain pressure, so that the drop of pressure in the tube is less than the difference of applied pressure to atmospheric pressure. Moreover, every exit from and entrance into an electrode space causes the formation of eddies, and in any case a slight drop of pressure exists even in the non-capillary portions of the tube. Being unable to state the amount of these losses, we calculated on the assumption that the applied pressure was entirely used up in the five capillary stretches, the dimensions of which have been given*. The amount of pressure used up in each portion

* The combined length of the three electrode spaces was 12 mm.

is consequently over-estimated, and we are by no means certain that this over-estimation produces merely an error consisting of an approximately constant factor. When these considerations are borne in mind the following figures will not seem unfit to support the formula:—*

A are the observed pressures between the ends of the tube corrected for the water column in the glass containing-vessel, B are the pressures calculated to exist between the two electrodes under consideration, from the law that the fall in pressure is proportional to the length along the tube. During the experiment, which lasted an hour, temperature varied between 18° and 19° C. The sensibility of the electrometer was such that 1 cm. on the reading scale corresponded to 0.05 volt.

A mm. Hg.	Portion of tube taken.	B mm. Hg.	E.M.F. volt.	E.M.F./B.
380.9	$\alpha\beta$	87.6	+0.593	6.76×10^{-3}
381.3	$\alpha\gamma$	224.5	1.400	6.23
382.2	$\alpha\delta$	321.3	1.914	5.96
380.7	$\beta\gamma$	136.8	0.815	5.96
382.0	$\beta\delta$	233.4	1.307	5.60
381.6	$\gamma\delta$	96.1	0.490	5.10
249.5	$\alpha\beta$	57.4	0.398	6.94
249.1	$\alpha\gamma$	146.7	0.971	6.62
248.3	$\alpha\delta$	208.7	1.351	6.47
249.8	$\beta\gamma$	89.8	0.571	6.36
249.6	$\beta\delta$	152.6	0.959	6.27
575.0	$\alpha\beta$	132.2	0.833	6.30
594.4	$\alpha\gamma$	350.0	2.100	6.00
567.7	$\beta\gamma$	204.0	1.197	5.87

Since when water is stored in a glass vessel and used continuously in an experiment of some duration, there arises the possibility that small changes may take place in the conductivity of the water, to avoid such an error experiments were carried out with very dilute solutions of potassium chloride. Of these the following are typical:—

Experiment 2. To test the effect of different pressures and duration of time.—A new tube was employed, containing two electrodes, 242 mm. apart. The whole length of the tube, excluding the electrode spaces, was 470 mm.†, and its diameter

* It may also be pointed out that the diameter of the tube was perhaps a little greater than those which strictly obey Poiseuille's law.

† Each electrode space was 5 mm. in length.

varied between the limits 0.816 and 0.823 mm. The solution employed was N/3000 KCl. The temperature varied between 20° and 20.5° C. One cm. on the electrometer scale corresponded to 0.025 volt. The experiment was commenced at 10.32 A.M. A and B have the same significance as before.

Time.	A.	B.	E.M.F.	E.M.F./B.
A.M.	mm.	mm.	volt.	
10.34	368.1	189.5	0.288	1.518×10^{-3}
41	361.5	186.1	0.282	1.514
47	360.5	185.6	0.286	1.544
50	707.9	364.6	0.422	1.158
52	707.5	364.2	0.428	1.176
55	203.0	104.5	0.144	1.377
58	208.6	107.4	0.159	1.478
59	360.4	185.6	0.270	1.458
11.01	366.1	188.5	0.279	1.480
11	412.1	212.2	0.316	1.490
16	422.4	217.4	0.327	1.504
18	178.7	92.0	0.157	1.703
19	172.1	88.6	0.154	1.737
21	609.8	314.0	0.471	1.500
24	602.1	310.0	0.456	1.471
26	472.2	243.1	0.352	1.448
28	463.3	238.5	0.347	1.454
38	397.8	204.8	0.300	1.464
39	326.6	204.2	0.297	1.452

If the figures are re-arranged according to pressure it is seen that for the extreme values there is a certain progression varying inversely as the pressure, but between the limits of 200 and 600 mm. the influence is small, and the simple form of the Helmholtz theory seems well supported by the experiment. The small remaining change within these limits seems unavoidable, and must be regarded as due to changes in the behaviour of the tube with time.

Experiment 3. To ascertain the effect of temperature.—A dilute solution of potassium chloride was used as in the previous experiment. It was heated to a convenient temperature before introduction into the glass bottle. There it cooled slowly, and measurements were made during the process. A thermometer was placed in the escaping stream of liquid. As a conceivable precaution against a larger "time-effect" a very slow stream of liquid was caused to flow continuously between the sets of readings. The tube used was the same as that in Experiment 2. One cm. on the electrometer scale corresponded to 0.025 volt. The experiment was commenced at 11.10 A.M.

Time.	A.	B.	Temp.	E.M.F.	E.M.F./B.	Mean.
A.M.			C.	volt.		
11.15	335.7	172.8	31.9	0.311	1.799×10^{-3}	
18	339.6	174.9	31.9	0.316	1.805	
24	341.7	176.0	31.6	0.318	1.808	1.804×10^{-3}
35	338.4	174.3	30.1	0.324	1.858	
38	348.7	179.5	30.1	0.338	1.882	
40	348.2	179.3	29.8	0.334	1.865	
45	341.7	176.0	29.7	0.318	1.806	
49	335.2	172.6	29.6	0.307	1.780	1.838
P.M.						
3.45	347.6	179.0	21.5	0.339	1.895	
49	380.7	196.0	21.4	0.361	1.840	
51	337.3	173.7	21.4	0.319	1.836	1.857

The change observed is only 3 per cent., *i. e.* is within the limit of experimental error previously found. Experiments were also made to ascertain the change of temperature caused by friction while the liquid was passing through the capillary. A thermometer was introduced into the tube connecting the bottle with the capillary, and a second placed in the stream of liquid issuing. A rise of temperature was observed which never exceeded $1^{\circ}.5$. For experiments at ordinary temperatures, therefore, slight temperature changes need not be considered.

Experiments with Dilute Solutions.

The question under discussion in this paper would be tested most advantageously if series of measurements could be made with different concentrations for different acids and alkalies. We attempted to make such measurements, but found many difficulties in the attempt to employ a large range of concentration. When the concentration is greater than N/1000 the E.M.F. obtained with the pressures employed becomes less than 0.1 volt. In this case there become of great importance the errors which arise from a difference of potential of both electrodes when the tube is filled with liquid at rest. We never succeeded in avoiding casual differences between the two electrodes, in this condition, of the order of several millivolts. In determining these values before and after the determination of the E.M.F. produced by flow of liquid, we found that often they change both in value and in sign. Devices such as connecting the electrodes before the measurement, insulating as well as possible the glass surface (both tube and flask) &c., proved insufficient remedies. It was therefore necessary

to make measurements only with such concentrations that the E.M.F. obtained was of a higher order of magnitude than the error. Hence high concentrations could not be used. On the other hand, concentrations of 1/10000 normality must also be avoided because impurities of the water can then exert too great an influence, and the value in the conductivity becomes too uncertain, so that of the many preliminary experiments those between N/5000 and N/1500 were found best; our final experiments were carried out between these concentrations in the following manner:—

The solutions used were prepared synthetically; in each case a definite amount of solution of known strength was added to a definite amount of distilled water in the containing vessel, and the whole mixed thoroughly. By addition of more water or solution a second concentration was obtained. The conductivity of each solution was determined immediately after the measurements of the potential. The experiments with each concentration lasted from one to one and a half hours. The capillary tube was cleaned with chromic acid mixture before each change of solution, and it was assumed that during the time of measurement the time effect was no greater than that previously observed.

If the conductivities are compared with those derived from the ionic mobilities of the solutions at these concentrations, it will be seen that differences exist. These are due to the fact that the distilled water was not of such a high degree of purity as that employed in exact conductivity measurements; larger quantities were required than could be obtained conveniently with water of such purity. The errors introduced from this source, however, were not large.

Kohlrausch has shown* that the temperature coefficients of the conductivity and of friction, for very dilute solutions, are very nearly the same, so that as the temperature at which the conductivity was measured was always very little different from that of experiment, the actual conductivity and coefficient of friction for that temperature were employed. The coefficient of friction was assumed to be the same as that for water †, and was calculated from O. E. Meyer's formula ‡

$$\eta = \frac{1.775}{1 + 0.03315t + 0.0002437t^2}$$

* Wied. *Ann.* vi. p. 193 (1879).

† This assumption is justifiable, since changes of concentration of one-eighth normality with these solutions only produce a difference of from 0.3–1.7 per cent. in the coefficient of friction.

‡ Wied. *Ann.* ii. p. 394 (1877).

Thus for 16° , $\eta=1.111$; 17° , $\eta=1.086$; 18° , 1.059 ; 19° , 1.033 , &c.

In every experiment the potential at zero pressure (*i. e.* with the capillary filled with solution at rest) was first measured, until constant, then that at one or more pressures, and finally that at zero pressure again.

In these experiments a new tube was used, with the following constants: total length, 518 mm., length between electrodes 256 mm., diameter 0.720 to 0.728 mm.*

The following table shows the results with dilute acids. A and B have the usual significance. It should be remembered that the concentrations are only approximate, and that an error of 3 or 4 per cent. is possible. E_1 and E_3 are the voltages found at zero pressure before and after the actual measurement (E_2) respectively. E_4 is the corrected voltage. In order to obtain $\phi_i - \phi_a$ in volts (Helmholtz's formula is given in absolute electrostatic units), the term $E_{1000}\kappa \dagger$ was multiplied by the factor $\eta \times 0.904 \times 10^5$, calculated from the formula. C is the equivalent of 1 cm. on the electrometer-scale in volts.

Solution.	Concentration.	A.	B.	C.	Temp.	E_1 .	E_2 .
HCl	N/5000	372 730	184 361	0.033	16° ...	-0.016	+0.098 0.191
		389 740	194 366	0.050	16° ...	-0.002	+0.105 0.173
CH ₃ COOH...	N/2000	762 384	377 190	0.033	15° ...	+0.053	+0.275 0.168

Solution.	E_3 .	E_4 .	$1/\kappa$.	$E_{1000}\kappa$.	$\phi_i - \phi_a$.
HCl	?	$+0.114 \pm ?$	0.1383×10^5	4.58×10^{-5}	$4.60 \pm ?$
		$0.207 \pm ?$		4.24	$4.26 \pm ?$
	0	$+0.107 \pm 2 \%$ $0.175 \pm 1 \%$	0.1341×10^5	4.09×10^{-5} 3.52	$4.11 \pm 2 \%$ $3.52 \pm 1 \%$
CH ₃ COOH...	+0.052	$+0.222 \pm 0.4 \%$ $0.115 \pm 0.8 \%$	0.1647×10^5	3.58×10^{-5} 3.70	$3.95 \pm 0.4 \%$ $4.20 \pm 0.8 \%$

The errors shown after the $\pm$ sign are calculated on the assumption that the flow of liquid produces an E.M.F. equal

* The total length of electrode spaces was 16 mm.

† E_{1000} is the voltage calculated for 1000 mm. pressure.

to the difference of the found value E_2 and a value between E_1 and E_3 . This assumption is certainly somewhat arbitrary, since we do not know whether these values E_1 and E_3 come from a disturbance which exists only when the liquid is in a state of rest. Thus the real uncertainty is larger than that given with the $\pm$ sign. In the case of acetic acid we think that some constant external influence came into play, and that the figures were not subject to a larger error.

In both the experiments with hydrochloric acid a more concentrated solution was also measured: in a further experiment a still more concentrated solution of N/1250 was employed. The results of these experiments are given in the following table.

Solution.	Concentration.	A.	B.	C.	Temp.	E_1 .	E_2 .
HCl	N/2500	760	376	0.033	16°	+0.012	+0.084
	N/2500	400 760	198 376	0.050	16° ...	+0.031	+0.073 C.111
	N/1250	900	445	0.050	17°	-0.009	+0.047

Solution.	E_3 .	E_4 .	$1/\kappa$.	$E_{1000\kappa}$.	$\phi_i - \phi_a$.
HCl	?	+0.072 $\pm$? %	0.0684 $\times 10^5$	2.79 $\times 10^{-5}$	2.80 $\pm$? %
	+0.032	+0.041 $\pm$ 1.5 %	0.0652	3.17	3.18 $\pm$ 1.5 %
		0.079 $\pm$ 1 %		3.21	3.23 $\pm$ 1 %
	-0.015	+0.059 $\pm$ 10 %	0.0314	4.20	4.22 $\pm$ 10 %

It would certainly be expected that all these values should lie close together, since the differences in concentration are so small that the differences in the E.M.Fs. arising from them are quite negligible. The differences between 2.80 and 4.22 may be taken as a sign of the degree of uncertainty connected with this second series. We think we are entitled to lay less stress both on the higher value for the N/1250 solution and on the smaller for the N/2500 solution. In both cases the ratio of E_1 and E_3 to the value E_2 is comparatively large, and the experiments are also in bad agreement with each other. Consequently we may conclude that the values for $\phi_i - \phi_a$ in the neighbourhood of 4 volts will not be far from the truth. In any case it seems evident from these results that the values for acids are less than 4.5 volts.

In the next table are given the values found for alkaline solutions.

Solution.	Concentration.	A.	B.	C.	Temp.	E ₁ .	E ₂ .
NH ₄ OH	N/5000	410	203	0.050	17°	+0.018	+0.496
		805	398		...		0.834
	N/2500	845	418	0.050	17°	+0.022	+0.599
	N/2500	405	200	0.050	17°	+0.027	+0.308
KOH.....	N/2000	400	198	0.033	16°	+0.003	+0.283
		760	376		...		0.489

Solution.	E ₃	E ₄ .	1/κ.	E _{1000κ} .	φ _i - φ _a .
NH ₄ OH	-0.005	+0.485 ± 5 %	0.4302 × 10 ⁵	5.55 × 10 ⁻⁵	5.45 ± 5 %
		0.823 ± 3 %		4.81	4.72 ± 3 %
	-0.016	+0.596 ± 6 %	0.2602	5.48	5.37 ± 6 %
	+0.009	+0.290 ± 6 %	0.2654	5.46	5.35 ± 6 %
KOH	-0.016	+0.289 ± 6 %	0.2028	7.20	7.23 ± 6 %
		0.483 ± 4 %		6.41	6.44 ± 4 %

It is at once apparent that these values are all greater than 4.5 volts*; the mean can be taken perhaps in the neighbourhood of 5.5 volts, so that there is a distinct difference between acid and alkali much greater than the error of experiment. In all cases the solutions were tested and the acidity and alkalinity were just perceptible.

The values obtained for water by Helmholtz from Quincke's experiments, and by Dorn, are both given as nearly exactly 4 daniell, which, when corrected to modern units, is equal to 5.07 volts. This value lies between our average values for acids and bases. The exact degree of acidity or alkalinity of such "pure" water, or of neutral solutions is of course uncertain. It is of interest, however, to note that an experiment which we carried out with a solution of potassium chloride also gave intermediate values.

* To try the influence of concentration, an experiment was carried out with N/50 ammonia which, although the value of E₄ was so small that little stress could be laid on it, at least supports the contention that the higher value for φ_i - φ_a is valid. The figures are:—A=920, B=455, C=0.060, temp. 18°; E = +0.035, E₂ = +0.087, E₃ = +0.023, E₄ = +0.057 ± 15 % volt; 1/κ = 0.02130 × 10⁻⁵; E_{1000κ} = 5.89 × 10⁻⁵, φ_i - φ_a = 6.24.

Solution.	Concentration.	A.	B.	C.	Temp.	E ₁ .	E ₂ .
KCl	N/2000	482 905	238 447	0.050	19° ...	+0.001	+0.360 0.684

Solution.	E ₃ .	E ₄ .	1/κ.	E _{1000κ} .	φ _i - φ _a .
KCl	+0.016	+0.352 ± 4 % 0.676 ± 2 %	0.3065 × 10 ⁵	4.82 × 10 ⁻⁵ 4.93	4.73 ± 4 % 4.84 ± 2 %

Discussion of Results.

The experiments just described lead to the conclusion that Helmholtz's and Dorn's value of five volts for the potential difference between glass and water is correct, and that when the liquid is made acid or alkaline the change is of the order of half a volt to the negative side in the first and to the positive side in the second case, the sign always applying to the liquid. If this change is compared with the theory quoted in the beginning of this paper, and if the smallness of the acidity and alkalinity is borne in mind, it is found that the sign of the change is the expected one, but that the magnitude is three times larger than the theory predicts. The supposition is suggested that an influence comes into play which enlarges all the values three times. Such a decrease would bring the value of the glass-water potential, as well as that of the acid-alkali change at the glass, to the magnitudes to be expected. The Helmholtz theory does not allow for such a factor. We are therefore led to look for some interfering phenomenon.

Knoblauch *, Perrin †, and Freundlich and Makelt have found facts which seem to have an important bearing on this question. Perrin especially, in an electro-osmotic investigation with powdered materials, as, for example, chromic chloride, and carborundum, found that the direction of the osmosis and therefore the sign of the charge at the surface of the powdered insulating material, always changed at the neutral point, the liquid becoming positive if alkali and negative if acid. From his results it must be concluded that pure water does not show any potential difference against the most different insulators. This view is in striking contrast

* *Zeitschr. f. phys. Chem.* xxxix. p. 225 (1902).

† *J. de chim. phys.* ii. p. 601 (1904); iii. p. 50 (1905).

with the results of Dorn and of ourselves ; but the change produced on making the solution acid or alkaline is, in sign, the same which we have found, and which is to be expected from the theory quoted in the beginning of this paper.

The theory of Perrin, also, is very different from that from which we start. It is akin to the views of Knoblauch, who found that when platinum was brought into contact with solid acids or bases, it obtained a positive or negative charge respectively. The fundamental point of these theoretical views is that the governing phenomenon is the diffusion of the ions. Knoblauch assumes that the platinum has an infinitely thin layer of water on it ; the solid acid or base is dissolved in this to a certain extent and diffuses on to the platinum ; the ions which diffuse faster first come in contact with the platinum, which accordingly becomes charged positively with acids, negatively with alkalies. Perrin, following some ideas of Langevin, thinks that the faster migrating ions are smaller ; they are thus enabled to come nearer to the surface of a solid insulator surrounded by electrolytically dissociated liquid than can the slower moving ions. In the case of such a liquid streaming along an insulator he thinks that the faster ions come within the last layer held in a state of rest by external friction at the surface, while the slower moving and, according to his view, larger ions remain in the moving layers of the liquid. This view is applicable to acids and alkalies because the hydrogen and hydroxyl ions are especially fast ; it is supported by Perrin's experimental data for these ions. It of course would be expected that it holds also in other cases where differences of the migration velocities exist, but Perrin could not verify this for lithium bromide, although he himself emphasized the fact that the lithium ion has only half the mobility of that of bromine*.

Contradictory as Perrin's results are to those of Dorn and of ourselves, so equally contradictory is his theory to that of Lamb, who thinks that the charge is rigidly confined to the first layer of solid and last layer of liquid. Helmholtz's theory of the phenomenon is so general that that it may be considered in agreement with either standpoint. The most important point, however, is that the diffusion theory cannot be assimilated up to the present time with the views which

* The standpoint of Perrin recalls some results obtained by Rutherford with ionized gases. He was able to show that such a gas when it passed through a narrow metallic tube produces a charge at the surface of the metal the sign of which is determined by that of the ion with higher diffusion coefficient.

have proved most successful in the explanation of electromotive forces, *i. e.* with the osmotic views put forward by Nernst *. Perrin's view leads him to the consequence that the electromotive force does not depend on the nature of the wall. The osmotic theory makes the potential difference always a function of the special nature of the properties of the two phases between which it takes place. Haber's theoretical deductions, from which we started, are grounded on the same standpoint.

The diffusion theory may account for differences in the phenomena caused by the use of different solids, by help of the assumption that a specific absorbing power for ions exists at solid surfaces, but the explanation becomes in this way still more indefinite, and does not show a clearer connexion with the osmotic theory.

Though the diffusion theory has up till the present time been unable to give more than qualitative results, whilst every step of the osmotic theory guides to quantitative results, certain difficulties arising especially in the electric behaviour of colloidal substances when considered from the osmotic standpoint, have led to some support being given to the former theory†.

Finally, the fact that the osmotic theory, according to our experiments, is able to explain qualitatively a result which till now seemed only explicable by the diffusion theory; and, on the other hand, the differences still remaining between the predictions of the osmotic theory and our results, suggest further experimental work in this direction; this will be carried out by one of us in this Institution.

This problem was undertaken by us at the suggestion of Professor Haber; and we desire to express our sincere thanks for the interest which he has shown and for the help which he has given throughout, both with the experimental difficulties and with the theory.

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* See Wied. *Ann.* lviii. Beilage, Heft 8, 1896.

† It may be remembered that Billiter (*Zeitschr. f. Electrochemie*, xv. p. 160, 1909) considers that the diffusion phenomena have only a secondary influence.

LXIV. *On the Retardation of Alpha Rays by Metals and Gases.* By T. S. TAYLOR*.

Introduction.

IN a preliminary paper † “On the Retardation of Alpha Rays by Metal Foils and its Variation with the Speed of the Alpha Particles,” the writer described some experiments which showed clearly that the air-equivalents of metal foils decrease with the range of the alpha-particles entering the foils ‡. By “air-equivalent” is meant the amount by which the range of the alpha-particles in air is cut down by their passage through the foil. It was shown that the change in the air-equivalents is small for thin foils of the lighter metals when the speed of the alpha-particles entering the sheets is high: but, when the speed of the particles is low for thin sheets or, when the sheets are thicker, the change becomes quite marked. A comparison of the change for sheets of different metals of nearly equal air-equivalents showed the rate of change to be in the order of the atomic weights of the metals. The results obtained in these experiments were not sufficient to furnish an explanation of the phenomenon; but the continuation of the experiments during the last year under somewhat different conditions has furnished results which do lead to conclusions of some interest.

Scattering of the Alpha Rays.

In the determination of the variation in the air-equivalents with the speed of the alpha-particle, as described in the paper cited above, the source of rays (polonium) with the metal sheet over it was set at such a distance from the ionization-chamber that some part of the top, or nearly horizontal, portion of the Bragg ionization-curve fell within the ionization-chamber. A slight increase in the range of the particle in this portion of curve corresponds to a considerable increase in the ionization. With the polonium set

* Communicated by Professor H. A. Bumstead.

† American Journal of Science, vol. xxvi. pp. 169-179, Sept. 1908.

‡ The phenomenon upon which this work was based was first observed by Mme Curie and has later been investigated by several others. Bragg & Kleeman (Phil. Mag. Sept. 1905, and April 1907) observed that the stopping power of a metal was not independent of the speed. Kucera & Masek (*Phys. Zeitschrift*, xix. pp. 630-40, 1906) and Meitner (*Phys. Zeitschrift*, viii. p. 489, 1907) ascribe the effect to a difference in the amount of scattering. McClung (Phil. Mag. Jan. 1906), Rutherford (Phil. Mag. Aug. 1906), and Levin (*Phys. Zeitschrift*, xv. pp. 519-521, 1906) obtained results which indicate that each successive layer of aluminium foil diminishes the range of the α -particle by the same amount.

at a definite distance from the ionization-chamber, it was found that, when the metal sheet was moved away from the polonium towards the ionization-chamber, the ionization increased. This increase in the ionization was attributed to the alpha-particle having a greater velocity (or range) upon entering the chamber when the sheet was near the chamber than it had when the sheet was at a distance from the chamber. Hence the metal sheet did not cut down the range of the particle so much when the sheet was at a distance from the polonium as it did when near the polonium. As a preliminary to more extensive experiments by this method two tests were made to ascertain whether a scattering of the rays could explain the increase in the ionization observed when the metal sheets were moved away from the polonium towards the ionization-chamber.

First test.—Any marked scattering of the rays by the foils would change the shape of the cone of rays, and especially the form of the top portion of the cone. The slope of the top, or nearly horizontal portion, of the Bragg ionization-curve, as well as the value of the maximum ionization, depends upon the form of the cone of rays arriving at the ionization-chamber. Thus, if scattering of the rays exists to a very marked degree, it might be expected that differences between the slope and form of the two Bragg curves obtained with and without the metal foil over the polonium could be readily detected. With polonium as the source of rays, numerous determinations of the Bragg curves, both with and without the various foils over the polonium, were made. A study of these curves showed them to run parallel to each other and to give the same value of the maximum ionization. The effect of putting the foils over the polonium was merely to diminish all the ordinates of the curves by the same amount.

Second test.—An iris diaphragm whose circular opening could be adjusted to any desired diameter between $\frac{1}{2}$ and $5\frac{1}{2}$ cm., was constructed of thin sheets of brass and placed directly below the ionization-chamber. The centre of the opening of the diaphragm was directly below the centre of the ionization-chamber. With the source of rays (radium C) at such a distance from the ionization-chamber that the chamber cut the top portion of the Bragg curve, the ionization was measured for various distances of the metal sheets above the source of rays; first with the diaphragm entirely open and then with the opening in the diaphragm of such diameter as to just limit the geometrical beam of rays, or to cut off the edge of the beam. For any given position of the sheet above the source of rays, the ionization was always

greater when the diaphragm was completely open than it was when the diaphragm just limited the beam. However, the difference between the ionization in the two cases was a constant value for all positions of the metal sheets above the source of rays. This difference would not be a constant quantity if the scattering of the rays was the occasion of the increase in the ionization produced by moving the metal sheets away from the source of the rays. On the contrary, the difference between the ionizations with and without the diaphragm limiting the geometrical beam of rays would be greater when the sheet is far away from the source of rays than when it is near the source of rays if scattering of the rays by the foils was the cause of the increase in the ionization. The fact that the ionization was greater with the diaphragm open than when it just limited the cone of rays signifies that more alpha-particles get into the ionization-chamber in the former than in the latter case, and therefore confirms the existence of the scattering of the rays by metal foils as found by Geiger *.

These two methods of investigation, although in the case of the latter showing the existence of the scattering of the rays, seem to be sufficient to preclude scattering as an explanation for the so-called decrease in the air-equivalents of the metal sheets as they are moved away from the polonium. By measuring the ionizations with and without the diaphragms limiting the cone of rays when there was not a metal sheet over the source of rays, it was found that the ionization was greater in the latter than in the former case, which shows that the rays are scattered by air as well as by metals. These methods, however, are not particularly suitable for measuring the amount of the scattering, and hence no comparison as to how much each metal scatters the rays was attempted. The important fact is that the effect under consideration is not influenced by the scattering of the rays.

Continuation of the Experiments.

In the first experiments polonium had been used as the source of rays, but in order to extend the study to alpha-particles of higher range, radium C has been used in the present experiments. This made it possible to use foils of greater thickness than had been previously used. A thin aluminium foil covered with a thin coating of lacquer was put directly over a capsule containing a thin film of pure radium bromide in order to prevent escape of the emanation. The hole in the brass plug over the radium bromide was of

* Proceedings of the Royal Society, Series A, vol. lxxxi. no. A 546 p. 174.

such dimensions that the cone of rays emerging from it fell well within the limits of the ionization-chamber. The radium bromide was set at such a distance from the ionization-chamber that a part of the top, or slightly inclined portion, of the Bragg ionization-curve due to the alpha-particles from radium C fell within the chamber, and the air-equivalents of the various metal sheets determined at various points in the path of the rays in exactly the same manner as that used in the first experiments*. The rays of shorter range than those of radium C had no effect upon the results since they did not reach the ionization-chamber. Beta and gamma rays are also given off by the radium bromide, but the ionization produced by them can be considered as a constant value over the part of the path used, and consequently the effect due to them is only a shifting of the curves to the right parallel to themselves which would have no effect upon the results under consideration.

In column 1, Table I., are given the different metal sheets used in the experiments. Column 2 contains the thickness in

TABLE I.

I. Metal Sheets.	II. Thickness in cms.	III. Air-Equivalent in cms.	IV. Ratio.
A Au.....	1.27×10^{-4}	0.719	5.67×10^3
B Au.....	1.74×10^{-4}	0.980	5.63×10^3
C Au.....	2.50×10^{-4}	1.375	5.50×10^3
D Au.....	3.50×10^{-4}	1.900	5.42×10^3
A Sn.....	3.86×10^{-4}	1.011	2.61×10^3
B Sn.....	7.99×10^{-4}	1.995	2.48×10^3
A Pb.....	2.84×10^{-4}	1.104	2.88×10^3
B Pb.....	4.11×10^{-4}	1.396	3.40×10^3
C Pb.....	6.95×10^{-4}	2.325	3.34×10^3
A Al.....	3.30×10^{-4}	0.597	1.79×10^3
B Al.....	6.77×10^{-4}	1.209	1.79×10^3
C Al.....	10.40×10^{-4}	1.803	1.73×10^3
D Al.....	16.10×10^{-4}	2.672	1.66×10^3
Hydrogen Sheets.			
A hydrogen	1.07	0.231	2.15×10^{-1}
B hydrogen	1.93	0.428	2.21×10^{-1}
C hydrogen	3.23	0.762	2.36×10^{-1}

* *Loc. cit.* pp. 173-175.

centimetres of the respective sheets. Column 3 has the air-equivalents of the sheets as measured directly when the sheets were nearest the source of rays. Column 4 contains the ratios of the air-equivalents to the thickness of the respective metals. The last three lines of the table will be referred to later. The air-equivalents of the sheets given in column 1, Table I., as determined by an improved method for any position of the sheets are given in Table II.

TABLE II.

Range in cms. of entering α -particle.	A Al.	B Al.	C Al.	D Al.	A Au.	B Au.	C Au.	D Au.
5.7	0.597	1.209	1.803	2.672	0.719	0.980	1.375	1.900
5.3	0.597	1.206	1.792	2.659	0.706	0.966	1.352	1.869
4.9	0.597	1.202	1.781	2.644	0.693	0.947	1.328	1.837
4.5	0.597	1.198	1.769	2.628	0.680	0.929	1.303	1.802
4.1	0.597	1.193	1.757	2.608	0.668	0.910	1.276	1.763
3.7	0.597	1.187	1.742	2.582	0.657	0.889	1.248	1.720
3.3	0.596	1.178	1.724	...	0.644	0.866	1.214	1.659
2.9	0.595	1.168	1.705	...	0.630	0.837	1.178	1.598
2.5	0.592	1.152	...	...	0.616	0.805	1.127	
2.1	0.584	1.131	...	...	0.600	0.757		
1.7	0.570	...	...	...	0.582			
1.3	0.555							

Range in cms. of entering α -particle.	A Sn.	B Sn.	A Pb.	B Pb.	C Pb.	Paper.	Celloidin.
5.7	1.011	1.995	1.104	1.396	2.325	1.020	0.520
5.3	1.002	1.980	1.085	1.371	2.296	"	"
4.9	0.993	1.960	1.064	1.343	2.260	"	"
4.5	0.984	1.938	1.042	1.320	2.220	"	"
4.1	0.971	1.914	1.020	1.289	2.180	"	"
3.7	0.957	1.884	0.999	1.263	2.124	"	"
3.3	0.941	1.845	0.977	1.232	2.046	"	"
2.9	0.923	1.795	0.950	1.200		"	"
2.5	0.902	...	0.923	1.163		"	"
2.1	0.882	...	0.888			"	"
1.7	0.853						"
1.3							

The aluminium foil over the radium to prevent the escape of the emanation cut down the range of the alpha-particles 0.46 cm. The height of the plug containing the radium was 0.9 cm. above the radium. Therefore the maximum available range of the alpha-particles entering the sheet is 5.70. The values of the air-equivalents for each of the metal sheets in Table II. represent the average results obtained from a series of from six to ten separate determinations, the details of which have been omitted for the sake of brevity.

Experiments were also made with sheets of paper and celloidin*. Two sheets of paper of about one and two cm. air-equivalents respectively, and three sheets of celloidin of air-equivalents of the order of 0.5, 1.0 and 2.0 cm. respectively were used. For these sheets of paper and celloidin, the ionization did not increase as the sheets were moved away from the radium but had the same value for all positions of the sheets and hence their air-equivalents remained constant.

The behaviour of the sheets of paper and celloidin, the atomic weights † of which are about the same as that of air, suggested the idea of undertaking to obtain sheets of some substance such as hydrogen, whose atomic weight is less than that of air. For this purpose a ring about one centimetre wide was cut from a brass tube six centimetres in diameter and two small brass tubes were put in the ring diametrically opposite each other. Thin films of celloidin were stretched across each side of the ring and held in place by universal wax. This formed a cell which could be filled with hydrogen and then used in the same manner as the metal foils. To be certain that the cell was always full of hydrogen a slight current of the gas was kept flowing through it all the time during an experiment. A current of air was kept circulating through the case surrounding the apparatus in order to prevent the hydrogen that might possibly leak from the cell from entering the ionization-chamber. The air-equivalent of the hydrogen cell, or sheet, when 0.9 cm. from the radium was determined by plotting the ionization-curve first with hydrogen and then with air in the cell. The ordinates of the latter curve were all increased by the thickness of the cell of hydrogen which gave the position of the curve if the cell had been evacuated. The difference between the ordinates of the two curves corresponding to a given abscissa was the air-equivalent of the hydrogen sheet.

When the cell containing the hydrogen was moved away from the radium, which was kept at a given position as in the previous cases, it was found that the ionization *decreased*, which signified that the total range in air of the alpha-particle was less when the hydrogen sheet was *far away* from the radium than when it was *near* the radium. Thus the amount by which the range of the alpha-particle was cut down by its passage through the cell was greater when the cell was at a distance from the radium than it was when it was near the radium. Consequently the air-equivalent of the

* Celloidin is a specially pure preparation of collodion.

† By atomic weight of air, paper, and celloidin is meant the average weight of the constituent atoms.

hydrogen cell *increased* as the range of the entering alpha-particle *decreased*. The particles had to pass through the celloidin sheets but this did not influence the effect because, as we have seen, the amount by which the range was cut down by the celloidin sheets was constant for all positions of the cell. Determinations of the air-equivalents in centimetres of three hydrogen cells given in Table I. were made for various distances of the cell from the radium and the results obtained are recorded in Table III.

TABLE III.

Range of the α -particle upon entering the hydrogen.	A Hydrogen.	B Hydrogen.	C Hydrogen.
5.2	0.231	0.428	0.762
4.8	0.235	0.434	0.776
4.4	0.241	0.442	0.791
4.0	0.247	0.451	0.807
3.6	0.254	0.460	0.831
3.2	0.262	0.470	0.861
2.8	0.271	0.483	0.896
2.4	0.283	0.499	0.938

The reason the maximum range here is 5.2 cm. instead of 5.7 cm. as it was in Table II. is because the air-equivalent of the lower film of celloidin must be subtracted since the alpha-particles must pass through it before entering the hydrogen. The air-equivalent of the lower film was 0.5 cm.

Although the air-equivalents of the celloidin sheets remained constant, it seemed probable, from the behaviour of the hydrogen sheets, that if the same experiments were performed in an atmosphere of hydrogen, the hydrogen-equivalent* of the celloidin sheet would not remain constant, but would decrease as the range of the alpha-particles decreased. To investigate this point the apparatus was enclosed in an air-tight sheet-iron case, which by several partial evacuations and refillings could be filled with practically pure hydrogen. With polonium as the source of rays the hydrogen equivalents in cms. of sheets of celloidin, aluminium, tin, and gold were determined for various distances of the sheets from the polonium. Only the results for the celloidin and A gold are given in Table IV. as they are sufficient to illustrate the point in question.

* The hydrogen-equivalent is the amount by which the range of the α -particles in *hydrogen* is cut down by their passage through the sheet.

TABLE IV.

Range in H of entering particle ...	13.0	12.6	12.2	11.8	11.4	11.0	10.6	10.2	9.8	9.4
Celloidin	23.20	23.08	22.96	22.80	22.64	22.48	22.32	22.16	21.88	21.60
A Gold	27.97	27.55	27.11	26.67	26.23	25.75	25.17	24.49	23.91	23.01

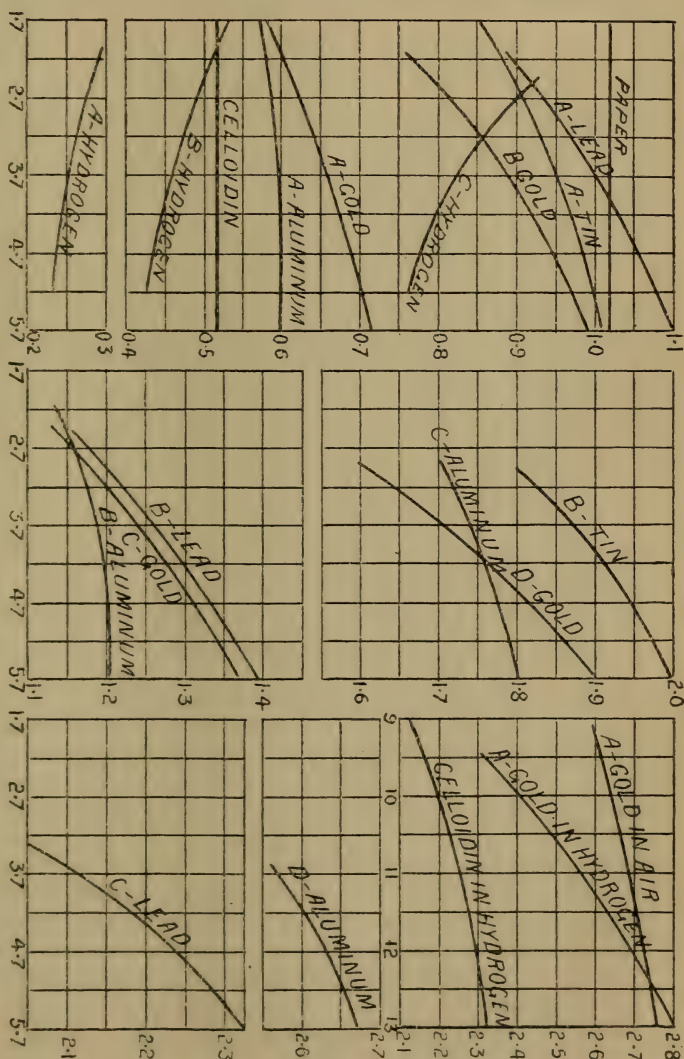


Fig. 1.

For the portion of the figure containing the curves designated as:—"A Gold in Hydrogen," "Celloidin in Hydrogen" and "A Gold in Air," the abscissas are the ranges in hydrogen of the alpha-particles when they enter the sheets and the ordinates are the hydrogen-equivalents of the sheets. For the other part of the figure the abscissas are the ranges in air of the alpha-particles when they enter the sheets and the ordinates are the air-equivalents.

The curves in figure 1 (p. 611) represent the results recorded in Tables II., III., and IV. By noting the slopes of the curves some comparison of the rates at which the air-equivalents of the various sheets change can be obtained.

Taking the general slope of each curve and dividing it into the air-equivalent of the corresponding sheet when 0.9 cm. from the radium, it is found that for a given metal the quotient thus obtained is nearly constant for all the sheets of the metal. This is shown in column 4, Table V. Thus for

TABLE V.

Sheets.	Slopes of Curves.	Air-equivalents.	Ratio.	$\sqrt{\text{atomic weight.}}$	Mean ratio $\sqrt{\text{atomic wt.}}$
A Au ...	0.032	0.719	2.24×10^1	14.05	23.80
B Au ...	0.051	0.980	1.92×10^1		
C Au ...	0.064	1.375	2.15×10^1		
D Au ...	0.100	1.900	1.90×10^1		
A Sn ...	0.032	1.011	3.16×10^1	10.91	34.34
B Sn ...	0.063	1.995	3.17×10^1		
A Pb ...	0.053	1.104	2.08×10^1	14.38	29.38
B Pb ...	0.071	1.396	1.96×10^1		
C Pb ...	0.110	2.325	2.11×10^1		
A Al ...	0.010	0.597	5.97×10^1	5.19	30.41
B Al ...	0.020	1.209	6.04×10^1		
C Al ...	0.033	1.803	5.48×10^1		
D Al ...	0.045	2.672	5.93×10^1		
A H ...	-0.020	0.231	1.16×10^1	1.17	
B H ...	-0.034	0.428	1.17×10^1		
C H ...	-0.064	0.762	1.19×10^1		

sheets of the same metal the rate at which the air-equivalent of each sheet changes with a change in the range of the entering alpha-particles, is proportional to its air-equivalent when nearest the radium. The approximately constant numbers in column 6, Table V., show that the percentage rate of change in the air-equivalent for any metal is nearly proportional to the square root of the atomic weight. The agreement of the values in columns 4 and 6, Table V., is as good as could be expected since the slopes of the curves in figure 1 could only be determined roughly. The proportionality is indeed only approximate, since the curve for any one sheet does not have a constant slope.

For the thin sheet of aluminium the air-equivalent is almost constant for the higher ranges or speeds, but as the speed of the entering alpha-particle decreases the air-equivalent decreases slowly, and in the lower ranges the decrease becomes quite apparent. For the thicker sheets of aluminium the change is more marked even for the higher ranges. The statements of McClung, Levin, and Rutherford that equal

successive layers of aluminium foil diminish the range of the alpha-particles by equal amounts seem to hold true for thin sheets of foil when the range is high; but when the metal sheet is thicker, or for thin sheets when the range is low, it does not hold. The slight difference, however, in the air-equivalent of the thin foil, when near and far away from the polonium, would scarcely be detected by measuring directly the air-equivalent in the two positions. This is probably the explanation of the above statements by McClung, Levin, and Rutherford.

Since the air-equivalent of a metal sheet decreases with the speed of the alpha-particle entering it, the ratio of the air-equivalent to the thickness of a given sheet of metal should be less than the same ratio for a thinner sheet of the same metal. This is shown to be true by the last column of Table I. For the hydrogen sheets on the contrary the same ratio should increase as the thickness of the cell or sheet of hydrogen increases. This is also confirmed by the last column of Table I.

While the air-equivalent of the sheet of celloidin remains constant the hydrogen-equivalent of the same does not remain constant but decreases as the range of the alpha-particle in hydrogen decreases. The curve "Celloidin in Hydrogen," fig. 1, which was plotted from the results recorded in Table IV., illustrates this point. It is to be noted also from the curve "A Gold in Hydrogen" (fig. 1) that the rate at which the *hydrogen-equivalent* of the A gold decreases is much greater than the rate at which its *air-equivalent* decreases. The curve designated "A Gold in Air" (fig. 1) is the portion of the "A Gold" curve in the same figure that lies to the left of the abscissa 3.0. The co-ordinates of that portion of the curve are magnified about $4\frac{2}{3}$ times so as to be plotted on the same scale as the curves obtained in the hydrogen atmosphere. $4\frac{2}{3}$ is the ratio of the thickness of a hydrogen sheet to its air-equivalent when near the radium. The slope of the curve "A Gold in Air" is practically the same as that of "Celloidin in Hydrogen" as can be seen from the figure. The angle which the curve "A Gold in Hydrogen" makes with the curve "A Gold in Air" is about the same as the angle which the curve "Celloidin in Hydrogen" makes with the axis of abscissas. The slope of the curve "A Gold in Hydrogen" is nearly $3\frac{3}{4}$ times the slope of the curve "Celloidin in Hydrogen." But $3\frac{3}{4}$ is the ratio of the square root of the atomic weight of gold to that of air

$$\left[\sqrt{\frac{197}{14}} = 3.75 + \right].$$

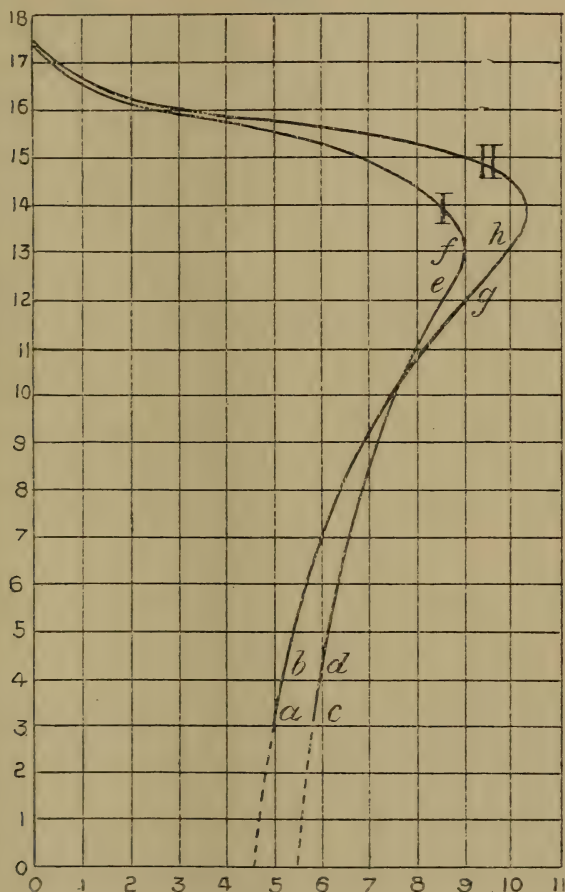
Hence the rates, at which the *hydrogen-equivalents* of the Gold and Celloidin sheets decrease with the speed of the alpha-particle entering the sheets, are proportional to the square roots of their respective atomic weights. Moreover the slope of the curve "Celloidin in Hydrogen" is numerically equal (but of opposite sign) to the slope of the curve "B Hydrogen" in air. The hydrogen-equivalent of the celloidin sheet was somewhat larger than the thickness of the "B Hydrogen" cell, but it seems entirely proper to conclude that the rate, at which the hydrogen-equivalent of the celloidin sheet *decreases* with the speed of the alpha-particle, is the same as the rate at which the air-equivalent of the "B Hydrogen" *increases* as the speed of the entering alpha-particle decreases.

The possibility that the observed variations in the ionization, which have been taken to be the measures of the changes in the air-equivalents, may be due to secondary rays is precluded by the fact that numerous direct determinations of the Bragg ionization-curves, with and without the metal sheets near the polonium, and again near the ionization-chamber, showed no irregularities in the curves as would be expected were secondary rays present in any appreciable amount. The behaviour of the air-equivalents of the hydrogen sheets in no way conforms to what might be expected to be produced by secondary rays.

The increasing of the air-equivalents of the hydrogen sheets and the decreasing of the hydrogen-equivalents of the celloidin sheets when moved away from the source of rays gave occasion for suspecting that some differences might be found to exist between the Bragg ionization-curves obtained in atmospheres of air and hydrogen respectively. To determine these curves, use was made of an apparatus constructed for Mr. F. E. Wheelock, of this Laboratory, which was similar to the one used thus far in the work, except that the vessel enclosing the main part of the apparatus could be completely exhausted. To make any comparison of the two ionization curves, it was necessary to determine them under similar conditions—*i. e.*, the same source of rays was used in the two cases, and the pressure of the air was so reduced as to make the range of the alpha-particles in air equal to their range in hydrogen at normal pressure. Polonium was used as the source of rays, and several Bragg curves were obtained in hydrogen at normal pressure and in air at a reduced pressure of about 17 cm. of mercury. Two of the curves are shown in fig. 2. The dotted portion of each curve is assumed to be of

the form it would take were it possible to move the polonium entirely up to the ionization-chamber. At all events these assumed portions of the curves can differ but little from what the actual curves would be.

Fig. 2.



The ordinates of the curves are the distances in centimetres of the polonium from the ionization-chamber. The abscissas are the deflexions in centimetres of the electrometer-needle per second. Curve I. was obtained in air at a reduced pressure of about 17 centimetres of mercury. Curve II. was obtained in hydrogen at normal pressure.

It is to be observed that the two curves in fig. 2 present slight differences in form. The probable interpretation of these differences will now be considered. Any given abscissa

of either curve is a measure of the ionization produced by the particles in the gas in the chamber when the polonium was at a distance from the chamber represented by the ordinate corresponding to the given abscissa. Consequently, the total area enclosed by the two axes of reference and either curve is proportional to the total ionization produced in the gas in which the curve was determined. By measuring these areas with a planimeter, it was found they were equal. This confirms the observations by Bragg* that the total ionization produced by the alpha-particle in air is the same as that in hydrogen. From the curves of fig. 2 it is seen that when the speed of the alpha-particle is high more ions are produced per centimetre of path in air than in hydrogen, but when the speed is low more ions are produced per centimetre in hydrogen than in air.

Let us suppose that for a given speed of the alpha-particle the amount of energy required to produce an ion is the same in all substances. Then for air we would have the relation

$$dI_a = -f(V) dE_a.$$

The corresponding relation in hydrogen is

$$dI_h = -f(V) dE_h.$$

Dividing the former by the latter, we have

$$\frac{dI_a}{dI_h} = \frac{f(V) dE_a}{f(V) dE_h},$$

which for a given speed V in each gas reduces to

$$\frac{dI_a}{dI_h} = \frac{dE_a}{dE_h}.$$

From this it is seen that for a given speed of the alpha-particle the ratio of the rates of the consumption of the energy in producing ions in air and hydrogen, is equal to the ratio of the rates at which the ionization is produced in the respective gases. On the basis of our hypothesis, let us consider the ratios of the energies consumed in the 4th and 13th centimetres (fig. 2) of the path of the particle in air and hydrogen. This ratio for the 4th centimetre of the path is proportional to $\frac{\text{area } cd \ 43 \ c}{\text{area } ab \ 43 \ a}$, since the areas are proportional to the ionizations produced in the gases. The corresponding ratio for the 13th centimetre is equal to $\frac{\text{area } e, f, 13, 12, e}{\text{area } g, h, 13, 12, g}$. The former ratio is seen from the figure to be greater than the

* Phil. Mag. March 1907, p. 333.

latter. Moreover it is also seen that the ratio of the energy of the alpha-particle absorbed by any given centimetre of air to the energy absorbed by the corresponding centimetre of hydrogen, is always greater than the corresponding ratio for the centimetre just beyond the given one. This is in agreement with the results obtained for the air-equivalents of the hydrogen cells; because the increase in their air-equivalents as the range decreases is due to the fact that the ratio of the energy absorbed by the hydrogen cell to the energy that would be consumed by the air which it displaces, continually increases as the cell is moved away from the source of rays. The thicker the cell the more rapid would be the rate of increase, as could be seen by comparing the areas which represent the ionizations in, say, 2 centimetres of air and hydrogen respectively in fig. 2 in two different positions. The increase in the ratio of the energies consumed in air and hydrogen respectively is in agreement also with the decrease in the hydrogen equivalent of the celloidin film.

Still making use of our hypothesis, the ratio of the energy consumed in the 9th and 10th centimetres of air at reduced pressure, to that consumed in the same centimetres of hydrogen at normal pressure, is expressed by the fraction $\frac{312}{305}$. The same ratio for the 13th and 14th centimetres is $\frac{77}{83}$. These ratios were obtained by measuring with a planimeter the areas in fig. 2. The former ratio divided by the latter gives 1.10. Since the hydrogen equivalent of the colloidin film is but slightly more than 2 centimetres, the ratio of its values at 9 and 13 cm. respectively from the polonium should be the same as the above ratio. The hydrogen equivalents of the film in the two positions (see fig. 1) are 2.320 and 2.120 cm. respectively; and the ratio of the former to the latter is 1.09, which differs little from the calculated ratio, 1.10, given above. Hence it is seen that the differences between the curves of fig. 2 are sufficient to account for the change in the hydrogen equivalent of the celloidin film, and consequently for the increase in the air equivalents of the hydrogen sheets when moved away from the source of rays. This agreement between the relative ionizations and the relative losses of energy of the particle in the two gases gives a considerable degree of probability to our hypothesis connecting the relation of the ionization produced to the energy consumed.

The experimental results show that the air-equivalents of the metal sheets decrease with the speed of the alpha-particle;

and hence the ratio of the energy of the alpha-particle consumed by its passage through a sheet of metal, to the energy that would be consumed by, say, 1 centimetre of air at the same point in the path of the particle, decreases as the range of the alpha-particle decreases. The behaviour of the metal sheets relative to the air is entirely analogous to the behaviour of the air, or celloidin relative to hydrogen. Consequently, if it were possible to measure the ionization produced by the alpha-particle at different points in the path of the rays in the metals, and if the ionization-curves were plotted on the same scale as those shown for air and hydrogen (fig. 2), it is probable that the curves for the metals would all present some such differences from the air curve as those existing between the air and hydrogen curves. Moreover, these differences might be expected to be such as to agree with the different rates at which the air-equivalents of the different metal sheets change. In the upper portion the curve for gold would probably lie within the air curve about the same amount as the air curve does within the hydrogen curve (fig. 2); and in the lower portion the curve for gold would probably lie without the air curve by the same amount as the air curve does without the hydrogen curve. At least some such differences would be in accordance with the square root law, since the square root of the atomic weight of air is a mean proportional between the square roots of the atomic weights of gold and hydrogen. The curves for the other metals would occupy intermediate positions between the curves for gold and air.

We have seen that for different metal sheets of about the same air-equivalents, the rates at which the air-equivalents decrease with the speed of the alpha-particle are proportional to the square roots of the atomic weights of the respective metals. Consequently, the rates of decrease of the ratios of the quantities of energy used up in the sheets to the energy that would be consumed by a centimetre of air at the same positions in the path of the particle decreases also as the square roots of the atomic weights of the respective metals. On the basis of our hypothesis that for a given speed of the alpha-particle the same amount of energy is required to produce an ion in all substances, and from the results in our experiments, it appears very probable that for the high velocities the alpha-particle loses its energy, in going through a substance, more rapidly the higher the atomic weight of the substance; but as the speed of the alpha-particle becomes less, this changes until for the low velocities the loss of the energy of the particle is more *rapid* the lower the atomic weight of the substance.

In conclusion, I wish to express my gratitude to Professor Bumstead, at whose suggestion these experiments were undertaken, for his valuable suggestions and interest in the work ; also to Professor Boltwood, who kindly prepared the polonium and secured the radium bromide for me, and gave me many valuable suggestions.

Summary of Results.

1. The air-equivalents of metal foils decrease with the speed of the alpha-particles entering them. The decrease is very small for thin foils of the lighter metals when the speed of the alpha-particle is high ; but when the speed is low for thin sheets, or when the sheets are thicker, the change becomes more marked. For different sheets of the same metal the rates of change are proportional to the air-equivalents of the sheets. For sheets of different metals of equal air-equivalents the rates of change are approximately proportional to the square roots of the respective atomic weights.

2. The air-equivalents of hydrogen cells or sheets *increase* as the speed of the entering particle decreases, while the air-equivalents of sheets of paper and celloidin remain constant.

3. The hydrogen-equivalents of sheets of paper, films of celloidin, and air do not remain constant, but decrease as the speed of the alpha-particle decreases. The rate at which the hydrogen-equivalent of a celloidin film decreases with the speed of the entering alpha-particle is numerically equal to the rate at which the air-equivalent of a hydrogen sheet of corresponding thickness increases.

4. The result obtained by Bragg, that the total ionization produced by the alpha-particle in air is the same as that in hydrogen, is confirmed by a more direct method.

5. It is very probable that for the high ranges the alpha-particle loses its energy, in passing through substances, more rapidly the higher the atomic weight of the substance ; but that this difference decreases slowly, until in the low ranges the loss of energy is the more rapid the lower the atomic weight of the substance.

6. A comparison of the Bragg curves for air and hydrogen indicates that the large ionization at low ranges (knee of the curve) is due at least in part to the fact that the particle loses its energy more rapidly in this part of the range, and not wholly to the higher ionizing efficiency of particles of low speed.

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LXV. *The γ -Rays of Uranium and Radium**. By FREDERICK SODDY, M.A., and ALEXANDER S. RUSSELL, M.A., B.Sc.†

1. *Introduction.*

HAVING at our disposal 50 kilograms of the purest commercial uranyl nitrate, provided by the generosity of a friend, we have been enabled to study the radioactivity of uranium X on a considerably larger scale than previously attempted. This body, which was discovered by Sir William Crookes (Proc. Roy. Soc. 1900, lxvi. p. 409), was early recognized (Rutherford and Soddy, Phil. Mag. 1903, v. p. 441) as a disintegration product of uranium producing the whole of the β -rays but none of the α -rays of that element. It has a period of average life of about 31 days. The period is long enough to allow of the separation of the greater part of the substance, even from a large quantity of uranium. When the activity of one product has decayed too far to work with, a fresh crop can be obtained by repeating the process. At each separation the uranyl nitrate is rendered purer, any fortuitous impurities present being eliminated, and as it is necessary in certain problems to be sure that no radioactive impurities are present initially, it is intended to carry out periodically a long series of such separations. The present work is concerned with the first three separations, carried out at intervals of about three months, and is devoted to a comparison of the γ -radiation of the active preparations separated, with the γ -rays of radium. We have found that, as in all other known cases, the γ -rays of uranium accompany the β -rays, both resulting entirely in the disintegration of uranium X. The results here recorded do not, however, bear out either of the rival views of the origin of the γ -rays, but rather indicate that the β -rays and γ -rays may be entirely distinct in origin.

Our knowledge of the γ -rays of uranium has previously been practically confined to the work of Eve, who examined the γ -rays of uranyl nitrate. Eve found in the first place that uranium only gave out about $\frac{1}{10}$ as much γ -radiation as thorium, though its β -radiation is, as is well known, very much more intense. Eve states that while the γ -rays of radium and thorium are of similar penetrating power, the γ -rays of uranium are much less penetrating, and are practically completely absorbed by 1 cm. of lead. Over a range

* A preliminary account of part of this work was published in 'Nature,' March 4th, 1909, p. 7, and *Phys. Zeit.* 1909, x. p. 249.

† Communicated by the Authors.

of from 0.26 to 0.94 cm. of lead he found that the absorption proceeded according to an exponential law, the value of the absorption coefficient, λ (cm.)⁻¹, being 1.4. For radium he gives the values from 0.57 to 0.46 over a range of from 0.64 cm. to 3.0 cm. of lead.

With our intensely active preparations of uranium X we have fully confirmed and extended Eve's statement with regard to the relative poverty of uranium in γ -rays as compared with the β -rays, and an accurate comparison of the ratio of the β - to γ -rays for uranium X and for radium C is detailed in this paper. We may state the result here that radium gives 50 times more γ -rays in proportion to β -rays than is the case with uranium. We have found, however, that the γ -rays of uranium are not absorbed exponentially until after about 1 cm. of lead, or an equivalent thickness of other substances, has been penetrated, and then they are similar in character, and only slightly inferior in penetrating power to the γ -rays of radium, the absorption coefficient being for most substances about 1.2 times greater over a range equivalent to from 1 to 5 cm. of lead. For thicknesses less than 1 cm. of lead the absorption curve is not exponential. The average slope of the curve, however, in a set of measurements with lead agreed approximately with Eve's value over a large part of the range he worked over. It is possible that a soft type of primary γ -rays is also present, but so far we have not been able to be sure of this, and this part of the subject is reserved for a future communication. In any case the γ -rays of uranium and radium *examined over the same range* are extremely similar in penetrating power and general character, and if a soft type of γ -rays from uranium exists, not represented in the γ -rays from radium, it is relatively feeble in character, and its existence is difficult to establish.

[The statement we made in preliminary communications that the absorption coefficient of the γ -rays of uranium was two and a half times greater than that of radium, which has unfortunately been quoted in abstracts, was based upon a published value for the radium rays which we have since found is altogether in error (Section 8). The ratios we ourselves have found for 13 substances are given in the last column of Table III. (p. 646).]

This being the case the great relative poverty in γ -rays of uranium compared to radium is a very remarkable fact. The β -rays of uranium are, like the uranium γ -rays, only slightly less penetrating than the corresponding radium rays. It is hardly possible to continue to regard the γ -ray as an

X-ray pulse generated by the expulsion of a β -ray electron from the disintegrating atom, a point of view which has been already seriously challenged by the alternative theory advocated by Bragg. But it is also difficult to accept the ready mutual convertibility of β - into γ -rays, and *vice versa*, which Bragg's theory requires, in face of the great similarity in nature and great difference in relative intensity which are shown by the β - and γ -rays of radium and uranium. The most natural conclusion is that the two types are not inter-dependent.

The work here described falls under two heads. First, the preparation of the uranium X and the comparison of its β - and γ -activity with that of radium under defined conditions will be described. Secondly, an account will be given of the absorption of the γ -rays of uranium and radium under comparable conditions, which has been studied in considerable detail. The γ -rays are much more difficult to measure accurately than the less penetrating types, and there is much that is mysterious and unexplained in the actions they produce. In particular we may mention that the γ -ray ionization itself is not in all cases a constant quantity, but may suffer a regular progressive increase in amount from a minimum to a maximum when consecutive observations with the same disposition are made. This variation, which may give rise to errors as great as ten per cent. in an observation, must be guarded against in all measurements in which the γ -rays are concerned. So far we have been concerned mainly to find a method of working which will eliminate these fluctuations, and which will give consistent results. In this we have been successful, but the cause of the variation is still to be worked out. So far as the determinations of the absorption coefficients are concerned they have been done under conditions as nearly alike as possible for uranium and radium. Not much weight is to be attached to the absolute values of the coefficients, for there is necessarily a certain amount of arbitrariness in the choice of the experimental disposition. We have tried a great variety, and our conclusion is that though many give curves which are practically exponential within the limits of error, it is difficult to repeat *ab initio* measurements of the absorption coefficient even under any one disposition with a variation of less than 5 per cent., while the actual value of the coefficient may depend considerably on the experimental disposition.

2. The Separation of Uranium X.

We have found the original methods of Sir William Crookes (Proc. Roy. Soc. 1900, lxvi. p. 409) the most useful in the separation of uranium X. Without entering in the present paper into any detailed description of the methods employed and of the numerous trials which have led to their selection, it is necessary to give in outline some description of the process of separation which has been found most suitable in dealing with these large quantities of material. Crookes found that when uranyl nitrate was crystallized from water the photographic activity, which is due solely to the uranium X (Soddy, Trans. Chem. Soc. 1902, lxxxi. p. 860), of the crystals was enfeebled, while that of the mother-liquor was correspondingly enriched. The same observation was made by Godlewski five years later (Phil. Mag. 1905, x. p. 51), who stated, without giving any details, that it was possible in one crystallization to obtain $\frac{2}{7}$ of the total quantity of uranium X in the mother-liquor. We found that the maximum separation of uranium X was effected when about two-thirds of the total quantity of uranyl nitrate crystallized out from the solution on cooling. The mother-liquor is then concentrated and the operation repeated several times until the amount of uranyl nitrate in the mother-liquor is reduced to about 100 grams. The uranium is then removed from this concentrate by adding excess of ammonium carbonate, in which uranium is soluble, leaving behind the uranium X and the impurities as a precipitate. An attempt to use the acetone method of Moore and Schlundt to separate the uranium X from a concentrate in the first series of separations failed owing to the considerable quantities of impurities present, and led to much difficulty and loss of time. Dealing with quantities of material such that $\frac{1}{10}$ per cent. of impurity represents an actual weight of 50 grams, Crookes' crystallization and ammonium carbonate methods are still very effective, but indeed it is doubtful if any of the methods subsequently proposed are any better than these original ones in ordinary circumstances. It may be stated that the known methods of separation take advantages of peculiarities in the properties of uranium rather than of uranium X. The latter does not appear to resemble any element very closely, and it is more difficult to separate it from admixture with other elements than from uranium.

The crystallization process employed may be illustrated from the case of the third separation after considerable experience had been acquired. Forty-seven kilograms of uranyl nitrate in four lots were dissolved in one-sixth of their weight

of water, and the solutions heated over water-baths till a density of 2.02 was attained. The solutions were then poured into large evaporating basins and allowed to cool over night. When crystallization had set in completely the mother-liquor contained about one-third of the total weight of salt. The mother-liquors from each set of crystals were drained off, concentrated over the water-bath to the same density as before, poured out and allowed to crystallize slowly. The resulting mother-liquor is similarly treated, and so on until the amount of uranyl nitrate in the mother-liquor did not much exceed 100 grams. Usually the whole of the crystals so obtained would again be recrystallized, the mother-liquor from the first batch of crystals being added to the next and so on, while the end fractions from this second recrystallization usually were still sufficiently rich in uranium X to be worth subjecting to a third recrystallization.

To test the amount of uranium X remaining in each batch of crystals a sample of about 40 grams is taken after thorough mixing. This is placed at a fixed distance below an electroscope provided with a base of aluminium foil 0.3 mm. thick, and the leak it produces is compared with that produced by a similar sample of standard uranyl nitrate.

In a single operation arranged so that about two-thirds of the salt crystallizes it was found that the crystals retained from 15 to 30 per cent. of their initial quantity of uranium X, so that of the total quantity of the latter all but from one-sixth to one-ninth is concentrated in the mother-liquor. One-seventh may be taken as an average fraction. In n similar successive crystallizations the quantity of salt in the last mother-liquor is $(\frac{1}{3})^n$ of the initial quantity, while the uranium X contained therein corresponds to $(\frac{6}{7})^n$ of the initial quantity. Thus after six successive crystallizations 50 kilograms would yield a final mother-liquor containing about 70 grams of uranyl nitrate together with the uranium X from 20 kilograms of uranyl nitrate. In the second and third series of recrystallizations two further similar fractions would result with the uranium X respectively of perhaps 10 kilograms and 5 kilograms. From the weights and activities of the various quantities of crystals obtained as the result of the process the fraction of the total uranium X separated could be deduced. In the second separation about 80 per cent. and in the third about 72 per cent. of the uranium X was removed by crystallization.

The next step in the operation consisted in the removal of uranium from the concentrates by means of ammonium carbonate. Each concentrate as it was obtained was added to

several litres of hot water, and treated with excess of a strong solution of ammonium carbonate containing ammonia. Too great excess is harmful, as uranyl ammonium carbonate is less soluble in ammonium carbonate than in water. Should all the uranium carbonate not redissolve the bulk of the supernatant liquor is drawn off and a further large quantity of hot water added. The liquor is then carefully filtered through an extracted filter on the pump and the precipitate washed with hot water, redissolved in dilute nitric acid and reprecipitated with excess of ammonium carbonate once or twice to ensure complete removal of the uranium. This process of separating uranium X is a very efficient one, and in some cases the separation was extraordinarily perfect. Thus on one occasion to the uranium filtrate from a precipitate containing the combined quantities of uranium X from about 40 kilograms of uranyl nitrate a little iron chloride was added and the original process repeated. The β -activity of this iron precipitate was practically inappreciable. When it is considered that the actual weight of uranium X in equilibrium with 50 kilograms of uranyl nitrate is below $\cdot 0005$ mg., the removal from the uranium solution of the whole of this, all except perhaps $\frac{1}{50000}$ part at most, in a single precipitation is a very remarkable operation. This is an exceptional case, but usually the amount of uranium X in the filtrate was relatively small. In the first separation the combined uranium-X-containing precipitates weighed about 4.8 grams, in the second it was much less, and in the third it was only 0.65 gram. These precipitates, which naturally have considerable chemical interest, have not yet been exhaustively examined. It was not difficult to recognize two groups of substances (in about equal quantity in the first batch), the one consisting of common earths, largely alumina, and the other consisting of a pale yellow body more closely allied to uranium, and possibly a member of the tungsten family. In the last separation the precipitate consisted mainly of the latter substance. The separation of the two groups was simply effected by solution of the precipitate in nitric acid, and by the cautious addition of ammonia, below the quantity required for neutralization, until a yellow colour made its appearance. On setting aside for a few hours, the yellow body precipitated out, containing nearly all the uranium X, leaving the alumina &c. in the still acid solution. In this way further concentration of the uranium X could be effected, but not much was gained by it. It has recently been noticed that this yellow compound is, like uranium carbonate, soluble in an excess of ammonium carbonate, but to a much

less extent, and this observation suggests a simpler method of removing it from uranium X than has yet been tried.

For other experiments it was necessary to obtain the uranium X as the least possible quantity of material in the form of thin films which could be examined in an intense magnetic field. In the first and second separations this was effected by a long series of chemical operations, in which the uranium X was removed from the solution of the precipitates in acid by successive additions of barium nitrate and sulphuric acid according to the method employed by Becquerel. The barium sulphate was then fused with alkali carbonates, and the well-washed barium carbonate, which contained the whole of the uranium X, was dissolved in acid. A little iron chloride was added and precipitated with ammonium carbonate, the precipitate containing the uranium X. In the second separation the filter-papers and precipitates were ignited, brought on to three strips of micro-cover-glass each 75 mm. long, 13 mm. wide, and 0.12 mm. thick. These were placed on a hot plate, a drop of hydrochloric acid added, and the solution made to cover the whole surface. The films were dried and heated till chlorine was given off, and were so obtained in a stable and tolerably coherent form. The weights of the films were 57, 211, and 11 mg. respectively, and their relative activity as 2 : 1 : 0.12. In this work they were kept in platinum trays placed side by side on a wood block. In the third separation the concentration was not pushed further after removing the uranium by ammonium carbonate. The burnt filter-papers were placed directly in the three platinum trays and treated with nitric acid (in one case hydrofluoric acid also to remove silica), the films formed being finally ignited at a red heat. They weighed 285, 200, and 152 milligrams respectively, with relative activities as 1 : 1 : 0.4. The time absorbed in the second separation from the start to the preparation of the products in the final form was twelve days. In the third separation the first film was produced in 3.3 days, the second in 7 days, and the last in 9 days from the start. To settle certain points it was desirable to reduce the loss of material and time of preparation, rather than the weight of the product, to a minimum, and this was done by omitting the long process of further concentration adopted in the first and second separations. The increasing purity of the uranium made this possible. An indefinite loss of active material occurs in the numerous operations required to remove the unidentified yellow body, whereas it was required to deduce for one series the exact percentage of the

equilibrium amount of uranium X contained in the preparations on the date of their comparison with radium.

Between the second and the third separations just described appeared the interesting paper by J. Danne, in *Le Radium*, 1909, vi. p. 42, on a new product in the uranium series, which he calls radio-uranium, intermediate between uranium and uranium X, and acting as the direct parent of the latter. Mr. J. T. Smith kindly undertook for us at this stage a careful series of measurements over a considerable period of the β -activities of various products separated in the course of this work, and of the whole of the fractions of uranyl nitrate resulting from the second separation, with the object of ascertaining whether in any of them any concentration or impoverishment of the parent of uranium X had occurred. The results show, up to the present time, that the whole of the preparations lost or regained their β -activity perfectly normally, and no alteration in the normal amount of the parent of uranium X had been effected. We may thus conclude that none of the varied chemical operations used in the work have affected the power of uranium to produce uranium X.

3. *Relative Intensity of the γ -Rays of Uranium and Radium.*

The bare uranium X films, the preparation of which has just been described, lit up an X-ray screen brightly in the dark-room, and indeed the glow could still be seen in a fully lighted room when the screen was held in the shadow of the observer. The three bare films from the second separation together produced about the same effect when held under the screen as 6.7 mg. of radium bromide in a sealed thin glass tube, though on account of the difference in the area of the glow, and the absorption that occurred in the glass of the tube in the case of radium, this is merely a very rough comparison. But the γ -rays from these films were accurately compared with the γ -rays from 0.47 mg. of radium bromide, eight days after the last crystallization of the uranyl nitrate was started. The substances were placed 7.5 cm. beneath the base of a brass electroscope, to which a thickness of 2.5 cms. of lead was clamped up to form a base. The γ -activity of the uranium X was only equal to that of 0.056 mg. of radium bromide. Owing to the loss of active material during the numerous processes of concentration it was not possible to deduce accurately the amount of uranium X in the preparations at the time of experiment, but the experiment clearly showed the great relative poverty of uranium in γ -rays as compared with β -rays.

In the third separation careful check was kept of the weight and initial activity of each batch of crystals after the operation. From the time of accumulation of uranium X in the crystals between the second and third separations it was deduced that the initial content of uranium X before separation was 98.5 per cent. of the equilibrium quantity. All but 28 per cent. of the equilibrium quantity of uranium X was separated initially, but at the date of the comparison of the uranium X with radium this had increased. From a curve of the regeneration of uranium X with time, the initial β -activity of the crystals and their weight, it was deduced that on that date 44.35 per cent. of the equilibrium quantity of uranium X was present in the crystals. The amount in the preparations was therefore 98.5 per cent. — 44.35 per cent., or 54.15 per cent. of the equilibrium quantity of 47 kilograms of uranyl nitrate. This is equal to the uranium X in 12.13 kilograms of metallic uranium in equilibrium. On a particular day, 8 days after the commencement of the last recrystallization, and 2 days after the finish of the separation, the γ -activity of the preparations was equivalent to the γ -activity of 0.094 mg. of radium bromide, measured through a thickness of 2.52 cm. of lead. Now it will be shown later that the absorption coefficient for lead, for a range of from 1 to 5 cm. of lead, for the radium γ -rays is 0.495, and for the uranium γ -rays is 0.725 (cm.)⁻¹. It is true that for thicknesses below 1 cm. of lead the absorption coefficients of the γ -rays, both of uranium and radium, appear to be greater than those given, but for present purposes no great error is likely to be introduced if we calculate the ratio of the initial intensities of the γ -rays of uranium and radium from the observed intensities by means of these coefficients. Thus the initial γ -activity of the uranium X in equilibrium with 12.13 kilograms of metallic uranium is equal to the initial γ -activity of 0.17 mg. of radium bromide, or that from 1 kilogram of uranium is equivalent to 0.138 mg.

It must be mentioned that the ultimate standard of comparison of radium in this case and throughout was a sealed tube containing 6.7 mg. of Giesel's radium bromide. The weight was taken with great care before sealing, but it is probable, since the compound was fairly old when weighed, that decomposition had occurred, and that the weight of radium therein was considerably greater than the formula-weight. Thus if the conversion into carbonate by the air had been complete the percentage of radium in the standard would be 79, instead of 53.5 as initially. Taking provisionally a mean value of 66.6 per cent. of radium in the sample the

γ -activity of 1 kilogram of uranium is equal to that of 0.009 mg. of radium, or radium gives about 10^8 times more γ -rays than uranium.

4. *The Relative Value of the Ratio of γ - to β -Rays for Uranium X and Radium C.*

It appeared very desirable to carry out an accurate determination of the relative value of the ratio of γ - to β -rays for uranium and radium under strictly comparable conditions. Two electroscopes were used, one for the measurement of β -rays and the other for the γ -rays. The former had a base of very thin aluminium foil, 0.095 mm. thick, and was set up so that the active preparations could be placed on the floor 119.5 cm. below the base. The γ -ray electroscope was the lead cylindrical electroscope with a base of lead 0.975 cm. thick described under section 10, and the preparations were placed 13.1 cm. beneath the base. As the source of the β - and γ -rays of radium was employed a film of radium C, prepared by exposing one side of a negatively charged copper disk to the radium emanation for 12.5 hours. After the exposure the disk was placed in a brass cell and a piece of 0.095 mm. thick aluminium foil was cemented over the cell to prevent any possible escape of adhering emanation. A similar sheet of aluminium covered the uranium X preparation. This consisted of one of the three preparations of the third separation, possessing 0.408 of the total activity. The date of these measurements was one week after those described in the last section.

Half an hour after the preparation of the film of radium C alternative measurements of the β - and γ -rays were taken over several hours, and from the decay curve obtained from the measurements the β - and γ -activity at any time could be readily deduced. Other similar measurements were also made of the β -rays when the base of the electroscope consisted of a sheet of aluminium foil 0.9 mm. thick, and of the γ -rays when the base of the electroscope consisted of plates of aluminium 4 and 6 mm. thick. The latter thickness is sufficient to reduce the β -radiation of both elements to an inappreciable amount. The former, however, was found to allow a little of the β -radiation of radium only to get through. For all dispositions measurements were also taken with the uranium X. In this way the relative ratio of the γ - to β -activity of uranium and radium under comparable conditions was ascertained. It may be assumed that 0.195 mm. of aluminium cuts down the β -rays of uranium and radium to practically the same extent, since the absorption is but small.

The initial values for the γ -rays before penetrating the 0.975 cm. of lead were deduced by means of the absorption coefficients referred to in the last section. It was found that under the conditions described the ratio of β -rays to γ -rays (corrected for absorption in 0.975 cm. lead) was for radium C 0.283 and for uranium X 14.05. The relative value of the ratio of β - to γ -rays for uranium X in terms of this ratio for radium C as unity is thus 49.7. Uncorrected for absorption this ratio is 62. It therefore appears that the initial γ -radiation of uranium as compared with its β -radiation is fifty times less than for the case of radium. When the β -rays were measured through 0.995 mm. of aluminium instead of 0.195 mm. the ratio was 0.58 times 49.7, which is to be expected, since the β -rays of uranium are rather less penetrating than those of radium. When the γ -rays through 6 mm. of aluminium were compared with the β -rays through 0.19 mm. of aluminium the result was 3.5 times more favourable to the γ -rays of uranium than through 0.975 cm. of lead. Under these conditions the γ -radiation as compared with the β -radiation is for uranium about 18 times less than for radium. This appears to be evidence that uranium gives a soft γ -radiation not given by radium, but the point will be discussed in more detail in a subsequent communication.

The γ -activity of a sealed tube containing 0.47 mg. of radium bromide was taken at the same time and with the same instrument as the other measurements. This gave an entirely independent estimate of the relative γ -activity of the uranium X preparations in terms of radium bromide through a much smaller thickness of lead than was used in the measurement a week previously. Nevertheless the result was nearly the same. The uranium X in equilibrium with 1 kilogram of uranium corresponded in initial γ -activity to 0.0150 mg. radium bromide, as compared with 0.0138 mg. found previously. The former value is probably the nearer to the truth.

5. *The Connexion between the β - and γ -Rays of Uranium X.*

On account of this great relative poverty of uranium in γ -radiation it was decided to test more closely the view that the γ -radiation accompanied the β -radiation in the disintegration of uranium X. In the first place the γ -activity of a kilogram of uranyl nitrate, the β -activity of which had been enfeebled to a known extent by crystallization, was compared with the γ -activity of another kilogram of uranyl nitrate in equilibrium with uranium X. For this purpose the substances were placed immediately below an electroscope, the

base of which consisted of a sheet of iron sufficient to absorb the β -radiation completely. The crystals just filled a cylindrical jar of about the diameter of the electroscope. It was found that the γ -radiation in the two substances was proportional to the β -radiation, indicating that the whole of the γ -radiation is derived from the uranium X. The effects measured were small, and naturally no great accuracy was attained.

The next step was to determine the rate of decay of the γ -radiation of uranium X as accurately as possible. Preliminary experiments with the preparations of the second separation had shown that the period of the decay was at any rate approximately the same as that of the β -rays. Measurements of each of the three preparations of the third separation have shown that the decay of the γ -radiation is exponential from the date of separation up to the present date (30 days). The value of the coefficient of decay was found to be 0.029 (day)^{-1} , which agrees as closely as is to be expected with the value found for the β -rays 0.031 (Rutherford & Soddy, *Phil. Mag.* 1903, v. p. 444).

Although these results, so far as they go, confirm the view that the γ -radiation accompanies the β -radiation in the disintegration of uranium X, there seems no escape from the conclusion that the γ -rays are a primary radiation due to the disintegration of the atom, and not a secondary radiation accompanying the expulsion of the β -particle. Since for two elements the β - and γ -radiations of which, although similar, each to each, in general character, yet differ in relative intensity in the ratio of 50 to 1, there seems no incongruity in contemplating the possibility of a β -radiation wholly unaccompanied by γ -radiation. One may even hazard the suggestion that possibly β -rays will be found to accompany the change of uranium X into radium while γ -rays accompany the change of uranium X into actinium. Although there is no evidence at present in favour of this there is equally nothing to be urged against it.

5 A.

[Later results for the decay of the γ -rays over a period of 80 days are now available. For the last 49 days the preparations have not been used in other experiments, or disturbed between the measurements. The value of $\lambda \text{ (day)}^{-1}$ over this period has been found to be 0.028 . This is about 10 per cent. smaller than the original value found for the decay of the β -rays, but it is possible that the discrepancy is due to an error in the determination of this latter constant. We are now investigating this point.]

6. *Absorption of Uranium γ -Rays by Matter. (First Series.)*

After preliminary trials with the uranium X of the first separation, a careful series of measurements was carried out with the preparations of the second separation on the absorption of the γ -rays by various substances. The selection of the particular experimental disposition employed involves necessarily a certain amount of arbitrariness, for as is well-known the ionization produced by γ -rays results largely from the secondary radiations it sets up in the walls of the measuring instrument. Again, one may, while keeping the distance between the active material and the electroscope constant, place the absorbing screens either directly over the active material, or directly under the electroscope, or at any intermediate position. Or, one may have the active material near to the electroscope and include a large cone of rays, or far away, and work with practically a parallel beam. It is curious that in all the published measurements of the absorption of γ -rays, only Wigger (*Jahr. Radioakt.* 1905, ii. p. 430) has given full and precise information on these important points.

The electroscope employed was a cylindrical brass one of internal height 12.8 cm., internal diameter 10.52 cm., and of wall thickness 0.145 cm. It was supported by four brass pillars 15.8 cm. above a permanent base-board. Most of the absorbing plates used were 11 cm. square, but lead and brass were in the form of circular disks, 11 and 12.7 cm. diameter respectively. They were clamped up tightly to the bottom of the electroscope forming its base. When the absorbers were insulators the upper surface was covered with aluminium foil, 0.0035 mm. thick. For light substances, with a density below 2, a plate of lead 1.26 cm. thick formed the base of the electroscope. The uranium X mounted on one of two wooden stands, of heights 1.2 and 7.55 cm. respectively, according to the nature and amount of the absorbing material under investigation, was placed underneath the electroscope. The following table gives the results. The first column gives the number of the experiment, the second the distance of the preparation from the base of the electroscope, the third the absorbing material, the fourth its density d , the fifth the range over which the absorption is exponential, the sixth the value of the absorption coefficient λ (cm.)⁻¹, and the seventh the mean value of the ratio $\lambda/d \times 100$. Some of these results are plotted as curves in fig. 1. The ordinates represent logarithms of the ionization leak, in divisions per minute corrected for natural leak, and the abscissæ are thicknesses in cm. To separate the curves in the figure from

one another an arbitrary constant has been added to the logarithm of the ionization leak in some cases. The numbers on the curves refer to the numbers in the first column of the table.

For mercury a different arrangement had necessarily to be adopted. A circular hole 11·5 cm. in diameter was cut in a block of wood 4 cm. thick, and to the bottom was fixed a circular plate of glass about 3 mm. thick. This was provided with levelling screws, and placed close over the preparations. It was levelled and sufficient mercury placed in it completely to cover the surface, and then weighed amounts of mercury were added or withdrawn. In experiments Nos. 1 and 2 the base of the electroscope was a plate of lead 1·2 cm. thick. In experiment No. 3 it was two sheets of tinfoil. The results in the latter case lie on a continuous curve convex to the origin, and no one value can be assigned to λ .

TABLE I.—Uranium X. γ -Rays.

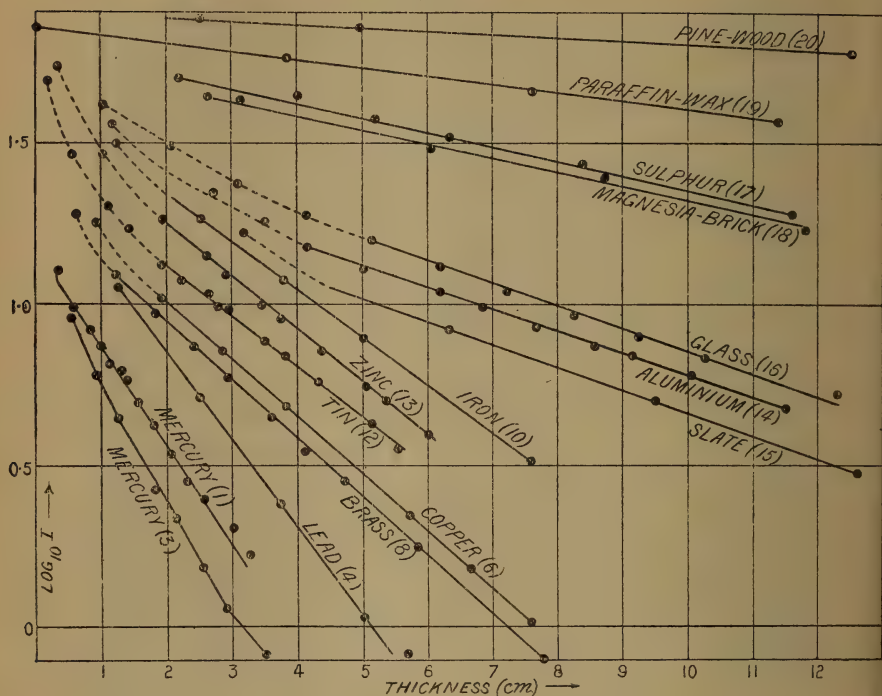
Brass Electroscope, absorbing plates forming base.

No.	Distance.	Material.	Density.	Range (cm.).	λ (cm.)—1.	$\lambda/d \times 100$.
1	8·25	Mercury.	13·59	·33 to 2·55	·687	5·05
2				·33 to 2·60	·760	5·59
3				Not exponential.		
4	"	Lead.	11·40	1·26 to 5·00	·634	5·56
5				1·9 to 5·00	·620	5·44
6				3·0 to 8·0	·413	4·69
7	"	Copper.	8·81	1·9 to 6·8	·430	4·88
8				1·8 to 6·0	·410	4·91
9				1·1 to 5·8	·443	5·30
10	"	Iron.	7·62	2·52 to 7·57	·335	4·40
11					·394	5·17
12					·351	4·84
13	14·63	Tin.	7·245	1·90 to 5·51	·351	4·84
14		Zinc.	7·07	1·9 to 6·0	·373	5·27
15		Slate.	2·85	6·0 to 12·6	·158	5·54
16		Aluminium.	2·77	5·0 to 11·5	·148	5·34
16	"	Glass.	2·52	5·2 to 10·3	·165	6·54
Below this the base consisted of 1·26 cm. of Lead.						
18	"	Magnesia Brick.	1·92	2·7 to 11·8	·1029	5·36
17		Sulphur.	1·79	2·2 to 11·6	·1046	5·84
19		Paraffin Wax.	0·862	3·8 to 11·4	·0576	6·68
20		Pine-Wood.	0·386	2·0 to 11·5	·0214	5·54

Mean value of $\lambda/d = \cdot 0536$.

It will be seen from the other curves that after a certain initial thickness of matter has been penetrated, corresponding

Fig. 1.



to 1 cm. of lead, or an equivalent thickness of other substances, the absorption follows an exponential law represented by the equation

$$\frac{I_{(t_1)}}{I_{(t_2)}} = e^{-\lambda(t_1 - t_2)},$$

where t_1 and t_2 refer to thicknesses in centimetres, $I_{(t_1)}$ and $I_{(t_2)}$ to the corresponding ionizations, and λ is a constant, which is usually termed "the absorption coefficient." Owing to the obliquity of part of the beam this absorption coefficient departs slightly from the true absorption coefficient corresponding to the λ of the theoretical equation

$$\frac{dI}{dt} = -\lambda I,$$

which represents the exponential absorption of a parallel homogeneous radiation passing through the absorbing plate normally to its surface. It can be shown, for example, to be

about 2.5 per cent. greater over the whole range for the disposition described in detail in section 10. For thicknesses less than 1 cm. of lead the curves are no longer exponential, and the value of λ apparently increases. The study of this part of the range forms the subject of a separate communication.

It will be seen that as a general result the experimental values of λ/d are nearly the same for all substances, the mean value being 0.0536. The departures of the values of λ/d from this mean are, however, greater than should be the case if they were due merely to errors of observation. Indeed the variations seemed not to be due so much to differences in the metals, as different results were obtained for different series of observations on the same metals, as to other unexplained causes. The extreme values for λ/d vary between .0668 and .0440, a difference of about 20 per cent. on either side of the mean value.

Beyond a thickness of 5 cm. of lead, or an equivalent thickness of mercury, the absorption appeared to be less than the normal. At this range, however, the effects produced are of the same order of magnitude as the natural leak of the instrument, and are too small to be measured with any certainty. Subsequent experiments with the more active preparations of the third separation failed to establish beyond doubt any alteration in the value of the absorption coefficient.

The disposition employed in this series is a troublesome one to work with in practice, but in our opinion the general results obtained are, from a theoretical point of view, of equal value to those obtained with another disposition detailed in section 11.

7. Variation of the Ionization produced by γ -Rays.

In order to compare directly the γ -rays of uranium with those of radium and to obtain further light on the variations referred to, experiments were now undertaken with radium. It was found, in the first place, that the errors of observation were reduced by removing the source of light a considerable distance from the electroscope, and interposing a sheet of ground glass between it and the electroscope. A thin cardboard cover also was placed over the electroscope which was removed only when the leaf system was being charged. This hood had the effect of maintaining a constant temperature around the wall of the electroscope while its presence did not affect the leak of the instrument. In the second place, a very curious effect was noticed, first in working with the more powerful γ -rays from radium. If the time for the leaf to cross the scale was taken and the experiment repeated a number

of times, the rate of leak gradually increased, considerably at first and then less rapidly, until a maximum rate was reached. This phenomenon was got with different electroscopes, different absorbing plates, and with both radium and uranium X. The sign of the charge on the leaf did not seem to produce any effect. The maximum rate of leak was found in all cases to be about 12 per cent. greater than the minimum. The cause of the steady increase in the rate of leak to a maximum remains unknown. We cannot at present give any explanation of it. In certain new forms of ionization-vessels we are now working with it is not shown. In this connexion may be recalled an experiment by G. Jaffé (*Ann. Phys.* 1908, xxv. p. 271), who found it to be impossible to obtain complete saturation in air at atmospheric pressure ionized by the γ -rays of radium, even with a potential gradient of 5000 volts per cm. Kleeman (*Phil. Mag.* 1907, xiv. p. 622) also called attention to gradual fluctuations in magnitude of 5 to 10 per cent. in the values of the leaks obtained by using a differential method with two ionization-chambers and a steady source of γ -rays. He suggested that the emanation might be coming off from the radium salt intermittently, causing alterations in the centre of origin of the γ -rays. This certainly does not apply to our case, for the effect is shown also by the γ -rays of uranium X, and the irregularities moreover are quite definite. The effect depends on the maintenance during successive observations of the charge on the leaf system; for if with everything in place the whole be left undisturbed for a quarter of an hour, and then a series of consecutive observations are taken without pause, the minimum leak is first obtained, and this gradually increases to the maximum. Thus in one experiment 6.7 mg. of radium bromide in a sealed tube was placed below a brass electroscope, the base of which consisted of 6.28 cm. of lead, and everything was left for ten minutes. Consecutive observations, then taken over 100 divisions of the eyepiece scale, were, in divisions per minute, 111, 114, 118, 120, 120, 122, 123.5, 124, 124.2, 125, 125, 125. If now the radium were held up to the side of the instrument for a minute without recharging, and the observations repeated, the minimum leak was again obtained, and this again increased to the maximum. In practice it is easier and quicker thus to obtain the minimum leak than the maximum leak, and we have so far been concerned to employ a method of working which shall avoid these errors and give consistent results. This is achieved in the following manner.

After each observation, the leaf being still partially charged, 6.7 mg. of radium bromide in a sealed tube is placed near

a window of the electroscope and the air within thoroughly ionized for about 40 seconds. The radium is then removed and the leaf charged up but slightly more than is necessary to get it onto the scale.

The leak so obtained is the minimum leak, and seems to be, if not the true, at least a consistent measure of the ionization of the γ -rays from the source used. The leak with the new electroscope of lead described later is quite constant for a particular disposition over any one day, leaks of 12 minutes duration rarely varying by more than three or four seconds. Also the values of λ obtained by this method for any one substance on different days do not usually differ by more than one or two per cent. After this method of working was found, the measurement of ionization leaks due to γ -rays became almost as simple as in the case of the other radiations, whereas before they were very uncertain and variable. Without some such precautions being employed γ -ray measurements can hardly be depended upon to 10 per cent.

8. Absorption of γ -Rays of Radium under Various Conditions.

The published values of λ for the γ -rays of radium show considerable differences among themselves, and we have carried out a large number of experiments to determine whether this value depends on the conditions. McClelland (Phil. Mag. 1904, viii. p. 70) and Eve (*Phys. Zeit.* 1907, viii. p. 183) agree fairly closely. The latter gives the value of λ as varying from 0.56 to 0.46 over a range of from .64 to 3 cm. of lead. Wigger, who worked over the range from 2.8 to 5 cm. of lead with a peculiar apparatus, found the value 0.241. Kleeman (Phil. Mag. 1907, xiv. p. 643) came to the conclusion, from experiments on the secondary cathode rays generated by γ -rays, that the latter must consist of three groups of rays. Madsen recently (Phil. Mag. 1909, xvii. p. 447) concluded that the γ -rays consisted of two homogeneous radiations possessing values for λ/d , independent of the nature of the absorbing substance, 0.028 and 0.12 respectively. Thus, for lead λ has the values 0.32 and 1.36. Since the present work was done Y. Tuomikoski has published (*Phys. Zeit.* 1909, x. p. 372) a determination of λ for γ -rays of radium over a range up to 18 cm. of lead. He gives the following values :—

Thickness { from (cm.) to	.4 1.0	1.0 2.2	2.2 5.4	5.4 12.0	12.0 15.8	15.8 18.0
λ (cm.) ⁻¹	.70	.58	.52	.50	.39	.25

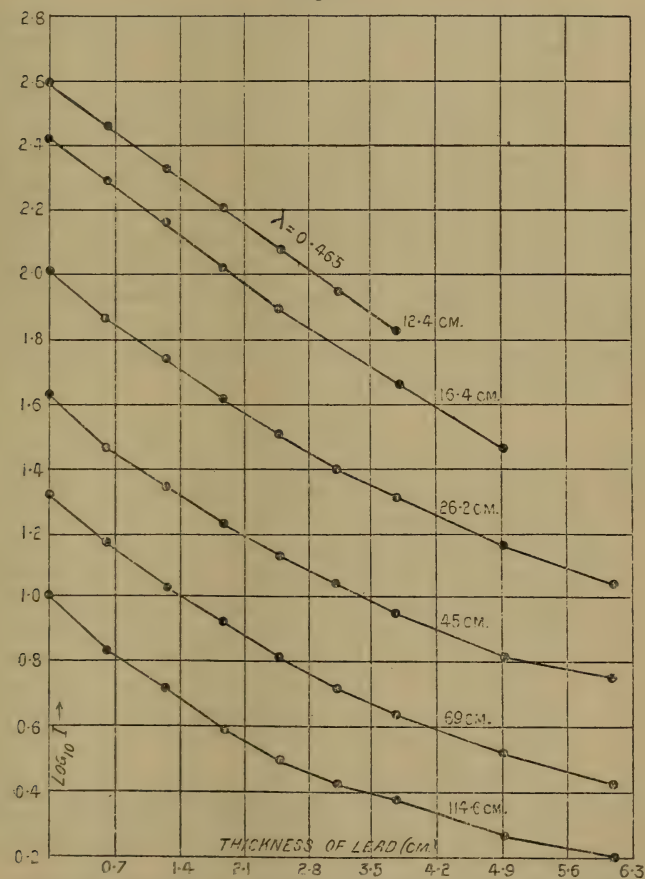
From the paper, which is merely a preliminary account, no conclusion can be formed, in view of the results to be given in this section, as to whether the convexity of the absorption curve to the origin, which these numbers disclose, is due to a real change in the value of the absorption coefficient.

We obtained for several dispositions results similar to those given by Eve and McClelland, and in no case near Wigger's lower value, so we repeated the latter's work as closely as possible. The apparatus consisted of a vertical brass cylinder 105 cm. long, 4.5 cm. inside diameter, 0.5 mm. wall thickness provided with a central electrode 95 cm. long passing through an insulator and directly connecting with a leaf system inside an ordinary brass electroscope. The lead plates were placed over the open top end of the cylinder and the radium was fixed at a definite height above. The capacity of the arrangement was 30 times greater than that of an ordinary leaf system. In an experiment with the radium 56 mm. from the top of the cylinder, as used by Wigger, the absorption proceeded exponentially from a range of 2.8 to 4.7 cm. of lead, the value of the absorption coefficient λ being 0.479, instead of 0.241 as stated by him. Wigger, however, appears to have calculated his results wrongly. He does not state the natural leak of his instrument; but neglecting this and recalculating from his own observations gave a value similar to what we obtained experimentally.

The effect of increasing the distance of the radium from the end of the cylinder to 78 mm. and 100 mm. was then tried. The leak fell off with great rapidity as the distance of the radium was increased, showing that only a few cm. of the top end of the long cylinder could be effective in contributing to the ionization. The curves obtained over the same range as before were quite exponential, but the values of λ were now different, being 0.438 at 78 mm. and 0.393 at 100 mm. This effect of the variation of λ with the distance of the source of radiation from the ionization-chamber was then investigated with a common form of apparatus. The brass electroscope, used in the measurements of section 6, was mounted on brass pillars on a wooden plank one end of which was supported on the slate bench while the other was supported by two long legs standing on the floor. A large hole was cut in the plank so as to allow free passage of radiations from beneath the table into the electroscope. The base of the electroscope was of lead 2.840 cm. thick clamped up as before. It was found that with 5 cm. of lead over the radium placed on the floor and 2.84 cm. as base the leak was

considerably less than if all the metal was clamped up to the base and the radium tube left bare. The difference in the leaks due to the two dispositions it would seem natural to ascribe to the generation of secondary rays in the slate bench and elsewhere sufficiently penetrating to get through the walls of the electroscope, which are produced in greater intensity in the first case than in the second. In consequence the lead, all except the 2.84 cm. forming the base, was placed directly

Fig. 2.



γ -Rays. Variation of Absorption with Distance of Radium from the Electroscope.

over the radium. The radium was placed below the electroscope at distances varying from 15.1 cm. to 114.6 cm. The absorption-curves obtained are shown in fig. 2. For the

smallest distance the curve is exponential ($\lambda = .465$); but as the radium is removed further and further away the curves become more and more convex to the origin. From 16.4 cm. to 114.6 cm. the preparation is below the level of the slate bench. Although at the time it was deemed impossible for any β -radiation, secondary or primary, to penetrate the walls or windows of the brass electroscope, we very probably, owing to our experience having been mainly with the β -rays of uranium, underestimated the penetrating power of the secondary β -rays of radium. From a mature consideration of the whole of the effects obtained during the work we thought the effect may have been due to secondary β -radiation, as in no other experiments had we been able to obtain any definite evidence of secondary γ -radiation, generated by γ -rays, more penetrating than the β -rays (compare Sections 8 A and 9). When the electroscope, on the other hand, has a *base* thin enough to allow secondary β -radiations to penetrate it, and the radium is placed a few centimetres beneath, it was noticed that the leak was always greater with the absorbing screens directly over the radium than when they were clamped up to the electroscope. This is to be ascribed to lateral γ -radiation generating secondary β -radiation at the point when it can enter through the electroscope base.

8 A.

[Since the paper was printed we have examined more closely the question as to whether secondary β -radiation caused the effect discussed in Section 8. The wall of the brass electroscope was thickened in places by external sheets of various metals. It was found that whereas the addition of brass up to a thickness of 3 mm. did not affect the actual value of the leak for a fixed disposition, even thin lead-foil cut it down appreciably. Beyond 2.2 mm. of lead no further reduction was noticed. At this stage for the particular disposition used the difference amounted to a third of the total effect. The leaks at the end of the lowest curve in fig. 2 were only one-half as great as in this case. These results taken in conjunction with those recorded in Section 10 show that the departure of the absorption-curves (fig. 2) from the straight line is to be ascribed to external secondary radiation more penetrating than the β -rays, so far as brass is concerned, but readily absorbed by lead. The choice of a thick lead electroscope in the final series of measurements described in section 10 was made from quite other considerations, but it now appears that secondary radiation effects were thus probably avoided, which might not have been the case if an

equally thick electroscope of another metal had been employed. The work of Eve just published (*Phil. Mag.* 1909, xviii. p. 175), taken into conjunction with that of Kleeman, Bragg and Madsen, led him to the view that lead is peculiar in that it absorbs secondary γ -radiation and generates little or none (compare concluding paragraph of section 10).]

9. *The Effect of Secondary Penetrating Radiation.*

The absorption of γ -rays by matter may be conveniently expressed with fair accuracy by an exponential law though small departures from this law are probably always present. We think that in an unsuitable form of apparatus secondary penetrating rays play a large part in causing these deviations. We have not been able by any theory of secondary radiation to predict beforehand what variation is likely to be obtained in any ordinary case. It suffices in the first instance to suppose that the γ -rays generate at each point in passage through matter a secondary radiation proportional in intensity to the primary, but with a different coefficient of absorption. If we assume that the rays, both primary and secondary, traverse the plate normally, the mathematical proposition is, reading thickness instead of time, formally identical to that of the theory of successive chemical changes, such, for example, as has been thoroughly worked out by Rutherford for successive radioactive changes. If Q_0 indicates the initial energy of the primary beam, Q_t the energy of the combined primary and secondary emerging from a plate of thickness t , λ_1 and λ_2 the absorption coefficients of the primary and secondary respectively, and κ is a coefficient of transformation, representing the fraction of the absorbed primary transformed into secondary, it can be at once shown that

$$\frac{Q_T}{Q_0} = (1 + A)\epsilon^{-\lambda_1 t} - A(\epsilon^{-\lambda_2 t}),$$

where

$$A = \frac{\kappa\lambda_1}{\lambda_2 - \lambda_1}.$$

As the ionizations, not the energies, are measured, and these are directly proportional to the absorption coefficients, this must be allowed for, and we obtain

$$\frac{I_T}{I_0} = (1 + B)\epsilon^{-\lambda_1 t} - B(\epsilon^{-\lambda_2 t}),$$

where

$$B = \frac{\kappa\lambda_2}{\lambda_2 - \lambda_1},$$

and I_T , I_0 refer to the ionizations.

We may merely state here that we have not yet been able to obtain any verification apart from the effects recorded in Section 8 A of the existence of secondary or reflected penetrating rays generated by the passage of γ -rays through matter in any of the dispositions employed. The above equations apply, of course, equally to the secondary β -radiation which is known to be generated; but the effect of this on the absorption-curve is only apparent over the same range as that of the primary β -rays, which is usually not directly investigable. Beyond this range the second term of the equation disappears, the absorption is exponential, and merely the absolute value of the ionization leak is multiplied by a constant factor. This state, by analogy to radioactive equilibrium, might be called "radiation equilibrium."

10. *The Form of Apparatus finally adopted.*

As the γ -radiation from uranium X is comparatively feeble, it was important to construct the electroscope of metal which for a given volume would give the largest effect. Lead was found to be easily the best for this purpose. This is curious as precisely the opposite effect was expected. For when the absorbing plates form the base of the electroscope the leak in the instrument for equivalent thicknesses (*i. e.* thickness $\times$ density) is least for lead. We have probably here an example of the difference discovered by Bragg and Madsen (Phil. Mag. 1908, xv. p. 663) between the incidence and emergence secondary radiation. The emergence radiation of lead is not greatly different from that of light metals like aluminium, whereas the incidence radiation is much greater than that from aluminium. Electroscopes of lead and brass with circular cross-section and electroscopes of lead, aluminium, zinc, and cardboard with square cross-section all of the same height were compared under identical conditions of experiment. It was found that in the two lead electroscopes the ionization produced by the γ -rays was proportional to the volume of the electroscope. Lead was easily the best, then came zinc and brass which gave about equal ionization. Cardboard lined with tinfoil came next, and lastly aluminium. The electroscopes were of varying thicknesses, but in no case was the ionization increased by increasing the thickness of the walls. The relative values of the ionization in the electroscopes were approximately:—Lead 100, zinc 75, brass 75, cardboard 68, aluminium, 57. To eliminate the possible effect of secondary rays and to secure the maximum ionization by γ -rays a cylindrical lead electroscope with small glass windows was constructed with the following dimensions:—Inside height

12.8 cm., inside diameter 9.0 cm., wall thickness 0.65 cm., thickness of top 1.18 cm., thickness of base 2.84 cm.

The electroscope was completely surrounded by two circular screens of lead of height 12.6 cm. and of thickness 0.244 cm., provided with small windows cut in them. The whole was fitted with a wooden stand and mounted on four brass pillars above the wooden table. With this new instrument the effect of varying the distance of the radium was again investigated in a manner precisely similar to that already described for the brass electroscope. *In every case the exponential law of absorption now held even up to distances of over a metre.* In contrast to the similar experiments with Wigger's apparatus, the value of λ also only varied slightly, if at all, with the distance. The value of $\lambda(\text{cm.})^{-1}$ from 2.8 to 8.9 cm. of lead in this series was 0.483 with the radium 13.1 cm. and 0.480 with the radium 113 cm. below the base of the electroscope. At intermediate distances λ was intermediate.

The base was too thick for work with the feeble γ -rays of uranium X, and it was replaced by a permanent base consisting of a plate of lead 0.975 cm. thick. With this alteration the instrument formed the standard one in all subsequent measurements. All the active preparations were placed at the uniform distance of 13 cm. from the under surface of the base, the absorbing plates were placed directly over the active material, and the air in the electroscope was thoroughly ionized between each observation.

Before giving the final results, it is of interest to refer to some observations which show that even in this method of work a certain amount of arbitrariness is introduced into the results by the particular disposition adopted. The effect on the absolute value of the leak in the electroscope of alternately placing the same plates over the active material and against the under side of the base of the instrument was examined. For the radium γ -rays the leak was slightly less both with copper and lead when the plates were clamped up to the base. For uranium X the same held true except for great thicknesses of lead when the converse held. Thus for uranium X the absorption curves for lead with the two dispositions actually cut one another. The effects so introduced, however, are not large, and the advantages in ease and quickness of working in placing the plates directly over the active material are very great as compared with the first method employed. At the same time we do not think that there is much to choose between the two methods from a theoretical point of view. Each is open to some objection.

10. *Comparison of the Absorption Coefficients of the γ -Rays of Uranium and Radium.*

With the method described in last section a series of comparable measurements of the absorption coefficients of the γ -rays of uranium X and radium were finally carried out. The only difference between them is that the uranium X was in the form of a surface of about 30 cm.² area, while the radium constituted practically a point source. The uranium X preparations employed were those of the third separation.

The results with radium are given in Table II. and fig. 3. They bear out the values of McClelland and Eve.

TABLE II.—Radium γ -Rays.

Lead Electroscope, base .975 cm. thick. Absorbing Plates laid directly over Radium, which was 13.0 cm. below under surface of base. 0.47 mg. of RaBr₂ used.

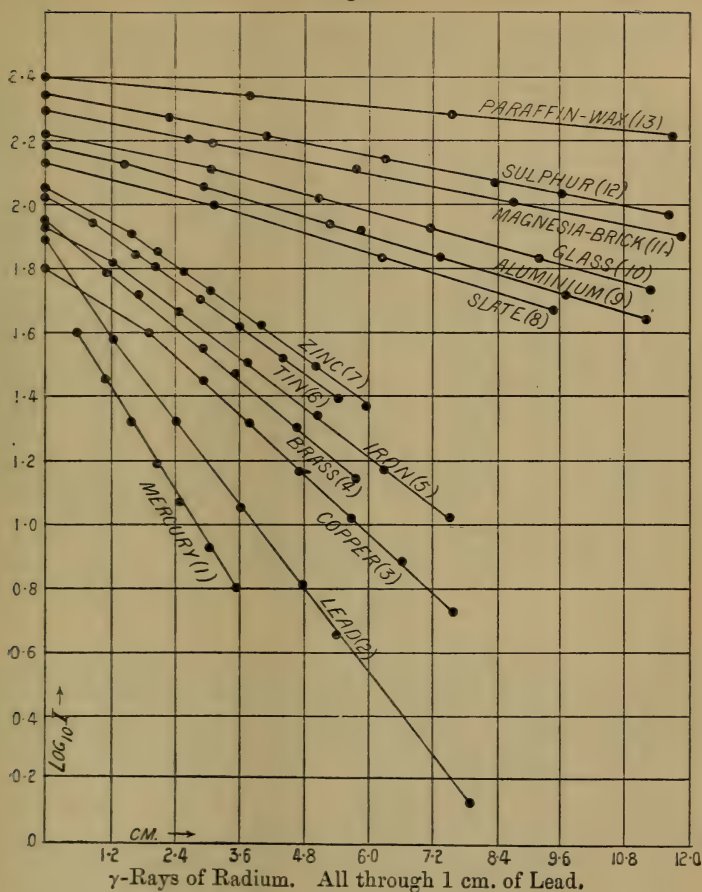
No.	Material.	Density.	Range (cm.).	λ (cm.) ⁻¹ .	100 $\times \lambda/d$.
1.	Mercury.	13.59	.34 to 3.32	.642	4.72
2.	Lead.	11.40	0 to 7.91	.495	4.34
3.	Copper.	8.81	0 to 7.60	.351	3.98
4.	Brass.	8.35	0 to 5.86	.325	3.89
5.	Iron.	7.62	0 to 7.57	.304	3.99
6.	Tin.	7.245	0 to 5.51	.281	3.88
7.	Zinc.	7.07	0 to 6.00	.278	3.93
8.	Slate.	2.854	0 to 9.44	.118	4.14
9.	Aluminium.	2.77	0 to 11.19	.111	4.01
10.	Glass.	2.52	0 to 11.26	.105	4.16
11.	Magnesia Brick.	1.92	0 to 11.86	.076	3.96
12.	Sulphur.	1.785	0 to 11.59	.0782	4.38
13.	Paraffin-wax.	0.862	0 to 11.39	.040	4.64

Mean value of λ/d (Nos. 3 to 11) = .0399.

The results with uranium X are given in Table III. and fig. 4 (pp. 646, 647).

From the curves (figs. 3 and 4) it can be seen how nearly the exponential law is followed in the case of all substances. Only the first points on the curve, with zero thickness, show slight irregularities. There is no evidence of the existence of the second negative exponential term, showing that if secondary rays are generated by the γ -rays of either uranium or radium, they are unable to penetrate 1 cm. of lead. Moreover the value of the absorption coefficient is for all substances of average density independent of the nature of the substance.

Fig. 3.



The mean value of λ/d for all substances having a density between the range of from 2.6 to 8.8 is for radium .040, and for uranium X .047. It may be noted that the deviations from the mean value are, considering the nature of the experiment, very small indeed. This is largely due to the precautions taken each time to secure the minimum leak referred to before.

Thus λ/d for uranium X for zinc gave a value of .0465 twice on different occasions. Repeating the experiment under exactly similar conditions, on the same day as one of the measurements, except that the leak was taken without thorough ionization of the air in the electroscope before each

TABLE III.—Uranium X γ -Rays.

Disposition as in Table II. except that the Uranium X occupied a surface of about 30 sq. cm.

No.	Material.	Density.	Range (cm.).	λ (cm.) ⁻¹ .	$100 \times \lambda/d$.	$\frac{\lambda \text{ Ur X}}{\lambda \text{ Ra}}$.
1.	Mercury.	13.59	3.43 to 3.535	.832	6.12	1.297
2.	Lead.	11.40	0 to 4.5	.725	6.36	1.465
3.	Copper.	8.81	0 to 7.6	.416	4.72	1.186
4.	Brass.	8.35	0 to 5.86	.392	4.70	1.208
5.	Iron.	7.62	0 to 7.57	.360	4.72	1.183
6.	Tin.	7.245	0 to 5.51	.341	4.70	1.212
7.	Zinc.	7.07	0 to 6.00	.329	4.65	1.183
8.	Slate.	2.854	0 to 9.44	.134	4.69	1.133
9.	Aluminium.	2.77	0 to 11.19	.130	4.69	1.169
10.	Glass.	2.52	0 to 11.26	.122	4.84	1.160
11.	Magnesia Brick.	1.92	0 to 11.86	.0917	4.78	1.207
12.	Sulphur.	1.785	0 to 11.59	.0921	5.16	1.178
13.	Paraffin-wax.	0.862	0 to 11.39	.0435	5.02	1.082
14.	Pine-wood.	0.386	0 to 12.51	.02926	7.58	

Mean value of λ/d (Nos. 3 to 9) .0470.

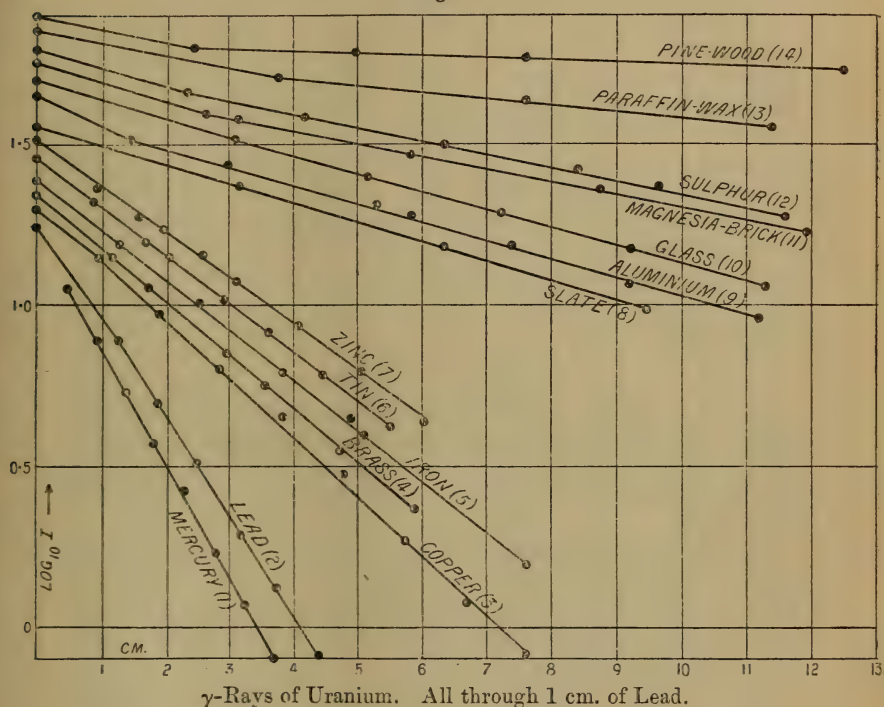
$$\frac{\lambda \text{ Ur X}}{\lambda \text{ Ra}} \quad , \quad 1.181.$$

observation, gave the value 0.0441 for λ/d , a value 5 per cent. less than the former value. Equally good experiments with the same metals in Table I. gave values for λ/d differing by 8 per cent. Now for uranium X the extreme values of λ/d for bodies within the density range specified are .0465 and .0472, which is only a variation of about 2 per cent. For radium the extremes are .0387 and .0414.

For substances of density beyond the limits 2.6 to 8.8 on either side the values of λ/d are considerably larger, the departures being more apparent for the case of uranium X than for radium. These departures seem to be genuine, for all the results showing them have been repeated, and the same or very similar results obtained. They probably have their origin in the disposition used. Thus in the original work with uranium X (Table I.), when the absorbing materials were clamped up, λ/d was, on the whole, fairly constant over the entire range from mercury to pine-wood, with a mean of .0536, while in the later work both with radium and uranium X, in which the materials are laid directly on the source of radiation, differences appear. The general arrangement of the experiment and the method of working in the former case, however, were much less accurate

than in the latter. A determination of λ for lead with uranium X with the new disposition and method of working

Fig. 4.



except that the metal was all clamped to the base of the electroscope gave a value 0.66, which is much nearer to the value given in Table I. than to that in Table III.

Considering now the ratio of the absorption coefficients for uranium and radium shown in the last column of Table III., it will be seen that, excluding the two heaviest substances, mercury and lead, and the two lightest, sulphur and paraffin-wax, the mean ratio is 1.182, and the actual values are wonderfully close to this mean. If we exclude also slate, which is an indefinite sort of material, the extreme variation from the mean value is only 2 or 3 per cent.

[It is very interesting to note that the division of the substances in Tables II. and III. into three groups, according to the values obtained for λ/d , is practically the same as the division made by Kleeman for totally distinct reasons in his study of the secondary radiations generated by γ -rays. (Compare *Phil. Mag.* 1908, xv. p. 644, Tables I. and II.)]

Lead is entirely anomalous with the value of the ratio 1.465, and this no doubt, as explained, is to be attributed partly to the disposition employed, and possibly also to the fact that the measurements were done in a lead electroscope. It is rather curious that lead, of all metals, seems in many ways to be the most unsuited for accurate measurements of the absorption of γ -rays, and yet it is the one which we, in common with previous investigators, have worked with most. We have noticed repeatedly that it is more difficult to obtain consistent results with lead as the absorber than with any other material. This is partially perhaps, but not wholly, due to its own high natural radioactivity, necessitating frequent redetermination of the natural leak. If we were repeating the work we should use by preference another metal in many of the measurements described in the first part of the paper.

Summary of Results.

1. The γ -radiation of uranium X, separated from 50 kilograms of uranyl nitrate, has been compared with the γ -radiation of radium, and it is concluded that the β - and γ -radiations are probably not, as hitherto assumed, interdependent.

2. The initial γ -radiation of the uranium X in equilibrium with 1 kilogram of uranium (element) is equal to that of 0.015 milligram of a particular sample of a radium compound, which may be provisionally regarded as containing 66.6 per cent. of radium.

3. The ratio of the β - to the γ -rays of uranium X is 62 when the same ratio for radium C is taken as unity, the γ -rays being measured through 1 cm. of lead. Correcting for the absorption in the lead, and assuming the rays to be homogeneous in each case, the value of the ratio for uranium is 50. When the γ -rays are measured through 0.6 cm. of aluminium, so as to admit any soft γ -radiation if present, the ratio for uranium is 18, not correcting for absorption. It is not yet decided whether a soft primary γ -radiation of uranium exists.

4. The γ -radiation accompanies the β -radiation of uranium X, and decays at approximately the same rate.

[4 *a*. Later results give 0.028 (day)^{-1} for the value of λ determined from the decay of the γ -rays. This is about 10 per cent. less than the known value derived from the decay of the β -rays.]

5. For thicknesses less than 1 cm. of lead, or its equivalent, the absorption of the uranium γ -rays does not follow any

simple law. For greater thickness the absorption is quite exponential, and nearly or wholly independent of the nature of the substance traversed, being proportional to its density. In one series the absorbing plates themselves formed the base of the electroscope, and the mean value of λ/d was 0.0536.

6. In a thick-walled lead electroscope with a base 1 cm. thick, the absorbing plates being placed directly over the preparation, the absorption both for uranium and radium was strictly exponential. The value of λ/d for all but the heaviest and lightest substances was 0.047 for uranium and 0.040 for radium, the ratio being 1.18. Lead appears exceptional under these conditions, and the value of λ/d is 0.0636 for uranium γ -rays, and 0.0434 for radium γ -rays, the ratio being 1.465. In the first disposition lead appears quite normal.

7. Some evidence of the existence of a secondary radiation more penetrating than β -rays, generated by the γ -rays, has been obtained, but almost certainly none exists able to affect measurements taken through 1 cm. of lead.

8. The γ -ray ionization is not in all cases constant, but suffers a progressive increase with certain dispositions, from a minimum to a maximum about 12 per cent. greater, when consecutive observations are made without pause. The effect depends upon the presence or absence of a charge on the leaf system; for if the charge and the ionization are maintained, the rate of leak tends towards a maximum; while if the ionization is maintained for a sufficient period after the charge has been dissipated the rate of leak becomes a minimum again.

Physical Chemistry Laboratory,
University of Glasgow.
June 1909.

[Additions made August 1909.]

LXVI. *The Kinetic Energy of the Positive Ions emitted from various Hot Bodies.* By F. C. BROWN, Ph.D.; Porter Ogden Jacobus Fellow, Princeton University*.

THIS paper describes an investigation of the kinetic energy of the positive ions from gold, silver, palladium, tantalum, nickel, platinum, aluminium phosphate, osmium, tungsten, and iron.

It was shown in a paper by Richardson and Brown† that if the kinetic energy of the ions is due solely to thermal

* Communicated by Prof. O. W. Richardson.

† Phil. Mag. [6] vol. xvi. p. 353 (1908).

agitation and is distributed among the different ions in accordance with Maxwell's law, then the current carried by those ions escaping from a hot portion of an infinite plane to a neighbouring parallel plane can be expressed by the formula

$$i = i_0 e^{-\frac{neV}{R\theta}},$$

where V is the difference of potential between the planes, ne is the quantity of electricity carried by half a cubic centimetre of hydrogen in electrolysis, i is the value of the current when the difference of potential is V , i_0 is the value for a potential-difference $V=0$, θ is the absolute temperature, and R is the ordinary gas constant in the equation $pv=R\theta$, taken for a unit volume of gas under standard conditions of pressure and temperature.

This formula was tested experimentally for the negative ions from platinum, carbon, and sodium-potassium alloy. The verification was quite satisfactory for platinum, but it was not so for carbon or the alloy. At that time it was considered that the failure of the experimental results to comply with the theory in the instances mentioned, was due to the disagreement between the experimental conditions and those required by the theory.

In a more recent paper* the author showed that the above formula was satisfactory for the positive ions from hot platinum. In that paper the hot platinum strip described was constricted slightly in the middle, so that only a small portion of it was heated by the electric current. It was estimated that no portion of the emitting surface differed from zero potential by more than ± 0.04 volt. Recently, while investigating the kinetic energy of the ions from tungsten and tantalum filaments, results were obtained which were not in agreement with those previously obtained with a platinum strip. It was not clear if the disagreement were mainly due to the hot body not having a plane surface or to the fall of potential over the conducting surface. It will be seen later in this paper that the errors arising from a small fall of potential over the strip can be neglected in comparison with errors arising when the emitting surface is not a plane, and parallel to the receiving surface.

* Phil. Mag. [6] vol. xvii. p. 355 (1909).

Theory of the Method.

Stated briefly the underlying theory of the method used in these investigations is the same as that presented in the paper by Richardson and Brown [*loc. cit.*]. The hot body, or thermionic radiator, comprised a plane of from 0.2 to 0.3 mm.² area, located in the centre of a disk of 13 mm. diameter. The ions emitted impinged on a parallel disk less than 1 mm. distant, which for reference we may call the upper disk.

On the view outlined above, the thermionic current * to the upper disk is expressed by the equation, ;

$$i = i_0 e^{-\frac{neV}{R\theta}},$$

which is equivalent to

$$R = \frac{ne(V_2 - V_1)}{\theta(\log_e i_1 - \log_e i_2)} \dots \dots \dots (2)$$

The working method consisted in applying different potentials to the upper plate and then measuring the rate of charging up. There are several advantages of this method over the one given in previous papers, which always required merely the recording of the potential to which the upper plate changed in a given time. The chief advantages of the present method are, that time is saved in the taking of the observations, the calculations are shortened and simplified, and inaccuracies due to any changes in the electrometer constant are avoided. From the data, curves were made showing the thermionic current for different potentials and also curves showing the logarithms of the current for these same potentials. In general the curves permitted a quicker and more accurate verification of formula (2) than was obtained without them. From the curves the value of the kinetic energy of the ions due to the velocity components normal to the surface of the thermionic radiator was readily determined, by making use of the methods previously given.

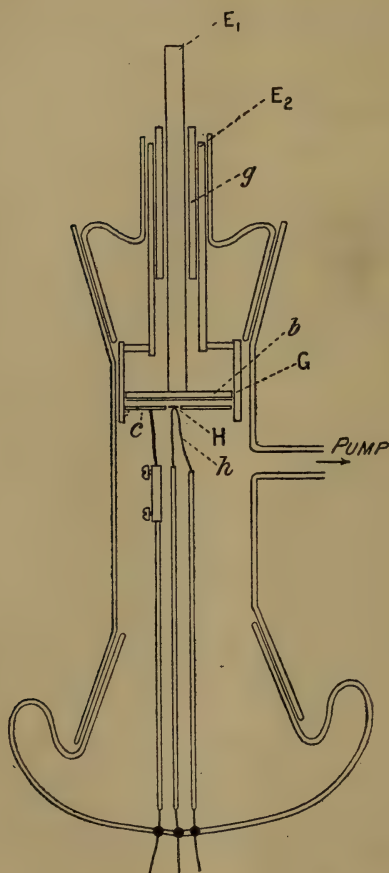
Experimental Arrangements.

The special part of the apparatus used in these investigations is shown in fig. 1. It was put together in three parts, a glass tube of 20 mm. diameter and two well-ground glass stoppers. Into the upper stopper were fitted a brass rod E_1 and a brass tube E_2 . The tube and the rod were separated and insulated from each other by glass tubing g . These parts were fixed and

* For justification for this and similar new terminology used in this paper, see paper by O. W. Richardson, Phil. Mag. June 1909.

made air-tight by sealing-wax covered over on the outside by soft wax. On the bottom of the rod was threaded a brass disk, *b*, to which was soldered a platinum disk of the same diameter. The guard-ring *G* was fastened to the bottom of the brass tubing, as shown in the diagram. It was an easy matter to clean the platinum disk on this upper plate before any series of observations, and thereby avoid any complications that might be due to the high potential effect. The nature of this effect will be explained further on in the paper.

Fig. 1.



The guard-ring arrangement made it impossible for any collected charges on the glass to leak to the upper plate or its electrical connexions.

The hot body *H* and the surrounding disk, *c*, were held in

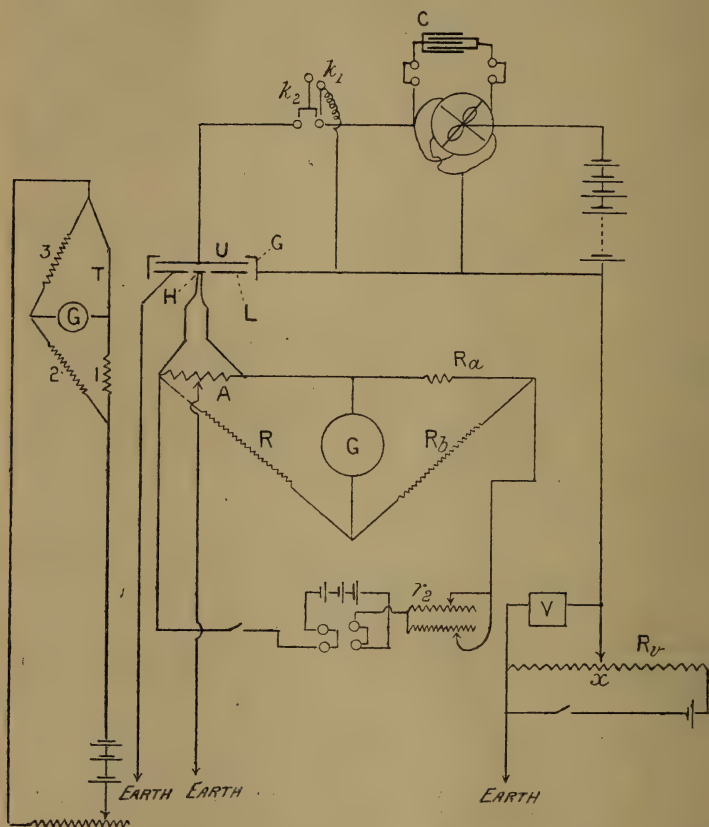
place by the stopper at the bottom. The disk *c* was of thin copper or platinum. It was adjusted to its place after the adjustment of the thermionic radiator *H*. This radiator in the experiments with gold, silver, palladium, aluminium phosphate, and iron, was in the form of a disk of approximately 3 mm.² area. In the other experiments the form was a strip or a wire. The radiator was attached directly or indirectly to two copper wires shown in the figure. The gold disk was welded to the uppermost point of a platinum-wire loop, in order that the temperature measurements might be checked by the measurement of the resistance of the platinum-wire loop.

The apparatus just described, together with the electrometer and its connexions, were placed inside a galvanized iron box 24×24×18 inches, in order to be free from moisture effects which are so aggravating in our climate during the summer months. The top of the box was in four parts. Each part turned down at the edge into a trough, so that when melted paraffin was poured into the trough, the top of the box became air-tight. This box arrangement was found quite satisfactory. Wires inside such a box do not need any additional shielding in order to be free from electrical disturbances.

The electrical connexions are shown in fig. 2 (p. 654). The surface of the thermionic radiator was placed at *H* and was in the same plane as the surface of the lower disk *L* and from 0.5 to 1.0 mm. below the upper disk *U*. It was attempted to have the two disks so near each other that only a comparatively small number of ions would escape to the guard-ring *G*. The upper disk was connected to one pair of quadrants of a Dolezalek electrometer and the condenser *C* through the keys *k*₂ and *k*₁. The other pair of quadrants are seen to connect electrically to the following: the key *k*₁, the guard-ring *G*, one terminal of the batteries which charged the electrometer needle, and to a variable point *x* in the resistance *Rv*. The last was a resistance of about 90 ohms across which there was a fall of potential of 2 volts. It was earthed at one end, as shown in the diagram. By adjusting the point *x* various potentials could be applied to the plate *U*, the values of which could be read on a voltmeter placed at *V*. By separating the quadrants by the key *k*₁, the rate of charging up of the electrometer indicated the current to the upper disk against the given potential. Usually about 240 divisions on the electrometer scale represented one volt. However, the method does not require that the sensibility be known accurately. The capacity of the electrometer system

was varied by inserting in parallel a standard condenser. This was carried out by a key manipulated from outside the air-tight box.

Fig. 2.



The thermionic radiator was heated, indirectly most often, by an electric current. The heater was merely a single loop of platinum wire or a loop of metal foil, a rough cross-sectional view of which is shown in the diagrams. The loop supported the radiator and the heat produced by the electric current flowed into the radiator by conduction. This device enabled the emitting surface to be kept at the same potential. It was quite essential that this potential should be zero. This was accomplished by shunting the heater with the resistance A, of about 20 ohms. The shunt resistance was earthed at such a point that the magnitude of the thermionic

current was unchanged when the direction of the heating current was reversed. The heating current was regulated by a large and a small resistance r_2 in parallel. The heating coil and shunt together formed one arm of a Wheatstone-bridge mesh. In some instances the temperature of the thermionic radiator was checked by the resistance of the platinum heater, and in others the resistance merely indicated whether or not conditions were unchanging during the observations.

The temperature was ordinarily determined by an optical method which within the working limits was perhaps subject to a possible error of 10 per cent. A standardized platinum wire of 2.1 cm. length was placed in a vacuum-tube. This wire was in one arm of a Wheatstone-bridge mesh (see fig. 2). The temperature of this wire was adjusted until it appeared to the eye to be at the same temperature as the thermionic radiator studied. Then the two bodies were taken to be at the same temperature, and this temperature was read from a temperature-resistance chart for the standardized temperature wire. In the calibration, the fiducial points were taken at the temperature of the room, at the temperature of the melting of potassium sulphate, 1066°C. , and at the temperature of melting platinum, 1745°C. [see paper by Richardson and Brown, *loc. cit.*].

Results of the Experiments.

After the thermionic body had been properly adjusted in position, there was usually not much difficulty in obtaining the necessary observations in order to arrive at the value of the kinetic energy. In no instance was there any trouble arising from a mixture of negative ions with the positive, such as was noted in the previous papers. This was mainly because the observations were taken at low temperatures and before the heating had been continued much over one hour. The heating never extended over more than five hours. All the materials studied showed a perceptible decay in the magnitude of the thermionic current after about 30 minutes' heating. Of course this rate of decay was a function of the temperature. No attempt was made to determine just how it decayed at different temperatures.

The pressure in the different experiments varied between 0.0001 and 0.1 mm. In a previous paper [*loc. cit.*] the author showed that within quite a large range the kinetic energy of the positive ions from platinum was practically independent of the pressure. Consequently, very little

attention was given to adjusting the vacuum to any particular degree, except when working with bodies that oxidized readily.

Air was the only gas let into the vacuum system previous to taking observations. The amount of absorbed gas given out by the different thermionic radiators varied considerably. As this was only secondary to the investigation, the amounts of gas were not observed carefully. However, it was noted that silver and tantalum especially gave out much less gas than platinum or palladium.

In carrying out the experiments a potential was applied to the upper disk, and the electrometer was allowed to charge up until there was a readable deflexion. The deflexion was noted and the current was taken as $i = \frac{\Delta V}{\Delta t}$. In case ΔV covered several divisions it was necessary to make a small correction, in order to allow for the variation in the potential that the ions went against. Often there was a difference in the magnitude of the thermionic current with the heating current direct and the heating current reversed. Presumably, this was because the thermionic radiator was not at zero potential, but slightly above zero in one instance and below in the other. However, it is evident that the thermionic current for zero potential should be between the values found with the heating current direct and the heating current reversed, and if the difference is not too large, the mean value cannot be far from the one desired.

Gold.

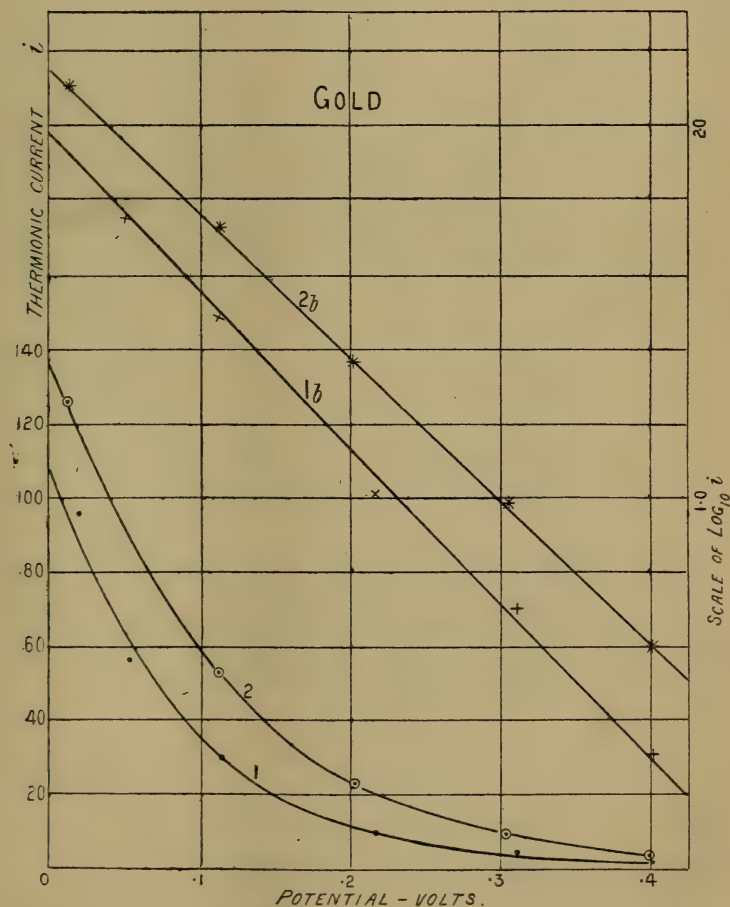
The gold used was in the form of a disk of about 3 mm.² area and 0.005 cm. thickness. It was welded to the top of a platinum-wire loop. The contact between the platinum and the gold probably extended over a length of 0.5 mm. If there was any fall of potential over the radiating surface of the gold, it was certainly less than 0.01 volt.

In this particular experiment there was a natural charging up of the electrometer amounting to 1.5 mm. in 30 seconds, when the heating current was off. This was not noticeable after completing this one set of observations. The correction for this is made in the accompanying table.

The temperature was first measured by the optical method previously mentioned. It was afterwards checked by measuring the resistance of the platinum-wire loop. The resistance at which the gold disk melted (1064° C.) was taken as a fiducial point. The table shows two sets of observations. For the lower temperature the optical method

gave 1030 degrees absolute and the resistance method gave 973 degrees. For the higher temperature the optical method gave 1190 degrees and the resistance method gave 1163 degrees absolute. In each case the mean of the values by the two methods was used in the calculation of the gas constant R . These determinations of the temperature are probably subject to less percentage error than that which might be expected to arise from the plane of the thermionic radiator not being in the plane of the disk.

Fig. 3.



From the data given in the table were plotted curves 1 and 2 (fig. 3) showing the values of the current for different

potentials on the upper disk. Curve 1 applies to the lower temperature of 1000° absolute and curve 2 applies to the observations at 1177° absolute, and is plotted on a scale 50 times smaller than is the other curve. That is, the absolute magnitude of the current is about 80 times smaller

Positive Ionization from Gold.

Capacity, microfarads.	Pressure, mm.	Temperature, absolute.	Initial Poten- tial, volts.	Electrometer Deflexions, mm.			Mean current.	Corrected value of current.	Mean Potential.	Gas Constant.
				Heating current.						
				Direct.	Reversed.					
·0001	·007	{ 973 1030	0	5 sec.	11, 9, 9, 10·5 14, 17	9, 9, 11, 10 17, 15	99 58	96 55	·02 ·05	4·2×10 ³
			0·1	10 sec.	6·4, 6·5, 6·8	6·0, 7·0, 6·2	33	30	·113	
			0·2	30 sec.	8·0, 7·8, 8·5	8·0, 6·2, 6·8	13	10	·216	
			0·3	30 sec.	5, 3·8, 6·0, 4·0	4·3, 4·0, 5·0	8	5	·31	
			0·4	30 sec.	3·5, 3·7, 3·1	2·5, 3·0, 2·0	5	2	·40	
·01	·01	{ 1163 1190	0	5 sec.	6, 6·5	7·0, 5·5	126	126	·013	3·9×10 ³
			0·1	10 sec.	5·0	5·7	53	53	·11	
			0·2	10 sec.	2·2	2·4	23	23	·2	
			0·3	30 sec.	3·2	2·7	9·6	9·6	·3	
			0·4	30 sec.	1·0	1·5	4·0	4·0	·4	
			0·5	30 sec.	0·6	0·9	2·0	2·0	·5	

Mean gas constant 4.0×10^3 .

at 1000° than it is at 1177° . This is not made clear in the table. The curves 1 *b* and 2 *b* show the corresponding logarithms of the current at the different potentials. From these curves and the other data the value of the gas constant *R* is easily calculated from the equation

$$R = \frac{neV}{\theta \log_{\epsilon} \frac{i_2}{i_0}}.$$

For $\theta = 1000^{\circ}$ ab. $R = 4.2 \times 10^3$

and for $\theta = 1177^{\circ}$ ab. $R = 3.9 \times 10^3$.

More reliance is placed upon the latter value, because the current was 80 times larger and was therefore capable of more accurate measurement.

Tantalum.

Tantalum was investigated under varying conditions. At first the thermionic current was measured from a filament of circular section 0.05 mm. diameter. This filament was taken from an incandescent lamp. The logarithm of the current when plotted against the potential was approximately a straight line which led to the value $R = 9.6 \times 10^3$ for the gas constant. The mean kinetic energy was therefore apparently larger than the predicted value.

A second filament was rolled into a strip with a thickness of 0.01 mm. and a width of 0.04 mm. This strip was bent over a form with square edges, so that it had the form .

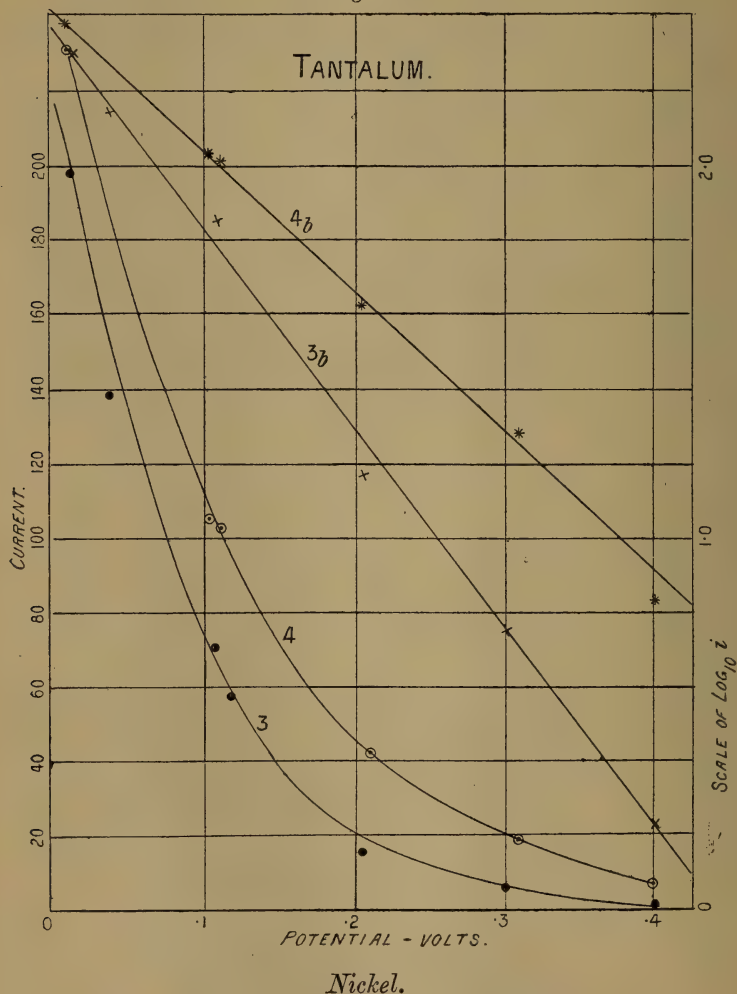
The lower disk surrounded the upper surface, leaving an area of about 0.2 mm.² free to emit ions to the upper plate. There was a fall of potential over this surface of probably 0.03 volt. The heating current was of the order of 0.05 ampere. The results are shown in the accompanying table. The curve 3 (fig. 4) is for the absolute temperature 1050 degrees and curve 4 is for the temperature 1273 degrees absolute.

Tantalum.

Capacity, microfarads.	Pressure, mm.	Temperature, absolute.	Initial Potent- tial, volts.	Mean deflexion.	Thermionic current.	Mean Potential, volts.	Gas Constant, R.
.0001	.001	1050	0	6.6 4.6	198×10^{-14} 138 "	0.014 0.037	3.0×10^3
			0.1	4.7	71 "	0.11	
			0.2	2.9	14.5 "	0.205	
			0.3	1.13	5.6 "	0.30	
			0.4	0.61	1.7 "	0.40	
0.01	.002	1270	0	4.6	230×10^{-13} 105 "	0.01 0.104	3.6×10^3
			0.1	2.1 6.2	103 "	0.113	
			0.2	2.55	42 "	0.21	
			0.3	2.3	19 "	0.31	
			0.4	.80	6.7 ,	0.40	
			0.5	.40	3 ,	0.50	

The logarithmic curves are 3 *b* and 4 *b*. Most of the points are seen to lie close to a straight line. The resulting values of R (3.0×10^3 and 3.6×10^3) are not very concordant. As in the case of gold more reliance is to be placed on the value 3.6×10^3 obtained with the larger current.

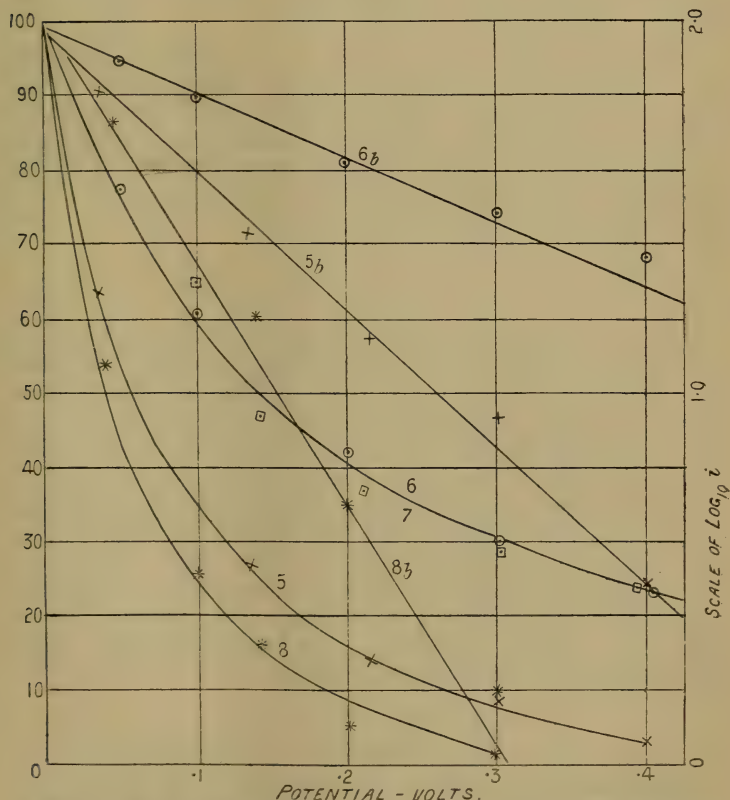
Fig. 4.



The thermionic current from nickel was investigated under essentially the same conditions as those last mentioned in connexion with tantalum. The ions were emitted from a nickel strip 0.004 cm. in thickness. The area of the emitting

surface was about 0.6 mm.^2 The fall of potential across this surface was approximately 0.05 volt , and no portion of it probably differed from zero potential by more than 0.025 volt . The effect of the fall of potential along the strip was investigated by shifting the potential of the centre of the strip from zero to $+0.02 \text{ volt}$. Thereby the thermionic current was increased by 10 per cent. This does not necessarily indicate that there would be an error as large as this in the value of R , because R depends on the way in which the current falls off as the upper plate acquires a potential. It is not easy to say how a small fall of potential would influence the distribution of energy.

Fig. 5.



From observations taken in the same way that the previous ones were, the current-potential and the logarithm of the current-potential curves were plotted. These are respectively 5 and 5b in fig. 5. The temperature was 1120 degrees

absolute. The value of the gas constant is calculated to be 3.6×10^3 .

Tungsten and Osmium.

Because of the difficulty of getting many of the materials in the form of a strip, it was thought advisable to try a round emitting surface, such as the surface of a wire or a filament. A platinum wire and tungsten, tantalum, and osmium filaments were substituted for the strip or the disk. At 1650 degrees absolute a platinum wire .11 mm. diameter gave the observations represented by curve 6 (fig. 5). For a tungsten filament similar observations at 1200° absolute gave the points marked [.], which are also shown roughly by curve 6. The corresponding logarithms of the current are shown in curve 6 b.

The value of the gas constant for the platinum wire was 6.2×10^3 and for the tungsten filament was 8.0×10^3 . We might readily ascribe the results to the form of the emitting surface were it not for the results from osmium, which show the value of the gas constant to be $R = 2.5 \times 10^3$. The observations in the case of osmium are shown by curve 8 and the corresponding logarithms in curve 8 b.

Silver.

The form of the thermionic radiator in the investigations with silver, palladium, aluminium phosphate, and iron was that of a disk. The silver disk was of the same dimensions as the gold disk previously described. It was welded to a palladium strip similar to the way the gold disk was welded to the platinum. Two series of observations were taken, using the same disk at different temperatures, in which the magnitude of the ionic current was 44 times as large in the one case as it was in the other. The mean energy was deduced from the curves in the ordinary way. It is given by $R = 3.0 \times 10^3$ and 2.9×10^3 . There is no evident reason why these results should not be fairly accurate. The mechanical adjustment seemed perfect.

Palladium.

Palladium gave quite a large ionization at first. One series of observations were taken from a palladium disk at a temperature of 1170 degrees absolute. The value of R was 3.4×10^3 . The amount of gas emitted was also large.

Silver.

Capacity, micro- farads.	Pressure, mm.	Temperature, absolute.	Initial Potential, volts.	Mean Current × Constant.	Mean Potential, volts.	Gas Constant.
·0001	·002	1020	0	16·3	·017	$3·0 \times 10^3$
			0·1	10·3	·046	
			0·2	4·5	·106	
			0·3	1·2	·2	
			0·3	·3	·3	
·01	·008	1150	0	680	·014	$2·9 \times 10^3$
			0·1	216	·112	
			0·2	93	·2	
			0·3	25	·3	
			0·4	7	·4	

Palladium.

·01	0·04	1170	0	520	·01	$3·4 \times 10^3$
			0·1	153	·11	
			0·2	63	·20	
			0·3	21	·30	
			0·4	9	·40	

Aluminium Phosphate.

·01	0·02	1230	0	1660	·018	$3·9 \times 10^3$
			0·1	620	·11	
			0·2	290	·20	
			0·3	180	·30	
			0·4	125	·40	
·01	0·006	1190	0	2080	·02	$3·4 \times 10^3$
			0·1	590	·11	
			0·2	250	·21	
			0·3	94	·30	
			0·4	35	·40	
			0·5	4	·50	

Aluminium Phosphate.

After the palladium was heated for somewhat more than an hour, the thermionic current at zero potential decreased to about 20 per cent. of its initial value. Then a water paste of aluminium phosphate was placed on the disk and the thermionic current was measured as noted in the table.

A second series of observations were made after a second coating of the phosphate had been put on by a slightly different treatment. The water was allowed to evaporate from the paste at room temperature, thus leaving an even white coating on the disk. The current was somewhat larger than it was in the first trial. The kinetic energy of the ions arising from the velocity components perpendicular to the surface of the disk, was, within the limits of error, the same as it was for the ions from the metals. That aluminium phosphate gives a larger positive ionization than do the metals was first observed by J. J. Thomson.

Iron.

Capacity, micro- farads.	Pressure, mm.	Temperature, absolute.	Initial Potential, volts.	Mean Thermionic Current in Arbitrary Units.	Mean Potential, volts.	Value of R.
·0001	—	1100	0 0·1 0·2 0·3 0·4	21·0 9·7 5·5 2·1 ·9	·02 ·12 ·21 ·31 ·4	4·6×10 ³
·0001	0·005	1100	0 0·1 0·2 0·3 0·4	16·5 8·6 5·1 2·4 1·1	·017 ·12 ·21 ·3 ·4	5·2×10 ³
·01	·01	1240	0 0·1 0·2 0·3 0·4	1400 810 360 170 63	·01 ·10 ·20 ·30 ·40	4·3×10 ³

Iron.

The positive ionization from iron was studied in essentially the same way as it was from the other elements. A silver disk was welded to a platinum heater and the iron disk was welded to the silver disk. This device was used mainly for convenience. The behaviour was somewhat irregular, but in every case the kinetic energy is seen to be larger than the normal value $3·7 \times 10^3$.

Summary of Results.

Radiator material.	Form of Radiator.	Temperature, absolute.	Current at zero potential $\frac{1}{10} \times 10^{12}$.	Vacuum Pressure.	Gas Constant. $R/10^3$.
Gold (1)	disk	$\left\{ \begin{array}{c} 1030 \\ 973 \end{array} \right\}$	1.0	.007	4.2
Gold (2)	disk	$\left\{ \begin{array}{c} 1190 \\ 1163 \end{array} \right\}$	60.0	.01	3.9
Silver (1)	disk	1020	.8	.002	3.0
Silver (2)	disk	1150	35.0	.008	2.9
Palladium	disk	1170	25.0	.04	3.4
Aluminium phosphate (1)	disk	1230	100.0	...	3.9
Aluminium phosphate (2)	disk	1170	120.0	.006	3.4
Nickel	strip	1120	2.5	.003	3.6
Iron (1)	disk	1100	1.0	...	4.6
Iron (2)	disk	1100	.8	.005	5.2
Iron (3)	disk	1240	...	.01	4.4
Platinum	wire	1695	...	...	5.1
Tungsten	filament	1150	1.0	.0003	5.1
Tantalum	filament	1050	.7	.0005	9.6
Tantalum	strip	1050	1.0	.002	3.0
Osmium	filament	1120	3.0	...	2.5

Discussion of Results.

In interpreting the results of the measurements it is necessary to consider to what extent they are reliable. In our discussion we have a right to discard the values obtained with tungsten, osmium, and with tantalum and platinum wires, because the form of the thermionic radiator is quite different from that required by the theory. There is, however, no obvious reason why the results with filaments should show such wide variations in the values of R . The observations with osmium were taken after several hours' heating, and negative ions may have been emitted simultaneously with the positive ions. This might account for the much lower value of R than those obtained with the other filaments. The other wires all gave values distinctly higher than those obtained with plane surfaces of metal, and it seems reasonable to attribute the discrepancy to the curvature of the surface used.

In the calculations of the mean energy certain hypotheses are assumed, and strictly speaking we are justified in these hypotheses and calculations only by virtue of the result. The theoretical value of the gas constant is taken from the equations $R\theta = \frac{1}{3}nmv^2$ and $R\theta = pv$, and is easily calculated to be 3.7×10^3 for a unit volume of gas under standard conditions of pressure and temperature. The lowest experimental

determination is $R = 2.9 \times 10^3$ which is for silver, and the highest value is given for iron, which is $R = 5.2 \times 10^3$. The mean of all the experimental values is $R = 3.8 \times 10^3$. This last fact is not inconsistent with the view that all the variations between the experimental and theoretical values are due to experimental errors. These errors may be largely due to the inherent structure of the thermionic radiator, such, for example, as might obtain in a very irregular crystalline surface. Considering the close agreement between the mean experimental value and the theoretical value of R , and that the disagreement is not very bad in any instance, we may state the following law to be true as a first approximation: The mean kinetic energy of the positive thermions emitted from hot bodies is independent of the material from which the thermions radiate and varies directly as its absolute temperature.

By referring to the summary it is noted that the variation in the values of R for any one material is decidedly less than the variation in the values when different materials are considered. For example, the mean variation of the values of R for each radiator separately is 5 per cent. from the mean value obtained, while the mean variation of the values of R for all the radiators together is about 16 per cent. from the value 3.8×10^3 . From these facts we may obviously make two deductions. First, the errors in the temperature determinations cannot be great enough to explain all the variations in the values of R . Second, there is no such change in the material with the time of heating or the intensity of heating as would warrant the variations obtained. Changes in the form or the size of the metallic crystals would be included in this. Of course these statements would not necessarily be true outside the limits of temperature and time prevailing in the experiments.

We must look elsewhere then for an explanation of the variation of our values of R . If the temperature were not uniform over the surface, the conditions would be such as the theory does not provide for. But as the area of the thermionic radiator was only 3 mm.², and set up as it was, there should have been very little difference of temperature at any instant. To the eye the surface appeared to be at the same temperature all over. Such errors as would arise from the edge of the disk being at slightly lower temperature than the middle would tend to give relatively too many ions with small kinetic energy. The author is quite certain that the errors arising from unequal distribution of temperature would not be sufficient to explain the variations obtained.

Without going into any further discussion here it is also improbable that the lack of uniformity in the electric potential of the disk could explain the variations.

After removing the silver disk it was noted that a spot of some impurity covered about one-fifth of the area of the disk. The thermions that penetrated this layer would probably have had their total energy diminished. Thus the relative number of low-energy ions over the whole surface would have been too great and the value of R would have been too low. This is the most plausible explanation for the low values given by silver.

In some instances at least some ions escaped to the upper disk from the heater by going between the radiating disk and the lower disk, or by going between the lower disk and the guard-ring. But the number was probably small. It is difficult to say just what correction should be applied for such conditions.

It was considered possible that the high values of R obtained in some instances might be accounted for in the fact that the disks do not fulfil the conditions of infinite planes which is required of them in order to apply the formula

$$\frac{i}{i_0} = e^{-\frac{neV}{R\theta}}.$$

Particularly in the experiments with iron, which were the last performed, there was the most probability of a large number of ions escaping to the guard-ring, because of the large distance existing between the thermionic radiator and the upper disk. Therefore the error that should accrue if the distance of the radiator to the upper disk were 2 mm. was calculated. In equation (8), as given in the paper by Richardson and Brown (*loc. cit.*), the current to the upper disk is expressed as

$$i = ne \int_{\sqrt{2\frac{eV}{m}}}^{\infty} F(u_0) du_0 \int_0^{\frac{1}{2}\frac{\rho_0}{x_0} \left(u_0 + \sqrt{u_0^2 - 2\frac{e}{m}} \right)} F'(W) dW,$$

where ρ_0 is the radius of the upper disk and x_0 is the distance between the upper disk and the thermionic radiator.

By substituting $\kappa mu^2 = x$ and $y = \sqrt{\kappa m}$, after putting in the values of $F(u_0) du_0$ and $F'(W) dW$, the above equation may be put in the form

$$i = \frac{ne}{\pi^{\frac{1}{2}}} \int_{2\kappa eV}^{\infty} e^{-x} \int_0^{\frac{1}{2}\frac{\rho_0}{x_0} (x^{\frac{1}{2}} + \sqrt{x + 2\kappa eV})} e^{-y^2} dy.$$

For different values of V and $\frac{\rho_0}{x_0}$ curves were plotted showing the relation between i and x . The area of any curve gave the total value of the current to the upper disk for the given potential on this disk. Such procedure showed that between 2 and 3 per cent. error should obtain in the value of the current at $V=0.1$, when the distance was $x_0=2$ mm., and that the error for the same or less distance with a potential of 0.1 volt on the upper disk should be less than 1 per cent. Errors arising from ions striking the guard-ring then should give an experimental value of R too large, but such errors could not account for values of R as large as were obtained with iron.

In addition to the sources of error which we have mentioned there are probably errors of a second order due to the photo-electric effect, to the electric and magnetic fields of the heating current, and to the high potential effect. In the photo-electric effect negative electrons should be given off from the upper disk of platinum. Ladenburg and Markau *, in their investigations, obtained currents from a small platinum disk of the order of 10^{-13} ampere, by the use of intense ultra-violet light from a mercury-vapour lamp. They also showed that the current was larger with the shorter wave-lengths of ultra-violet light. As the positive thermions were obtained at very low temperatures it is quite improbable that the number of photo-electrons emitted could have furnished a current as large as 10^{-13} ampere, which should lead us to expect errors of a very small order in our results owing to this cause.

High Potential Effect.

In the paper by Richardson and Brown (*loc. cit.*) mention was made of a peculiar effect, for which at that time there seemed to be no reasonable explanation. It was named the high potential effect, because of the way in which it first appeared. When a potential of 200 volts was applied to the upper plate, thereby drawing off thermions, the sign of which depended on the sign of the potential, there was, after the removal of the high potential, a thermionic current seemingly of the opposite sign, and this current would seemingly go against a potential as high as 75 volts. Often the imposed effect would remain 24 hours practically undiminished, providing the thermionic radiator were not heated, but when ions were being given off from it the effect decayed rapidly,

* *Verh. der deutschen Phys. Gesell.* x. no. 14, p. 562 (1906).

the rate depending chiefly on the temperature of the radiator and the potential on the upper plate.

It was found later that the quantity of electricity in this effect was not only of the opposite sign to that drawn from the radiator, but that it was also equal in amount. Many other experiments were undertaken which seemed to lead to the unreasonable view that the ions drawn off by the high potential acquired additional energy from the high potential and held it in a potential form on the upper plate, and when the high potential was removed and the radiator was glowing this energy became kinetic.

The effect was ultimately found to be due to the presence of an insulating layer which had developed on the upper disk. The special parts of the apparatus were enclosed in a glass tube fitted on to a brass plate. It was made air-tight by sealing-wax covered over by soft wax. No doubt after a time some of the soft wax condensed on the upper plate, forming an insulating layer thereby, thus making a condenser with the conducting layer on one side missing. The high potential pulled the ions on to this insulating layer and held them, and then when the high potential was removed, these held the ions on the opposite side of the insulating layer. This left a difference of potential between the one side of the condenser and the hot radiator. Thus ions of the opposite sign were pulled across from the hot body until the bound ions on the condenser were neutralized, *i. e.* until the so-called high potential effect had decayed. In one instance the quantity of electricity collected on this condenser in this way was sufficient to charge up a capacity of .2 microfarad to a potential of 1 volt. This required the thickness of the insulating layer to be about that of 20 hydrogen molecules.

The capacity of this condenser was measured directly by the method of mixtures by bringing up to the surface of the upper plate a globule of mercury on the end of a copper wire. The mean of several determinations showed the capacity to be 2.1×10^4 E.S.U. per sq. cm. of surface.

To establish that the insulating layer was something condensed on the upper plate and not due to an oxide or impurity emitted from the platinum itself, several other experiments were carried out. Entirely new apparatus showed the effect after continued heating. When the upper plate was cleaned it would disappear for a time, but would always reappear in time, even when the purest resistance platinum was used as a thermionic radiator. Also when the surrounding atmosphere was hydrogen there was no essential

difference in the results. The apparatus mentioned here was encased entirely in glass with ground joints. The joints were made air-tight by vaseline. The vaseline is no doubt what formed the insulating layer. For when an entirely new apparatus was made of platinum and glass and the whole was boiled in nitric acid, it was impossible to obtain the effect, even after several days' intermittent heating.

In the work described in our former paper there was probably a greater chance for errors arising from the high potential effect than there was in this work. In some of the series of observations we noted that the current fell off more rapidly than the theory warranted. This would have resulted had the condenser been charged almost to its capacity. Had the capacity been much smaller the errors might have proved serious.

The Charge carried by the Thermions.

It is interesting to note that we can determine from the experimental curves the charge carried by the thermions. To do this, however, we must assume that the translational kinetic energy of the ions has the same mean value as would the molecules of a gas at the same temperature.

In curve 2, to which we have already referred, is given a current-potential curve for the thermions from gold. The area of this curve gives the total energy of all the ions which are emitted at zero potential. By dividing this value by the number of ions, the mean kinetic energy is obtained in the form

$$e\bar{V} = e \times 11 \text{ volt,}$$

then

$$\frac{1}{2}nm\bar{v}^2 = n\bar{V}e, \quad (1)$$

where n is the number of molecules of a gas in unit volume at 0° centigrade and one atmosphere pressure.

By the well-known formula of the kinetic theory of gases

$$nR\theta = \frac{1}{2}m\bar{v}^2, \quad (2)$$

where

$$R = 9.6 \times 10^{-17} *.$$

From equations (1) and (2) we find the value of the

* See Jeans, 'Dynamical Theory of Gases,' p. 113.

charge to be

$$e = \frac{3}{2} \cdot \frac{R\theta}{\bar{V}} = 1.55 \times 10^{-20} \text{ E.S.U.}$$

This value of the charge should be in error by not more than 20 per cent. It is gratifying to know that it is within about 20 per cent. of the values of the charge carried by ions from other sources. However, it must be recognized that our observations do not furnish an absolute measurement of the charge, because in the determination of the kinetic energy, the value of ne was assumed to be of the same value as in electrolysis. On the other hand, if our interpretation of the kinetic energy of the thermions is accepted, then the value of the charge must also be accepted.

The Nature of the Positive Thermions.

The evidence furnished in this paper does not give us any proof as to the nature of the positive ions. However, we may point out certain conclusions which are consistent with the results. The facts that the kinetic energy and the ratio

of $\frac{e}{m}$, as measured by J. J. Thomson *, O. W. Richardson †,

and Mr. E. R. Hulbirt ‡, are approximately the same, lead us to infer that the positive thermions are essentially of the same structure, if not identically the same. In addition it is seen from the summary table that for the same area of emitting surface and at the same temperature, the thermionic current has about the same value in all cases in a freshly heated surface. The first mentioned facts may be in agreement with the view suggested by Prof. O. W. Richardson, that the positive thermions may be some impurity such as the sodium atom. However, it does seem a little unlikely that such an impurity in all matter should occur in about the same quantity in all metals.

The fact that the thermionic current decays with heating in all the elements studied is added evidence against the view that the positive thermions are not positive electrons essential to the structure of the atom. Probably the strongest evidence heretofore proposed against such a view is in the observations

* 'Conduction of Electricity through Gases,' 2nd ed. p. 148.

† Phil. Mag. Nov. 1908, p. 740.

‡ Paper not yet published.

of Prof. Richardson*, showing that the positive ionization can be greatly renewed in an old wire by the presence of oxygen, whereas the negative ionization is unaffected.

The author considers the gas theory, also suggested by Prof. Richardson †, as promising as any in sight. In order to agree with his determinations of $\frac{e}{m}$ he suggests the possibility of the positive ions being N_2 , CO , and O_2 .

There is no certain proof that these ions might not receive their charges from chemical action, which takes place when the metal is heated.

It may be possible by studying the products given off by a freshly heated body to determine which are the positive thermions. One method would be to separate the charged particles in sufficient quantity from the uncharged particles and to examine the spectrum. Another method would be to hunt for those spectral lines which are common to the products emitted by all the different thermionic radiators. The chief objection that may be urged against the last method lies in the fact that most of the metals, as shown by Delachanal ‡ give out many gases.

In conclusion I again take this opportunity to express my highest regard for the encouragement and help received from Prof. O. W. Richardson.

Palmer Physical Laboratory,
Princeton University.

LXVII. *A Search for the Heavier Gases of the Helium Group in Minerals.* By C. F. HOGLEY, A.R.C.Sc., B.Sc., Imperial College of Science §.

PART I. *Argon in Malacone.*

UP to the present there has been considerable uncertainty as to the amount of argon contained in the mineral malacone. Several experimenters || have found that the quantity of argon is appreciable and in excess of the quantity of helium

* Phil. Trans. Series A, vol. 207, p. 1 (1906).

† Phil. Mag. ser. 6, Nov. 1908, p. 740.

‡ Science Abstracts, vol. xii. p. 249 (1909).

§ Communicated by the Hon. R. J. Strutt, F.R.S.

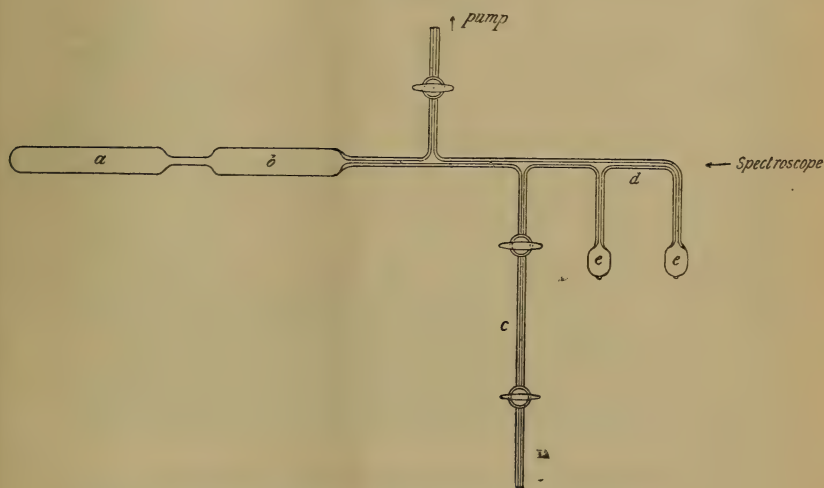
|| (i.) Von Antropoff, *Zeitsch. f. Elektroch.* 1908, vol. xiv.

(ii.) Kitchin & Winterson, Chem. Soc. Trans. vol. lxxxix. 1906, p. 1570.

contained in the mineral, a result which Prof. Strutt has failed to confirm*. The experiments described in this paper were designed with the intention of obtaining some evidence on the subject which may be considered conclusive. The great source of error likely to occur in dealing with argon is by contamination of the gas with air, and the apparatus used was constructed to eliminate the possibility of this error. The liberation, purification, and examination of the gas took place in the same tube, from which air was entirely excluded.

The apparatus shown in fig. 1 was exhausted until only a

Fig. 1.



- a.* Hard glass tube containing mineral and copper oxide.
- b.* Phosphoric oxide to absorb moisture.
- c.* Inlet tube for air. Volume between taps 0.10 c.c.
- d.* Vacuum-tube.
- e-e.* Bulbs containing liquid alloy of sodium and potassium.

very feeble discharge would pass through the vacuum-tube, showing a faint spectrum of hydrogen. (The hydrogen was probably due to decomposition of the hydroxides of sodium and potassium, formed in sealing on the bulbs containing the electrodes, or from the glass itself.) The pump was closed when it seemed certain that nothing but hydrogen remained

* The Hon. R. J. Strutt, Proceedings Roy. Soc. A. vol. lxxx. 1908, note p. 593.

in the apparatus, and the mineral was heated for about an hour. Any hydrogen was taken up by the copper oxide and the water was absorbed by the oxide of phosphorus. The remaining gases, excepting the inert elements, were absorbed by the sodium-potassium electrodes. In all the experiments, the residual gas showed the brilliant yellow helium glow in the vacuum-tube, and the spectroscope indicated little or no trace of argon, the gas being practically pure helium. (The bright orange argon line could be distinguished only faintly, whilst the red and blue lines were invisible in most cases.) To obtain some idea of the quantity of argon present with the helium, a cubic millimetre of argon was admitted into the apparatus containing the residual gas. This was done by admitting 0.1 c.c. of air by means of the side tube, shown in fig. 1, and absorbing all except argon by the sodium-potassium electrodes. It was found that this small quantity of argon gave a spectrum extending beyond the helium spectrum into the red, and also showing well defined lines in the violet, clearly proving that the quantity of argon present in the helium before admitting the air was much less than 1 cub. mm. The quantity of mineral used for each experiment was 10 grams, and at the end of the experiment the helium was pumped off and measured. The volume was roughly 0.10 c.c. Thus the volume of argon was shown to be less than one per cent. of that of the helium obtained from malacone.

In one experiment the helium was liberated by decomposing the mineral with potassium bisulphate, and in the others by simple heating, but there was no appreciable difference in the results.

The results of the investigation show conclusively that argon is contained in this mineral in only minute quantities even in comparison with the helium. It is true that the gases are not entirely expelled by simple heating, but this would not account for the great discrepancies between the results of the various experimenters, and can scarcely affect the general conclusion.

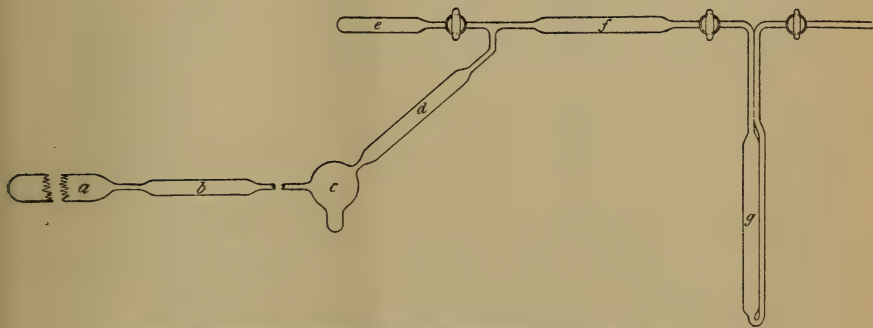
PART II. *Krypton in Fergusonite.*

It is mentioned in a paper by Travers, Senter, and Jaquerod (Phil. Trans. A. vol. 200. 1903, p. 174) that an appreciable quantity of krypton has been found in the mixed helium obtained from cleveite. If the presence of this gas were in any way connected with the radioactive properties of the mineral, there should be no reason to suppose that it would be present in one strongly active mineral rather than in another. The mineral used for this investigation was

fergusonite, which gives off about 1 c.c. of helium per gram on heating, and is more easily obtained than cleveite.

The apparatus used (fig. 2) was exhausted whilst the charcoal bulb was continuously kept hot to expel all traces of air. It was then rinsed out with oxygen supplied by a side tube containing potassium permanganate, and again exhausted. The hard glass tube containing the mineral (about 140 grams) was heated in an ordinary combustion-furnace, and the gases conducted over suitable absorbents and

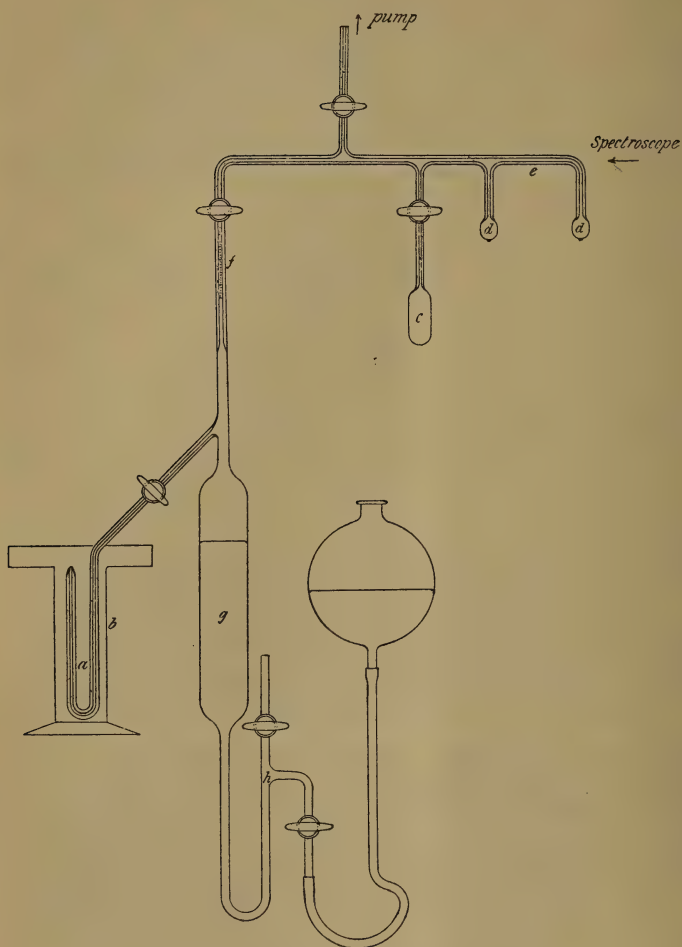
Fig. 2.



- a.* Hard glass tube containing 140 grams of fergusonite.
- b.* Hot copper oxide for absorbing hydrogen.
- c.* Bulb for receiving condensed moisture and drainings from *d.*
- d.* Potassium hydroxide to absorb carbon dioxide.
- e.* Side tube for producing oxygen.
- f.* Phosphoric oxide to absorb last traces of water vapour.
- g.* Bulb containing charcoal for absorbing gases at low temperatures.

finally passed through charcoal cooled by liquid air. This absorbed all but the helium which was pumped away. When no more helium could be obtained, the heating was stopped and the charcoal heated to expel the absorbed gases, which were collected through the pump. The nitrogen was removed by sparking with excess of oxygen over hydrated potassium hydroxide for about half-an-hour. The excess of oxygen was removed by melted phosphorus and the residue introduced into the apparatus shown in fig. 3 (p. 676), where the last traces of all gases, except the inert gases, were removed by means of the sodium-potassium electrodes. Spectroscopic examination showed the presence of helium and argon in the tube. Before the presence of krypton could be investigated with certainty, the helium had to be removed. This was done by absorbing the argon, and heavier gases if present, with charcoal cooled by liquid air and pumping away the helium. After closing the pump and warming up the

Fig. 3.



- a.* Tube for introducing gas into apparatus.
- b.* Mercury trough.
- c.* Charcoal bulb.
- d-d.* Sodium potassium electrodes.
- e.* Vacuum-tubes.
- f.* Graduated capillary for measuring gas,
- g.* Large bulb into which gas is allowed to expand. The gas is compressed into "*f*" and measured.
- h.* Air trap.

charcoal, the tube was found to contain argon only. No lines but those due to argon could be observed in the spectrum of the gas. The volume of the argon was measured by the McLeod gauge and found to be 3 cubic mm., a quantity which could be well accounted for by contamination with air in the course of transference from one vessel to another.

As krypton is very easily absorbed by charcoal in liquid air, it was thought advisable to make sure that none had remained in the large bulb used in the first part of the investigation. This bulb was maintained at a temperature of 220° C. for about 6 hours by means of an electric heater, and the expelled gas collected. The gas was practically all carbon dioxide, and after absorbing most of it with hydrated potassium hydroxide, the residue was introduced into the vacuum-tube and the discharge passed until all trace of carbon dioxide was removed. The spectrum of the remaining gas was compared with that of krypton, but no agreement was found in any of the lines.

The amount of helium given off by the mineral was in round numbers 100 c.c.s. Had there been present with it $\frac{1}{10}$ of a c.c. of krypton, the spectrum of the latter would have been conspicuous when the residue was examined: hence the experiment points to the conclusion that krypton, if present at all, which is indeed doubtful, must amount to less than one part in a million of the helium content. As opportunity affords it is intended to carry on the investigation with respect to other strongly radioactive minerals, as it is not to be expected that definite results can be drawn from the behaviour of one mineral.

In conclusion I beg to express my thanks to Professor Strutt for suggesting this research and for his valuable advice during its procedure.

LXVIII. *Radioactive Minerals in Common Rocks.* By J. W. WATERS, B.Sc., Imperial College of Science, South Kensington*.

IT has been shown by Rutherford and Strutt that a knowledge of the amount of uranium and also the amount of helium contained in a rock or mineral enables us to form a minimum estimate of its age. In making accurate measurements of these quantities it is much more convenient to experiment upon substances comparatively rich in radioactive constituents. Prof. Strutt has found that the phosphatic nodules and phosphatized bones which occur in

* Communicated by the Hon. R. J. Strutt, F.R.S.

sedimentary rocks afford suitable material for the purpose. The object of the following investigation was to find in what minerals the radio-activity of igneous rocks is most concentrated, and, if possible, to separate them with a view to their use as material for finding the age of the rocks in which they occur.

The first material that was treated was rotted Cornwall granite from which kaolin and mica had been removed at a China-Clay Works. This was passed through a fine sieve and then washed with a gold-miner's pan to reject the lighter constituents, such as quartz and felspar. When the bulk was sufficiently reduced the lighter minerals which remained were removed by means of a heavy liquid (bromoform, spec. grav.=2.9), those minerals whose specific gravity exceeds 2.9 sinking in the bromoform while the lighter ones float on the surface.

The heavy and light constituents were tested for radio-activity by finding their effect on the rate of leak of a gold-leaf electroscope when spread over a tray placed inside the electroscope.

Normal rate of leak	=	4.0	scale divs. per hour.
Increase due to heavy minerals	=	2.5	„ „
„ „ light	=	0	„ „

It thus appears that the radioactivity is concentrated in the heavier minerals; the lighter constituents were therefore rejected and further separation of the heavier minerals attempted. A little magnetite was removed with a permanent magnet and found to be non-radioactive. The residue was mainly tourmaline. This was removed by a comparatively powerful magnetic field produced by an electromagnet. The first part of this that was extracted was not active, but on removing the last traces of magnetic matter by a very powerful field a powder was obtained in which the radioactivity appeared to be concentrated. By concentrating the feebly magnetic mineral by magnetic fractionation a powder was obtained for which the corrected rate of leak was 16 scale divs. per hour. On examination under the microscope this was seen to be distinguished by the presence of a number of small opaque fragments in some of which an hexagonal shape could be detected. The other minerals present, being of density not much greater than 2.9, were washed away by agitating the powder in bromoform. Advantage was also taken of the small size of the particles to obtain the active mineral pure by sifting.

About $\frac{1}{2}$ gram of the mineral was obtained from 2 or 3 lb.

of the original material. The leak due to the final product was 2.5 scale divs. per minute or nearly 40 times the normal leak of the electroscope. Spectroscopic examination of the mineral showed that titanium was present in large quantity. The mineral was therefore considered to be anatase or rutile.

Some of the mineral was tested for helium, and was found to contain about 1.5 c. mm. per gram.

The same methods were applied to a gneiss from the Inner Hebrides and the radioactivity was found to be concentrated in a very heavy, non-magnetic mineral, occurring in very small crystals. Examination under the microscope showed this to be zircon; this was confirmed by spectroscopic examination which showed zirconium to be present. About $\frac{1}{10}$ gram of the mineral was obtained from 1 lb. of the rock.

These experiments are being continued on other rocks.

I desire to thank Prof. Strutt for suggesting these experiments and for his interest and advice throughout.

LXIX. *Proceedings of Learned Societies.*

GEOLOGICAL SOCIETY.

[Continued from p. 440.]

March 10th, 1909.—Prof. W. J. Sollas, LL.D., Sc.D., F.R.S.,
President, in the Chair.

THE following communications were read:—

1. 'Some Notes on the Neighbourhood of the Victoria Falls (Rhodesia).' By Thomas Codrington, M.Inst.C.E., F.G.S.

The author gives an account of the way in which the basalt lies in the valley of the Zambesi below and above the Victoria Falls, and points out how that has determined the features of the river. The basalt through which the Batoka Gorge has been cut appears in the course of the Zambesi for 2 miles above the Victoria Falls, causing rapids over rocky bars between many islands. It then disappears, and the river above flows quietly between alluvial flats for 5 miles, the basalt being traceable here and there below the water until above Candahar Island it again rises and constitutes the bed of the river from bank to bank, causing rapids that extend for many miles up the river. Attention is called to a wide tributary valley which joins the main Zambesi Valley on the east at the Falls; and to the Maramba River flowing down it, and its fluvial deposits. The discovery of stone-implements and artificially-worked stones in the gravel and the bed of the Maramba is noted.

The difference, observable in the artificially-worked stones from defined areas on both sides of the Zambesi, is shown by a comparison of numerous specimens collected by the author. The very large majority of flakes and flaked stones having no trace of design, over those that can be considered as implements, however rough, suggests that the manufacture of stone tools on a large scale was here carried on for use in the sand-covered country on both sides of the Zambesi, where there is no stone; and the difference, in the proportion of sharp unworn specimens to those apparently waterworn, seems to show that while many were made on the ground, the flakes and refuse of the making of many more were brought down from the tributary valley on the east.

The author considers that there is no evidence as to the age of the implements found near the Zambesi; none have been discovered in gravel that belongs to that river, and the presumption appears strong that they were made from quartzite found on the surface since the Batoka Gorge was eroded.

2. 'A Contribution to the Petrography of the New Red Sandstone in the West of England.' By Herbert Henry Thomas, M.A., B.Sc., F.G.S.

The paper is supplementary to that which dealt with the mineralogical composition of the Pebble-Bed, and was published in vol. lviii. of the *Quarterly Journal* (1902) p. 620. The results obtained are based on the microscopic investigation of a large number of samples taken from various New Red horizons and localities in Devon and Somerset.

The author gives a list of minerals identified, and tables showing their distribution. In the description of certain mineral species, it is suggested that anatase occurs both as detrital crystals and as crystalline groups formed in the rocks since their deposition. The forms presented by grains of staurolite as well as certain crystals of tourmaline with an unusual habit are described.

It is recognized that the various divisions of the New Red Sandstone, namely the Lower Breccias and Sandstones, the Lower Marls, the Pebble-Bed, and the Upper Marls and Sandstones, although linked together by a general similarity of mineralogical composition, present physical and mineralogical differences indicative of variations in the source of supply and conditions of deposition.

The rounding of the grains is most complete (millet-seed) in the sandstones of the Lower Breccias and Sandstones, a feature continued, but in a less striking manner, into the overlying division.

With regard to the vertical and horizontal distribution of minerals, it is found that staurolite is abundant in the Lower Breccias and Sandstones of the extreme south of Devon, but less plentiful northwards; that garnet is present in all the New Red rocks of North Devon and Somerset, but in South and Central Devon only occurs in the Lower Marls and in the Upper Marls and Sandstones.

FIG. 7.

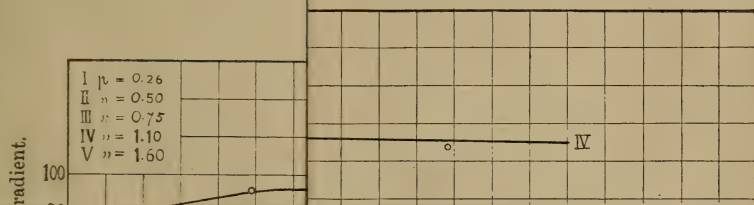


FIG. 4.

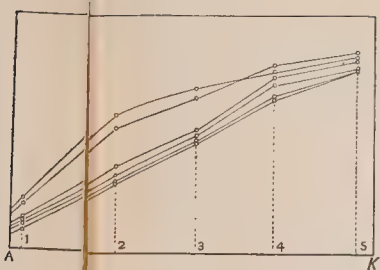


FIG. 5.

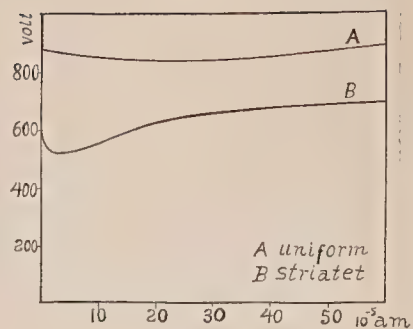


FIG. 6.

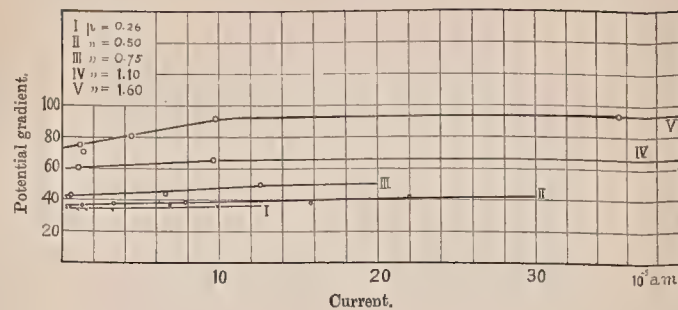


FIG. 7.

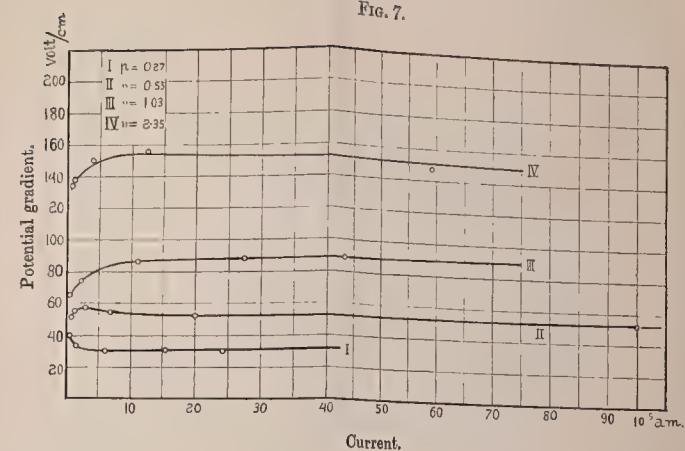
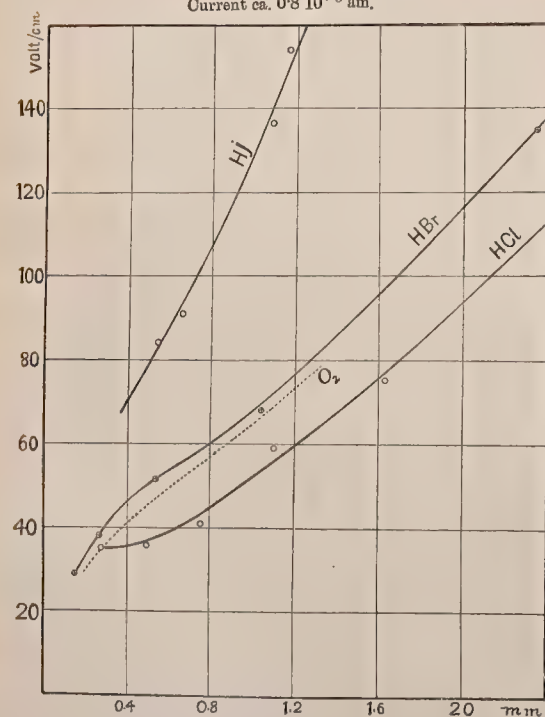
FIG. 8.
Current ca. $0.8 \cdot 10^{-5}$ am.

FIG. 9.

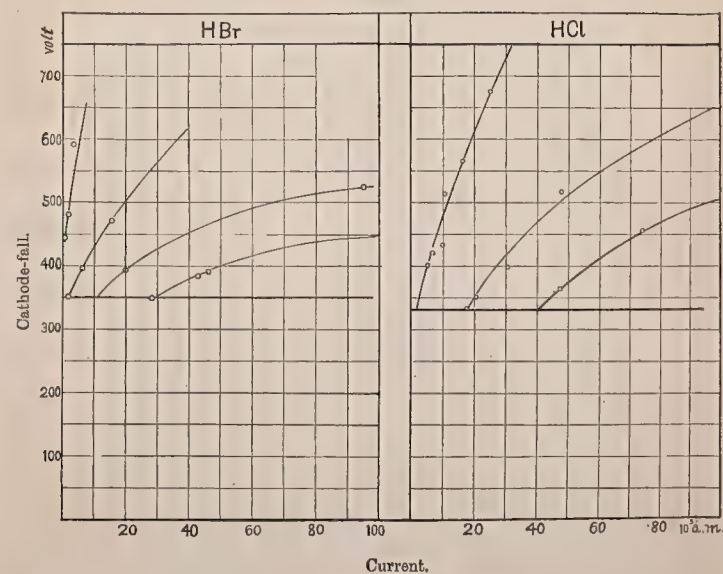


FIG. 10.

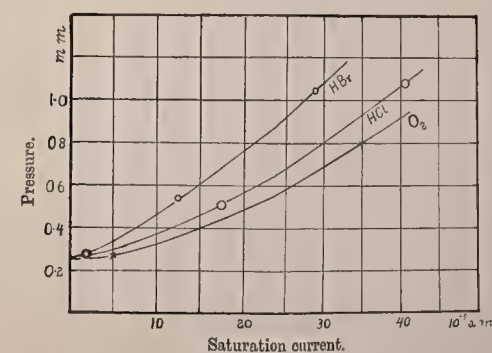


FIG. 11.

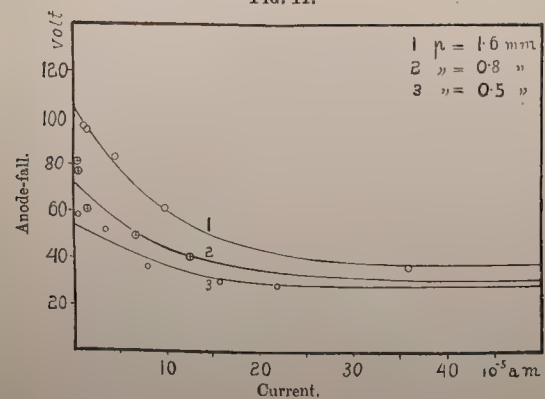


FIG. 12.

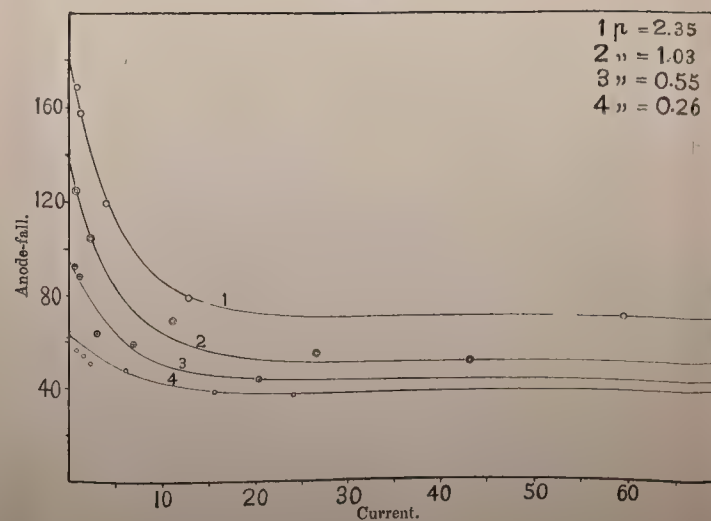


FIG. 13.

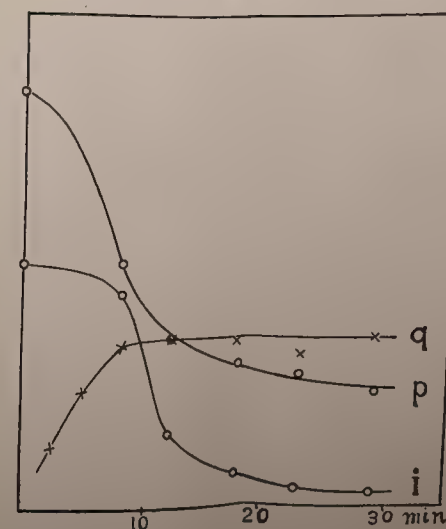


Fig. 4.

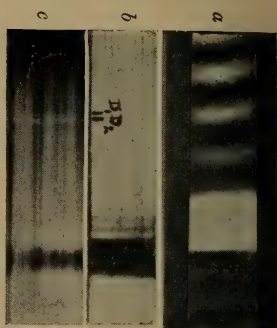


Fig. 1.



Fig. 2.

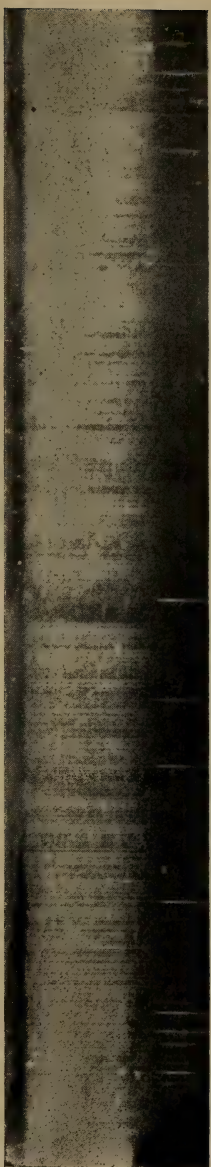


Fig. 3.



RED.

D.D.

GREEN.

BLUE.

IRON.



FIG. 1.

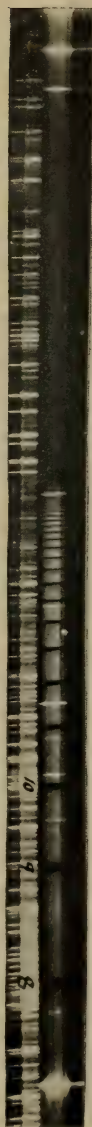


FIG. 2.

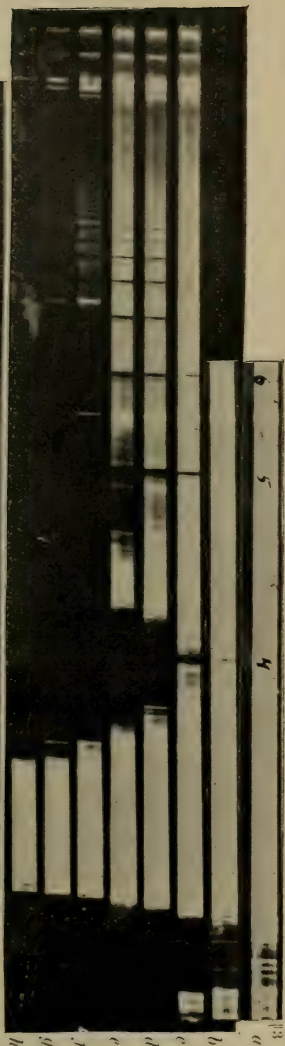


FIG. 3.

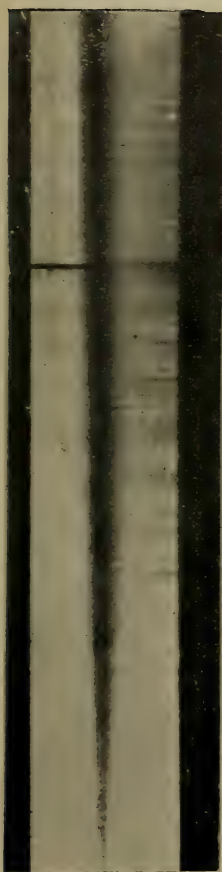
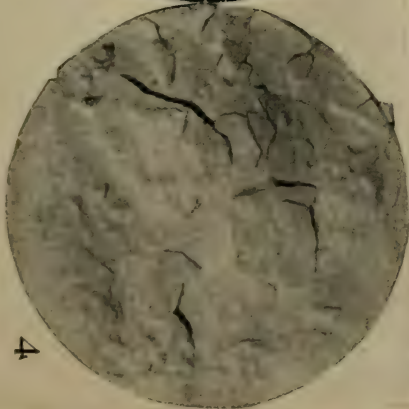
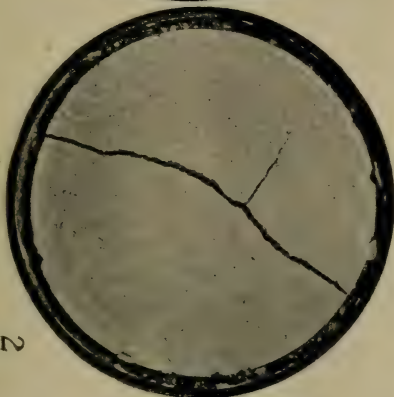
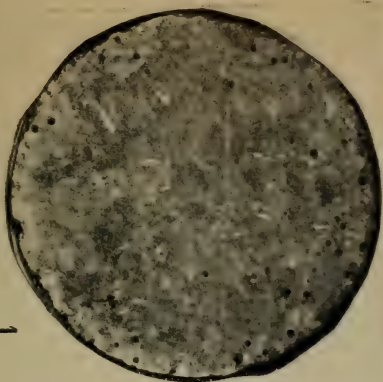
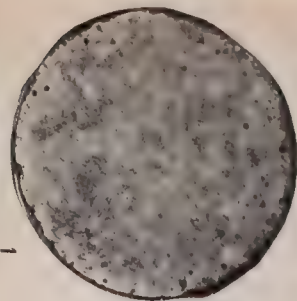
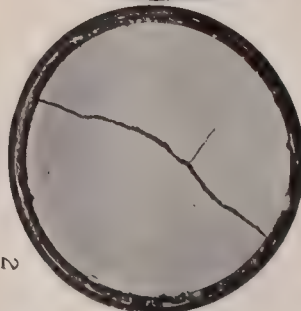


FIG. 4.

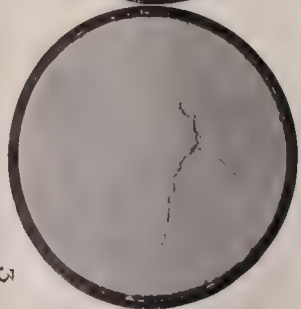




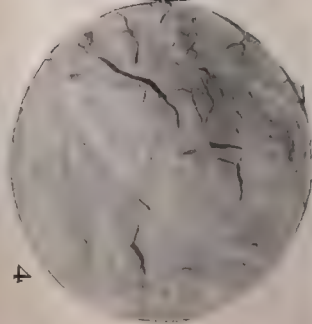
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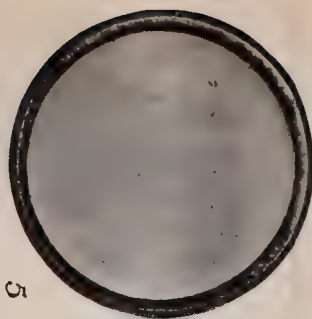
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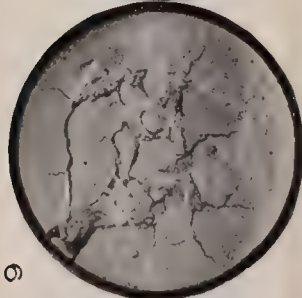
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7



8

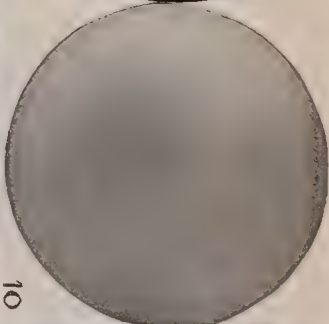
WILDERMAN.



10



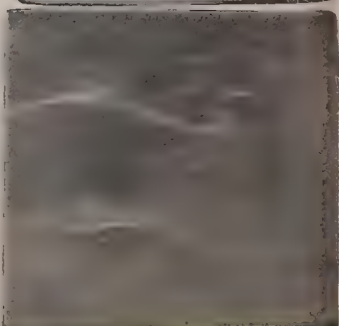
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12



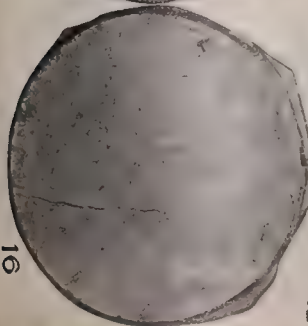
13



14



15

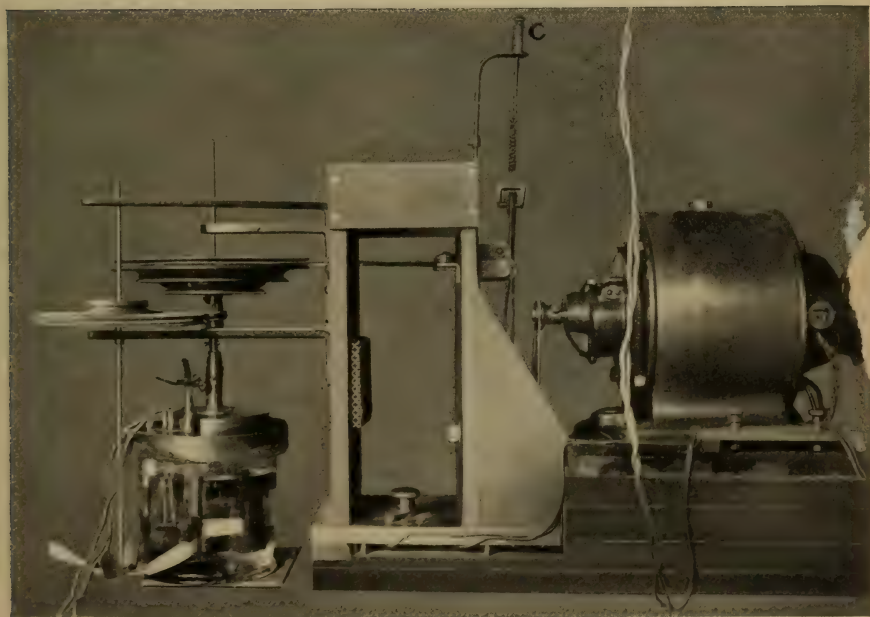


16

FIG. 1.



FIG. 2.



THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[SIXTH SERIES.]

NOVEMBER 1909.

LXX. *The Kinetic Energy of the Ions emitted by Hot Bodies.*
—II. By O. W. RICHARDSON, M.A., D.Sc., *Professor of Physics, Princeton University**.

IN the first part of this paper† the writer showed how valuable information concerning the initial kinetic energy of the thermions could be obtained from observations of the way in which they spread out in a uniform electric field. The method employed was to apply the electric field between two parallel conducting planes. In one of these lay a narrow straight platinum strip which could be heated by means of an electric current. The opposite plane was movable by small measurable amounts in a direction perpendicular to the length of the strip. It contained a narrow slit whose edges were parallel to those of the strip, and thus perpendicular to the direction of motion. The proportion, which passed through the slit, of the total number of ions received by the plane was measured for a series of positions of the former.

It was pointed out that the apparatus used in the above investigation had been designed for another purpose, and was far from fulfilling several of the geometrical conditions laid down by the theory. The results obtained could, therefore, only be regarded as approximate. The present experiments have been carried out with a new apparatus involving the same principles, but made larger and constructed so as to satisfy accurately the geometrical conditions. It is believed

* Communicated by the Author.

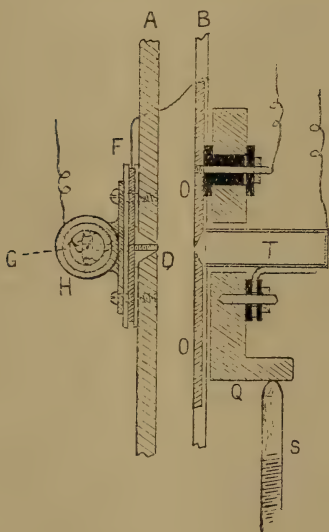
† Phil. Mag. [6] xvi. p. 890 (1908).

that difficulties of that character have been overcome, and the theory has been subjected to a much more exact test than before. The following experiments form a very satisfactory confirmation of the theory, but only under certain rather restricted conditions. It will be seen that under certain other conditions, which will be considered later, the curves obtained are not at all what was expected. The discrepancy, however, is probably due to causes lying outside of the present theory.

The Apparatus used.

The apparatus previously used has already been fully described*, so that it will only be necessary to specify the changes which have been made. The essential parts are shown in detail in figs. 1 and 2, in which the same parts are

Fig. 1.

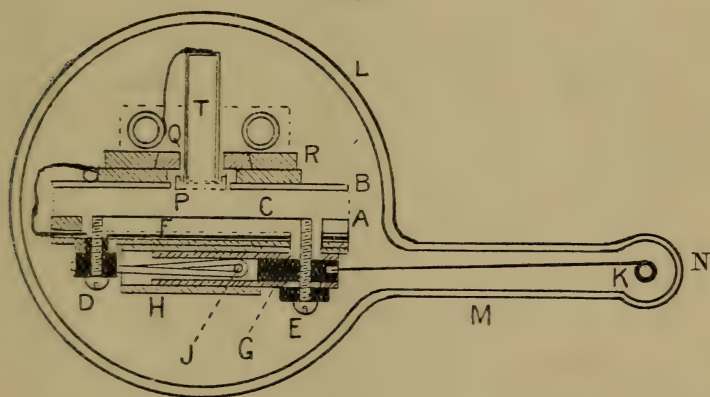


denoted by common letters. These figures are drawn to scale, natural size, and represent respectively sections by vertical and horizontal planes perpendicular to the conducting planes A and B. The latter were the surfaces of brass plates of the thickness shown; their length, which is not shown by the figures, was about 20 cm., and they were insulated from each other by hard rubber blocks, into which they were

* Phil. Mag. [6] xvi. p. 740 (1908).

screwed, at the ends. The platinum strips C (fig. 2) were cut by means of a dividing engine, so that their edges were accurately parallel. They were 0.02 cm. broad in some experiments and 0.04 cm. in others. In order to cut the thin platinum-foil it was cemented on to plate-glass with shellac and afterwards loosened with alcohol. The strips were welded to stout platinum plates about 2 mm. square, and subsequently boiled in nitric acid and distilled water. The plates were then soldered to the screws D, E. As the plates were of substantial thickness compared with that of the strip it was believed that this method would prevent the solder from diffusing into the hot strip during the experiments. When thus mounted the strip lay in the front of a narrow slit in the plate A of the shape shown by the transverse section in fig. 1. The narrowest part of this slit was about 0.05 cm. wide, and was a little behind the plane surface (about 0.01 cm.) as the edges were cut away, so as to give a

Fig. 2.



very slight taper for about 0.1 cm. distance. The strip was set so as to be exactly parallel to the plane A, but the least bit behind it, probably about 0.005 cm. The object of these precautions was to prevent the ions emitted by the sides and back of the strip from straying into the field between the plates A and B. To the same end the slit was backed by an insulated plate F, which could be connected by the wire shown with the plate B. This ensured that whenever there was a field between A and B there was a field of the same sign and of somewhat greater magnitude tending to drag back the ions from the back of the strip.

Trouble was experienced at first in keeping the strip C in

the plane in which it was supposed to lie, on account of the thermal expansion of the platinum. This was ultimately quite overcome by the device shown (chiefly) in fig. 2. One end of the strip was held by the fixed screw D insulated from the plate A by vulcanized fibre shown black in the figures. The screw E which held the other end was rigidly fixed in a bit of vulcanized fibre driven into the brass cylinder G, from which it was insulated as shown by the figure. This cylinder slid with a pin and slot guide in the outer brass cylinder H which just fitted it. It was pulled back by a light indiarubber band which passed over the pin J. This passed through the cylinder and also over a hook (not shown) attached to the back of the plate A. The cylinder could be pulled forward by a thread passing over the brass rod shown in section at K. The whole of this part of the apparatus was enclosed in a wide vertical glass tube L and the thread passed down a horizontal side tube M. The rod K passed up another vertical tube N, which carried a mercury sealed ground stopper to which the rod K was rigidly attached. By turning the stopper the string could be wound or unwound round the rod, and so the strip C could be kept just tight from outside the apparatus. Whenever the heating current was reduced it was of course necessary to unwind some of the string, otherwise the strip would snap in cooling.

The opposite plane consisted of a fixed plate B, which contained a rectangular opening P (fig. 2). This extended the whole length of the plate B in a direction perpendicular to the section in fig. 2. The opening contained two plates O, O (fig. 1) which were accurately in the same plane and flush with the fixed plate B. They were almost as wide as the opening, but not in contact with it at any point. The part of them which is shown in fig. 1 is distinguished from the plate B by shading. They were rigidly and metallically connected together and insulated from the plate B. They were supported by the rigid brass carriage Q (both figures) which was pushed by the screw S along the ways R (fig. 2) which were attached to the fixed plate. The edges of the slit between the plates O, O were cut away as shown, so that all the ions which passed through the slit should pass into the copper box T. The number of ions reaching the plates and reaching T were measured simultaneously in the manner previously described*. It will be observed that only the ions reaching the middle centimetre of the plate B were received by either the testing plates or the slit, so that the error arising from the finite length of the hot strip would be

* Phil. Mag. [6] xvi. p. 740 (1908).

small. The ions received by the testing system also were emitted by the hottest and most uniformly heated part of the strip, and irregularities in the electrostatic field due to the mounting of the ends of the strip were eliminated. The strip was shunted by a high resistance, and the potentials were applied at a sliding contact near the mid-point of this. The adjustment could be tested by the change in the thermionic current under zero potential when the heating current was reversed*.

The glass tube L was closed at each end by brass plates cemented with sealing-wax. The necessary connexions to the exterior were made in an obvious kind of way and presented no novel features. To exhaust the apparatus a Gaede mercury-pump was used and worked continuously. This enabled the screw S to be connected to the head outside by an axis passing through a mercury-sealed rubber and brass joint, in which it turned freely. The air leakage was always small compared with the gas evolved when the strip was heated. The connexions to the pump and McLeod gauge were made with short wide tubes. Even at the highest temperature the pressure could always be kept below $\cdot 004$ mm., and in some of the experiments at lower temperatures the pressure registered on the McLeod gauge did not exceed $\cdot 0004$ mm. The evolution of gas when the strip was heated soon reached a steady condition, when further heating seemed to produce little or no diminution in the rate at which it was given off. The screw head, which was below the main apparatus, was provided with a toothed wheel device for registering the whole number of turns, and was also divided into tenths so that fractions could be measured. One complete turn was equal as before to $\cdot 0635$ cm.

Zero Electric Field.

It was pointed out in the first part of this paper† that in the particular case when the plates A and B are at the same potential, so that there is no electric field between them, the value of the ratio of the current through the slit to that which reaches the plates is given by the very simple relation

$$I/I_0 = \left(\frac{z^2}{z^2 + x^2} \right)^{\frac{3}{2}}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where I is the ratio when the slit is in any position distant x from the central symmetrical position, for which I has the maximum value I_0 , and z is the distance between the strip

* See Richardson and Brown, *Phil. Mag.* [6] xvi. p. 353 (1908)

† *Loc. cit.* Equation (14), p. 908.

and the slit and plates. I_0 is thus the value of I when $x=0$. This formula presumes that the widths of the strip and slit respectively are negligibly small, a condition which was satisfied in the experiments.

For the deduction of formula (1) it will be sufficient to refer to the former paper. It follows directly from the assumption that the electrons leaving the metal, do so with a distribution of velocity, which is identical with that given by Maxwell's law for the particles of a gas of equal molecular weight leaving the same surface at the same temperature. The fact that the normal component of the velocity of the escaping electrons is that required by Maxwell's law has already been proved by Richardson and Brown*. If the present experiments confirm formula (1) it will be legitimate to conclude that the velocity components parallel to the emitting surface also obey Maxwell's law, and the observed disagreement will set a superior limit to the amount of deviation from the requirements of the theoretical law.

The apparatus used in this part of the investigation was not quite the same as that shown in figs. 1 and 2. The slit D was not cut in the plate A. Instead of lying in the slit the strip C was 1 mm. in front of the conducting plane A, and it was, therefore, only 4 mm. distant from the plane B. Otherwise the arrangements were the same as those described above. It was intended at first to have the strip mounted in front of the plane in this way throughout the investigation, but it was found that when a difference of potential was applied between A and B the experimental curves instead of having only one maximum had wings on each side of it. These were clearly due to ions from the back of the strip being drawn out by the electric field. The arrangement with the slit D was adopted to avoid this effect and was found to be successful. In the case where no electric field was applied there could be no objection to the strip being in front of the plane. The results of the experiments with the apparatus arranged in this way, which were made with great care, have been retained, and are those which are described in detail below. In order to be quite sure, however, fresh experiments were made after the slit had been constructed, and the position of the strip changed. These were found to agree satisfactorily with the results obtained with the first arrangement.

The temperature of the strip was regulated by keeping its resistance constant. The observed resistances also served to measure the temperature, the resistance corresponding to

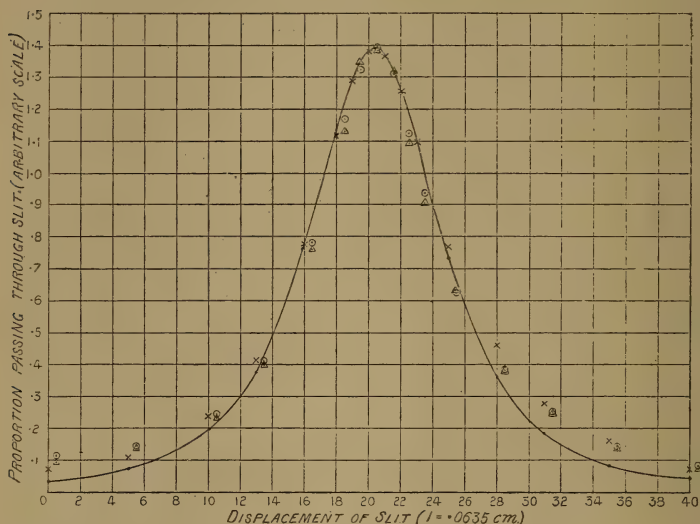
* Phil. Mag. [6] xvi. p. 353 (1908).

the melting-point of potassium sulphate being subsequently observed and used as a fiducial point. It was not possible to vary the range of temperature over which observations were taken much; since at lower temperatures complications were caused by the presence of positive ionization in amounts comparable with that of the negative, whilst at temperatures slightly higher than those used the negative ionization became too big to be measured on the electrometer when all the available capacity was inserted. In addition to this the theoretical conditions are vitiated in the case of large currents as the theory neglects the dynamical action of the electrons on each other after they have left the metal.

The writer never succeeded in getting the conditions so that the proportion of the thermionic current which passed through the slit was independent of the direction of the primary heating current. No matter how the earthing-point in the shunt was adjusted the ratios were all invariably from ten to twenty per cent. greater with the heating current in one direction than in the other. There are two effects depending on the direction of the heating current which are not altogether eliminated in the apparatus used. These are the effects of the electrostatic force parallel to the length of the strip arising from the fall of potential required to drive the heating current and the magnetic force in a perpendicular direction arising from the same current. Both of these will give the ions a drift parallel to the slit which will reverse in direction with the current. This ought not to make any difference provided the slit, the plates, the box-shaped electrode, and the strip are perfectly symmetrical about a vertical plane down the centre of the apparatus. These conditions were satisfied, so far as was possible with an apparatus of this kind in a reasonable time, but there was always a small amount of asymmetry arising from defective mechanical construction. There is no doubt that this explanation is correct, as there was no disagreement within the limits set by the errors of observation in the relative values of the ratios for different positions of the slit in the different experiments. This will be brought out quite clearly when the actual experimental data are tabulated (see fig. 3, p. 688). Formula (1) in fact only involves the relative values of the ratios, so that it will hold whether asymmetry of this kind occurs or not. For this reason the further refinement of the mechanical structure of the apparatus was not proceeded with, as it was felt that it would involve an amount of labour and time which would be quite incommensurate with the benefit to be expected. The point is that this asymmetry, after all, is not

very big, and there are many other points of a much more pressing character which can be satisfactorily investigated with the apparatus in its present state of development.

Fig. 3.



The results of a typical experiment will now be given:— The electrometer suspension was a quartz fibre, and the needle was charged to a potential of 80 volts. The instrument then gave a deflexion of 437 divs. for a volt, and the spot was a fine line about one-fifth of a millimetre wide, so that tenths of a division were read. It is necessary to use a sensitive electrometer, otherwise the back electromotive force due to the charging up of the plates vitiates the results. The capacity attached to the slit-electrode was 0.01 microfarad, that attached to the plates 0.3 microfarad, and the electrometer and its connexions had a capacity of 180 cm. The capacity for the slit-readings was therefore $9 \times 10^3 + 180$ cm., and that for the plates and slit together $27 \times 10^4 + 9 \times 10^3 + 180$ cm. The temperature when the readings were taken with the current direct was 1034°C. , and the pressure about .009 mm. of mercury. When the current was reversed the temperature was 1069°C. , and the pressure varied from .003 to .005 mm., the average pressure being about .004 mm. The temperature estimations are probably only accurate to within about 20° . The values found are given in the subjoined table.

1. Position of Screw. Turns.	Current Direct.			Current Reversed.		7. Means of (4) and (5).	8. Ratio theoretically predicted.
	2. Slit Deflexion.	3. Deflexion for Slit and Plates.	4. Ratio of Slit to Total.	5. Slit Reading.	6. Ratio of Slit to Total.		
0	7.5	100	.075	10.7	.144	.11	.04
5	11.0	99	.111	8.6	.145	.145	.084
10	23.6	98	.241	12.6	.178	.27	.22
13	40.0	96	.416	19.1	.302	.46	.425
16	73.4	94.2	.779	30.0	.509	.87	.86
18	105	94.4	1.114	53.0	.964	1.28	1.27
19	119.5	92.8	1.288	83.5	1.446	1.46	1.44
20	126.2	91.8	1.377	86.4	1.635	1.55	1.54
21	125	91.6	1.365	84.5	1.719	1.49	1.52
22	114.2	91	1.258	78.8	1.618	1.32	1.41
23	98	89.4	1.095	57.7	1.389	1.125	1.22
25	68.5	89	.770	39.4	1.154	.773	.827
28	41.0	88.6	.463	24.5	.776	.47	.40
31	25.8	88.8	.293	16.2	.478	.296	.21
35	24.8	86.5	.280	8.6	.313	.17	.097
40	14.0	87.6	.162	4.5	.177	.087	.045
	6.6		.075		.099		

Columns 1 to 4 refer to a set of readings with the current in one direction, called direct, and columns 5 and 6 to a set in the opposite direction. Column 1 indicates the relative displacement of the slit as measured on the screw-head. Columns 2 and 5 give the readings in scale-divisions due to the thermionic current through the slit in 30 seconds. Column 3 gives the deflexion of the electrometer after the condenser attached to the plates has been connected with the electrometer. The numbers are therefore proportional to the charge received by both the slit and the plates in the same time-interval. The numbers in column 4 are obtained by dividing the numbers in 2 by the corresponding numbers in 3. Column 6 gives the corresponding quantities for the experiment with the current reversed. These ratios are therefore proportional to the fraction of the total current to the plates which passes through the slit; they are not identical with this on account of the capacities being different. To get the actual fractions it is necessary to multiply the values given by the ratio of the two capacities used in the two cases. In these experiments this multiplier was $102/3102$.

It will be observed that the values in column 6 are throughout somewhat greater than those in column 4. The causes of this want of symmetry have already been discussed. To eliminate it as far as possible the mean of the two series of values are given in column 7, whilst column 8 contains the values of I/I_0 calculated from formula (1) for the particular value, $z = .04$ cm., of the distance between the strip and the plates which occurred in these experiments. It will be observed that the formula does not involve any arbitrary constant except in so far as I_0 , the value of the ratio of the two currents at the central maximum, may be regarded as such. Thus all the other points are determined as soon as the height of the central maximum is given. The agreement between columns 7 and 8 is very satisfactory near the middle of the table when the values of the ratio are considerable; but it is not so good further away from the centre where the ratios are small. The extent of the agreement can be gauged better when the results are exhibited graphically; and it is convenient to postpone further discussion of the results until that has been done.

In order to exhibit the results graphically it is convenient to multiply each series of ratios by a common factor, so that they all have the same maximum point. This enables us to test the assertion made above that the differences which arise when the current is reversed disappear when relative values only are considered. For in that case if each series is brought

to the same maximum by multiplying all the terms in it by a common factor, then all the points of all the different series should lie on a common curve which should be identical with that predicted theoretically. To test this the results of the experiments recorded in the foregoing table have been plotted in fig. 3. The method adopted has been to first plot the numbers in column 4 against those in column 1. The points $\times$ thus obtained are found to lie on a smooth curve with a maximum ordinate of 1.39 at 204 turns. The theoretical curve

$$I = I_0 \left(\frac{z^2}{z^2 + x^2} \right)^{\frac{3}{2}} = 1.39 \left(\frac{z^2}{z^2 + x^2} \right)^{\frac{3}{2}}$$

which passed through this point was then constructed, the distance z during the experiments being 0.4 cm. This is the continuous curve shown in fig. 3. In addition to the set of observations corresponding to the current direct, those with the current reversed are also shown in fig. 3. The values in column 6, however, have not been taken as they stand, but have all been multiplied by a common factor so as to reduce the maximum to 1.39. They can thus be compared with the current direct at the same time. They are shown by the points marked thus $\odot$ in fig. 3. The series of points marked thus Δ correspond to another set of observations also with the current reversed, in which a somewhat higher temperature and larger capacities were used than in the preceding. These again have all been multiplied by an appropriate factor to bring the maximum to the value 1.39 corresponding to the first set of numbers.

The exact data corresponding to fig. 3 are:—

- (1) Full curve represents the theoretical curve

$$I = 1.39 \left(\frac{z^2}{x^2 + z^2} \right)^{\frac{3}{2}}.$$

- (2) $\times$ Current Direct. Temperature = 1034° C. Capacities used = .3 microfarad and .01 microfarad. Mean deflexion for plates and slit 93 divs. in 30 secs.
- (3) $\odot$ Current Reversed. Temperature = 1069° C. Capacities .3 and .01. Mean deflexion 53 divs. in 30 secs.
- (4) Δ Current Reversed. Temperature = 1098° C. Capacities 1 microfarad and .04 microfarad. Mean deflexion for slit and plates 68 divs. in 30 secs.

The mean deflexions have been given so that an estimate may be made of the total thermionic current to the slit and the plates.

Reverting to fig. 3, we see that the points Δ and $\odot$ are identical within the limits of experimental error. This justifies the reduction to a common scale by multiplying each series of observations by a suitable factor. We also notice that a smooth curve drawn through the points $\odot$ and Δ would also include the points $\times$ within the limits of experimental error. This proves that the percentage change in the ratios produced by reversing the heating current is independent of the position of the slit ; or, in other words, the ratios I/I_0 of the thermionic current I through the slit in any position to its value I_0 in the central position is independent of the direction of the heating current. This fact has already been stated in a rather different manner.

It will be observed that the different observations shown not only agree with one another, but they also agree fairly well with the theoretical curve. In fact, in the central part of this figure from $x=12$ to $x=26$ turns the agreement is within the limit of the errors of observation. Outside these limits the deviation is too great to be explained in this way, and it is found that in this case the observed current through the slit is invariably greater than the calculated.

There are several ways in which the current at some distance from the centre might become too big. The normal thermionic current is much greater near the centre, and scattering of the ions by the small quantity of gas present would have the effect of making the current through the slit too big at points further off. However, a comparison of the experimental results failed to reveal any connexion between the divergence of the curves and the gas pressure, which varied considerably in the different experiments. It is important to notice also that after the platinum strips have been heated for a little while their surfaces are no longer plane, owing to a recrystallization of the metal. Under the microscope the originally smooth rolled foil is found to have developed a well-defined crystalline structure, which begins to make its appearance after the foil has been heated a very short time. It is difficult to decide how much ought to be allowed for this ; it will evidently become much more important when there is a normal electrostatic field between the two planes. The writer is inclined to the view that the effect arises from the scattering of the electrons by the plates rather than the gas. It seems quite possible that a much more marked effect when a normal electric field is present arises in the same way, and the question will be reconsidered later on in that connexion.

Our general conclusion, then, from the results exhibited in

fig. 3 is that the agreement between the observed results and those predicted theoretically * is a satisfactory one. We may therefore infer that the electrons emitted by hot platinum have the distribution of velocity required by Maxwell's law to a very considerable degree of accuracy.

It has already been stated that similar results were obtained after the slit had been cut in the plate A and the strip arranged so that it was a little behind but almost flush with the surface of A. Under these circumstances the value of z , the distance between the strip and the plates, was increased to 0.5 cm. The mean of two series of observations taken with the heating current in opposite directions at an average temperature of very nearly 1100° C. (measured as 1102° C.) gave the following values of the ratio of the charge passing through the slit to the total at the distances, expressed in turns of the screw, stated :—

Distance	0	5	10	13	15	17	18
Observed Ratio ...	·12	·17	·25	·55	·88	1·16	1·35
Calculated Ratio ...	·07	·155	·34	·61	·90	1·23	1·44
Distance	19	20	21	22	23	24	40
Observed Ratio ...	1·59	1·68	1·68	1·57	1·38	1·08	·03
Calculated Ratio...	1·58	1·64	1·64	1·56	1·40	1·22	·09

It will be seen that the results exhibit a satisfactory agreement with the new calculated values corresponding to $z=0.5$ cm. The pressure recorded in these experiments only varied between .0021 and .0027 mm.

If we refer to the previous paper we see that formula (13) (*loc. cit.*) may be written

$$I_0/n_1 = \frac{\xi}{2z} \dots \dots \dots (2)$$

In this formula z is the distance of the strip from the plates, ξ is the width of the slit in the latter, and n_1 is the total current received by the plates and slit. If the slit extended the whole way across the plates and the box behind were set so as to catch every ion which passed through the latter, then I_0 would be the current through the slit. In the present experiments the width of the plates was .854 cm. and that of the box only .60 cm. This would indicate that the experimental values of I_0 ought to be multiplied by .854/.6 before being substituted in the above formula. The correction will, however, be somewhat less than this, as some of the ions which passed through the slit beyond the outer edge of the

* Phil. Mag. [6] vol. xvi. p. 909 (1908).

box would probably reach the outside of it. As the exact amount of this correction is somewhat uncertain, it will be most satisfactory to see first what is obtained when it is disregarded altogether. Let us calculate the values of I_0/n_1 for each of the three sets of experiments recorded in fig. 3. For the observations with the current direct marked $\times$ the capacities were $\cdot 01$ microfarad for the slit and $\cdot 3$ microfarad for the plates. The ratio of the deflexions at the centre of the scale was $1\cdot 39$. The value of I_0/n_1 is therefore

$$= 1\cdot 39 \times \frac{9 \times 10^3 + 180}{27 \times 10^4 + 9 \times 10^3 + 180} = \cdot 0458.$$

With the current reversed and the same capacities, points marked $\odot$, the maximum was $1\cdot 39/81$, so that

$$I_0/n_1 = \cdot 0564.$$

With the current reversed and the capacities $\cdot 04$ microfarad and 1 microfarad respectively points marked Δ the maximum was $1\cdot 30$, so that

$$I_0/n_1 = \cdot 0502.$$

The mean of the two values of I_0/n_1 with reversed current is $\cdot 0534$, and of this with the value with the current in the opposite direction $\cdot 0496$.

The value of $\xi/2z$ in this experiment was $\cdot 04/2 \times \cdot 4 = \cdot 050$.

Thus without allowing for the correction considered above, there is complete agreement between the two sides of the equation. As the correction will probably amount to 20 per cent. and possibly somewhat more than that, it seems clear that for some reason or other the slit is receiving more than its share of the thermions as compared with the plates. This result is readily explained on the supposition, that some of these electrons are reflected when they strike the plates, which was made to explain the divergence between the experimental values and the calculated curve in fig. 3. These results would suggest that from 20 to 40 per cent. of these low-speed electrons are reflected when they strike the plates, but the arrangement used was not suitable for obtaining an exact numerical estimate.

Finite Electric Field.

A great many experiments have been made with this apparatus with a difference of potential applied between the plates A and B. As regards agreement with theory, the results obtained do not show any improvement over those recorded in the previous paper. In fact, with the improved apparatus

the negative electrons show a very definite spreading out in excess of what is required by the theory. This was indeed shown by the older experiments; but one could not be so certain of it, owing to the limitations of the apparatus then used. The matter is being investigated further on the view that it is due to reflexion of the electrons from the plates, and it would be unprofitable to discuss it further until more experiments have been made. The experiments in this direction have not been confined to platinum, but have included an infusible alloy called nichrome as well.

Conclusions.

The chief point of this paper is to be found in fig. 3, which exhibits a very close agreement between the distribution of the electrons from a hot platinum strip, in the absence of an external electric field, as found experimentally, and that which was predicted in the former paper on the hypothesis that the electrons emitted from the strip left it with a distribution of velocity equivalent to that required by Maxwell's law.

In conclusion I wish to thank my assistant Mr. Cornelius Bol for his assistance in carrying out these experiments and for drawing the diagrams of the apparatus used.

LXXI. *Notes on the Kinetic Theory of Matter.* By O. W. RICHARDSON, M.A., D.Sc., Professor of Physics, Princeton University*.

THE present communication is a brief discussion of the expression for the probability $F(u)du$ that a molecule leaving any surface in a gas should have a velocity component perpendicular to that surface which lies between u and $u+du$. The value of this probability is

$$F(u)du = 2kmue^{-kmu^2}du \dots \dots (1)$$

where m is the mass of the molecules and $\frac{3}{4k}$ their mean translational kinetic energy. This result was stated in a paper by Richardson and Brown† on the Kinetic Energy of the Negative Electrons emitted by Hot Bodies. It was given there without deduction as the writers were under the impression that it was a familiar result. A number of persons, however, who have taken an interest in the paper

* Communicated by the Author.

† Phil. Mag. [6] xvi. p. 357 (1908).

have had difficulties with it, so that it may not be out of place to give the deduction here.

It is important to remember that the above expression does not represent the probability that the velocity of a molecule selected from a given volume at any instant should lie within given limits. The selection has to be made from among those molecules which leave a given surface in a given time. It will probably conduce to clearness to get rid of the notion of probability altogether and to use instead the number $N(u)du$ which leave unit of surface with a velocity component perpendicular to the surface between u and $u+du$. If formula (1) is true it is clear that

$$N(u)du = 2kmN e^{-kmu^2} du \quad . \quad . \quad . \quad (2)$$

where N is the number of all sorts which leave unit area in unit time. It is necessary therefore to prove formula (2). It is proved in practically every text-book of the Kinetic Theory of Gases that if n is the total number of molecules of any gas per cubic centimetre at any instant, the number which leave unit area of surface in unit time with components of velocity perpendicular to that surface between u and $u+du$ is

$$N(u)du = n \left(\frac{km}{\pi} \right)^{\frac{1}{2}} u e^{-kmu^2} du \quad . \quad . \quad . \quad (3)$$

Whence

$$N = \int_0^{\infty} N(u)du = \frac{n}{2(\pi km)^{\frac{1}{2}}}$$

Substituting for n in (3) in terms of N we get

$$N(u)du = 2kmN e^{-kmu^2} du,$$

whence (1) follows.

It is interesting to observe that this expression is unchanged when the geometrical surface to which we have so far confined our attention is replaced by a physical boundary in escaping through which the molecules have to do work. The author has shown* that the velocity component u_0 normal to the surface after escape is related to the corresponding quantity u_1 impinging on the surface by the equation

$$u_0^2 = u_1^2 - \frac{2}{m} \Phi$$

where m is the mass of the molecule and Φ is the change in the work function in passing through the surface. The

* Phil. Trans. A. vol. 201, p. 501 (1903).

number of particles escaping per second for which u lies between u_0 and $u_0 + du_0$ will be the same as the number of those striking the surface per second inside for which u lies between u_1 and $u_1 + du_1$, where

$$u_0^2 = u_1^2 - \frac{2}{m} \Phi \quad \text{and} \quad u_0 du_0 = u_1 du_1.$$

This is, from (3),

$$\begin{aligned} N_0(u_0) du_0 &= n \left(\frac{km}{\pi} \right)^{\frac{1}{2}} u_1 du_1 e^{-km u_1^2} \\ &= n \left(\frac{km}{\pi} \right)^{\frac{1}{2}} u_0 du_0 e^{-km(u_0^2 + \frac{2}{m} \Phi)}. \end{aligned} \quad (4)$$

where n is the number of molecules per c.c. inside the boundary. The total number N_0 which escape per second will be those for which u_0 is greater than zero, *i. e.*

$$\begin{aligned} N_0 &= n \left(\frac{km}{\pi} \right)^{\frac{1}{2}} \int_0^\infty u_0 du_0 e^{-km(u_0^2 + \frac{2}{m} \Phi)} \\ &= \frac{n}{2} \frac{e^{-2k\Phi}}{(\pi km)^{\frac{1}{2}}}. \end{aligned} \quad (5)$$

Substituting for n in (4) we therefore have

$$N_0(u_0) du_0 = 2km u_0 N_0 e^{-km u_0^2} du_0,$$

whence

$$F(u_0) du_0 = \frac{N(u_0) du_0}{N_0} = 2km u_0 e^{-km u_0^2} du_0 \quad (6)$$

It follows that the distribution, among the particles leaving any surface, of the component of velocity normal to the surface, is independent of the amount of work done by the particles in crossing the surface, including the usual case where the work has the particular value zero.

The application of this result to the evaporation of electricity, or thermionics, has been considered in numerous previous papers. It is, however, equally applicable to ordinary molecular evaporation. If we suppose, as in the theory of surface tension, that the molecules of a liquid, in order to escape into the vapour, have to get through a skin in which they are acted on by forces which on the average are directed along the inward normal: then it follows from the above calculations that the proportion of the molecules escaping in a given time, for which u lies between u and $u + du$, is $2kmue^{-km u^2} du$. It is therefore independent of the

forces in the surface layer which determine the latent heat of evaporation, and is the same for the molecules leaving the liquid as for any other surface drawn in the vapour at the temperature of the liquid.

The distribution of the components of velocity parallel to the surface will be unaltered by the passage of the particles through it. This is clear even if, as is presumably the case, the component Φ_s of the work function parallel to the surface deviates, in individual instances, from its average value zero. For, for each class of paths for which Φ_s has a given positive value, there will be an equal number for which it has a negative value, so that the change introduced by those which are accelerated parallel to the surface will just be compensated for by others which are retarded.

The distribution of velocity among the particles leaving any surface will therefore be completely independent of the work done in passing through the surface.

As a particular case it follows that the distribution of velocity among the molecules escaping from a liquid by evaporation will be just what is proper to a gas of the same molecular weight at the temperature of the liquid (or its vapour). The escaping molecules will be in complete statistical equilibrium with those in the surrounding vapour.

It is necessary to lay stress on this point as there is a widespread heresy to the effect that, since the molecules have to do work in escaping from the liquid, their average kinetic energy in the vapour will be less than in the liquid; so that the surface layer, with its force directed towards the interior, will act in some sort like Maxwell's Demon, letting only the quicker molecules through. When this argument is kept to the qualitative plane it may be used to demonstrate a violation of the second law of thermodynamics. The preceding considerations show that there is nothing in this.

It may be permissible to add that both the case of a liquid and the surrounding vapour and that of a conductor and the escaped thermions which are in equilibrium with it are particular examples of a general theorem in the kinetic theory of matter which states that the mode of distribution of kinetic energy among the ultimate particles of a system is independent of the potential energy of the part of the system in which they occur (Jeans' Dynamical Theory of Gases, p. 78). The potential energy only affects the concentration of the ultimate particles in the different parts of the system, the average value and mode of distribution of the kinetic energy being unaffected.

LXXII. *Alternating-Current Spark Potentials and their Relation to the Radius of the Curvature of the Electrodes.* By JOSEPH DE KOWALSKI, Ph.D., Professor of Experimental Physics at the University of Fribourg, Switzerland, and ULRICH J. RAPPEL, Ph.D., Assistant in Physics at said University*.

[Plate XIX.]

Introduction.

THE phenomena of the disruptive discharge, and the measurement of the respective spark potentials, have long occupied the attention of physicists. In an excellent work by J. J. Thomson†, we find an extensive enumeration of the principal works on this subject. A series of accurate measurements have been made by Lord Kelvin, Paschen, Baille, Heydweiller, Orgler, de Kowalski, Voigt, Algermissen, and C. Mueller, all with direct current. The measurements carried out up to the present time with alternating current were more or less of a technical character, with the possible exception of Steinmetz‡. But with our present improved methods these can to-day be carried out more accurately, and this has furnished the motive for the following work. We have endeavoured to find a method of the greatest accuracy. It seemed to us that a method should give results of a higher degree of accuracy than those carried out up to the present (1 per cent. to 2 per cent.), in order to justify theoretical deductions.

In the first part of the work, the methods applied for obtaining this accuracy are set forth, with the results of the direct measurements. In the second part, these figures are utilized in examining more closely the theory of the relation of spark potentials to the radius of curvature of the electrode, as expounded by the leading physicists.

* Communicated by the Authors. Paper presented on the 3rd of May at the Imperial Academy of Sciences, at Cracow.

† J. J. Thomson, 'Conduction of Electricity through Gases,' p. 366 (1906).

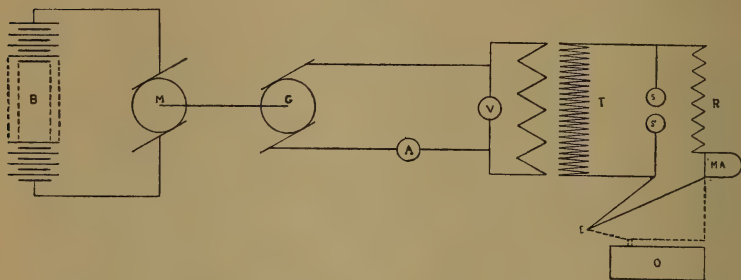
‡ C. Steinmetz, Trans. of Amer. Inst. of Elec. Eng. vol. xv. p. 281 (1898).

PART I.

Method and Apparatus.

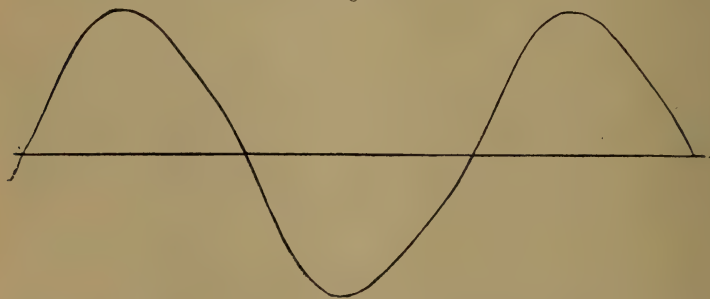
The circuit was set up in accordance with the diagram presented herewith (fig. 1). B represents a storage-battery,

Fig. 1.



capable of giving from 110 to 130 volts. M is a direct current motor. Suitable rheostats are inserted in the circuit of the armature and fields. Coupled directly with the motor is a 3.5 k.w. monophasé generator G, which was specially constructed for these measurements by the Comp. él. de Genève. Its voltage curve very nearly approaches a sine-wave, as is evident from the oscillogram reproduced in fig. 2.

Fig. 2.



The current from the generator is fed through a rheostat to a step-up transformer T, constructed by the firm Brown, Boverie & Co., of Baden, Switzerland. The indicated ratio of transformation is from 70 to 50,000 volts.

V and A indicate respectively a voltmeter and an amperemeter, which were simply used for orientation purposes, and not for the measurements.

The high-voltage terminals are connected with the discharger, represented on Plate XIX. This piece of apparatus was specially constructed for this series of experiments by the Société Gènevoise pour la Construction d'Instruments de Physique, according to the plans furnished by us.

Of the two marble columns carrying the electrodes one is attached to a block of ebonite, which in turn is firmly bolted to a cast-iron frame; the other is similarly constructed, but is bolted to a rider, capable of being displaced horizontally by means of a threaded screw. This screw was constructed by the Société Gènevoise with their customary care, and its pitch guaranteed to be exact to the hundredth part of a millimetre. Parallel to the screw is a metre-scale along which slides a little brass plate attached to the rider. This little brass plate carries a fine scratch, enabling one to read off on the metre rod the number of entire millimetres. Attached to the outer end of the screw is a circular disk of brass, having its circumference divided into 100 equal parts, enabling one to read exactly to the hundredth part of a millimetre. Joined to the screw of the discharger, by means of a ball and socket joint, is a cylinder of ebonite, having a diameter of 4 cm. and a length of 85 cm. The other end of this ebonite rod is furnished with a disk for measuring exactly to the hundredth part of a millimetre, and also with an ebonite crank for turning the screw. This appliance enables one to move the screw and take the exact measurements, without danger to the operator from the high potential of the discharger. The marble columns contain two metallic cylinders carefully centred: these are furnished with screws, upon which the various spherical electrodes are to be screwed.

Most of the previous determinations of spark potentials were made by setting the electrodes a given distance apart, and then raising the voltage slowly until the spark jumped. This method of procedure rendered the exact reading of the difference of potential very difficult because of the failure of the instruments in responding quickly enough to the change of potential; thus the individual readings differed greatly. To eliminate this source of error we proceeded in the following manner. The ends of the wires leading to the spherical electrodes were joined to a precision-milliamperemeter of Siemens and Halske by means of suitable resistances. One of the electrodes was kept grounded. When the milliamperemeter readings were definite and constant the electrodes were brought closer by moving the one that was grounded. In this way we were enabled to obtain results whose maximum

difference for the same reading of the milliamperemeter was 0.2 per cent. The phenomena of dilatation, observed by Warburg* and others, was eliminated by the use of an arc-lamp.

The alternating current milliamperemeter was carefully calibrated. It was capable of indicating directly to 30 milliamperes, but the region of the scale between 10 and 27 was exclusively used, because the readings here could be more exactly taken to the hundredth of a milliampere.

The resistances used were of manganese wire and free from self-induction. Measurements undertaken previously with the pendulum of Helmholtz proved that the self-induction and capacity of these resistances were so small that they could practically be neglected. The resistances were repeatedly calibrated by comparison with normal resistances, the latter having been calibrated by the Phys.-Technische Reichsanstalt. The total value of the resistances at our disposal was 1298800 ohms, and was capable of carrying a current of 30 milliamperes for a length of time without being appreciably heated.

The spheres employed as electrodes were most carefully made by the Société Gènevoise. There were 6 pairs of spheres with the following diameters:—2, 3.85, 10, 15, 20, and 30 cm. Measurements taken with the cathetometer showed that the irregularities in the construction did not exceed 0.2 mm. We make use of this opportunity to thank the Société Gènevoise for the splendid execution of this difficult task. At each series of experiments the zero for the distance of the spheres from each other was determined, likewise the barometer and thermometer readings.

Evidently the number of amperes in the high-tension circuit, multiplied by the number of ohms, gives the value of the effective voltage, whereas to determine the maximum voltage one must know exactly the curve of tension. For this purpose an oscillograph, constructed on the principle of Blondel, by the firm Siemens and Halske, was utilized. The high periodicity of the galvanometers (frequency to 6000 per second) permits an exact reproduction of the curve with an alternating current whose frequency is 33. In spite of the grounding of one of the poles, the greatest care must be exercised because of the extreme danger connected with the work, which must of necessity be performed in the dark. From the oscillograms and the milliampere readings the maximum voltage is deduced according to the method in common use.

In the following table are found the values determined for the ratio of the effective to the maximum voltage:—

* Warburg, *Ann. d. Phys.* v. p. 811 (1901).

Diameter of spheres in cm.	No. of Resistance-Boxes.	Ratio.	Average Ratio.
2	3	1.442	1.44
	6	1.439	
	7	1.438	
	9	1.439	
3.85	3	1.440	1.44
	5	1.443	
	7	1.441	
	9	1.437	
10	4	1.430	1.43
	6	1.432	
	8	1.430	
	10	1.431	
15	3	1.432	1.43
	4	1.430	
	6	1.429	
	8	1.429	
20	9	1.429	1.43
	3	1.432	
	6	1.428	
	7	1.430	
30	8	1.431	1.43
	2	1.432	
	3	1.429	
	5	1.428	
	6	1.433	

In the following section are given the tables of the striking-distances, with the corresponding voltages, both effective and maximum :—

TABLE II.—Diameter of spheres 2 cm. Ratio 1.44.
Frequency of alternating current 33. Barometric height
715 mm. Temperature 20° C.

Explosive Distance in cm.	Effective Volts.	Maximum Volts.
0.0850	2730	3931
0.1364	3884	5593
0.1767	4733	6816
0.2200	5680	8179
0.3163	7607	10954
0.3723	8764	12620
0.4936	11094	15975
0.5370	11876	17101
0.6724	14349	20663
0.8752	17802	25635
0.9533	19028	27400
1.0689	20830	29995
1.3536	24383	35112
1.5316	26332	37918
1.7473	28047	40388
2.2151	31207	44938

TABLE III.

Diameter of spheres 3.85 cm. Ratio 1.44. Frequency of alternating current 33. Barometric height 720 mm. Temperature 19° C.

Explosive Distance in cm.	Effective Volts.	Maximum Volts.
0.1030	3106	4472
0.1452	4026	5797
0.1812	4779	6882
0.2379	5982	8614
0.3110	7442	10716
0.3871	8992	12948
0.5070	11291	16259
0.6903	14674	21130
0.8425	17492	25188
0.9293	19055	27439
1.0416	20966	30191
1.2526	24555	35359
1.3703	26482	38134
1.4735	28194	40599
1.6597	31032	44686

TABLE IV.

Diameter of spheres 10 cm. Ratio 1.43. Frequency of alternating current 33. Barometric height 716 mm. Temperature 19° C.

Explosive Distance in cm.	Effective Voltage.	Maximum Voltage.
0.1119	3279	4689
0.1549	4180	5977
0.1981	5094	7284
0.2475	6140	8780
0.3057	7295	10432
0.4030	9220	13185
0.5185	11454	16379
0.6425	13722	19622
0.8632	17761	25398
0.8848	18161	25970
0.9902	20050	28674
1.2365	24259	34692
1.3404	26071	37280
1.4360	27805	39761
1.5932	30368	43429
1.7800	33450	47834

TABLE V.

Diameter of spheres 15 cm. Ratio 1.43. Frequency of alternating current 33. Barometric height 714 mm. Temperature 20° C.

Explosive Distance in cm.	Effective Volts.	Maximum Volts.
0.1154	3318	4744
0.1534	4118	5888
0.1900	4868	6961
0.2365	5823	8326
0.2927	6940	9924
0.3766	8600	12298
0.4627	10231	14630
0.5426	11755	16809
0.6560	13882	19851
0.8172	16737	23933
0.9410	18995	27162
1.0109	20306	29037
1.2380	24201	34607
1.3546	26245	37530
1.4793	28268	40423
1.6514	31085	44451
1.8730	34683	49597

TABLE VI.

Diameter of spheres 20 cm. Ratio 1.43. Frequency of alternating current 33. Barometric height 716.4 mm. Temperature 21° C.

Explosive Distance in cm.	Effective Volts.	Maximum Volts.
0.1148	3290	4704
0.1504	4063	5810
0.2357	5845	8358
0.2883	6896	9861
0.3817	8731	12485
0.4717	10436	14923
0.5444	11822	16905
0.6546	13888	19859
0.8123	16684	23858
0.9961	19993	28589
1.1088	21967	31412
1.2419	24252	34680
1.3540	26202	37468
1.4430	27756	39691
1.6310	30969	44285

TABLE VII.

Diameter of the spheres 30 cm. Ratio 1.43. Alternating current frequency 33. Barometric pressure 716.9 mm. Temperature 19° C.

Explosive Distance in cm.	Effective Voltage.	Maximum Voltage.
0.1144	3273	4680
0.1493	4029	5761
0.2311	5758	8233
0.2846	6801	9725
0.3777	8644	12360
0.4666	10335	14779
0.5380	11700	16731
0.6485	13783	19709
0.8040	16540	23652
1.0360	20618	29483
1.1166	22100	31600
1.2470	24324	34783
1.3595	26311	37624
1.4606	28023	40072

A direct comparison of our figures with the measurements undertaken by other physicists is hardly possible. Spark potentials for alternating current have seldom been determined under well-defined conditions. The researches of Steinmetz, which up to the present were the most reliable, do not make allowance for tardiness or dilatation, which is apt to give values that are too high, as has been sufficiently proved by Warburg* and Starke†. Consequently, our figures are smaller than those found by Steinmetz. As Steinmetz ‡, in his series of researches, allows a divergence of from 4 per cent. to 6 per cent., it seems clear that these differences are traceable to this source. We are able, however, to make an indirect comparison with the values found by other physicists, who employed the direct current. According to M. Toepler§ the following holds true:—Taking as unity the spark potential corresponding to $\frac{x}{d}=1$, where x =the explosive distance and d the diameter of the spheres, the spark potentials corresponding for the same distances are independent of the values of d .

* Warburg, *Ann. d. Phys.* v. p. 811 (1901).

† Starke, *Wied. Ann.* lxvi. p. 1009 (1898).

‡ Russell, *Phil. Mag.* vol. xi. [6] p. 265 (1906).

§ M. Toepler, *Elektr. Zeitsch.* p. 998 (1907).

In the following table is set forth the comparison of our determinations with those of other physicists:—

	Freyberg.		Heydweiller.		Kowalski & Rappel.	
d in cm.	1·0	2·0	1·0	2·0	2·0	3·85
$x : d = 0·1$	0·18	0·19	...	0·18	0·18	0·18*
0·2	0·37	0·32	0·32	0·32	0·32	0·32
0·3	0·45	0·44	...	0·45	0·44	0·44
0·4	0·54	0·55	0·53	0·58	0·55	0·53
0·5	0·65	0·65	...	0·69	0·66	
0·6	0·75	0·74	0·74	0·79	0·75	
0·7	0·82	...	...	...	0·83	
0·8	0·90	...	0·90	...	0·90	
0·9	0·98	...	...	...	0·96	

* 0·18 was taken as the basis in these calculations.

The table shows that the general results obtained agree very well with those found by other physicists. Nevertheless the absolute values are from 4 to 6 per cent. smaller than those which the direct current yields (application of the law of Paschen). Whether the cause lies in the elimination of dilatation or in the nature of determinations with alternating current is difficult to determine. Researches in this direction are now in progress.

PART II.

Relation of the Spark Potential to the Radius of Curvature of the Electrodes.

Recently A. Russell† has endeavoured to establish a theory on the relation of the spark potential to the curvature of the electrodes. Previously it was commonly accepted that the spark would jump when the maximum electric intensity between the electrodes had passed certain definite limits. Since this theory failed to explain all observed facts A. Russell has added an hypothesis of his own. He assumes that for explosive distances greater than 0·1 cm. a certain value $\mathcal{E}$ is to be deducted, in the determination of the maximum stress, which he terms lost volts. For air he takes this value to be 800 volts. The following reflexions are to serve as a test for the proposed theory of Russell.

† A. Russell, *Phil. Mag.* vol. xi. [6] p. 237 (1906).

Formulae for the Electrical Intensity between Spherical Electrodes.

The formulæ for the electrical intensity between two spheres of the same radius have been given already by Kirchhoff*, and have been lately brought into a more elegant form by Russell.

If the two spheres have respectively a potential of V_1 and V_2 the following formula applies:—

$$\mathcal{E}_{\max.} = \frac{V_1}{a} \cdot \frac{(1+q)^2}{1-q} \sum_1 \frac{1-q^{4n-3}}{(1+q^{4n-3})^2} q^{2n-2} \\ - \frac{V_2}{a} \cdot \frac{(1+q)^2}{1-q} \sum_1 \frac{1-q^{4n-1}}{(1+q^{4n-1})^2} q^{2n-1},$$

where a is the radius of the sphere, and q defined by the following equation

$$2q = \frac{A}{a} - \frac{\sqrt{A^2 - 4a^2}}{a},$$

where A represents the distance between the centres of the spheres. In case $V_1 = -V_2 = \frac{V}{2}$ we have

$$\mathcal{E}_{\max.} = \frac{V}{x} \cdot f,$$

where

$$f = \frac{x}{2a} \cdot \frac{(1+q)^2}{1-q} \sum_1 \frac{1-q^{2n-1}}{(1+q^{2n-1})^2} q^{n-1},$$

when x represents the shortest distance between the centres of the spheres.

When $V_1 = V$ and $V_2 = 0$, as is the case in our problem, we have

$$\mathcal{E}_{\max.} = \frac{V}{x} f_1,$$

where

$$f_1 = \frac{x}{a} \cdot \frac{(1+q)^2}{1-q} \sum_1 \frac{1-q^{4n-3}}{(1+q^{4n-3})^2} q^{2n-2}.$$

* Kirchhoff, Crelle's Journal, p. 89 (1860).

To determine f_1 we have made use of the following formulæ which possess a degree of exactness greater than 0.001:—

$$\frac{x}{a} < 0.1, \quad f_1 = 1 + \frac{1}{3} \frac{x}{a} \quad (1)$$

$$\frac{x}{a} < 0.2, \quad f_1 = 1 + \frac{1}{3} \frac{x}{a} + \frac{1}{45} \frac{x^2}{a^2} \quad (2)$$

$$\frac{x}{a} < 0.3, \quad f_1 = 1 + \frac{1}{3} \frac{x}{a} + \frac{1}{45} \frac{x^2}{a^2} + \frac{73}{53760} \frac{x^3}{a^3} \quad (3)$$

$$0.3 < \frac{x}{a} < 0.7, \quad f_1 = f + \Delta$$

where

$$f = 1 + \frac{1}{3} \frac{x}{a} + \frac{1}{45} \frac{x^2}{a^2} + \frac{73}{53760} \frac{x^3}{a^3}$$

and

$$\Delta = \sqrt{\frac{x}{a}} \cdot \frac{x+4a}{2a} \cdot \frac{k k' K^2}{\pi^2}$$

Here k , k' , and K are determined from the following :

$$\sqrt{\frac{2K}{\pi}} = 1 + 2 \sum_1^{\infty} q^{n^2} = 1 + 2(q + q^4 + q^9 + \dots)$$

$$\sqrt{\frac{2Kk'}{\pi}} = 1 + 2 \sum_1^{\infty} (-1)^n q^{n^2} = 1 + 2(-q + q^4 - q^9 + \dots)$$

$$k = \sqrt{1 - k'^2};$$

$$2q = \frac{A}{a} - \frac{\sqrt{A^2 - 4a^2}}{a}$$

represents the distance between the two centres.

$$0.7 > \frac{x}{a} > 1$$

$$f_1 = f + \Delta,$$

where

$$f = \frac{1}{2} \left(\frac{x}{a} + 1 \right) + \frac{1}{\frac{x}{a} + 2} + \frac{\frac{x}{a}}{2 \left(\frac{x}{a} + 2 \right)^3} + \frac{\frac{x}{a}}{2 \left(\frac{x}{a} + 2 \right)^4} + \frac{\frac{x}{a}}{2 \left(\frac{x}{a} + 2 \right)^5} - \frac{\frac{x}{a}}{\left(\frac{x}{a} + 2 \right)^7} - \frac{2 \frac{x}{a}}{\left(\frac{x}{a} + 2 \right)^8}, \quad (5)$$

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and has the same value as in formula (4),

$$\frac{x}{a} > 1, f_1 = \frac{x}{a} + \frac{1}{\frac{x}{a} + 1} + \frac{1}{\left(\frac{x}{a} + 1\right)\left(\frac{x}{a} + 2\right)^3} \dots \quad (6)$$

In the following tables we have collected the results of these calculations for testing the theory of Russell.

Diameter of spheres : 2 cm.

Explosive Distance in cm.	$\frac{x}{a}$	f_1	Maximum Intensity in $\frac{\text{volt}}{\text{cm.}}$	According to Russell in $\frac{\text{volt}}{\text{cm.}}$
0.085	0.085	1.0285	47568	37888
0.1364	0.1364	1.0459	42871	36739
0.1767	0.1767	1.0596	40856	36063
0.2200	0.2200	1.0744	39944	36038
0.3163	0.3163	1.1131	38549	35689
0.3723	0.3723	1.1372	38549	36106
0.4936	0.4936	1.1956	38696	36759
0.5370	0.5370	1.2184	33801	36986
0.6724	0.6724	1.2960	39827	38285
0.8752	0.8752	1.4290	41853	40547
0.9533	0.9533	1.4837	42645	41400
1.0689	1.0689	1.5690	44028	42854
1.3536	1.3536	1.7897	46423	45366
1.5316	1.5316	1.9356	47918	46907
1.7473	1.7473	2.1183	48960	47991
2.2151	2.2151	2.5303	51332	50418

Diameter of spheres : 3.85 cm.

Explosive Distance in cm.	$\frac{x}{a}$	f_1	Maximum Intensity in $\frac{\text{volt}}{\text{cm.}}$	According to Russell in $\frac{\text{volt}}{\text{cm.}}$
0.1030	0.0535	1.0178	44196	36291
0.1452	0.0754	1.0251	40929	35281
0.1812	0.0941	1.0314	39159	34607
0.2379	0.1236	1.0415	37713	34211
0.3110	0.1615	1.0541	36332	33620
0.3871	0.2011	1.0679	35716	33509
0.5070	0.2634	1.0893	34934	33216
0.6903	0.3586	1.1310	34620	33310
0.8425	0.4377	1.1676	34908	33800
0.9293	0.4827	1.1900	35137	34113
1.0416	0.5411	1.2206	35385	34448
1.2526	0.6507	1.2742	35970	35156
1.3703	0.7118	1.3216	36780	36009
1.4735	0.7654	1.3567	37383	36647
1.6597	0.8622	1.4201	38235	37551

Diameter of spheres : 10 cm.

Explosive Distance in cm.	$\frac{x}{a}$	f_1	Maximum Intensity in $\frac{\text{volt}}{\text{cm.}}$	According to Russell in $\frac{\text{volt}}{\text{cm.}}$
0.1119	0.0223	1.0075	42226	35021
0.1549	0.0309	1.0103	33986	33769
0.1981	0.0396	1.0132	37154	33063
0.2475	0.0495	1.0165	35038	31753
0.3057	0.0611	1.0204	34820	32150
0.4030	0.0806	1.0269	33596	31558
0.5185	0.1037	1.0348	32689	31093
0.6425	0.1285	1.0432	31860	30562
0.8632	0.1726	1.0582	31151	30171
0.8848	0.1769	1.0597	31101	30143
0.9902	0.1980	1.0669	30892	30030
1.2365	0.2473	1.0838	30683	29982
1.3404	0.2680	1.0937	30419	29767
1.4360	0.2872	1.1012	30480	29867
1.5932	0.3186	1.1343	30919	30349
1.7800	0.3560	1.1553	31046	30527

Diameter of spheres : 15 cm.

Explosive Distance in cm.	$\frac{x}{a}$	f_1	Maximum Intensity in $\frac{\text{volt}}{\text{cm.}}$	According to Russell in $\frac{\text{volt}}{\text{cm.}}$
0.1154	0.0153	1.0051	41335	34366
0.1534	0.0204	1.0068	38649	33399
0.1900	0.0253	1.0068	36945	32700
0.2365	0.0315	1.0105	35578	32160
0.2927	0.0390	1.0130	34339	31571
0.3766	0.0502	1.0167	33200	31041
0.4627	0.0617	1.0205	32661	30897
0.5426	0.0723	1.0241	31725	30216
0.6565	0.0874	1.0291	31144	29889
0.8172	0.1089	1.0365	30358	29344
0.9410	0.1254	1.0421	30082	29197
1.0109	0.1347	1.0453	30025	29198
1.2380	0.1650	1.0556	29507	28825
1.3550	0.1806	1.0609	29393	28697
1.4793	0.1972	1.0665	29144	28568
1.6514	0.2202	1.0744	28921	28401
1.8730	0.2497	1.0846	28722	28259

Diameter of spheres : 20 cm.

Explosive Distance in cm.	$\frac{x}{a}$	f_1	Maximum Intensity in $\frac{\text{volt}}{\text{cm.}}$	According to Russell in $\frac{\text{volt}}{\text{cm.}}$
0.1148	0.0115	1.0038	41118	34126
0.1504	0.0150	1.0050	37705	32361
0.2357	0.0236	1.0078	35733	32313
0.2833	0.0288	1.0096	34526	31725
0.3875	0.0387	1.0127	33120	30998
0.4717	0.0472	1.0157	32133	30411
0.5444	0.0544	1.0181	31615	30119
0.6516	0.0655	1.0218	30999	29751
0.8123	0.0812	1.0270	30167	29156
0.9961	0.0996	1.0332	29653	28823
1.1088	0.1109	1.0372	29384	28636
1.2419	0.1242	1.0417	29089	28428
1.3540	0.1354	1.0455	28932	28315
1.4430	0.1443	1.0486	28842	28261
1.6310	0.1631	1.0549	28645	28128

Diameter of spheres : 30 cm.

Explosive Distance in cm.	$\frac{x}{a}$	f_1	Maximum Intensity in $\frac{\text{volt}}{\text{cm.}}$	According to Russell in $\frac{\text{volt}}{\text{cm.}}$
0.1144	0.0076	1.0025	41012	34002
0.1493	0.0099	1.0033	38707	33331
0.2311	0.0154	1.0051	35802	32324
0.2846	0.0189	1.0063	34387	31559
0.3777	0.0252	1.0084	33000	30865
0.4666	0.0311	1.0104	32000	30268
0.5380	0.0358	1.0119	31468	29964
0.6485	0.0432	1.0144	30831	29580
0.8040	0.0536	1.0178	29944	28932
1.0360	0.0690	1.0230	29115	28325
1.1160	0.0744	1.0248	29049	28315
1.2470	0.0831	1.0277	28666	28007
1.3595	0.0906	1.0302	28511	27905
1.4606	0.0973	1.0324	28328	27763

These tables show that the claims of Russell are not justified regarding the constancy of this maximum stress. We see rather a certain regularity in the results, and for a definite explosive distance a minimum seems to exist. The theory of Russell is, therefore, no satisfactory explanation of the question.

LXXIII. *Musical Arc Oscillation in Coupled Circuits.* By E. TAYLOR JONES, D.Sc., Professor of Physics in the University College of North Wales, and MORRIS OWEN, B.Sc., Isaac Roberts Student of the University College of North Wales, Bangor*.

[Plate XX.]

IN a recent communication† one of us has shown that if the primary of a pair of coupled circuits, in series with a condenser, forms the shunt to a Duddell musical arc; and if the secondary coil is connected to another condenser, either of two notes may under certain circumstances be heard. It was also shown that the frequencies of the electrical oscillations in the circuits corresponding to the two notes are generally much lower than those calculated for the circuits from the inductances of the coils and the capacities of the condensers, the resistances being neglected.

In the present paper the influence of the resistances of the circuits is considered. It is shown that the difference between the observed and calculated frequencies cannot be mainly attributed to the resistances, and a different explanation is suggested.

One of the chief difficulties attending experiments on this subject is that of obtaining really simultaneous values of the two frequencies. It is of little use to sound the two notes one after the other and determine their frequencies. The conditions in the arc can change so rapidly that the constants of the circuits cannot be regarded as identical on two occasions, even if separated by only a short interval of time.

A number of cases are considered in the present paper in which the two oscillations were recorded simultaneously. The method consists usually in arranging matters so that one oscillation corresponds to the octave or a higher harmonic of the other. Some cases are also considered in which, in addition to the two ordinary arc notes, a *third* note is produced; and the view is put forward that this is a *difference-tone* of the other two, arising, however, from an electrical oscillation of the same frequency in the circuits.

The oscillations were studied by means of the short-period electrometer, or electrostatic oscillograph, described by one

* Communicated by the Authors.

† E. T. Jones, Phil. Mag. p. 42, January 1909. This paper will be referred to as "*l. c.*"

of us *. This was connected to the terminals of the condenser in the secondary circuit; the photographs obtained therefore represent the waves of potential at the plates of this condenser.

In most of the experiments the secondary condenser was of the parallel-plate type and of variable capacity, with heavy paraffin-oil as the dielectric. The primary capacity consisted of a paraffin-paper condenser, or a number of these in parallel. In all cases the primary coil was of self-inductance $\cdot 004619$ henry, and resistance $1\cdot 038$ ohms: the secondary coil had self-inductance $L_2 = 70\cdot 15$ henry, and resistance $R_2 = 14022$ ohms. The mutual inductance M of the two coils was $\cdot 2192$ henry.

The carbons used for the arc were solid and 9 mm. in diameter.

The total capacity C_2 in the secondary circuit includes the capacity of the secondary condenser, of the oscillograph, and of the secondary coil. The capacities of all the condensers were determined by measuring the periods of their electrical oscillations when connected in series with coils of known self-inductance.

Full details as to the methods employed in the determination of periods of oscillation and of the constants of the circuits were given in the previous papers, referred to above.

(1) *Resonance of the Octave.*

Case I. C_1 = primary capacity = $9\cdot 55$ microfarad.

C_2 = secondary „ = $\cdot 000275$ † „

This case was referred to in the previous paper ‡, where it was pointed out that the octave, which is generally present when the lower arc note is sounding, becomes in this case very much intensified. The photograph showing this was also given in the paper referred to §. The frequency determined from the photograph was 613. The frequencies calculated from the inductances of the coils, and the capacities of the condensers, the resistances being neglected, were $1301\cdot 2$ and $723\cdot 1$, which do not satisfy the octave relation or agree with the observed frequency.

In order to find the influence of the resistances of the circuits the frequencies n_1, n_2 were calculated for the above value of R_2 and for various values of R_1 , the resistance of the

* E. T. Jones, Phil. Mag. ser. 6, vol. xiv. p. 238, August 1907.

† In the previous paper the value of this capacity was given as "about" $\cdot 000275$ microfarad. Subsequent measurements showed that this value is correct to within one part in 300.

‡ *L. c.* p. 43.

§ *L. c.* Plate I. fig. 11.]

primary circuit. In these calculations the following formulæ, given by Drude *, were employed :—

$$\left. \begin{aligned} T^2 + T'^2 - 2\beta^2 &= T_1^2 + T_2^2 - \frac{1}{2}(\theta_1 - \theta_2)^2, \\ 2\beta(T^2 - T'^2) &= -(\theta_1 - \theta_2)(T_1^2 - T_2^2), \\ (T^2 - T'^2)^2 - 8\beta^2(T^2 + T'^2) &= (T_1^2 - T_2^2)^2 \\ &\quad - 2(T_1^2 + T_2^2)(\theta_1 - \theta_2)^2 + 4M^2C_1C_2, \end{aligned} \right\} \dots (1)$$

where

$$\begin{aligned} \theta_1 &= \frac{1}{2} R_1 C_1, & T_1^2 + \theta_1^2 &= L_1 C_1, \\ \theta_2 &= \frac{1}{2} R_2 C_2, & T_2^2 + \theta_2^2 &= L_2 C_2, \\ T &= \frac{1}{2\pi n_1}, & T' &= \frac{1}{2\pi n_2}, \end{aligned}$$

and β is a constant which vanishes when the resistances of the circuits are neglected.

The elimination of T and T' between these equations leads to a cubic for β^2 . In each of the cases examined the cubic had only one positive root (giving real values for β), and β^2 being thus determined the above equations allow T and T' , and hence the frequencies n_1 and n_2 , to be calculated.

Table I. contains the values of β^2 , n_1 , and n_2 , calculated in this way for various values of R_1 , the other constants having the values given above, and L_1 being taken as the self-inductance of the primary coil.

TABLE I.

R_1 in ohms.	$\beta^2 \cdot 10^3$.	n_1 .	n_2 .
0	·0005	721·8	1309
3	·002112	725·1	1300·5
5	·006669	729	1296·7
10	·02964	747·5	1285·4
15	·071804	781·8	1264·9
32·995	·472159	1187·8	1187·8
38	·665507	1643·6	1178·1
40	·750912	2113·5	1173·6
41	·795478	2596·6	1172·2
50	1·250994	Imaginary	1163·2

* Drude, *Ann. der Physik*, xiii. p. 534 (1904).

From the last two columns it will be seen that as the value of R_1 is increased the frequency of the slower oscillation *increases*, and that the ratio of the two frequencies at first deviates more and more from the octave relation.

It is clear, therefore, that the observed frequency 613 of the musical arc note and the intensification of the octave of this note cannot be accounted for by merely assuming that the arc has any constant resistance during the oscillations and including this in our calculations.

It seems desirable to examine next the hypothesis that the arc behaves as if it had self-inductance. This supposition is not a new one. It has been employed * to account for the well-known fact that the alternate-current arc has a power-factor different from unity. It has also been pointed out by M. La Rosa † that oscillations may be produced by the musical arc even when the shunt circuit connected to it has no natural period of oscillation. Very intense high notes may in fact be produced by connecting the carbons to a paraffin-paper condenser by well-twisted insulated wires, no coil being included in the circuit.

Assuming, therefore, that the arc has self-inductance which is to be added to that of the circuit connected to it, the problem in the present case is to find what value of L_1 will make the frequency of one oscillation of the system twice that of the other. Neglecting the resistances of the circuits, the frequencies of the system are given by Oberbeck's formula

$$8\pi^2 n^2 = \frac{1}{1 - \frac{M^2}{L_1 L_2}} \left[\frac{1}{L_1 C_1} + \frac{1}{L_2 C_2} \pm \sqrt{\left\{ \left(\frac{1}{L_1 C_1} - \frac{1}{L_2 C_2} \right)^2 + \frac{4M^2}{L_1^2 L_2^2 C_1 C_2} \right\}} \right]. \quad (2)$$

Substituting the above values of L_2 , C_1 , C_2 , M , and putting $n_2 = 2n_1$, we have a quadratic equation for L_1 , the roots of which are $\cdot 003811 \cdot 10^9$ and $\cdot 0066795 \cdot 10^9$. The former value is less than the self-inductance of the primary coil ($\cdot 004619 \cdot 10^9$), and is therefore discarded. Taking, therefore, $L_1 = \cdot 0066795 \cdot 10^9$ cm., and substituting in (2) we find for the frequencies of the system 617 and 1234.

It was pointed out in the previous paper that at the time when the photograph was taken, the amplitude of the wave was not quite at its maximum; so that the frequency corresponding to the exact octave relation is slightly different from the measured value 613. It is in fact very difficult to obtain a photograph during strong resonance, since the

* Cf. Ayrton and Sumpner, Proc. Roy. Soc. vol. xlix. p. 424 (1891).

† Science Abstracts, p. 697 (1907).

oscillations are easily stopped by "brushing" or sparking at the terminals of the secondary circuit.

Further, in arriving at the above calculated values the resistances of the circuits are not taken into account. These considerations are probably sufficient to account for the small difference between the observed and calculated values of the frequency, while the close agreement between these values gives support to the views that during these oscillations the arc may be regarded as having a constant self-inductance, and that when the two frequencies of the system are nearly an octave apart, and when the lower note is sounding, the octave of this note produces forced oscillations of great amplitude in the system.

Case II.—This case is similar to the last. The coils were the same, and the capacities were $C_1=11.63$, $C_2=.0003911$ microfarad. The lower note only* was heard, and at a certain length of the arc the octave resonance occurs. Several photographs were taken of the curve of secondary potential, but in none of them was the amplitude quite at its maximum. Two curves showing the intensified octave were measured, the observed frequencies being 490 and 546.

Assuming as before that $n_2=2n_1$ the values of L_1 deduced from (2) are $.0019362 \cdot 10^9$ and $.0080879 \cdot 10^9$ cm., the former of which is inadmissible. Taking, therefore $L_1=.0080879 \cdot 10^9$, (2) gives for n_1 the value 510.5, which does not differ greatly from either of the observed values.

Case III.—Another example of the same kind is given by $C_1=16.98$, $C_2=.0004771$ microfarad. Several photographs were taken showing the intensified octave, and the curve having the greatest amplitude was found to have a frequency of 452.9.

With $n_2=2n_1$, equation (2) gives $L_1=.006471 \cdot 10^9$, and $n_1=469.9$.

The difference between the observed and calculated values is here again probably within the limits of error explained under Case I.

Case IV.—This case differs in one respect from the others. The capacities were $C_1=14.62$, $C_2=.0002898$ microfarad. If the condition $n_2=2n_1$ is inserted in (2), the two values of L_1 obtained are less than the self-inductance of the primary coil, and are therefore inadmissible. For all possible values of L_1 n_2 is greater than $2n_1$.

The question therefore arises whether the octave relation may not be satisfied by the two frequencies if the resistances of the circuits are taken into account. A few trials showed

* Cf. l. c. p. 41.

that this was the case. If L_1 is taken as $\cdot 006 \cdot 10^9$ cm., equation (2) gives $n_1 = 528\cdot 7$, $n_2 = 1205\cdot 4$. With $R_2 = 14000$ ohms, $R_1 = 15$ ohms (*i. e.* nearly 14 ohms being assumed for the arc), equations (1) give $n_1 = 572$, $n_2 = 1194$: while $R_1 = 25$ ohms makes n_2 less than $2n_1$.

The observed frequency in this case was 573; and it is clear that this condition and the octave relation are satisfied by assuming L_1 to be slightly greater than $\cdot 006 \cdot 10^9$, and R_1 rather greater than 15 ohms.

The length of the arc during the octave resonance was about 1.5 mm., the current in the arc about 1.7 ampere. From the results given by Duddell* it seems probable that an arc of this length conveying so small a current might have a resistance of over 14 ohms.

The photographs for Cases II.-IV. are similar in character to that given in the previous paper for Case I., and are not reproduced here.

(2) *Resonance of Higher Harmonics.*

Case V.—With a large capacity in the primary circuit and a small secondary capacity, other notes, lower than that which gives the octave resonance, become prominent, and are accompanied by high potentials at the terminals of the secondary condenser. These appear to be due to resonance of the third and higher harmonics of the arc note.

Thus with $C_1 = 14\cdot 62$, $C_2 = \cdot 0002898$ microfarad (as in Case IV.), if the arc-length is made slightly greater than the value which gives the octave resonance, the note falls gradually in pitch, but becomes fairly stable and prominent when about a fifth below the value given in Case IV. The photograph showing the curve of secondary potential for this case is given in Plate XX. fig. 1. The curve appears to consist chiefly of a prominent third harmonic superposed upon the fundamental. The frequency of the latter, determined by comparison with the curve given by a 768 tuning-fork photographed simultaneously, was found to be 359.4.

If we insert the relation $n_2 = 3n_1$ in equation (2), we find $L_1 = \cdot 011589 \cdot 10^9$ cm., and hence, by (2), $n_1 = 385$. The difference between the observed and calculated values is within the limits that may be expected from the considerations explained above.

Case VI.—With $C_1 = 19\cdot 03$, $C_2 = \cdot 00029$ microfarad, resonance of the fourth harmonic of the arc note may be obtained.

* Duddell, Phil. Trans. vol. ccciii. (A) pp. 323, 326 (1904).

The curve of secondary potential is shown in Plate XX. fig. 2. The frequency determined from the photograph is 285.2.

If we assume $n_2 = 4n_1$ in equation (2), we find

$$L_1 = .016289 \cdot 10^9 \text{ cm.}, \text{ hence } n_1 = 285.4.$$

Case VII.—Using a still larger capacity in the primary circuit, $C_1 = 21.32$ microfarad, and the same secondary condenser, a photograph was obtained showing a prominent fifth harmonic. This is shown in Plate XX. fig. 3. The observed frequency is 224.5.

With $n_2 = 5n_1$ equation (2) gives $L_1 = .023088 \cdot 10^9 \text{ cm.}$ Hence $n_1 = 226.7$.

Case VIII.—With the same capacities as in Case VII. and a slightly longer arc, a still deeper note, about a minor third below the last, can be recognized as producing resonance. We did not succeed in obtaining a photograph of the curve for this case. If, however, the relation $n_2 = 6n_1$ is inserted in equation (2), we find $L_1 = .03361 \cdot 10^9 \text{ cm.}$, giving $n_1 = 188$, which is very nearly a minor third below the note of Case VII.

(3) "Difference Tones" produced in Coupled Circuits by the Musical Arc.

Case IX.—It was pointed out in the previous paper* that if a certain relation exists between the primary and secondary capacities the two notes of the arc are equally stable. This is the case with $C_1 = 16.98$, $C_2 = .001098$ microfarad, and the two notes are separated by an interval of about a fifth. If, with these two condensers, the lower note is sounding and the arc-length is gradually diminished (the pitch gradually rising), at a certain stage the note suddenly drops by an octave. This deep note is fairly rich in quality, and it differs from the ordinary arc notes in that it is perfectly definite in pitch. Any alteration in the arc-length made when the deep note is sounding either causes the sound to stop altogether, or changes it into one of the two primary notes. The deep note was not very stable in this case, and several exposures had to be made before a photograph was obtained showing the curve of secondary potential while this note was sounding. The photograph is shown in Plate XX. fig. 4. The photograph in fig. 5 happened to be taken just as the note was changing from the lower primary into the deep note an octave below, and covers the whole period of

* *L. c.* p. 41.

this change. The photographs show that the deep note corresponds to an electrical oscillation of the same frequency in the circuits, and is not merely produced in the air, or in the ear, as is sometimes the case when the difference-tone of two musical notes sounding simultaneously is heard.

The deep note appears to be the difference-tone of the two primary notes, and is due to an electrical oscillation in the system which is only produced when the two primaries are equally stable and (in the present case) when the ratio of their frequencies is exactly $3/2$.

If we assume $n_2 = \frac{3}{2} n_1$ in equation (2) we obtain the two values $\cdot 004633 \cdot 10^9$ and $\cdot 007586 \cdot 10^9$ for L_1 . Taking the smaller value (since the arc was very short when the difference-tone was sounding), and substituting in (2) we find $n_1 = 484 \cdot 73$, $n_2 = 727 \cdot 09$. Hence $n_2 - n_1 = 242 \cdot 36$.

The frequency of the difference-tone measured from the photograph was $237 \cdot 5$. The observed frequency of the deep note therefore agrees closely with the difference of the frequencies of the two primes calculated on the assumption that the interval between them is a perfect fifth.

Case X.— $C_1 = 14 \cdot 62$, $C_3 = \cdot 0008722$ microfarad. In this case also the two primary notes are equally stable, and the deep note, an octave below the lower primary, is produced at a certain arc-length.

Assuming as before $n_2 = \frac{3}{2} n_1$ in (2), the values $\cdot 004734 \cdot 10^9$ and $\cdot 006540 \cdot 10^9$ cm. are obtained for L_1 . Taking the former value as being more appropriate for a short arc and inserting it in (2), we find $n_1 = 529 \cdot 7$, $n_2 = 794 \cdot 5$. Hence $n_2 - n_1 = 264 \cdot 8$.

This agrees closely with the frequency of the deep note determined from the photograph, which was $263 \cdot 8$.

Case XI.— $C_1 = 5 \cdot 64$, $C_2 = \cdot 0002886$ microfarad. In this case the difference-tone was exceptionally stable and powerful, quite as loud, in fact, as either of the two primary notes, and more easily produced and maintained than in the two previous cases. The lower primary note was often heard simultaneously with the difference-tone, but we are not sure that the upper primary could be distinguished when the latter was sounding. The length of the arc when the difference-tone was sounding was about $0 \cdot 75$ mm., the current in the arc generally about $2 \cdot 5$ amperes. Several photographs were taken with the difference-tone sounding. These (as well as those of Case X.) are very similar in character to the curve shown in Plate XX. fig. 4, and are not reproduced here.

The frequency of the difference-tone determined from the photographs was 438.6.

With regard to the calculated frequency the assumption $n_2 = \frac{3}{2}n_1$ in equation (2) leads to no real values of L_1 in this case. The resistances must therefore be taken into account. After a number of trials Table II. was drawn up showing the values of the two frequencies, calculated from equations (1), for various values of the resistance, R_1 , of the primary circuit, the self-inductance of the primary circuit being taken as $.0049 \cdot 10^9$ cm.

TABLE II.

R_1 ohms.	R_2 ohms.	n_1 .	n_2 .	$\frac{n_2}{n_1}$.	$n_2 - n_1$.
0	0	869.2	1328.3	1.528	459.1
5	14000	873.4	1325.7	1.517	452.3
8	„	880.1	1322.5	1.503	442.4
10	„	888.7	1311.9	1.476	423.2
20	„	969.8	1238.1	1.277	268.3

It is clear, therefore, that if the primary circuit has a resistance of slightly over 8 ohms (*i. e.* if the resistance of the arc is about 7 ohms), and a self-inductance of the above value, the ratio of the two frequencies will be exactly 3/2, and their difference will agree closely with the observed frequency of the deep note.

From the measurements given by Duddell* it appears that 7 ohms is not an impossible value for the resistance of the arc under the circumstances of this experiment.

(4) Conclusions.

From the above results it may be concluded :—

- (1) That the great differences between the observed frequencies of the electrical oscillations in coupled circuits produced by the musical arc and the values calculated from the inductances of the coils and the capacities of the condensers, cannot be accounted for merely by allowing for the resistances of the circuits.

* W. Duddell, Phil. Trans. vol. cciii. (A) pp. 323, 326 (1904).

- (2) That the principal cause of the difference is the fact that the arc behaves as if it has self-inductance, which is constant during any particular train of oscillations (though different under different circumstances), and which must be taken into account in calculating the frequencies.
- (3) That any one of the harmonics of the arc-note up to the sixth can be strongly reinforced by suitably adjusting the ratio of the two frequencies of the system.
- (4) That if the two notes of the arc are equally stable, and if the interval between them is a perfect fifth, a third note may be heard, which corresponds to an electrical oscillation of frequency equal to the difference of the frequencies of the two primes.

Bangor, July 1909.

LXXIV. *Flexural Vibrations of Thin Rods.* By GEORGE GREEN, M.A., B.Sc., Assistant to the Professor of Natural Philosophy in the University of Glasgow*.

§ 1. **T**HE main object of this paper is to point out a method of applying hydrodynamic solutions already obtained to the solution of problems relating to flexural vibrations of thin elastic rods.

The results found apply to rods not subjected to permanent tension, and vibrating so that one principal axis of each transverse section lies in the plane of vibration. The central line of the rod is assumed to remain unaltered in length; and particles lying in a plane transverse section, when undisturbed, remain always in a plane normal to the central line. For convenience in what follows we may here derive the equation of motion of a rod subject to these conditions. Taking ω as the area of each section, $\kappa^2\omega$ its moment of inertia, q as Young's modulus, dx as the length of the central line of a small portion, R its radius of curvature, and N as the total tangential force acting on the cross-section at x , we obtain the equation of angular motion of the element

$$q\kappa^2\omega \frac{\partial}{\partial x} \frac{1}{R} + N = \rho\kappa^2\omega \frac{\partial^3 y}{\partial x \partial t^2}, \quad \dots \dots (1)$$

where y is the vertical displacement of the element and ρ is its density. The equation of vertical motion is

$$\frac{\partial N}{\partial x} = \rho\omega \frac{\partial^2 y}{\partial t^2} \dots \dots \dots (2)$$

* Reprinted from Proc. Roy. Soc. Edin. vol. xxix. 1909, from a separate copy communicated by the author.

As pointed out by Lord Rayleigh*, whose notation we have adopted, terms depending on the angular motions of the sections of the bar may be neglected in the above equations; accordingly, by eliminating N from (1) and (2) we obtain finally the equation

$$\frac{\partial^2 y}{\partial t^2} + \kappa^2 b^2 \frac{\partial^4 y}{\partial x^4} = 0, \quad . \quad . \quad . \quad . \quad (3)$$

in which we have put $\frac{\partial^2 y}{\partial x^2}$ for $\frac{1}{R}$ and b^2 for $\frac{g}{\rho}$.

§ 2. Consider now frictionless liquid in a straight canal with vertical sides. Take the origin of coordinates at point O , at a distance h above the undisturbed level, and draw OX parallel to the canal and OZ vertically downwards. Let the motion be infinitesimal and let ξ , ζ , be the displacement components at any time t of any particle of water whose undisturbed position is x , z . If the motion of the water be started primarily from rest by pressure applied to the free surface, the hydrodynamical equations of motion take the well-known form

$$\dot{\xi} = \frac{\partial}{\partial x} \phi(x, z, t); \quad \dot{\zeta} = \frac{\partial}{\partial z} \phi(x, z, t); \quad . \quad . \quad (4)$$

from which we obtain by integration

$$\xi = \frac{\partial}{\partial x} \phi(x, z, t); \quad \zeta = \frac{\partial}{\partial z} \phi(x, z, t). \quad . \quad . \quad (5)$$

In virtue of the incompressibility of the fluid we have also the equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0, \quad . \quad . \quad . \quad . \quad (6)$$

which shows that if ϕ be known at all points of the free surface, and if it be zero at points infinitely distant, its value can be determined at all points throughout the fluid. The equation to be fulfilled at the free surface, according to a result given by Cauchy and Poisson, is

$$\left(\frac{\partial \phi}{\partial z} \right)_{z=h} = \frac{1}{g} \left(\frac{\partial^2 \phi}{\partial t^2} \right)_{z=h}, \quad . \quad . \quad . \quad . \quad (7)$$

where g denotes gravity.

§ 3. Remembering now that every function derived from ϕ by differentiations or integrations with respect to x , z , t ,

* 'Theory of Sound,' vol. i.

satisfies all the equations satisfied by the function ϕ , we see that we have

$$\frac{\partial^2 \phi}{\partial z^2} = \frac{1}{g^2} \frac{\partial^4 \phi}{\partial t^4} \quad \dots \quad (8)$$

at a free, or level, liquid surface. Equations (6) and (8) combined give us the relation

$$\frac{\partial^2 \phi}{\partial x^2} = -\frac{1}{g^2} \frac{\partial^4 \phi}{\partial t^4}; \quad \dots \quad (9)$$

and when we put y in place of ϕ , κb in place of $\frac{1}{g}$, and interchange x and t , this equation may be written in the form

$$\frac{\partial^2 y}{\partial t^2} + \kappa^2 b^2 \frac{\partial^4 y}{\partial x^4} = 0, \quad \dots \quad (3)$$

which is the equation already found for flexural vibrations of an elastic bar, when terms depending upon the angular motions of the sections of the bar may be neglected. Accordingly, equation (9) proves that every hydrodynamical potential function, with x and t interchanged, is a solution of equation (3) above. (The converse is not always true.) In relation to the flexural-waves problems z is in general to be treated as a constant.

§ 4. It may be of some interest to note that, in all solutions for elastic bars obtained by applying the above result, the displacement y at each point of the bar may be regarded as derived from a single function $V(t, z, x)$ such that

$$y = \frac{\partial V}{\partial x} \quad \dots \quad (10)$$

Further, it follows from the relationship established between such a function V and the hydrodynamic potential function, that every function derived from V by differentiations or integrations with respect to t, z, x , is a solution of the differential equation (3). Thus any solution of (3) may be a displacement potential, or a displacement, or a velocity. If the function V represents displacement, we readily find from equation (7) that the curvature at each point x of the bar can be obtained by a single differentiation with respect to z , being given by the equation

$$\frac{1}{R} = \frac{1}{\kappa b} \frac{\partial V}{\partial z}, \quad \dots \quad (11)$$

from which the potential energy can easily be obtained.

§ 5. Solutions for vibrations of a rod of finite length l are derived directly from the hydrodynamic solutions for waves in a canal of length l by simple interchange of x and t ; and they are applicable to all cases, whether the rod has its ends free, or clamped, or "supported." In particular, the normal functions are easily obtained in this way.

In the case of an infinite rod, all mathematical results relating to surface waves in a canal infinitely long and infinitely deep become immediately useful; space-curves in the hydrodynamical waves problems becoming time-curves for the flexural waves, and *vice versa*.

§ 6. In this connexion it may be useful at a later time to examine some of the numerous hydrodynamical solutions relating to surface waves and groups of waves. A number of curves are shown in papers on Water-Waves* by the late Lord Kelvin, illustrating results derived from particular hydrodynamic solutions comprehended in the following general expression, given in his last Waves paper:—

$$\{\text{RS}\} \text{ or } \{\text{RD}\} \frac{\partial^{j+k+l}}{\partial t^j \partial x^k \partial z^l} \frac{1}{\sqrt{(z+ix)}} e^{-\frac{gt^2}{4(z+ix)}}.$$

In this, $\{\text{RS}\}$ denotes a realization by taking half the sum of what follows it with $\pm i$; $\{\text{RD}\}$ denotes a realization by taking the difference of what follows it with $\pm i$ divided by $2i$. As an example of flexural waves in an infinite elastic rod, arising from a given initial displacement, we may take the solution

$$\begin{aligned} y &= \{\text{RS}\} \frac{1}{\sqrt{(z+it)}} e^{-\frac{x^2}{4\kappa b(z+it)}} \\ &= \sqrt{\frac{1}{T}} \cos\left(\frac{x^2 t}{4\kappa b T^2} - \frac{1}{2} \tau\right) e^{-\frac{x^2 z}{4\kappa b T^2}}. \end{aligned} \quad (12)$$

where $T = \sqrt{(z^2 + t^2)}$, and $\tau = \tan^{-1}\left(\frac{t}{z}\right)$

In what follows, z is taken as 1 and κb as $\frac{1}{4}$ in order to allow us to use Lord Kelvin's hydrodynamical results in our present problem.

§ 7. Taking the origin of coordinates at the middle of the bar, the initial configuration is given by

$$y = e^{-\frac{x^2}{4\kappa b}}. \quad (13)$$

Almost immediately after the commencement of motion an

* Proc. Roy. Soc. Edin. vol. xxv. Feb. and June 1904; vol. xxvi. Oct. 1906.

infinite number of waves are formed along the bar, with amplitudes diminishing according to the law $e^{-\frac{x^2}{4\kappa b T^2}}$ and with the distances from zero to zero becoming shorter and shorter as we pass from the middle toward the ends of the bar. The zeros come into existence at the ends of the bar, and begin travelling inwards to the middle. The first zero formed comes almost instantaneously to the middle region, and the others follow it in their order of formation. The inward progress of the zeros soon ceases, the first zero never quite reaching the centre; in a short time they begin to move in the opposite direction and continue to do so for ever, and the amplitudes at any point x ultimately fall off according to $\frac{1}{t^{\frac{1}{2}}}$. The middle point of the rod subsides to its

undisturbed position nonvibrationally, while the distance from it to the first zero on either side continually increases after a certain time, being given by the equation $x^2 = 3\kappa b \pi t$.

§ 8. The seven curves given by Lord Kelvin, as space curves for water-waves, on page 191 of his paper (Proc. Roy. Soc. Edin. vol. xxv. Feb. 1904), show the condition of things in our present problem at seven different points near the middle of the bar, as t increases from 0 to ∞ , provided the curves be continued to meet the axis of t asymptotically at infinity (see fig. 1 and § 9 below). These curves show

Fig. 1.



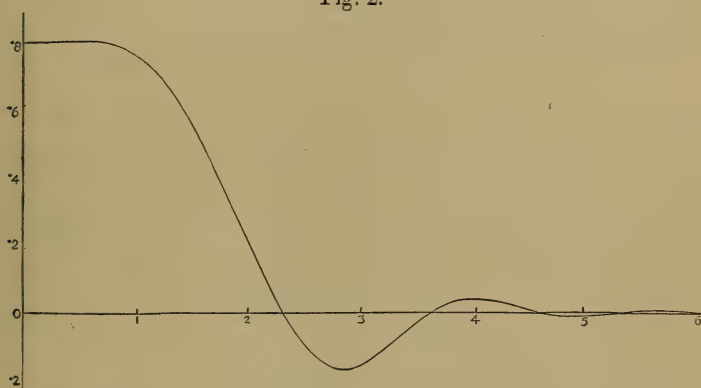
Abscissas represent space for water-waves, time for flexural-waves.

that points very near the origin never pass through their initial positions, but fall back nonvibrationally to them; points farther from the middle rise slightly and then behave in the same way. At points along the bar more and more

distant from its middle point the disturbance consists of a larger and larger number of waves which travel inwards past the point considered, before $t=1$, and after that recross the point in the opposite direction one by one. When the first zero recrosses the point it subsides gradually to its original place of rest, only reaching it, however, after an infinite time. The successive maxima of displacement increase very slowly at first, then more quickly, and then diminish finally according to $\frac{1}{t^{\frac{1}{2}}}$ as already stated.

§ 9. The diagrams of figs. 1 and 2 are taken from Lord

Fig. 2.



Abscissas represent time for water-waves, space for flexural-waves.

Kelvin's paper referred to in § 8, and they are reproduced without change of the lettering applicable to them as water-wave diagrams. To make them correspond exactly to the flexural-waves problem solved by equation (12), we must reduce the ordinates in both figures in the ratio $\sqrt{2}:1$; then in fig. 1 replace t by x on each of the seven curves, and take ordinates as representing displacement and abscissas as representing time. As a water-waves diagram fig. 2 represents the vertical displacement of the water at point $x=2$, from $t=0$ to $t=\infty$; as a flexural-waves diagram it represents the shape of the right-hand half of the rod, $x=0$ to $x=\infty$, at time $t=2$, corresponding to the initial configuration given by equation (13).

These curves are useful chiefly as illustrations of the propagation of waves in dispersive media from a given initial disturbance confined in the main to the neighbourhood of the origin. They show clearly the distinctive features of wave-propagation in the two cases where the wave-velocity varies

directly as the square root of the wave-length and inversely as the wave-length respectively, for an infinite succession of regular sinusoidal waves. It is interesting to observe that in both cases the wave-disturbance is ultimately spread throughout the entire medium, but that in the case of water-waves the wave-length of the disturbance at any time increases continuously, and in the case of flexural-waves it diminishes continuously, as we pass outwards from the middle point of the initial disturbance. In the case of water-waves also, Lord Kelvin's investigations show that each individual wave lengthens and increases in speed as it advances, while in the case of flexural-waves each wave lengthens and diminishes in speed, as we shall see by equations (15) and (16) below for the solution we have chosen for illustration.

§ 10. At a moderate time after the motion has commenced τ may be put equal to $\frac{\pi}{2}$ in equation (12), and the argument of the cosine varies then only with $\frac{x^2 t}{4\kappa b T^2}$, so that it is easy for us to trace the outward progress of any particular maximum or zero of displacement along the bar. Thus the position of any zero is determined by an equation of the form

$$\frac{x^2 t}{4\kappa b T^2} = c = \frac{4n+3}{4}\pi, \quad (14)$$

and the velocity of the zero is given by

$$\frac{dx}{dt} = \frac{8\kappa b c t - x^2}{2tx} = \frac{2\kappa b c}{x} (15)$$

When t is large, so that we may put $\frac{t}{T^2} = \frac{1}{t}$, equation (14) enables us to write (15) in the following approximate form,

$$\frac{dx}{dt} = \frac{x}{2t} (15')$$

It can easily be verified that when t is very small and x great the velocity of any zero is given by

$$\frac{dx}{dt} = -\frac{x}{2t} (16)$$

In each case, what we obtain in these equations is also the wave-velocity for an infinite train of waves of wave-length equal to that maintaining in the immediate neighbourhood of the point x at time t .

§ 11. To obtain the velocity of the group of waves of wave-lengths approximately equal to λ —that maintaining in the neighbourhood of x at time t —we put

$$\frac{x^2 t}{4\kappa b T^2} - \frac{(x-\lambda)^2 t}{4\kappa b T^2} = 2\pi, \quad \dots \quad (17)$$

which enables us to write down the group-velocity thus:—

$$\frac{dx}{dt} = \frac{8\pi\kappa b t - \lambda x}{\lambda t}. \quad \dots \quad (18)$$

When t is very small and x great, the right-hand side of (18) is approximately equal to $-\left(\frac{x}{t}\right)$. If in (17) we take the wave-length λ as small compared with x , we can obtain the following approximate expression for λ when t is great:—

$$\lambda \doteq \frac{4\pi\kappa b t}{x}. \quad \dots \quad (19)$$

With this value for λ , the right-hand side of (18) becomes $\left(\frac{x}{t}\right)$.

Equations (15)–(19) show that in the two cases, when t is small and x great, and when t is great, the group-velocity is twice the wave-velocity; which is in accordance with the theory of group-velocity given by Osborne Reynolds and extended by Lord Rayleigh.

LXXV. *A Gyrodynamical Solution of the Problem of the Amagnetic Mariner's Compass.* By J. J. TAUDIN CHABOT*.

[Plate XXI.]

THERE is an ever-increasing desire to replace the geo-magnetic ship's-compass by an instrument, which would be independent of the earth's magnetism, as it seems possible to do, in particular at the self-rotating planet's surface, by the use of rotating masses, whose axes of rotation move only in a plane, unalterable relatively to the rotating planet itself.

The precursor of all constructions, ever to be executed for this purpose, is Bohnenberger's machine, dating from 1817, in Tübingen. It enabled us, for the first time, to show approximately by experiment some results of the works published over half a century before by d'Alembert, Euler, and especially by Lagrange, on rotation round axes with

* Communicated by the Author.

only one point fixed, so that since then we distinguish apart from a theoretical dynamics of rotation still a kinematics of rotation of a more experimental kind, and treatises by Ampère, Poincot, Thomson (Lord Kelvin), &c. on the one side, as well as such by Sang, Foucault, Sire, &c. on the other.

We must throughout take note as to whether the axis of rotation is fixed at one point only, viz., the centre of mass of the rotating system, so that it can take up any position in space, or is fixed in one plane, wherein, it is true, it may take up any azimuth, but which it does not leave.

The first case indicates the way to obtain an axis of rotation of unchangeable direction in absolute space, like a finger (so to speak) of the great clock, that is Earth. Here, however, we are not able to succeed on account of the unavoidable friction on the pivot—in default of a directing force to overcome this.

But there is fortunately a directing force to hand on the earth's surface, where it is of value in practical arrangements made according to the second mode, whereby to overcome the bearing-friction; and a definite direction can be secured of the axis of rotation, which is no longer unvariable in relation to absolute space, but in relation to the earth alone.

Accordingly the conditions appear to be given, by which we may succeed in setting up an amagnetic compass, as one has endeavoured to do in very many ways by gyroscopic* contrivances for nautical use, trying to determine the meridian on board ship by means of the axis of rotation of a gyrostat*, held horizontally in Cardani suspensions.

The whole group of previous investigators have endeavoured to succeed in this manner, but evidence is forthcoming, that no experimentalist has given a general and rigorous account of the actual behaviour and the necessary conditions. And yet for this it is only necessary to analyse the system of forces, which enters into the performance of a gyrostat mounted on board ship in the manner described.

It appears then, first of all, that the vertical axis, round which the turning into the meridian should take place, is only an imaginary one. In practice we have to deal with an axis, which moves over the surface of a cone; and the axis of *this* cone only is the ideal vertical axis, round which constructors have hitherto considered their gyrostats to rotate. Now we can make the apex of the cone small, very small indeed, but never zero, a certain amount of

* A *gyroscope* means the whole of the arrangement including the device for observation (hand, microscope, or the like), whilst *gyrostat* is the rotating mass in its frame alone. So the gyrostat can be called a gyroscope (as is generally done by French and German authors) with no more sense than, for instance, a spectrum can be called a spectroscopist.

play in the upper (or, in the cases of suspensions, in the lower) bearing, and therefore a tilting of the horizontal axis of the gyrostat, so soon as forces arise, which turn the upper (or, in the case of suspensions, the lower) end of the vertical axis ever so little in its bearing, being unavoidable.

The smallest forced nutation of a gyrostat's axis, namely, is enough, as Fessel and Plücker showed with their so-called precession apparatus*, and as one could demonstrate with Bohnenberger's machine†, to excite a force-component, which constrains the whole system to rotate round a third axis, perpendicular to both that of the gyrostat and that of nutation. This last axis, in the case of a ship's gyrostat, is unfortunately exactly the vertical axis, round which it should turn for indicating the course. Therefore, every acceleration of the ship's motion—whether it be rectilinear or curvilinear—acts so as to deflect the gyrostat; and we are not, in general, in a position to separate this deflecting effect from the visible deviations, by which a change in the ship's course should be revealed. The greater the gyrostatic moment of inertia, the smaller will be the influence of the disturbances, but the greater will be the time-period of the oscillation of the amagnetic compass,—far above the limit permitting one to distinguish at every instant between oscillations of the compass-card and rotations of the ship.

So some years ago (1904) I entered upon a completely new mode of treating the problem, after it appeared to me, that above everything the instrument itself should indicate continually, whether its action is to be depended upon,—and this in the simplest manner, so that it should indicate either with unfailing correctness or not indicate at all; consequently the fact that it is in operation might at the same time be a guarantee, that it operates correctly, and that, further, it is to be recommended, that the would-be vertical axis should, if feasible, be altogether done away with.

This consideration led me to a principle of arrangement, which up till now had been most carefully avoided: a disturbance of the position, but one, which is periodic, with a fresh adjustment always following it. The playing forces arising from a complexity of rotations, which must be given *a priori*, the nature of the new total configuration is characterized as of a higher order, of which the gyrostat constitutes but an organic member: in distinction from previous

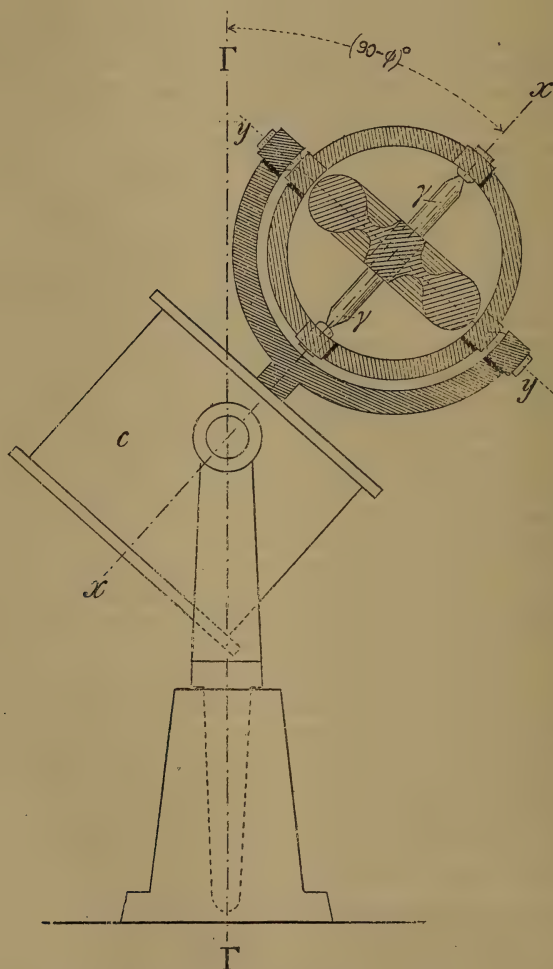
* Fessel's precession apparatus, described by Plücker and by Poggen-dorff, see *Annalen der Phys. u. Chem.* vol. xc., 1853.

† See my communication, "Simple Diagram connecting the various motions in the so-called Bohnenberger's machine," *Phil. Mag.* ser. 6, vol. ix., 1905.

forms tried, it is gyrodynamic ; I might therefore call the instrument, in distinction from the *gyrostatic compass*, a *gyrodynamical compass*.

The development of the idea obviously transforms the ill-fated vertical axis into a horizontal axis, which experiences no complete rotation as before, but only periodic oscillations of extremely small amplitude. Further details are afforded by the following explanation, with the help of the description of a model.

Fig. 1.



The principal axis Γ of the model (see fig. 1) cuts at an angle $(90 - \phi)^\circ > 0^\circ$ the axis of rotation, x , of the forked-

bearing of a gyrostat, perfectly balanced about an axis y , normal to the axis x , and having an axis of rotation γ , perpendicular to y . Rotations can take place round the axis Γ , the axis x , and the axis γ ; and that too in this order, in general, with increasing angular velocity: $\omega_{\Gamma} < \omega_x < \omega_{\gamma}$; correspondingly, the slowest rotation, round the axis Γ , can be excited perhaps by hand; that round the axis x by means of the clockwork C; the most rapid rotation, round the axis γ , by means of a cord, wheelwork, or electromotor. Further possibilities, that the model may be arranged to exhibit—viz. velocities graded otherwise as $\omega_{\gamma} > \omega_x > \omega_{\Gamma}$ —will here remain undescribed.

By observations is obtained the state of equilibrium of the gyrostat, *i. e.* of the system round the axis y , while none, one, or more of the rotations round the axis Γ , round x , or round γ are excited. Thereby we distinguish between 8 principal cases, partially subdivided, according to the given azimuth of y or γ :

	According as the system			and the angle		will be the system
	round Axis Γ	round Axis x	round Axis γ	between Axis y & Plane Γx	between Axes γ & Γ	
1.	rests	rests	rests			astatic
2.	turns	"	"			"
3.	rests	turns	"			"
4.	turns	"	"			"
5.	rests	rests	turns			"
6.	turns	"	"	$= 0^{\circ}$		"
				$= 90^{\circ}$	$= 0^{\circ}$ (syntropic motions)	stable
					$\neq 0^{\circ} \text{ \& \; } \neq 180^{\circ}$	negatively stable
					$= 180^{\circ}$ (antitropic motions)	unstable
7.	rests	turns	"		$= (90 - \phi)^{\circ}$	stable
					$\neq (90 - \phi)^{\circ} \text{ \& \; } \neq (270 - \phi)^{\circ}$	negatively stable
					$= (270 - \phi)^{\circ}$	unstable
8.	turns	"	"			oscillating

Of these cases Sire specially dealt with No. 6 *b*, Gilbert with Nos. 6 *a* and 6 *b*, without, however, the gyrostat in the state of rest being an astatic system. Sire arrived at the stability of his gyroscopic pendulum* in the state of rest merely in that he laid the axis γ a good deal lower, than the axis y , Gilbert, in his barogyroscope†, in that he kept both axes in one plane, balanced the system most carefully about the axis y and then added an adjustable weight. Gilbert studied the special case of identity of the axis Γ with the axis of rotation of his planet (Earth)—getting, therefore, ω_F as the angular velocity of the planet's rotation,—with the consequence, that his model scarcely showed more than a gyrostat with a permanently horizontal axis y in a forked-bearing, rotative in a foot; whereas Sire kept the axis Γ as the principal axis of his model, which, according to the relative direction of rotation round the axis γ , was approximated (even in opposition to the so-called centrifugal force) or fled by the gyrostatic pendulum.

Instead of giving the unbalanced gyrostat a position of rest by means of a gravitational moment, we can make the balanced gyrostat stable in any chosen position by means of an elastic force (acting about the axis y), and then observe the deflexion, brought about as a consequence of rotation at the same time round Γ . The cases Nos. 6 and 7 are identical, as is evident, if $\phi = 90^\circ$, viz. the axes x and Γ coincide.

In case No. 8, being here of special interest, the pendulum-motion of the gyrostat round the axis y results, as ordinarily, from the intermittent action of a deflecting force against a directing one, which every time brings back the pendulum system into its position of stable equilibrium. The directing force arises from the simultaneous rotation (which also might be considered as a rotary oscillation) round the axis x , or by means of a control-spring; whilst, in an altogether similar manner, a force acting around the axis y arises from the rotation round the axis Γ , which, purely a directive force, in this case means a deflecting one, every time then (and thus intermittently) growing to a maximum, when the y -axis passes the azimuth normal to the Γx plane. The y -axis describes in consequence a conoid-surface round x with about the spiral-ellipse (similar to the Lissajou-figure for oscillation-ratios of 1 to 2 for 0° and 180° or 90° and 270° phase-difference) as generating-line and its apex in the intersection of the axes x and y . Moreover, the chief

* Sire, Pendule gyroscopique, *Archives des Sciences physiques et naturelles*, vol. i. 1858.

† Gilbert, Barogyroscope, *Comptes rendus*, vol. xciv., 1882.

conditions to be realised here are : exact balancing about the axis y , by adjusting the centre of the rotating mass (and *not* by an additional weight, as Gilbert did in his barogyroscope*), and practically no slack-motion of the axis y in its bearings.

Let us now identify the principal axis of rotation, Γ , of our model with the axis of rotation of the planet, so that as tangible of the whole merely remain the gyrostat, lying in its bearings, with the clockwork, C , which imparts the angular velocity ω_x , syntropic with ω_γ ; and let this apparatus be mounted with the x -axis in suitable Cardani-suspensions on board a ship, where it, when in the geographical latitude ϕ , takes up the position, indicated in the figure relative to the earth's axis Γ , whenever the y -axis passes through the meridian. At this moment the gyrostat behaves astatically with respect to the simultaneous rotation round Γ (*cf.* above, No. 6 *a*), and responds exclusively to the directing force, due to the rotation round x , or else to an elastic device, which tends to turn the γ -axis into the vertical, *i. e.* into the extension of the axis of rotation x . But at the next instant the y -axis has passed through the meridian, experiences again the deflecting force of the rotation round Γ , which comes into play, and increases with the sine of the angle of rotation up to a maximum for 90° rotation of the axis y round x (*cf.* above, No. 6 *b*). Were the just-named directing force not available, so would the deflexion of the γ -axis from the vertical reach the value $(90-\phi)^\circ$, *i. e.* the γ -axis would set itself parallel to the planet's axis, but as the directing force continues, the deviation remains in general less than $(90-\phi)^\circ$, with the exception of the case (treated further later on) of a resonance between the oscillations round y and the rotation round x . After a further 90° rotation of the y -axis round x the elongation will be again zero, and the γ -axis again stand vertical in the extension of the axis of rotation x . A model allowing to observe these movements most beautifully according to the theory—built by Messrs. Hartmann & Braun—is shown in fig. 2, Pl. XXI.

Of special importance, as already emphasized in the introduction, is now an analysis of the system of forces, acting in our gyrostat. Besides the practically constant gravitational force, the remaining inconstant forces may be classified as translatory- and rotatory-ones. All translatory forces, resulting from the inertia-reaction, can be separated into vertical and horizontal components, in consequence

* *Rarissima avis!* I saw one only once, and of average workmanship, at the Collège de France in Paris.

either of the rolling and pitching of the ship (see Poggendorff's falling machine *, of date 1854), or else horizontal as a result of rectilinear accelerations on her way and their variations; they have no influence on the position for the time being of the balanced gyrostat. Translatory forces along an arc (or circular translatory forces) in consequence of the rotation of y round x , which tend to give the bearings of the gyrostat a rotation— ω_x , exist only whilst the gyrostat is started and soon vanish; they are of no further importance.

Rotary forces enter in two ways into the phenomena; the intermittently acting deflecting force, already considered, arising in consequence of the rotation round the planet's axis Γ , and the permanent directing force, likewise referred to, resulting from spring tension or from rotation round x . In the last is also merged any disturbance, arising from an alteration of the ship's way, which, taking place in a horizontal plane, has the same character as a rotation about a vertical axis, that is round x , or, what means the same thing, round an axis parallel to x ; therefore, an increase of ω_x only can influence the directing force dependent on and proportional to it—viz. to a vanishingly small extent—what solely means a variation (of practically unassignable value) of the maximum elongation of the gyrostatic oscillation and in no way will disturb the frequency of the pendulum-motion.

And this frequency only is of importance to us; it is equal to that of the rotary motion of the forked-bearing, and necessarily goes through a to-and-fro motion round y during each rotation round x . If now the rotations on board ship are counted by reading a mark, fixed rigidly to the ship, it will happen, that apparently less or more, than a complete pendulum oscillation about y corresponds to a single rotation round x . This phenomenon is easily explained: the ship altering her course, she alters her angular position to the same extent as the phase-change between the oscillation and the rotation of the gyrostat. Indications of this phase-change imply then indications of the variation of the ship's way, or rather of the way in general, in so far as only the passing of the y -axis through definite azimuths at the planet's surface affects the oscillation, and these indications can easily be obtained by any stroboscopic method, with the help of electromagnetic sending- or contact-contrivances, which would be controlled by the rotation or by the oscillation.

The last one is allowed to arise not with the largest

* Poggendorff's Falling Machine, *Annalen der Phys. u. Chem.* vol. xcii., 1854.

possible amplitudes, but playing only with an arm of the pendulum-system between two stops placed near together, and the breaking of an electric circuit is brought about by the motion of the γ -axis through the vertical *in* (so to speak) *statu nascenti*. If we keep then, for example, a vacuum-tube lighted whilst it rotates round an axis normal to its own axis, synchronously with the axis of the fork x (thus at the same time with angular velocity ω_x), so will the ship's way and its variations be indicated by the position and the angular deviations of the luminous line of the tube. Thence arises obviously the possibility also of indicating it at a distance and at several distant points of the ship at the same time,—not only by means of vacuum-tubes of course, but also by means of indicator-plates (scales) or of the ordinary compass-cards.

The basis of determining the numerical relations of an instrument constructed for this purpose, is obtained as follows:—We take two planes of reference, one through the equator of the rotating planet, and one through the meridian of the point in question, perpendicular to its surface and normal to the equatorial plane. At this point the axis x is the vertical. The deviation of the axis x from the equatorial plane, *i. e.* the geographical latitude, we take as ϕ , the angle between the now-horizontal axis y and the meridian-plane as χ , the angle between the axis γ and the horizontal plane as ψ . Further we denote by I_y the moment of inertia of the system M round y , by I_γ that of the rotating mass m round γ and by ω_n the greatest value of the angular velocity of an alteration of the ship's way, for which the instrument shall be required to perform, wherefore we do not need the value of the most rapid turning as the largest angular velocity in general, but it evidently is enough to insert in the calculation the greatest value of ω_n for which at any time will be steered from the instrument alone.

The azimuths $\chi=0^\circ$ and $\chi=180^\circ$ of the y -axis correspond then to the value $\psi=90^\circ$, as is shown in the figure of the model, whilst for $\chi=90^\circ$ and $\chi=270^\circ$ the axis γ shows the greatest possible deviation, but only, in general (as above pointed out), without ψ taking up its theoretically smallest value ϕ .

At the instants when $\chi=0^\circ$ and $\chi=180^\circ$ the pendulum-system is able to follow unlesened a directing force F_x , called into play by the rotation round x or an elastic force F_r , which one will have adjusted so as to render the x -axis stable in the vertical. So in F_r we have to deal with an independent variable, which can be measured as circumstances

require ; in F_x , on the other hand, with a dependent variable, directly proportional to ω_F , ω_γ , I_γ , and $\cos \psi$. Whilst $\chi = 90^\circ$ or 270° , ψ results from the combined action of F_x or F_r and that deflecting force, F_F , which arises from the simultaneous rotation round the planet's axis Γ , exactly as F_x arises from that round x , and hence is proportional to ω_F , ω_γ , and I_γ , with a maximum value for $\psi = 90^\circ - \phi$, and a minimum value for $\psi = \phi$.

It is further necessary to choose $\omega_x > \omega_n$, lest ω_x , in general, in relation to the surface of the planet, might become $\leq$ zero. The rotating-velocity ω_x , and the oscillating-one t_y , must be such as to suit one another, that is, such, that they synchronize (a to-and-fro motion round y corresponding to a complete rotation round x), whence the amplitude of this oscillation may be small. One recalls further the fact, that the time-period of an oscillation, $2\pi I^{\frac{1}{2}} F^{-\frac{1}{2}}$, will be determined by the magnitude of the moment of inertia, I , and the directing force F , this one as shown previously, the other built up by the factor I_γ , and a gyrostatic term, which is a quadratic function essentially of I_γ and ω_γ .

The described arrangement with horizontal bearings for the gyrostat introduces the important advantage over all those with vertical gyrostatic axes, that a construction is possible, which permits this axis to be regarded as very nearly indeed rigidly fixed, and at the same time to content with a very small directing force, viz. inertia ; the small angular motion, instead of complete rotation, allows one the use of the finest cut bearings obtainable, or possibly of magnetic setting or adjustment of the pressure on the bearings, so that in consequence the directing forces to be chosen (or the impulses $I_\gamma \omega_\gamma$) can be smaller than is usual with gyrostats, this meaning right away a shortening of the time-period of the oscillations. And for a completely rigid axis the period of oscillation would be simply inversely proportional to the square-root of the restoring force in the actual model, subject then to a small correction for the non-absolute rigidity of the material of which it is made.

Valuable is a comparison of the formulæ given formerly at Montreal, however in another connexion, by Lord Kelvin *, who saw still the principal contents of the present communication.

The synchronism being obtained between the rotation round x and the oscillations about y , we generally get forced oscillations, that is, oscillations, which are differently possible between the limits, determined by the selected values of

* Reports of the British Association, liv., 1884.

the separate factors, and thus can be forced into synchronous motion. This oscillation-region involves also the so-called free oscillation-period ; if we determine ω_x correspondingly, resonance will arise between the rotation and the oscillation, and thereby result the strongest tendency to oscillate. This shows us the manner, in which, even when large gyrostatic moments exist, short periods of oscillation may be obtained, namely, by compensating the inertia by simple stiffening of the control-spring (F_r), because in the case of resonance the greatest possible amplitude of oscillation is the resultant of the directing and deflecting forces' combined action ; the ratio ω_x/ω_n is here to be taken accordingly large. By means of suitable transformations (electromagnetic synchronous motors or the like) the ratio ω_y/ω_x may then be forced to an equally constant magnitude as that of the once-for-all determined magnitudes of mass and moment of inertia of the regarding parts of the constructed instrument.

For practical needs we might employ two or more gyrostats of smaller construction, oscillating in one frame together round a common axis y , rather than a single larger gyrostat, so as to combine two or more such systems with axes y , shifted round through equivalent angles with respect to one another, their bearings either over one another in a single framework, rotating about the vertical x , or again each in a forked bearing rotating individually, the synchronous motions of all the bearings maintained with a constant phase-difference between them in the same manner as in the control of electrical clocks, equally at distant points of the ship, wherever these synchronously-acting instruments may be installed, —whilst their paths of connexion *count* amongst the numerous nervous fibres of that gigantic creature, the modern ocean-runner.

Hohenwaldau, near Stuttgart.

February 13, 1908.

LXXVI. *Multiple Atomic Disintegration. A Suggestion in Radioactive Theory.* By FREDERICK SODDY, M.A.*

THE cause of atomic disintegration remains unknown. It is difficult to construct any model of the disintegrating mechanism, chiefly on account of certain features in connexion with the process. In particular may be mentioned the fact that the period of average life of the atoms disintegrating is the same whether newly formed atoms or those which have

* Communicated by the Author.

already survived many times the average period are considered. What may be termed the inevitableness of the process, and its entire independence of all known conditions, suggests that the cause of disintegration is apart from the atom. It is difficult to believe that the cause is resident in space external to the atom. It seems more probable that it exists within the atom and at the same time is uninfluenced by it. The question about to be discussed is whether necessarily only one mode of instability can exist within the atom at the same time.

In certain problems in radioactivity it is becoming necessary to take into account that the same element, for example uranium, may give rise to two different series of disintegration products, represented for example by actinium and radium. Rutherford has suggested in order to account for the small quantity of actinium, relatively to radium, in minerals that actinium may be formed from uranium as a side or branch chain, not in the mainline of descent. The suggestion in this paper is that multiple modes of disintegration may be proceeding simultaneously and independently within the same atom. There is nothing in this idea opposed to all that is known of the character of atomic disintegration. On the other hand it will be shown that such disintegrations would simulate closely in many respects simple disintegration.

It has been established that the law of simple disintegration really expresses the operation of the law of probability for a process selecting the particular individual atoms disintegrating without reference to the character of the individuals, but solely according to the total number in existence. That is to say, the period of actual life of the individual remains indefinite up to the moment of disintegration, and no process of sorting a collection of atoms into long-lived and short-lived varieties is even conceivable. Similarly for multiple disintegration the view advocated is that the law of simple disintegration applies to each mode of disintegration exactly as if it were the only one in operation. It follows that the question by which particular mode any individual atom will disintegrate remains indefinite up to the moment of disintegration and no process is conceivable which would sort a collection of atoms according to the mode by which eventually they will disintegrate and to the product which eventually they will produce.

If for the same atom two modes of disintegration can occur independently, the constant of the first mode being represented by λ_A and that of the second by λ_x , the quantity

Q of the substance will diminish according to the differential equation

$$\frac{dQ}{dt} = -\lambda_A Q - \lambda_X Q$$

$$\text{or} \quad \frac{Q_t}{Q_0} = e^{-(\lambda_A + \lambda_X)t},$$

where Q_t is the quantity remaining at time t and Q_0 the initial quantity.

That is to say, for a radio-element undergoing a dual disintegration the quantity would diminish exponentially with the time as in a simple disintegration. The radioactive constant of the multiple change would be the sum of the separate constants. If, for example, α -rays were given out in one of the modes of disintegration, and β -rays in the other, each type of radiation would decay at the *same* period, namely, the sum of the two separate periods. The same holds true whatever the number of modes of disintegration. So that multiple disintegration would not be easy to detect so far as ordinary criteria go. But with regard to the products formed in the various modes there will be constancy of proportionality between them. To suggest a concrete case, let us suppose the disintegration of uranium X is dual, one mode producing ultimately radium, the other ultimately actinium, and let the constant λ_A apply to the radium mode and λ_X to the actinium mode of disintegration. Then the ratio between the number of uranium atoms forming radium and the number forming actinium is given by $\frac{\lambda_A}{\lambda_X}$. If the further disinte-

grations of both these elements are simple the same ratio will express the relation between the quantities of the final inactive products of the two series. This suggests at once a method of obtaining evidence of the process and at the same time of determining the nature of the final products. If a constant ratio were found between any two inactive elements present in uranium minerals it would be very strong evidence from the present point of view.

According to the researches of Boltwood (Am. Journ. Sci. 1908, p. 296) on the proportionate α -radiation contributed by the separate constituents in equilibrium in uranium minerals, it is possible to deduce (Phil. Mag. Oct. 1908, p. 517) that of eight atoms of uranium seven go to form radium and one to form actinium. The ratio $\frac{\lambda_A}{\lambda_X}$ is therefore equal to 7.

This is also the ratio between the quantities of the end-products of the two series in minerals on the assumption stated.

Proceeding with the idea that the disintegration of uranium X is dual, it makes no difference whether we regard the β - and γ -rays as derived from one only or from both of the modes of disintegration. Or, we may even suppose that the β -rays come from the radium mode, while γ -rays come from the actinium mode of disintegration, which appears to have something to recommend it as it would explain the relative poverty of uranium in γ -rays (Soddy and Russell, *Phil. Mag.* Oct. 1909, p. 631). Both types of rays would decay at the same rate. The apparent value of the radioactive constant of uranium X (about $\cdot 031$ (day) $^{-1}$) would be the sum of the two separate constants, one of which is seven times the other. The radium mode would thus have the constant $\cdot 027$ and the actinium mode of disintegration, $\cdot 004$.

This point of view explains perfectly the known relations between uranium, actinium, and radium, preserving the inevitableness of the phenomenon, without the necessity of supposing, as on Rutherford's theory of a side-chain, that actinium is not a lineal descendant of uranium in the same sense as radium is ('Radioactive Transformations,' p. 177). Considering the long series of successive disintegrations passed through by the radioactive atoms it would be rather remarkable from one point of view if multiple disintegration did not sometimes occur. The relations between thorium and uranium in minerals would receive perhaps a better explanation on the view that thorium is one product of a multiple disintegration in the uranium series. The evidence has been discussed by Boltwood (*Am. Jour. Sci.*, 1905, xx. p. 256) who concluded that it was most probable that thorium was a disintegration product of uranium. The period of thorium is probably five times as long as that of uranium, and hence thorium cannot be intermediate between uranium and radium as in this case there should be a constant ratio between thorium and radium in equilibrium and not between uranium and radium. On the present idea the ratio of thorium to uranium and to the end-products of radium can be calculated but the expressions involve the age of the mineral. These expressions are

$$\frac{T}{U} = \frac{\lambda_A}{\lambda_B - \lambda_A - \lambda_X} (1 - e^{-(\lambda_A + \lambda_X - \lambda_B)t})$$

$$\frac{T}{E} = \frac{\lambda_A(\lambda_A + \lambda_X)}{k\lambda_X(\lambda_A + \lambda_X - \lambda_B)} \left(\frac{e^{-\lambda_B t} - e^{-(\lambda_A + \lambda_X)t}}{1 - e^{-(\lambda_A + \lambda_X)t}} \right)$$

where t is the age of a mineral, assumed to contain no thorium

or end-product initially; T, U, and E, the existing quantities of thorium, uranium, and end-product of radium; λ_A , λ_x the constants of the thorium and radium modes in the multiple disintegration of uranium; λ_B the constant of thorium itself, and k is the fraction of the number of uranium atoms undergoing the radium mode of disintegration which eventually become atoms of the end-product in question. Were all subsequent disintegrations simple k of course would be unity. The effect of the intermediate bodies between uranium and the end-product has been neglected. So far as is known this would be small. Unfortunately on account of the long period of thorium and the isomorphism of uranium and thorium oxides it is doubtful if much evidence can be derived from the composition of minerals. Thorianite is regarded as an extremely old mineral geologically yet it contains so great a percentage of thorium that from the point of view of radioactivity it must be a recent formation. As Boltwood has pointed out (*loc. cit.* p. 262), Hillebrand's analyses of uranium minerals show generally that the percentage of total rare earths (less perfectly of the thorium) increases as the lead increases but there are numerous exceptions. It is probable that geological time is all too short to enable the necessary evidence to be obtained from the composition of minerals.

Before concluding a special case may be referred to which has some points of interest and might very probably occur. If one of the modes of disintegration were a relatively mild form of rearrangement, rather than an explosive disintegration, it is conceivable that the ray-producing change might occur with the same period independently of whether the rayless change had or had not occurred, expelling the same character of ray in each case. On this view actinium would proceed from rearranged but undisintegrated uranium X, radium from uranium X disintegrating directly, or *vice versa*. If λ_A represents the constant of the rayless and λ_x that of the ray-producing mode of disintegration, it can be shown for this case that the rays will now follow the exponential changes with their own true period, λ_x , rather than with the $\lambda_A + \lambda_x$ period. This holds true whether the rearranged uranium X is present or absent initially. The calculation which gives this simple result is lengthy and may be omitted. If γ -rays accompanied the β -rays in the direct mode of change of the uranium X, and not in the change of rearranged uranium X, the γ -rays would decay with the $\lambda_A + \lambda_x$ period, the β -rays according to the λ_x period. The difference in period would be small and might well be overlooked.

At present there is no experimental evidence either for or against the view that the disintegration of uranium X is multiple, although such evidence has been sought for in this laboratory for some time past. It must be understood that this particular case has been taken for purposes of illustration only.

Physical Chemistry Laboratory,
University of Glasgow,
Sept. 13th, 1909.

LXXVII. *On the Active Deposit from Actinium in Uniform Electric Fields.* By W. T. KENNEDY, B.A.*

[Plate XXII.]

I. *Introduction.*

IN a number of experiments which have been carried on with the emanations and the emanation products from the radioactive substances, it has been shown by Rutherford † that with thorium emanation the amount of activity imparted to a rod charged negatively was independent of the pressure until a pressure of 10 mm. was reached, and that below this pressure it decreased as the pressure in the containing vessel was lowered. At 1/10 mm. pressure it was only a small fraction of its maximum amount. Makower ‡ has also obtained similar effects with the excited activity from radium emanation. Further, Rutherford § experimenting with radium emanation, found that at atmospheric pressure the greater part of the active deposit went to the cathode, while only about 5 per cent. went to the anode. From these results he has drawn the conclusion that while most of the active deposit particles of radium are positively charged, some at least must carry a negative charge, inasmuch as they are drawn to the anode in electric fields.

More recently Russ || showed that when positively and negatively charged electrodes were placed in a vessel containing either air, sulphur dioxide, or hydrogen charged with the emanation from radium, the relative amounts of the active deposits obtained on the two electrodes varied with the pressure at which the exposures were made. With all three gases the

* Communicated by Professor J. C. McLennan, and read before the Royal Society of Canada on May 26, 1909.

† Rutherford, *Phil. Mag.* Feb. 1900.

‡ Makower, *Phil. Mag.* Nov. 1905.

§ Rutherford, *Phil. Mag.* Jan. 1903.

|| Russ, *Phil. Mag.* May 1908.

active deposit on the negative electrode gradually decreased as the pressure was lowered, while that obtained on the positive electrode showed a corresponding increase, until ultimately at the lowest pressure investigated, the amounts of the active deposit obtained on the two electrodes were approximately equal.

In a second paper Russ* gives an account of a similar set of observations made with the emanation from actinium. In these experiments positively and negatively charged electrodes were again exposed in a vessel filled with air and containing a quantity of actinium. In this case as the pressure of the air was lowered the amount of the active deposit obtained on the negative electrode gradually increased, passed through a maximum value at a certain critical pressure, and ultimately fell away again at the lowest pressure investigated. On the other hand, in these experiments Russ found that the active deposit obtained on the positive electrode steadily decreased as the pressure of the air was lowered.

In his second paper Russ also describes a series of experiments in which exposures were made in air when the distance between the actinium and the electrodes was gradually increased. The results which he gives show that with the air at 760 mm. the amount of the deposit obtained on the cathode steadily decreased as the salt was placed at distances varying from 2 to 50 mm. from the electrodes. Under similar circumstances the amount obtained on the anode at first increased as the salt was removed, and finally, after passing through a maximum value, fell away again at the longer distances. With the air at 2 mm. pressure, however, the amount obtained on the cathode steadily increased as the distance of the salt from the electrode was varied from 2 to 42 mm., but the active deposit obtained on the anode with the same variation of distances gradually decreased.

In discussing his results Russ points out that Debiere had found that the amount of emanation obtained at atmospheric pressure from a uniform layer of actinium fell to half value in going .55 cm. from the layer, and that consequently one would expect to find a decrease in the activity of the electrodes as the salt was removed, and possibly, too, a continuance in the ratio of the activities of the deposits obtained on the two exposed terminals. The results given by him, however, show that this was far from being the case.

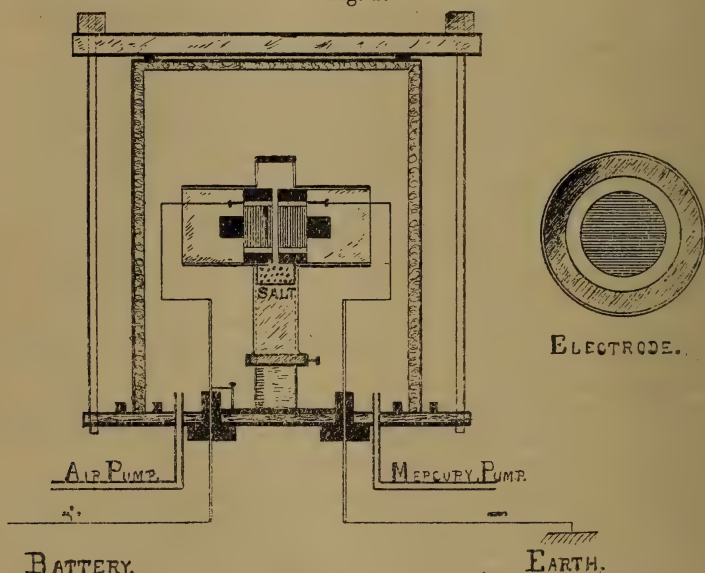
* Russ, *Phil. Mag.* June 1908.

In the various experiments referred to above on the active deposits from radium, thorium, and actinium, the different investigators—with the exception of Debierne—do not appear to have taken any precaution to study the behaviour of the active deposits with uniform electric fields, and as it was thought that some points which are more or less obscure in connexion with these active deposits might be cleared up if they were studied in this manner, it was decided to apply this method to the investigation of the active deposits from actinium, which on account of the short life of its emanation is peculiarly suitable for the purpose.

II. *Apparatus.*

The apparatus consisted of a metallic cylinder about 5.5 cm. in diameter, which was supported horizontally in an air-tight chamber. Into this cylinder (as shown in fig. 1) there were

Fig. 1.



fitted two electrodes provided with guard-rings. The salt was carried in a small tray which could move freely up and down in a vertical tube (1.5 cm. in diameter), which led into the cylinder. The tray could be clamped in position at any desired distance from the electrodes, and the latter, which were capable of easy motion, could readily be placed in the exposing cylinder at any selected distance apart. The air-tight chamber was also provided with tubes for the

admission and removal of the different gases used, and through its base wires suitably secured were led, for the purpose of charging the electrodes. The electrodes, which were circular, were 2.5 cm. in diameter, and the guard-plates which surrounded them were each .5 cm. in width.

III. *Measurements.*

In the various experiments which are to be described later, the sample of actinium used was obtained from the Chininfabrik, Braunschweig. In making the exposures the electrodes were exposed in every case to the action of the emanation for two hours before being removed from the exposing vessel for measurement. The activities of the electrodes were tested by an ordinary α -ray gold-leaf electroscope, and all the values which are quoted in the paper represent the activities of the electrodes 10 minutes after the exposures ceased. In making the measurements of the activities of the two electrodes, observations were continued for a period of forty or fifty minutes. From these observations, of which fig. 10 (Pl. XXII.) is illustrative, the rates of decay of the deposits on both anode and cathode were found to be the same, and to be approximately about 39 minutes. In making all the exposures the electrodes were charged to a potential of approximately 250 volts.

IV. *Active Deposits and Distance between the Electrodes.*

In commencing the study of the active deposits from actinium a set of measurements was made on the active deposits obtained on the two electrodes when the salt was placed in the vertical tube at a constant distance from the cylinder, and the electrodes were gradually separated. In a particular set of observations the plates were placed vertically 1 mm. apart, and the salt was brought as close to them as the construction of the apparatus would permit. With this arrangement the salt, which was always covered with a layer of thin filter-paper, was at a distance of 11 mm. from the lower edge of the plate electrode.

Observations were made at atmospheric pressure on the active deposits obtained on both electrodes for distances 1, 2, 3, 4, 5, 6, and 8 mm., and the results are all recorded in Table I. A curve illustrating them is shown in fig. 2 (Pl. XXII.). From these results it will be seen that as the electrodes were separated the activity obtained on both plates steadily decreased. With distances apart greater than 3 mm. no measurable activity was obtained on the anode, but with the greater distances, viz. 8 mm., the active deposit obtained

TABLE I.—Atmospheric pressure.

Separation of Electrodes.	Activity on the Cathode.	Activity on the Anode.
1 mm.	17.0	.15
2 "	13.7	.1
3 "	11.5	.0
4 "	10.3	.0
5 "	9.0	.0
6 "	8.7	.0
8 "	8.6	.0

on the cathode was still about one-half that obtained for a separation of only 1 mm. on the same terminal.

V. Active Deposits and Distance of Salts from the Electrodes.

Measurements were also made on the active deposits obtained on the electrodes at pressures of 760, 120, 25, and 5 mm., when the distance between the electrodes was maintained at 2 mm., and the salt placed at a series of distances from the electrodes varying from 1.1 cm. to 5 cm. These results are recorded in Table II., and curves illustrating

TABLE II.

Pressure.	Distance of Salt from Electrodes.	Activity on the Cathode.	Activity on the Anode.
Atmospheric	1.0 cm.	39.5	.8
"	1.3 "	26.5	.3
"	2.0 "	8.5	.1
"	3.0 "	2.75	.0
"	4.0 "	1.0	.0
"	5.0 "	.5	.0
120 mm.	1.1 cm.	73.0	5.5
"	2.0 "	36.8	3.2
"	3.5 "	16.5	1.7
"	4.0 "	6.3	.7
25 mm.	1.1 cm.	63.5	27.0
"	2.0 "	41.0	17.5
"	3.5 "	25.3	11.8
"	5.0 "	15.7	6.7
5 mm.	1.1 cm.	32.3	29.5
"	2.0 "	27.0	23.7
"	3.5 "	17.7	16.0
"	5.0 "	13.5	11.3

them are drawn in figs. 3, 4, 5, and 6 (Pl. XXII.). From the numbers given and from the form of the curves it will be seen that for all pressures the activity obtained on both electrodes steadily decreased as the distance between the salt and the electrodes was increased. This result, it will be seen, is somewhat different from that obtained by Russ with his apparatus, for, as stated above, he found at certain pressures that the active deposits slightly increased as the salt was removed, attained maximum values at certain distances, and finally fell away as the salt was still further removed.

VI. Activity of Deposits in Air dependent upon Pressure.

The next variation made in the experiments was to keep the electrodes at a constant distance apart, 2 mm., and the salt at a fixed distance 11 mm. from them, while the pressure of the air was gradually lowered, and the activity of the deposits corresponding to different pressures was measured. The range of pressures investigated was from 750 to .5 mm. of mercury. Table III. gives the results for air, and

TABLE III.

Pressure.	Activity on the Cathode.	Activity on the Anode.
.5 mm.	11.7	11.3
1.0 "	14.5	14.0
2.0 "	18.5	17.0
7.5 "	37.5	29.0
12.0 "	49.0	33.0
25.0 "	62.5	28.5
42.0 "	82.3	17.7
72.0 "	92.0	10.0
92.0 "	91.0	7.5
120.0 "	76.0	5.5
135.0 "	73.0	5.0
162.0 "	68.0	4.0
192.0 "	66.5	3.0
320.0 "	43.0	1.15
500.0 "	31.0	.70
750.0 "	20.5	.45

figs. 7, 8, and 9 (Pl. XXII.) were drawn from them, and illustrate the manner in which the deposits occurred at the various pressures. The curves for both electrodes, it will be seen, follow similar laws. The activities on both electrodes steadily rose as the pressure fell, both passed through maximum values, and both fell away again and approached equality at the lowest pressure.

The maximum activity for the cathode was obtained at a pressure of 80 mm., while that on the anode was not obtained until a pressure of 17 mm. was reached. The maximum cathodic deposit, it will be seen, too, was only about 2.75 times that obtained on the anode.

From time to time decay curves were drawn for the active deposits obtained on the two electrodes, and it was always found that these deposits decreased in activity to half value in about 39 minutes, irrespective of the electrodes upon which they were obtained. This fact, combined with the similarity of form in the activity curves for the two electrodes, goes to show that with both electrodes the deposit always consisted of the same transmutation product or products, and that the difference in the amounts obtained on the two terminals must be traceable either to differences in the charge acquired by the deposit particles in their passage, by diffusion or otherwise, through the air, or to the phenomenon of recoil recently noted by Otto Hahn *.

VII. Activity of Deposits obtained in Carbon Dioxide at different Pressures.

In order to study how the deposits might be affected by a modification in the conditions of their diffusion a set of measurements, similar to those carried out with air, was made with carbon dioxide at pressures varying from 750 mm. to 1 mm. of mercury, and the values of the active deposits obtained are recorded in Table IV. From these results

TABLE IV.

Pressure.	Activity on the Cathode.	Activity on the Anode.
1.0 mm.	15.6	14.7
8.0 "	37.5	28.5
18.0 "	60.5	29.5
30.0 "	75.5	19.6
40.0 "	79.5	16.0
73.0 "	81.6	9.0
105.0 "	72.0	7.0
123.0 "	66.5	5.8
235.0 "	46.7	3.0
480.0 "	22.0	2.2
750.0 "	11.7	.75

curves in figs. 11 and 12 (Pl. XXII.) have been drawn. It will be seen that these curves also follow laws similar to those

* *Deut. Phys. Ges.* xi. Jahr. no. 3.

obtained with air, and that the deposits on the two electrodes gradually increased as the pressure was lowered. Both passed through a maximum, and for still lower pressures the active deposit on both plates decreased and approached equality for the lowest pressures investigated. The maximum activity on the negative electrode was obtained at a pressure of 60 mm., while the maximum active deposit on the positive terminal was not obtained until the pressure was 14 mm. The maximum active deposit on the cathode was about 2.68 times the maximum deposit on the anode, and this, it will be seen, is not very different from the ratio which was found for the maximum activities obtained in the case of air. Care was taken in the experiments with carbon dioxide and air to repeat all the observations a number of times, and the curves indicated in figs. 7 and 11 (Pl. XXII.) were uniformly obtained for the two gases under the conditions described.

VIII. *Activity of Deposits obtained in Hydrogen at different Pressures.*

Another series of measurements was made with hydrogen, under conditions similar to those already described with air and carbon dioxide, and the active deposits obtained at the various pressures are given in Table V., and a curve representing them in fig. 13 (Pl. XXII.). From these observations

TABLE V.

Pressure.	Activity on the Cathode.	Activity on the Anode.
6 mm.	21.0	20.0
46 „	52.0	43.0
117 „	83.0	43.7
235 „	106.0	27.7
457 „	87.3	15.0
760 „	78.3	8.3

it will be seen again, that as the pressure was lowered, the active deposits on both electrodes steadily increased, passed through maximum values, decreased, and on the decrease approached equality at the lowest pressures examined. In hydrogen the maximum active deposit on the cathode was obtained at about 250 mm., but the maximum active deposit on the anode was not obtained until about 80 mm. pressure was reached. The maximum active deposit on the negative

electrode was about 2·3 times the maximum active deposit on the positive terminal. Repeated measurements have not been made to confirm these results, but the same precautions were taken with hydrogen as with air and carbon dioxide. It will be seen from the values given, that the ratio of the maximum activities for the two terminals in the case of hydrogen is only slightly less than the corresponding ratios for air and carbon dioxide.

IX. Comparison of the Active Deposits in Air, Carbon Dioxide, and Hydrogen.

For purposes of comparison the active deposits obtained under the various circumstances are collected in Table VI. From this Table it will be seen that the pressures at which

TABLE VI.

Gas.	Active Deposit on the Cathode.	Pressure.	Active Deposit on the Anode.
Air.	20·5	750 mm.	·45
"	92·0 (maximum)	80 "	
"		17 "	33·0 (maximum)
"	11·7	·5 "	11·3
Carbon Dioxide.	11·7	750 mm.	·75
"	82·0 (maximum)	60 "	
"		14 "	31·0 (maximum)
"	15·6	1 "	14·7
Hydrogen.	78·3	760 mm.	8·3
"	108·0 (maximum)	250 "	
"		80 "	46 (maximum)
"	21·0	6 "	20

the maximum cathode deposits were obtained for the different gases are—carbon dioxide 60 mm., air 80 mm., hydrogen 250 mm. These pressures are approximately in the ratio 1 : 1·33 : 4·2. Now, since the coefficient of diffusion of a gas is inversely proportional to the molecular weight of a gas into which it is diffused; and since, further, the coefficient of diffusion is inversely proportional to the total pressure of the two diffusing gases, it follows, since in this case the diffusing transmutation product must be exceedingly minute, that the coefficient of diffusion for the product will be inversely proportional to the pressure of the gas into which it is passing.

Further, since the maximum activities on the cathode for the different gases are approximately the same, *i. e.* 82, 93, and 108, we may look upon the results as due to the diffusion

of a maximum effect in the three gases, and consequently for the critical pressures given above, deduce an approximate estimate of the ratios of the coefficient of diffusion of the active product or products into the three gases at atmospheric pressure. These follow directly from the argument just presented, and are given in Table VII.

TABLE VII.

Gas.	Pressure for maximum Active Deposit on the Cathode.	Calculated Ratios of the Coefficient of Diffusion of the Active Product into the different Gases at Atmospheric Pressure.
Carbon Dioxide.....	60	1
Air	80	1.3
Hydrogen	250	4.2

Following the same line of argument in case of the anode deposits, since the maximum effects were obtained at pressures 14 mm., 17 mm., and 80 mm., for carbon dioxide, air, and hydrogen respectively, it follows that the ratios of the coefficients of diffusion of the active products concerned were as 1 : 1.21 : 5.7, *i. e.* the relative coefficient for air was slightly lower and that for hydrogen slightly higher than the values deduced from the behaviour of the cathode deposits.

The ratios of the coefficients of diffusion of the active product concerned, as deduced from the cathode deposits, are practically the same as some values given in a paper by Russ*; and this agreement goes to show that it is a diffusion phenomenon which is the paramount one in the present investigation.

The interpretation of the maximum effect obtained on each electrode with the three gases, however, presents some difficulty. One should have expected, with the active deposits in the experiments in which the salt was placed at different distances from the electrodes, that in the case of the lowest pressures a maximum value would have been obtained for a certain critical distance of the salt from the plates. But, as the curves in figs. 5 and 6 (Pl. XXII.) show, no such maximum values appeared.

* Russ, Phil. Mag. March 1909.

X. *Active Deposits in the Absence of Electric Fields.*

In the experiments described up to the present the active deposits were all obtained with a potential-difference of approximately 250 volts between the electrodes; with these conditions, however, it was impossible to draw any definite conclusion as to the relative quantities of charged and uncharged deposit particles involved in any particular measurements. With the object of throwing some light on this point an additional set of observations was made on the deposits obtained in air at different pressures with the electrodes uncharged, and at a distance of 2 mm. apart.

The activities obtained on the two electrodes in these experiments were added together, and the numbers representing them are given in Table VIII. For purposes of comparison the total activities obtained with air at different pressures under a field of 250 volts are also inserted in the Table, and curves representing both sets of values are shown in fig. 14 (Pl. XXII.)

TABLE VIII.

Pressure.	Total Deposit with Field of 250 volts.	Total Deposit. No Field.
9 mm.	46.1	21.6
43 „	64.0	52.0
80 „	65.3	54.4
127 „	50.5	50.4
422 „	23.0	24.9
755 „	13.4	10.8

From an inspection of the two curves it will be seen that the total active deposit was practically the same with and without the field at all pressures above the critical one. At and below this pressure the deposits obtained with the electric field applied, as the figure shows, were somewhat in excess. From this experiment it would seem that the deposit particles in very great measure go to the walls of the vessel in which they are produced whether an electric field be applied or not.

The manner in which they are carried there, however, is not evident. It is possible that a certain proportion of the deposit particles are uncharged, and that these reach the walls by ordinary diffusion.

Then again these deposit particles may be electrically charged, some of them being of one sign, and some of the

opposite, and diffusion again may be the chief factor in producing the deposit; or further, if the disintegration recoil phenomenon is the determining factor, it is possible that, with the plates close together, the deposits are made by reason of the velocity of expulsion alone. If this latter be the explanation, the sign of the charge carried by the deposit particle would not then exert any considerable influence except in the most intense fields.

In all the measurements made the cathode deposit was, except at the very lowest pressures, considerably in excess of that obtained on the positive terminal. This goes to show that part at least of the deposit particles carry a positive charge and reach the electrode under the influence of the field.

The manner in which these particles gain their positive charge, however, is not clear.

From experiments by Logeman* and others it is known that a plate of copper on which polonium is deposited emits a copious stream of delta particles. These, it is also known, are beta particles of low velocity, which are very probably ejected either from the copper or from the polonium or its transmutation product under the bombardment of the alpha particles. It is possible too that the delta radiation may be caused by and accompany the α particle in its expulsion from the parent atom. Such a plate of copper as that mentioned above is known to acquire a positive charge when placed in a very high vacuum, which shows that an excess of negative electricity leaves it as a result of the action of the various radiations.

It is possible then that some of the deposit particles from actinium or other active emanations may gain a positive charge under the action of the alpha radiations present in much the same way as the copper plate in the polonium experiments. This would then account for the positive charge on the particles, and consequently for their removal under the field to the negative terminal.

On the other hand, some experiments recently made by H. W. Schmidt† have brought out a parallelism between the amount of active deposit obtained in air under different electric fields and the intensity of the ordinary conduction current through the air under the same fields. This would seem to show that the charges are acquired by the deposit particles as a result of collision with the gaseous ions present in the same vessel.

* Logeman, Proc. Roy. Soc. A, lxxviii. p. 212 (1907).

† *Phys. Zeit.* ix. pp. 184-187, March 1908.

Many of the results obtained in the present investigation go to support this view, and one in particular is of special interest.

In this experiment the electrodes were 2 mm. apart, the salt was placed 1.3 mm. below the electrodes in the exposing vessel, and the exposures were made under different voltages in air at a pressure of 9 mm. The exposures were made first with the two electrodes at the same potential, and then with them at different potentials ranging in extent up to 1150 volts.

The results of the various activity measurements are given in Table IX., and curves drawn from these numbers are shown in fig. 15 (Pl. XXII.).

TABLE IX.

Pressure 9 mm. ; Electrodes 20 mm. apart ; Distance of Salt 1.3 cm.		
Voltage.	Activity on Negative Electrode.	Activity on Positive Electrode.
0	11.0	10.6
83	12.1	10.0
242	14.9	10.2
467	15.5	10.3
523	15.8	11.2
545	18.6	14.2
605	17.4	13.0
641	35.8	25.0
683	19.2	13.1
794	30.7	19.2
928	30.7	25.0
955	38.7	21.6
1149	29.4	19.2

Note.—Sparking potential for pressure 9 mm. and distance 2 mm. = 460 volts.
(Carr, Phil. Trans. Roy. Soc. vol. cci. pp. 403–433.)

From these results it will be seen that when the potentials of the two plates were the same the activities of both were alike ; but that for all potential-differences the activity of the negative terminal was greater than that of the positive. For potential-differences from 200 to 450 volts the activities were practically independent of the potential-difference applied. From 460 volts upward, however, the activities of both electrodes increased, and finally at about 800 volts again became independent of the applied potential.

From Carr's results * it is known that 460 volts is the spark potential for a pressure of 9 mm. with the plates 2 mm. apart. Consequently for all voltages above 450 the exposures were made with a current passing between the plates. This would mean that a large number of ions were present between the terminals during the exposure; and it is interesting to note that the presence of these ions resulted in a considerable increase in the activity of the two plates. But just how this result is brought about is difficult to explain as the exact relation which exists between the number of ions present in a gas and the active deposit particles is still obscure, and it will be necessary to make additional experiments before the question can be cleared up.

One of the main questions left open in the present investigation is the cause of the decrease in activity of the electrodes in the three different gases at the low pressures.

It seems fair to conclude from the results that in the region of low pressures there was a gradual decrease both in the excess of positively charged deposit particles, and also in the total number of the particles present in the space between the electrodes.

The experiments, however, do not show whether this decrease was due to a falling off in the amount of emanation coming into this space or in the amount of emanation and deposit particles passing directly through it into the outer chamber of the apparatus. From the fact that at the low pressures as well as the high ones the activity fell away as the distance of the salt from the electrodes was increased, it would seem that the decrease mentioned above was due to decrease in the amount of emanation entering the space between the electrodes. The matter, however, is not clear, and consequently the explanation of the decrease must be deferred until the scope of the investigation can be extended.

In conclusion I desire to thank Professor McLennan for the selection of the subject, and for the very helpful suggestions he has offered from time to time during the investigation.

Physical Laboratory,
University of Toronto.
May 26th, 1909.

* Phil. Trans. cci. pp. 403-433.

LXXVIII. *Talbot's Fringes and the Echelon Grating.* By
R. W. WOOD, *Professor of Experimental Physics, Johns
Hopkins University**.

[Plate XXIII.]

SO much has been written in explanation of Talbot's fringes and the curious circumstance that they appear only when the retarding plate is introduced from the side upon which the red of the spectrum appears (in the case where the plate is in front of the objective of the spectroscope) that it may appear at first sight that anything further must be superfluous. Personally I never felt satisfied that I fully understood the physical explanation of this circumstance until Schuster's explanation appeared, which is quite satisfying and very easily grasped. On looking into the matter a little more fully, and subjecting some of the conclusions drawn from theoretical considerations to the test of experiment, it has seemed to me that this explanation does not account for the failure of the fringes to appear when the plate is introduced from the wrong side. I have accordingly been forced to work out another explanation, which I believe to be more complete and to present less difficulty than the previous ones. The conclusions arrived at are doubtless contained in the elaborate mathematical treatments which have been given by Airy and Stokes, but there seems to be a good deal of difficulty in forming a definite idea of just what happens in each case from an inspection of these equations. Incidentally we shall find that the echelon grating is a special case of Talbot's plate, the aperture with its retarding plate being simply an echelon grating of two elements.

The treatment given by Walker (*Phil. Mag.* April 1906), while correct as far as it goes, makes no mention of the splitting of certain monochromatic elements of the continuous spectrum into double lines, which I pointed out in my '*Physical Optics*,' and is in addition a little difficult to follow. An explanation to be perfectly satisfactory should enable us to form a clear picture of exactly what happens to each monochromatic element of the continuous spectrum, and how these elements are re-arranged so as to give us a spectrum traversed by black bands.

The following brief treatment enables us to understand the action of the echelon grating and all of the peculiarities of the Talbot fringes. Assume plane waves of monochromatic light incident upon a rectangular aperture, brought to

* Communicated by the Author.

a focus by a lens. The first diffraction minima to the right and left of the central maximum will lie in directions such that the path difference between the disturbances coming from the edges of the aperture is a whole wave-length. If the right-hand half of the aperture is covered with a transparent plate which retards the wave-train one half wave-length, we shall have zero illumination at the centre, and two bright maxima of equal intensity in the position previously occupied by the first minima. It is clear that the path difference for the left-hand maximum is one wave-length, and for the right-hand one zero. We may therefore call the former the spectrum of the first order, and the latter the spectrum of zero order, considering the aperture as a grating of two elements. If we consider the change as taking place progressively, starting with a transparent lamina of infinite thinness which gradually thickens, it is clear that the central maximum moves to the right, decreasing in intensity, while its neighbouring maximum to the left increases in brightness moving in the same direction. When the thickness corresponds to a half-wave retardation these two maxima are of equal intensity, and occupy the positions previously occupied by the minima to the right and left of the central maximum. If we increase further the thickness of the plate the maxima drift further to the right, the left-hand one increasing in brilliancy, the right-hand one decreasing. When the retardation becomes one whole wave-length we again have a single bright maximum at the centre, as with the uncovered aperture. In this case, however, it is the spectrum of the first order which is at the centre. As we go on increasing the thickness of the plate the march of the fringes continues, new ones coming into view on the left and disappearing on the right, and the order of spectrum increasing. The condition when we have a pair of bright maxima will be recognized as the condition which obtains when an echelon grating is set for the position of double order, the single order position corresponding to the case when we have a single bright maximum at the centre.

Suppose now that we have a thick retarding plate and gradually decrease the wave-length of the light which illuminates the slit from which the plane waves are coming. We shall have the same march of the fringes as before, the gradual increase in retardation resulting from the decrease in wave-length. If our source emits two wave-trains of different wave-length, and the retardation for one of the trains is an odd, and for the other an even number of half wave-lengths, it is clear that the single and double maxima

exist simultaneously, and all trace of the maxima and minima will disappear owing to the form of the intensity curve; in other words, the lines are not resolved.

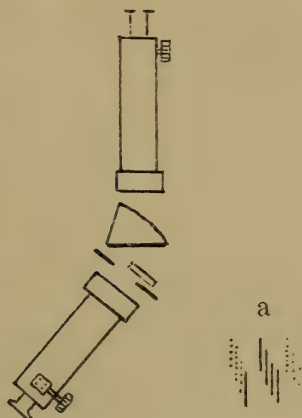
If, however, we increase the number of retarding plates, placing them so as to divide the aperture into a number of vertical strips of equal width, the width of the maxima decreases in proportion to the distance between them, as in the case of the ordinary diffraction-grating when we increase the number of lines, and we have resolution of the lines. Our aperture is now acting as an echelon grating, set in position of single order for one train of waves and double order for the other. With very thick plates, as in the case of echelons such as are actually used, it is evident that a very minute change in wave-length will be sufficient to change the retardation by the amount necessary for resolution. We are now in a position to discuss the effect of the thickness of the plates, their number, and the width of the steps. Decreasing the width of the steps increases the width of the space between the spectra, in other words, forms the system of maxima and minima on a larger scale. Increasing the thickness decreases the change in wave-length necessary in passing from the condition of single order to that of double order, in other words the greater the thickness the greater will be the shift of a maximum for a very minute change in λ , and in consequence the greater the resolving power. An increase in the number of plates merely reduces the width of the maxima, without affecting the distance between them, or the amount of shift produced by a given change of λ . This then is the whole theory of the echelon in a nutshell (a coco-nut perhaps). Personally I have found that a class can be made to understand the echelon much more clearly by this method than by prolonged study of the mathematical treatment. I have already given a very simple treatment of the decrease in the width of the principal maxima, and the increase in the number of the secondary maxima in the case of the ordinary diffraction-grating as the number of lines is increased (Phil. Mag. vol. xiv. p. 477 (1907)).

We are now ready for the Talbot fringes.

Consider the slit of a spectrometer illuminated with monochromatic light, the prism, aperture, and retarding plate arranged as shown in fig. 1. If the plate retards the stream an even number of half wave-lengths, we shall see in the instrument a single bright line with accompanying faint maxima and minima. If we now decrease the wave-length a trifle the line will move a little to the right, if we disregard the action of the prism. We can verify this experimentally

by removing the prism and viewing the slit directly with the telescope. The slit can be illuminated with a monochromatic illuminator (spectroscope with narrow slit in place of the eyepiece), the slit of the instrument being substituted for the slit of the first spectroscope.

Fig. 1.



As we gradually decrease the wave-length by turning the prisms of the illuminator we shall see the line become double and single in succession, the doubling being accomplished by the march of the fringes to the right and their fluctuating intensity in the manner already described. It is a little difficult to put into words the changes which accompany the alteration of wave-length. The appearance is much like that presented by a picket fence in motion viewed through a narrow vertical aperture, *i. e.* the pickets are only visible as they pass across a narrow vertical region. If we imagine the visibility of the pickets to be a maximum as they cross the centre of the aperture and least when at the edges, we shall have a fairly accurate conception of the appearance of the moving maxima. Suppose that a change of 100 Ångström units in the wave-length is sufficient to change the single line to the double line. The changes which take place in the focal plane of the telescope (prism removed) as we decrease the wave-length over this range are indicated in fig. 1 *a*. To avoid confusion I have represented each successive appearance a little lower down in the figure.

Now consider the prism in place, turning the telescope so as to view the deviated image. As we decrease the wave-length there will be a shift to the *right* as before resulting from the retardation of the plate, and a shift to the *left* due

to the increasing deviation produced by the prism. If the compensation is perfect the prismatic dispersion will shut up the picket-fence arrangement of lines in the diagram into single lines; in other words, if we illuminate the slit simultaneously with all of the wave-lengths in the given range of 100 A.É. we shall not see a short continuous spectrum, but single bright lines, to the intensity of which all the waves contribute. These lines are of course much narrower than the corresponding range of the continuous spectrum which would be formed in the absence of the plate. We thus see that the retardation has the effect of compressing a narrow region of the spectrum into a much narrower one which constitutes one of the bright Talbot bands. If we consider what happens to the entire spectrum, we can perhaps obtain a still clearer idea of how the bands are formed.

Consider a large number of equidistant monochromatic constituents of the spectrum: call these the lines A. Mid-way between them is another set of lines B. The distance from a line A to B is such that the difference in the retardation for the two lines is half a wave-length. If the retardation for the A lines is an even number of half waves, they will remain single and fixed in position. The lines B will, however, double and fall upon the neighbouring lines A. The regions previously occupied by the lines B are the dark Talbot fringes. It may at first sight appear as if the same result would be obtained regardless of the side on which the retarding plate was introduced. It obviously would for the wave-lengths A and B; but if we consider the elements of the spectrum between the A and B lines, we shall see that in one case they are shifted so as to fall upon the stationary lines A, while in the other they are moved into the region between. In the latter case we have a continuous spectrum if we consider the widths of the regions A and B infinitely narrow. In the foregoing treatment we have considered the condition as that of the "best thickness" which obtains when shifts due to retardation and prismatic dispersion exactly compensate. It is clear also that to obtain the bands the aperture must be somewhat restricted so as to obtain an appreciable doubling of the lines for which the retardation is an odd number of half waves.

I have verified all of these points experimentally to make sure that no slip had been made. To show that the different monochromatic constituents were differently affected by the retarding plate, three spectra were photographed in coincidence. The first (No. 3) shows the Talbot bands with white light illuminating the slit, the second (No. 2) with the iron

arc spectrum under the same conditions, and the third (No. 1) the iron arc without the retarding plate. These spectra are reproduced as negatives on Pl. XXIII. fig. 1. At some points in the spectra the iron lines appear the same, in other points they appear doubled. These latter points are indicated by arrows.

We thus see that the Talbot bands can be fully accounted for without in any way making use of the idea utilized by Schuster, that a stream of waves which is already behind another stream cannot interfere with the latter, if it is put still further behind by retardation. The advanced portion of the train must be retarded if interference is to take place. While this explanation is not open to criticism, if it is true that the grating constructs the monochromatic elements of the spectrum from the assumed irregular pulses which are supposed to constitute white light, it appears to me that we can account for the fringes without making this supposition. In other words, Talbot's fringes would be formed even if the two streams could interfere regardless of which one was retarded, and this being the case it does not seem as if Schuster's explanation can be accepted as the correct one.

We can, in fact, form Talbot's fringes under conditions which preclude the factor considered by Schuster, and we find that they fail to appear when the retarding plate is on the wrong side, though interference is still taking place.

We find that if we take highly homogeneous light to start with, such as the iron spectrum, the lines are doubled as before even when the plate is introduced from the wrong side. This is what we should expect, for we now have much longer trains of consecutive disturbances, and one train can interfere with the other regardless of whether it is advanced or retarded. The same will hold true if we take a band spectrum of very narrow and close lines which appears continuous in our spectroscope. Talbot's bands can be seen in the nitrogen band spectrum only when the plate is introduced from the proper side, though we must consider interference as still taking place when the plate is put on the wrong side.

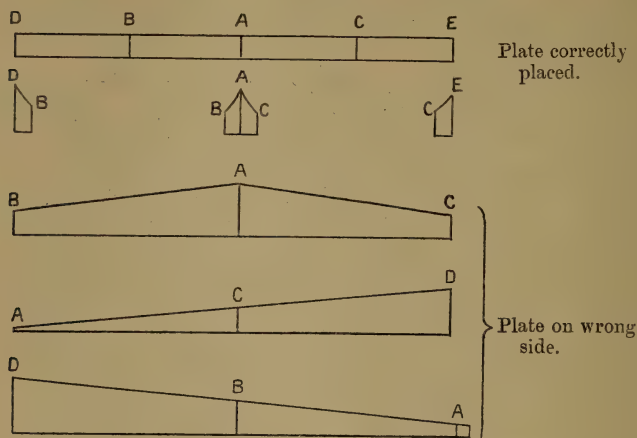
Supplementary Note.

The interference in this case is not to be considered as introducing dark regions in the spectrum, but as redistributing the wave-lengths; in other words, decreasing its purity.

Let us calculate what happens to the continuous spectrum for the two cases involving the correct and incorrect placing

of the plate. This may be done by taking the limited region of the spectrum DE (fig. 2).

Fig. 2.



Assume it to be of infinite purity, and consider that the narrow regions at D, A, and E remain single when the retarding plate is introduced, while those at B and C become double, in the manner previously described. Furthermore, consider that they also become double when the plate is introduced on the wrong side, as is the case when we use homogeneous light. I have made the calculation for the wave-lengths D, B, A, C, E, and also a number of intermediate values, by drawing the spectrum on coordinate paper, and calculating what happens to each element, as a result of the retardation. The lines between those which are lettered become double, the components, however, are of unequal intensity and are unsymmetrically placed with respect to the original position of the line. With the retarding plate in the right position we find that the elements of the spectrum are pushed together into the position of the lines D, A, and E, forming the bright Talbot fringes (see fig. 2).

In the case of the incorrect position, the regions of the spectrum between AB, AC, &c., instead of being squeezed together are pulled out, the intensity distribution being as shown in fig. 2 (superposition of 3 lower figures). It is clear that no dark bands appear in this case, and yet at any point in the spectrum we shall find in general a number of different wave-lengths, as a result of the interference which we have assumed to take place. It seems to me, therefore, that we can account for the failure of the bands to appear,

even if we *assume* interference between the retarded and unretarded portions of the wave-train. The question now suggests itself "Does interference take place, in the manner supposed, in the case of the continuous spectrum formed from white light?" I have attempted to test this matter in the following way:—The Talbot fringes were focussed upon the slit of a second spectroscope. If the slit fell upon a bright band, the band appeared much drawn out, owing to the presence in it of a number of different wave-lengths. If it fell upon a dark band the image in the eyepiece appeared much fainter.

If the slit of the second spectroscope was opened wide the bands appeared, but they were very faint, owing to the widening of the bright bands by the dispersion, which caused them to overlap the dark regions. The slit was now narrowed again, and the retarding plate introduced from the wrong side. If interference took place in the manner assumed we should expect three or more wave-lengths (lines separated by dark spaces) to appear in the eyepiece. In other words, if we put the slit at the point A in the spectrum we should expect A to appear very bright, and C and B as well, with about half the intensity of A. This was found to be the case. Three bright lines, separated by very black regions, appeared in the eyepiece.

I next removed the mica plate from the aperture. In the eyepiece there now appeared in monochromatic light the diffraction maxima and minima of the 1st class, a central one very bright bordered by others much fainter. On introducing the mica plate, again on the wrong side, the central maximum became furrowed by black minima, breaking up into the three bright lines previously alluded to. This seems to show that interference takes place even when we retard the wrong half of the train. Very likely I have fallen into some pitfall, and I am sure that I hope that Prof. Schuster will discover it. The subject seems to me to be a tricky one to deal with, so many factors have to be taken into account, such as the decrease of purity resulting from the limited aperture, the presence of the second spectroscope, &c. It is probable that the second spectroscope can be shown to be responsible for the interference, as when showing fringes of high order in the case of the so-called direct interference of white light. There seems at first sight to be no escape from the conclusion, that if a spectroscope shows three bright lines when placed at a given point in the spectrum, these three wave-lengths must be present. We must remember, however, that spectroscopic analysis of white light, interfering

with large path-difference, might be erroneously interpreted, as indicating the presence of long trains of monochromatic radiation in the white light, whereas the thing is in reality produced in the spectroscope.

This paper must not be considered in any way as an attack upon the impulse theory of white light.

The only point which I wish to question is this: If we can account for the presence and absence of the Talbot bands in the two cases, by the rearrangement of the wave-lengths, *assuming interference as taking place in each case*, are we justified in saying that their absence in one case can be explained by the *inability* of the retarded portion to interfere with the unretarded? Would they not be absent just the same even if interference did take place? I am not to be understood as claiming that interference does take place in the case when we start with white light, in spite of the experiment which appears to show that it does, for I cannot imagine how a wave-train, already behind another train, could be made to interfere with it by further retardation, unless perhaps we explain it in the same way (by resonance) as Schuster explains the formation of interference fringes with white light when no spectroscope is employed.

The object of the present paper has been merely to "visualize" the manner in which the fringes are formed. I have attempted to carry the thing a point further, for it has occurred to me that even with a source giving what appears to be a continuous spectrum in a spectroscope of low power (*e. g.* the band spectrum of nitrogen), the line constituents of which are highly monochromatic, and hence capable of interfering regardless of which half of the beam is retarded, an infinitely narrow linear (vertical) element of the spectrum is made up of more than one wave-length, on account of the limited resolving power. I have not, however, been able to see just how this will affect the case just considered.

Very likely Professor Schuster will be able to defend his position, and I am sure that I hope he will, for his explanation is a very neat one, and I dislike to feel that we must abandon it.

Talbot's Fringes produced by an Echelon.

If, instead of covering the aperture with a single thin plate, we cover it with a pile of thin plates arranged in steps, we find that Talbot's fringes appear as before. Their appearance is however quite different, for the minima are now very broad and the maxima very narrow and bright.

Our pile of plates forms an echelon grating, and the diffraction maxima are much narrower, consequently the light of the continuous spectrum is squeezed into regions which are narrow in comparison to the width of the dark regions.

The mica echelon, which I described in a previous paper (Phil. Mag. vol. i. 1901, p. 627), answers admirably for the production of these fringes, a photograph of which is reproduced on Pl. XXIII. fig. 2.

Fainter secondary maxima appear in the dark regions, their number, position, and the distribution of the intensity among them depending upon the angle at which the echelon is set. We find very unsymmetrical distribution of intensity when the echelon is quite oblique, a circumstance which is of interest in connexion with the recent work of Raman on the diffraction by oblique apertures (Phil. Mag. Nov. 1906).

LXXIX. *What is Interference? A Rejoinder to Professor Wood.* By ARTHUR SCHUSTER, F.R.S.*

THROUGH the courtesy of Professor Wood I am enabled to add a few words of comment to the foregoing paper. The question at issue is the explanation of the observed fact that when a spectrum is viewed with the pupil partly covered with a thin transparent plate, it is found to be crossed by dark bands only when the plate is introduced on that side on which the violet end appears. In the complete theory of these bands which has been given by Airy and Stokes, the characteristic feature, that the bands do not appear when the plate is introduced from the red end, requires a complicated mathematical analysis, which I have shown to become unnecessary when white light is treated as a collection of impulses. It is then at once seen to be obvious that in order to bring two series of impulses to interference, we must retard the front one or accelerate the one that is in the rear. The explanation, which I have based on this principle, throws a new light on the phenomenon, but was not intended to invalidate the correctness of the older theory. The two representations of white light (by homogeneous waves or by impulses) are not mutually exclusive; they represent two points of view, and we may adopt either one or the other in different problems according to convenience.

Professor Wood in his experimental investigation of Talbot's bands approaches the subject by building up his

* Communicated by the Author.

continuous spectrum from a discontinuous one, and the treatment of Stokes becomes therefore to him the more natural one. With his usual skill, he devises instructive experiments illustrating the intermediate steps; and those who prefer to adhere to the older view of white light may find in these experiments sufficient excuse to skip Stokes's rigorous mathematical treatment without too great a strain on their conscience. My own investigation, so far as I can see, remains untouched, as I dealt with white light alone.

Professor Wood now throws doubt on my treatment of the subject by an argument which may be summarized as follows:—(1) Schuster's explanation of the absence of the bands when the plate is introduced on the wrong side is based on the principle that there cannot be interference when there is no overlapping. (It is admitted that the principle is correct.) (2) Interference effects can be observed with homogeneous light even though the plate is introduced on the wrong side. (It is admitted that there is overlapping in this case.) (3) White light can be built up with homogeneous light; and if this is done, the plate still being on the wrong side, the bands disappear, while interference persists. (4) The absence of interference cannot therefore be made the criterion for the disappearance of the bands; and hence "it does not seem as if Schuster's explanation can be accepted as the correct one."

The fallacy here lies at the end of the third step where it is asserted that interference persists even though the effects of interference have disappeared. Matters of definition should be kept clear of arguments on the validity of a certain reasoning, and if two discussions which both explain observed facts satisfactorily, lead to different results as to whether there is "interference" or not, the only deduction that can correctly be made is, that there is disagreement in the use of the word. What indeed is "interference"? Every writer on Optics uses the expression, and yet we may search the literature of the subject without finding a definition which can be applied to test whether in any particular case we can say that there has been interference or not. As long as one is satisfied to use the term generally and somewhat vaguely to classify a certain group of phenomena, no harm is done, and it is sometimes useful to retain a word which by common consent has a plastic meaning. But if it is to be made the test by which an explanation is to be judged, then we must have recourse to some adequate definition. In Mascart's treatise on Optics it

is stated that the principle of interference is identical with the principle of superposition. This is clear and definite, but it would force us to abandon the word altogether, because every optical phenomenon would become a phenomenon of interference, and we should have to admit that rays from independent sources of light could and always would interfere with each other. In order to retain a meaning which shall be definite and concordant with the present usage, I have introduced a definition into the second edition of my 'Optics,' which I think specifies what I believe to represent the consensus of opinion on the subject.

If the observed illumination of a surface by two or more pencils of light is not equal to the sum of the illuminations of the separate pencils, we say that the pencils have interfered with each other and class the phenomenon as one of interference.

Stress should be laid on the adjective "observed." It means that the effect must be observable in the actual case considered and not only be capable of being made visible by some additional appliance. Adopting this definition, the only way in which it can be decided experimentally whether interference takes place when the plate in Talbot's experiment is introduced on the wrong side, would be to observe each part of the aperture separately, and see whether the combined effect is equal to the sum of the separate effects. There is no indication in Professor Wood's paper that he has found any changes of illumination which would indicate that there is interference when the bands are not visible.

Professor Wood's difficulty can be brought to an issue by means of a much simpler case than the one treated by him. Consider the formation of a spectrum by a grating. I assume it to be generally known how an "impulse" of white light is converted into a succession of impulses by the grating, and how at each point of the spectrum more or less homogeneous light is formed by such a succession. Shall we say in this case that a grating acts by interference? My answer is "No," if the impulses are sufficiently short not to overlap. If instead of white light we use homogeneous light, there is undoubtedly interference because the illumination of the whole grating is not equal to the sum of the illuminations due to each line separately. Let now white light be formed by a large number of homogeneous waves spread all over the spectrum. If we argue that the interference of the homogeneous waves is preserved when they are sufficiently crowded together to form a continuous spectrum, we should have to draw a distinction between two kinds of white light, one of which is

white *ab initio* and the other white through the overlapping of homogeneous waves. Though the distinction would only be a verbal one, it is obviously inconvenient. If we adopt the above definition of interference the matter becomes definite. We should have to interpret the result of experiment by saying that interference has disappeared during the process of forming white light. In effect interference is not like a substance which, if present in the several constituents of a mixture, must also be present in the mixture itself. It may be destroyed as well as produced by overlapping. If this be accepted difficulties such as those raised by Professor Wood will cease to trouble us.

The case of Talbot's bands with the retarding plate introduced on the wrong side is best represented by a grating ruled in two parts which are separated by an opaque portion. With white light we should then have at each point of the spectrum a succession of impulses followed first by a quiet interval and then by another succession of impulses. There cannot be a question of interference here ; but if we apply a second grating to investigate the spectrum formed by the first, then if the resolving power be sufficient, we can make the second group of impulses overlap the first and interference would result. For this reason Professor Wood's second spectroscope complicates the phenomenon by introducing the conditions necessary for interference and the observations made with it have no bearing on the question whether there has been interference in the original spectrum or not. These questions were fully considered and settled during the discussion on the nature of white light which it is not necessary to revive now. The manner in which Professor Wood tries to prove that there is interference with the plate in Talbot's experiment on the wrong side, is identical with that by which the regularity of vibration in white light used to be thought capable of being proved by means of experiments with spectroscopes, which themselves manufactured the regularity they were supposed to disclose.

I have entered into these elementary matters because I think that under present circumstances a more scientific use of the word interference would help students over difficulties. At any rate, the point at issue between Professor Wood and myself can only resolve itself into a question of the proper use of words, so long as he cannot point to some experimental and observable distinction between light which is white *ab initio* and light which is made white by mixing homogeneous waves.

LXXX. *On the Formation of Stria in a Dust-Tube by an Electric Discharge.* By THOS. JAS. RICHMOND, B.Sc. (Lond.); Research Student at University College, Nottingham*.

[Plates XXIV.-XXVI.]

IN 1898 H. V. Gill, S.J., in a paper in the 'American Journal of Science,' propounded a theory to account for the striation of the electric discharge in vacuum-tubes. In this paper the author draws attention to the fact that, under suitable conditions, lycopodium powder placed in a glass tube arranges itself in well-marked striæ under the influence of an electric discharge. This suggested the following experiments on the various factors involved in the formation of the striæ, the special object being to ascertain whether the frequency of the electrical oscillations could be obtained from observation of the striation of the powder under their influence.

The nature of the striation might conceivably depend on the nature of the powder used; that is on the size and form of the grains; on the electrical conditions; and on the dimensions of the tube containing the powder. It might also depend, perhaps, on whether the further end of the tube were open or stopped. All these questions have been considered in the following, and quantitative experiments were made in which the frequency and wave-length of the electrical oscillations were measured by means of Fleming's Cymometer, and the striation in each case observed; and although no definite relation between the frequency of the electrical oscillations producing a certain striation and the distance between the striæ was detected, some results were obtained which appear of interest and worth recording.

As regards the effect of the structure of the powder, it is to be observed that an aggregation of spherical particles tends, under the influence of fluid motion, to arrange itself as striæ across the direction of the motion. An example of this effect is afforded by the well-known method due to Kundt for the determination of the velocity of sound. When the note is first elicited from the sounding rod, the lycopodium forms itself into a series of fine close lines along which a series of nodes and antinodes is more or less clearly discernible, but these nodes are not at first sufficiently clearly defined to allow an accurate value of the half-wavelength to be obtained, and it requires more or less continued stroking of the rod to cause the lycopodium to assume that very distinct pattern from

* Communicated by Prof. E. H. Barton.

which the value of the half-wavelength can be satisfactorily obtained. A similar phenomenon was observed in the following experiments especially in the wide tubes.

Preliminary Experiments. Variation of figures with diameter of the tubes.

The first experiments were made with a large Wimshurst machine giving an intense spark about two centimetres long. A length of glass tubing was taken and carefully cleaned by means of a cotton-wool plug, and dried. A small quantity of carefully dried lycopodium was then poured into the tube, the latter being shaken in order to get the particles of powder as evenly distributed in the tube in the form of a cloud as possible. The powder was then allowed to settle, and by gently tapping the tube when horizontal the powder was obtained as a practically uniform line along the tube, which was now placed on supports with one end near the spark-gap of the machine and perpendicular to the discharging-rods. The tube was then rotated very carefully about its axis until the line of powder was just on the point of slipping down the side of the tube. Under these conditions, one or two sparks usually sufficed to give the pattern to the lycopodium, but continued sparking usually added to the distinctness of the striation. It was noticed that at the passage of each spark the powder jumped and fell again at the cessation of each spark.

In the case of the first tube, which was 1.5 centimetres radius, the figure consisted of very distinct close parallel lines filling the whole length, about 60 centimetres, of the tube. It was also noticed that the striæ were curiously bifurcated at places. This bifurcation was found to be removed almost completely by continued sparking. In this and other wide tubes, the first effect of the discharge was to produce a series of very fine interlacing lines which by further sparking gave the final system of parallel lines more or less bifurcated.

A second tube was now taken about one centimetre in diameter, and using the same spark as before the striation of lycopodium was obtained. The pattern formed in this case differed in several respects from that of the previous one. In the first place the distance between successive striæ was greater than in the case of the larger tube; also the lines were of the nature of irregular heaps compared with the remarkably clear regular lines obtained in the first case. Another difference consisted in the lack of bifurcation of the lines in the narrower tube. This freedom of the lines from bifurcation in the narrower tube was found to render the use of such tubes more suitable for quantitative experiments than

wider ones on account of the greater certainty as to the number of lines contained in a certain length of the tube.

The difference in the distances between the lines in the above two cases suggested the investigation of the relation between the striæ-distance—*i. e.*, distance between successive striæ—and the diameter of the tube in which the pattern was formed. A set of five tubes ranging in diameter from 1 centimetre to 4.6 centimetres was taken, and using the same spark in all cases the figure assumed by the lycopodium was observed and measured, care having been taken to ensure the cleanness and dryness of the tubes. This somewhat rough set of readings indicated a turning value in the variation under investigation, as shown in figures given in Table I. and graphically in Curve I. fig. 1 (p. 775).

TABLE I.—Variation of figures with diameter of tube.

Diameter of tube.	Striæ-distance.
·9 cm.	·77 mm.
1·0 „	·77 „
1·6 „	1·00 „
2·6 „	·77 „
4·6 „	·55 „

From these figures it appears that the striæ-distance has a maximum value, viz. 1.00 mm. in the 1.6 cm. tube. In confirmation of the existence of this turning value a second set of readings was taken. The tubes having been carefully cleaned and dried and the pattern in each case obtained as clear as possible, a vernier microscope was used to measure the intervals. The tubes were in turn laid on the stage of the microscope, a piece of black paper being placed underneath to facilitate the reading of the microscope. By this means the figure as viewed through the microscope appeared very distinct and the separate lines could be counted with ease. The striæ-distance was obtained by observing the distance through which the microscope travelled in order that say 10 lines should pass across the intersection of the cross-wires of the eyepiece. It was noticed that some uncertainty existed in some cases as to whether one or two lines had passed across the intersection of the cross-wires owing to the bifurcation of the lines as previously mentioned. This most probably explains the several high values obtained in the figures for the 4.4 centimetres tube shown in Table II. There were

important differences in the character of the striation in the various tubes. Thus in some tubes, particularly in one about 3 cms. in diameter, as many as 50 to 150 lines were clearly formed and measured, while in others only about 20 were formed at all and some of these indistinctly. The readings of this set of experiments are shown in Table II. and graphically in Curve II. fig. 1.

A further set of readings is given in Table III. and graphically in Curve III. fig. 1.

TABLE II.

Diameters of tubes (mm.).				
Readings for striæ-distances.	3.5.	9.	16.5.	44.
	.72 mm.	.93 mm.	.75 mm.	.52 mm.
	.80 "	.94 "	.72 "	.55 "
	.82 "	.95 "	.73 "	.58 "
	.75 "	.94 "	.77 "	.59 "
			.76 "	.55 "
				.53 "
				.59 "
				.54 "
				.64 "
				.57 "
				.63 "
				.55 "
Means ...	.77 mm.	.94 mm.	.76 mm.	.56 mm.

TABLE III.

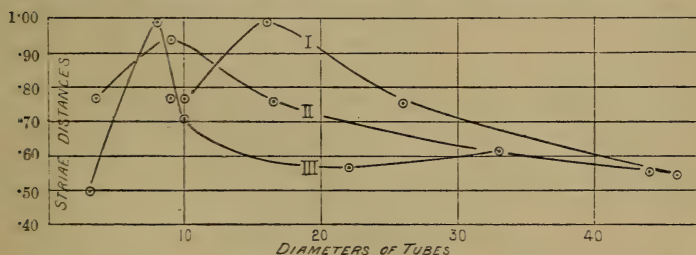
Diameters of tubes (mm.).					
Readings for striæ-distances.	3.	8.	10.	22.	33.
	23.6...98	93.1...10	12.2...10	.2...10	152.1...10
	72.8	83.5	18.5...10	5.9...10	158.4...10
			25.1...10	11.7...10	150.3...10
			32.7...10	17.4...10	156.8...10
		83.5...50	41.1...10	23.4...10	160.6...10
		32.2	47.8...10	28.9...10	160.0
			54.7...10	33.8...10	128.7...50
			70.0...20		122.1...10
					115.8...10
					109.2...10
					103.0...10
			71.7...10		96.9
			79.2...100		78.4...80
			147.7		26.2
Means ...	.50 mm.	.99 mm.	.71 mm.	.57 mm.	.64 mm.

From these figures we observe that the distance between successive striæ for a given spark varies with the diameter of the tube in which the pattern is obtained; and while the striæ in wider tubes are distinctly more crowded than in narrow tube, there is a particular diameter for which the striæ-distance is a maximum. This maximum for different sparks is found to occur in tubes of different diameters (see fig. 1).

This variation of striæ-distance with diameter of tube is shown in fig. I. (Pl. XXIV.).

The figures given in Table II. refer to sparks obtained from the Wimshurst having a battery of condensers in parallel with its condensers: those in Table III. to sparks from the machine without the battery, *i. e.*, for an oscillation of higher frequency. The figures formed in the latter case were remarkably well-defined and regular.

Fig. 1.—Variation of striæ-distance with diameter of tubes.



Very wide tubes were now taken—about 8 to 15 cms. in diameter. In these, the final striations did not differ much from each other or from those obtained with the wider tubes used above. The first stage of the striation was a system of curiously bifurcated fine close lines. Repeated sparking caused the striation to change somewhat in appearance, the final effect being a set of lines about .5 millimetre apart.

Using tubes of the same diameter but of various lengths, it was found that change of length had no observable effect on the striæ-distance for a given spark.

Tubes having the end remote from the spark-gap stopped by a cotton-wool plug were also used. In these cases, the striation obtained did not differ from that obtained with open tubes. Also a tube was taken, and the striation for a given spark was obtained. The powder was observed to jump quite a millimetre high at each spark as previously pointed out. The end near to the spark-gap was then stopped by a plug of cotton-wool. The passage of sparks now left the

powder absolutely unmoved. The pattern was then destroyed by knocking the tube and no amount of sparking would cause the striation of the powder. This shows that the formation of the striation is due to aerial motions.

Variation of figures with nature of powder.

The effect of the nature of the powder was now looked for. Using the same electrical arrangements as above in all the cases striæ were obtained in tubes about 1 cm. in diameter with various powders, the figures in Table IV. were obtained.

TABLE IV.—Variation of striation with powder.

Name of Powder used.	Striæ-distance.	Nature of Powder: form and size.
Lycopodium	·84 mm.	Spheroidal particles.
Egg-powder	·73 "	Round particles of smaller size.
Starch	·53 "	" " " "
Pepper	·81 "	Spheroidal " particles. " "
Boric Acid	·70 "	Round particles.
Zinc Chromate	—	Very light & very small particles.
Powdered Charcoal	1·20 "	Long narrow needles.
Bath-Brick	·52 "	More or less approaching spheroidal.
Carbonate of Iron	·70 "	Irregular.

From these figures it appears that the nature of the powder has some influence on the striæ-distance. A noteworthy feature of these figures is that lycopodium and pepper are not very different in their behaviour. This similarity in behaviour of pepper and lycopodium is perhaps to be expected since the two powders were very similar in form and size of grain.

For figure exhibiting the variation of the striation with various powders see fig. II. From this figure it is observed that the striations in case of lycopodium and pepper are very similar in appearance. It is also evident that lycopodium gives the most definite figures, it being much easier to measure the striæ-distance in this case than in case of starch (*c*, fig. II.) or boric acid (*e*, fig. II.).

A factor which appeared to enter largely into the formation of a distinct figure is the density of the powder. Thus some of the powders seemed too light to give a clear figure. As indicated in Table IV., no striation could be obtained with chromate of zinc, the particles, which were very fine, being caused to jump at each spark, but at the cessation of a spark

they appear to have fallen very haphazard and not in the regular manner necessary for the formation of striæ. With the other lighter powders the figure obtained was unsatisfactory when compared with that with lycopodium which gave by far the finest figures; and while in the case of the latter successive sparks added to the definition of the figure, with the lighter powders the passage of a spark often destroyed the distinctness that had been obtained by previous sparks.

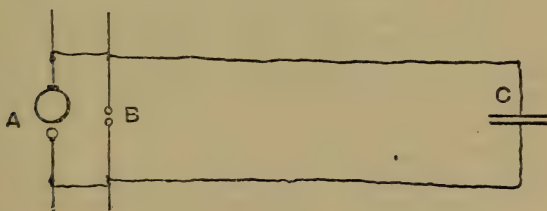
In the case of charcoal powder, which was obtained from soft wood and having the form of long narrow needles, the striation was well defined, the separate striæ consisting of irregular heaps. The striæ-distance, as shown in Table IV., was somewhat greater than for lycopodium, but the distance varied very irregularly along the tube. With bath-brick, which had been ground in a mortar, producing more or less spherical particles, and also with carbonate of iron and with starch, a fairly well-defined figure was obtained.

For the subsequent experiments lycopodium was always used for the reason just indicated.

Variation of figures with electrical conditions.

For the investigation of the variation of striation with change of electrical conditions, a discharging circuit as represented in fig. 2 was used. A denotes the discharging

Fig. 2. — Wimshurst Circuit.



knobs of machine; B the spark-gap; C a rectangular wire circuit containing various condensers. The frequency of the electrical oscillations was varied by changing the condensers and the rectangular circuit.

It was not possible to measure the frequency of the oscillations by means of the cymometer, the frequencies in these cases being outside the range of the instrument, viz.: 42 to 30 millions per second.

A set of five figures was also obtained using the sparks from the machine directly putting various condensers in

parallel with its jars. These figures are reproduced in fig. III. (Pl. XXV.), in which *a*, *b*, *c*, *d*, &c. are in descending order of frequency. The quantitative results of this set of observations are shown in Table V.

TABLE V.—Variation of striæ-distance and electrical conditions.

Description of Sparking Circuit.	Striæ-distances.
<i>a.</i> Machine; and rectangular circuit 200×30 cms..	·68 mm.
<i>b.</i> Machine; rectangular circuit; and one large leyden-jar in parallel with the jars of the machine	·61 „
<i>c.</i> Machine; rectangular circuit; and six jars of various capacities in parallel with the jars of the machine	·84 „
<i>d.</i> Machine; rectangular circuit; and twelve jars in parallel with jars of the machine ...	·80 „
<i>e.</i> Machine; rectangular circuit; and a battery of 6 large jars together with 7 other jars in parallel with jars of the machine ...	1·1 „
Spark 2 cms. long in all cases.	

Variation of striæ-distance along the same tube.

In the earlier experiments it was thought that a variation in the distribution of the lines forming the pattern for a given spark was observed along the length of the same tube. To investigate this question tubes 120 cms. long and 1 cm. in diameter were taken, and the figure for various sparks was obtained.

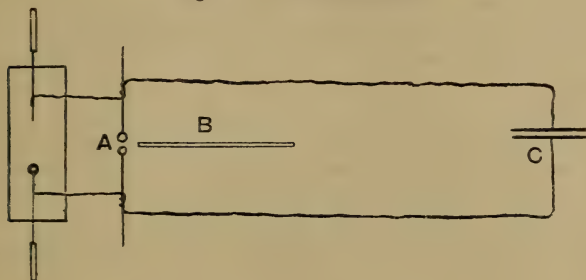
The figures in these cases (see fig. V., Pl. XXVI.) were remarkably well-defined. In addition to this, there were in the various tubes, four of which were used for different sparks, distinct periodic variations along the lengths of the tubes in the striæ-distance. In these cases the tubes were supported by a clamp near one end. The periodic variations just mentioned were visible from the beginning, but became more in evidence by long continued sparking. The distance between two successive places of rarefaction or compression of the lines is seen from figs. V. *a* & *b* to be approximately constant and somewhat different for the various tubes, being between 3 and 4 centimetres in V *a*...

The figures obtained in these tubes are shown graphically in fig. 4 (p. 781).

Experiments with the induction-coil.

A series of experiments was now made using an induction-coil for the discharge. A spark-gap A in fig. 3 insulated by

Fig. 3.—Coil and Circuit.



sticks of vulcanite passing through corks in glass bottles was arranged from the terminals of a 5-inch induction-coil through the primary of which the current from 5 secondary cells was passed. The spark-gap was a centimetre long. The discharging circuit was the standard inductance provided with the cymometer and various leyden-jars and oil-condensers. The tube containing the lycopodium was placed at B in fig. 3. The striation was obtained in a tube 30 cms. long and 1 cm. in diameter, using the standard inductance and an oil-condenser in the discharging circuit.

The figure obtained in this case was remarkably different from those obtained previously (*cf.* figs. I. & IV.).

In the first place, the lines of powder were singularly straight and parallel and equidistant and very clear. Also the striæ-distance was very small, smaller than in any of the previous cases. It is to be remarked here that the oscillations in this case were of higher frequency than those previously used. Another peculiarity was observed. The first few sparks caused the powder to form the striæ very clearly, but as the sparking continued the whole system of striæ was caused to move bodily away from the spark-gap until a centimetre or so of the tube near the spark-gap was cleared of powder. The powder was observed to jump at the passage of a spark, and the rapidity of the sparks and the nearness of the lines of powder caused the striation to be invisible during the sparking.

Various other frequencies for the oscillations were obtained by adding one or more turns of cable to the standard inductance and the striation in each case obtained. Reproductions of these figures are shown in fig. IV. There does

not appear to be any difference in the striæ-distances commensurate with the difference of frequency.

Quantitative Experiments.

The previous set of experiments appear to show very little dependence of striæ-distance on electrical conditions. A series of figures was now obtained with four circuits, the frequency of the oscillations produced varying from $\cdot 58$ to $2\cdot 1$ millions per second, as measured by the cymometer. These frequencies extended over the whole range practically of the cymometer, so it was not possible to make quantitative experiments with circuits of other frequencies. The figures, reproductions of which are shown in fig. IV., do not differ much among themselves although the extreme frequencies are in the ratio of 4 : 1 nearly.

The quantitative results are given in Table VI. It may be remarked, as may be seen from figs. IV. *f* & *k*, that the striation did not usually extend the whole length of the tubes, usually only for 10 or 15 cms. This was very probably due to the crowding up of the lines as mentioned above.

TABLE VI.

Oscillation Frequency.	Striæ-distance.
$\cdot 58 \times 10^6$ per sec.	$\cdot 46$ mm.
$\cdot 79 \times 10^6$ „	$\cdot 38$ „
$1\cdot 40 \times 10^6$ „	$\cdot 40$ „
$2\cdot 10 \times 10^6$ „	$\cdot 33$ „

Discussion of Figures (figs. I.-V.).

Examination of the figures shows that whatever cause or combination of causes governs the formation of the striation in different cases distinctly different patterns are obtained. These differences are perhaps more strikingly seen in the actual tubes than the reproductions given in figs. I. to V. An example of this difference is afforded by comparison of fig. III. *e* and fig. IV. *k*, in which cases the striæ-distances are 1.1 mm. and $\cdot 33$ respectively.

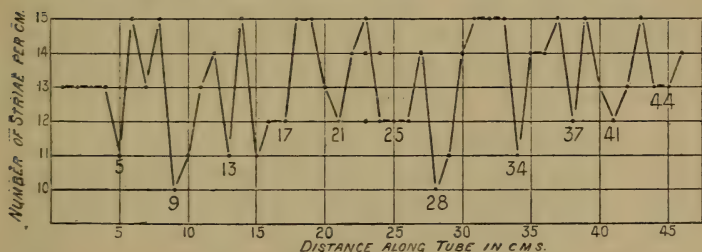
In the first place, it was found that one particular diameter of tube gave a maximum striæ-distance for a given circuit. Again, it appears that the nature of the powder has an

influence on the pattern obtained; thus, referring to fig. II., we have, using starch and lycopodium with the same spark, striæ-distances $\cdot 58$ mm. and $\cdot 76$ mm. respectively. This is no doubt a purely mechanical effect, a finer powder such as starch taking up a finer pattern than the coarser grained lycopodium, while pepper of very similar size and form to lycopodium gives a pattern very similar to that with lycopodium.

Also, it appears that for circuits of different frequencies certain differences are observable among various figures. These variations, however, seem irregular and not in any obvious proportion to frequencies of the electrical oscillations. Thus in fig. III. a variation in the striæ-distance in the manner to be expected was found, *i. e.* in the same direction as the electrical wave-length. Again in fig. IV., which refers to oscillations of known frequencies, no obvious law of variation for the striæ-distance with electrical wave-length is exhibited. Thus, while the ratio of the striæ-distances for fig. IV. *g* & *k* is $1\cdot 4$, that of the corresponding electrical wave-lengths is $3\cdot 6$.

Further, in figs. V. *a* and V. *b* we have instances of variation along the same tube. The explanation of this variation is not obvious. In fig. 4 the variation of the number of striæ

Fig. 4.—Variation of striæ-distance along a given tube.



per centimetre with distance along the tube is exhibited, and as is apparent on examination of figs. V. *a* and V. *b* this shows very approximately equidistantly distributed maxima and minima. The distance between successive minima in fig. V. *a* is indicated by the figures on the curve in fig. 4, and appears about 3 or 4 cms.; and from this it seems permissible to infer the existence of some periodic disturbance whose wave-length is of this order. The same phenomenon is seen in fig. V. *b*, which corresponds to a spark of higher frequency. This might perhaps be due to the resonance of the tube to some component of the sound of the spark.

In conclusion we have to admit that the various phenomena observed have not been reduced to any simple law, but subsequent researches will perhaps throw further light on the matter and clear up the present obscurities.

Finally, I have to express my thanks to Prof. E. H. Barton, at whose suggestion the above experiments were undertaken.
University College, Nottingham,
April 27, 1909.

LXXXI. *On a Method of Testing Photographic Shutters.*

By A. CAMPBELL, B.A., and T. SMITH, B.A.*

(From the National Physical Laboratory)

[Plate XXVII.]

I. *Principle of the Method.*

THE importance of having accurate methods of testing photographic shutters is well shown by the number and variety of the methods that have from time to time been proposed and used for this purpose. The method described herein is intended to provide a rapid test, while ensuring the maximum of accuracy. The total duration of exposure for low speeds (say longer than $\cdot 1$ sec.) is determined with an error not exceeding $\cdot 005$ sec., and for high speeds not exceeding $\cdot 0005$ sec. For exceptionally fast exposures, measurements can probably be made to within $\cdot 0001$ sec. Attention has been specially directed to the elimination of all calculations, and a permanent record is obtained of each test.

The essential principle of the method consists in photographing on a moving plate a narrow beam of light reflected from a mirror which is forced to make angular vibrations of known frequency about an axis parallel to the direction of motion of the plate. There is thus obtained on the plate a sine curve, and if the light on its way to the plate passes through the shutter, the length of the exposure can be found by counting the number of vibrations recorded on the plate. The method has been rendered practicable by the fact that a Vibration Galvanometer affords a suitable means of imparting the necessary oscillations to the mirror; with this instrument both the amplitude of the vibration and the frequency are under control, and these are points of great importance. Owing to these conditions not being satisfied, and for other reasons, a tuning fork is not nearly so suitable for imparting vibrations directly to the mirror. When it is desired to find

* Communicated by the Physical Society : read May 14, 1909.

the total duration of the exposure only, the general arrangement is shown in fig. 1. Light from a point source A is

Fig. 1.



reflected by the mirror M of the vibration galvanometer G, through the shutter S on to the photographic plate P. The mirror makes oscillations in a horizontal plane, and the amplitude is such that the vibrating beam nearly fills the horizontal diameter of the shutter aperture. The plate falls vertically, and in doing so sets off the shutter. In practice, instead of a point source of light a Nernst lamp is used with its filament vertical, and in front of the plate is placed a narrow horizontal slit T to limit the width of the record on the plate. Lenses L_1 and L_2 are employed to regulate the intensity of the light and the amplitude of the vibration recorded as well as for focussing.

II. Description of the Special Apparatus employed.

(a) Camera.

Fig. 2.

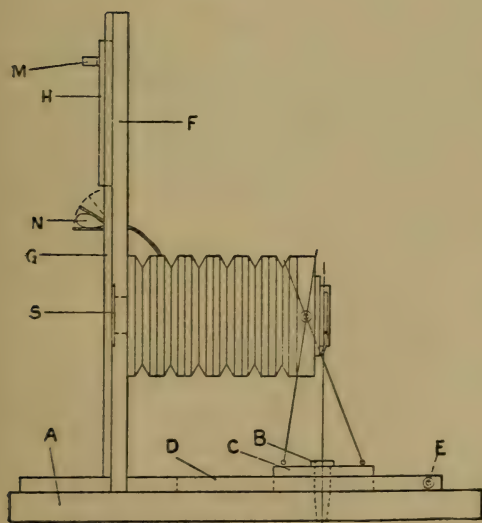
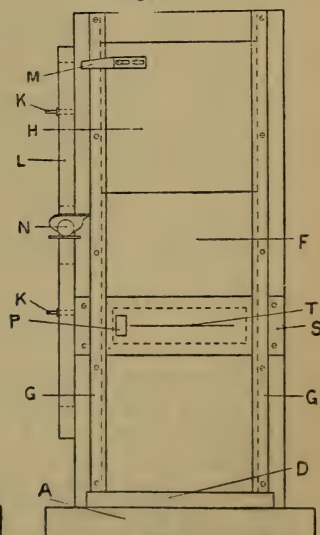


Fig. 3.



A special camera has been constructed to hold the shutter S, the lens L_2 , and the slit T (fig. 1). It is provided with

brass ways down which the plate-holder and plate slide, and an adjustable mechanism for setting off the shutter. A side elevation is shown in fig. 2, and a view of the back in fig. 3. A is the base carrying the whole apparatus, and into it is fixed a vertical pin B. C is a piece of wood in which is bored a hole through which B passes freely. C slides between two portions of the base D of the camera, and can be held in any position by tightening the nuts E. The front of the camera, with the shutter and lens L_2 , is supported by struts from C, and set so that the axis of the pin B, if produced, would pass through the centre of the shutter. By this means the whole vibrating beam of light will be able to pass through the shutter though the whole camera is rotated through a few degrees about B. The back F of the camera is attached rigidly to the base D. The metal plate S carries a narrow horizontal slit T extending the whole width of the plate, and a window P is provided to facilitate focussing. G, G are the grooved metal plates down which the plate-holder H slides. This holder carries a projection M which engages with the upper plate of a ball compressor N; when the holder falls this upper plate is forced down and the pneumatic bulb of the release is compressed. On the return upward journey of the holder the projection passes the compressor freely. The pneumatic release is carried on the moveable part L and may be fixed at any desired height by the butterfly screws K, K.

A number of records are taken side by side on one plate. This is secured by rotating the camera about B between each exposure. With this apparatus, ten or more records can be made side by side on a 5×4 in. plate. The base A has grooves cut in it to serve as a guide to the amount of rotation to be given to the camera. In making tests it is possible to make on one plate ten records of the various speeds given by a shutter in one minute.

A view of the front of the camera is shown in fig. 9 (Pl. XXVII.).

(b) *Electrical Apparatus.*

The vibration galvanometer used is of the moving coil type (as already described by one of us*), and can be readily tuned to frequencies of 50 or $500 \sim$ per second. As its mirror is of fair size (50 to 80 sq. mm.), it gives with a Nernst lamp a sufficient amount of light to avoid "tailing off" in the records for quick exposures with rapid plates. The source of current may be an ordinary lighting circuit,

* Proc. Phys. Soc. vol. xx., and Phil. Mag. Oct. 1907.

the current through a lamp as resistance being made periodically intermittent by means of a wire interrupter, an electrically maintained tuning fork, or, for the higher frequencies, a microphone hummer*. The wire interrupter, which is merely an electrically maintained monochord, can be set to the desired frequency by the help of a tuning-fork; the maintained fork and the hummer give constant and known frequencies, and do not require setting. The pulsating current is led through a primary coil of 50 or 100 turns, and over this is placed a movable secondary coil connected directly to the vibration galvanometer, which is tuned to resonance with the source of current. The amplitude of its vibration can be brought to the desired amount by changing the position of the secondary coil relatively to the primary.

The distance from mirror to plate is usually about 100 cms.

III. Determination of the Efficiency.

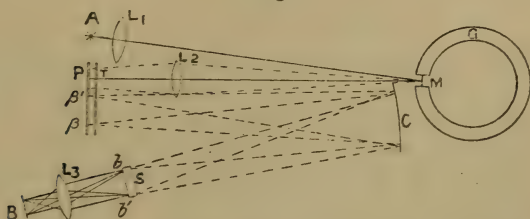
For a more complete test the efficiency, in addition to the total duration of exposure, is determined. If τ denote the total duration of the exposure, T the equivalent exposure, a the area of the shutter aperture at the instant t , and A the maximum opening, we have the relation

$$AT = \int_0^{\tau} a \, dt ; \text{ the efficiency} = \frac{T}{\tau} = \frac{\int_0^{\tau} a \, dt}{A\tau}.$$

Thus it is necessary to obtain a record of the area of the shutter opening at every instant of the exposure.

For the determination of the efficiency the method employed is essentially that proposed by Sir William Abney, who uses a siren to measure the time. We proceed as follows:—The

Fig. 4.

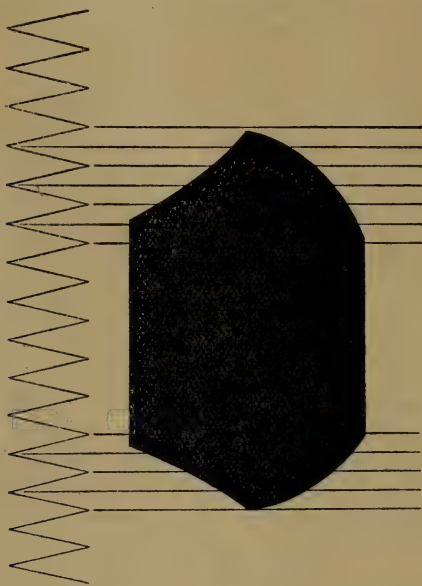


shutter is removed from the vibrating beam of light (see fig. 4) so that a continuous sine curve is recorded over the

* Proc. Roy. Soc. A. vol. lxxviii. p. 208 (1906).

whole length of the plate, and this curve serves now simply as a time record. A line source of light B is focussed by a lens L_3 on to a narrow horizontal slit $b b'$ placed in a diametral plane of the shutter as close as possible to the shutter-leaves. An image $\beta \beta'$ of this slit is formed by the concave mirror C on the photographic plate by the side of the vibrating beam of light. As the plate falls the shutter is opened as before, and a record is obtained giving at every instant of time the length of slit through which light was passed by the shutter leaves (fig. 5). Measurements are then taken of

Fig. 5.



the area of shutter aperture corresponding to a number of lengths of the slit opening. Fig. 6 shows some stages in the opening of a particular type of shutter, the white line representing the position of the slit. From a pair of records such as shown in figs. 5 and 6 a curve is drawn showing at every instant of the exposure the area of the shutter aperture through which light can pass to the plate. It is now evidently a simple matter to find the amount of light cut off owing to the finite length of time taken by the shutter leaves to open and close, and hence to calculate the efficiency.

IV. *Examples of Records.*

A number of shutters of different types have been tested with the apparatus. Figs. 7 and 8 (Pl. XXVII.) are examples of the records obtained, the former at 50 and the latter at 500~ per second. Figs. 5 and 6 are copies of records obtained in testing the efficiency.

Fig. 6.



For general use the most convenient frequencies to work with are 50 and 500~ per second, with possibly 2000 for special work ; or perhaps 100 and 1000~ per second would be suitable. The full advantages that the method offers can only be secured by the use of round numbers, such as the above, for the frequencies of the oscillations.

In conclusion, we wish to thank Dr. Glazebrook for the kind interest he has taken in the working out of the method.

LXXXII. *On the Form of the Pulses constituting Full Radiation or White Light.* By ALBERT EAGLE, B.Sc., A.R.C.S., Imperial College of Science and Technology*.

ACCORDING to the modern theory of White Light founded by Gouy, and subsequently developed by Lord Rayleigh, Schuster, and others, White Light does not consist of periodic wave-trains of all wave-lengths, which are simply separated or "dispersed" when it is drawn out into its spectrum, but consists essentially of a succession of non-periodic pulses emitted independently of one another. It is out of such pulses that the spectro-scope builds up the periodic wave-trains we observe in the spectrum.

If these pulses are all similar and follow one another

* Communicated by the Physical Society: read June 11, 1909.

quite at random, it follows that the distribution of energy in each pulse must be the same as the distribution of energy in the total succession of pulses. The distribution of energy in the spectrum obviously depends on the shape or form of the pulses making it. Lord Rayleigh has shown how* the distribution of energy in a pulse of any given form could be calculated, and calculates the distribution of energy for a pulse of the form $f(t) = e^{-c^2 t^2}$. Other suggestions as to the form of the pulse have been made by other writers, and the distribution of energy which would be obtained from them has been calculated. In no case, however, has a form been hit upon which gave a distribution in accordance with fact.

The inverse problem—viz., from the distribution of energy, to find the form of pulse which would give rise to it—has not, as far as I know, been published, and its solution is the object of the present paper. We ought, however, to state that the problem is not one which admits of a definite solution, as the distribution of energy in the spectrum is independent of the relative phases of the infinitesimal harmonic components out of which the pulse may be considered to be built up; whereas its form must obviously depend as much on the relative phases of the components as upon their relative amplitudes.

Let $y = f(x)$ denote the form of the pulse in space. Lord Rayleigh has given† the now well-known relation

$$\int_{-\infty}^{\infty} f(x)^2 dx = \frac{1}{\pi} \int_0^{\infty} (A^2 + B^2) d\alpha, \quad (1)$$

where

$$A = \int_{-\infty}^{\infty} f(\mu) \cos \alpha \mu d\mu,$$

and

$$B = \int_{-\infty}^{\infty} f(\mu) \sin \alpha \mu d\mu.$$

The left-hand side of (1) is clearly proportional to the whole energy of the pulse, and the equation is to be interpreted as implying that the energy belonging to the waves comprised between α and $\alpha + d\alpha$ is proportional to $(A^2 + B^2) d\alpha$. The wave-length λ is of course $\frac{2\pi}{\alpha}$. Hence,

* Phil. Mag. vol. xxvii. p. 465 (1889), or Collected Works, vol. iii. p. 268.

† *Op. cit.*

if we are given that the energy between α and $\alpha + d\alpha$ is proportional to $F(\alpha) d\alpha$, we have

$$F(\alpha) = \left[\int_{-\infty}^{\infty} f(\mu) \cos \alpha \mu d\mu \right]^2 + \left[\int_{-\infty}^{\infty} f(\mu) \sin \alpha \mu d\mu \right]^2. \quad (2)$$

This equation solved for f will be the solution of the problem.

Two particular solutions may very easily be obtained, first when the pulse is an even function, and second when it is an odd function. In the first case (2) reduces to

$$\int_0^{\infty} f(\mu) \cos \alpha \mu d\mu = F(\alpha)^{\frac{1}{2}}, \quad (3)$$

and in the second case to

$$\int_0^{\infty} f(\mu) \sin \alpha \mu d\mu = F(\alpha)^{\frac{1}{2}}. \quad (4)$$

Now, in Fourier's Theorem

$$\phi(x) = \frac{1}{\pi} \int_0^{\infty} d\alpha \int_{-\infty}^{\infty} \phi(\lambda) \cos \alpha(\lambda - x) d\lambda,$$

let $\phi(x)$ be equal to zero if $x < 0$, and be equal to $f(x)$ if $x > 0$. Then we have

$$\frac{1}{\pi} \int_0^{\infty} d\alpha \int_0^{\infty} f(\lambda) \cos \alpha(\lambda - x) d\lambda \quad (5)$$

is equal to $f(x)$ for positive values of x and equal to zero for negative values of x . Hence

$$\frac{1}{\pi} \int_0^{\infty} d\alpha \int_0^{\infty} f(\lambda) \cos \alpha(\lambda + x) d\lambda \quad (6)$$

is equal to zero for positive values of x .

Adding and subtracting (5) and (6), we obtain

$$f(x) = \frac{2}{\pi} \int_0^{\infty} \cos \alpha x d\alpha \int_0^{\infty} f(\lambda) \cos \alpha \lambda d\lambda \quad (7)$$

$$= \frac{2}{\pi} \int_0^{\infty} \sin \alpha x d\alpha \int_0^{\infty} f(\lambda) \sin \alpha \lambda d\lambda \quad (8)$$

for positive values of x .

In much of what follows we shall frequently have an equation involving a cosine with a similar one involving a sine.

In order to prevent having to duplicate such equations, we will write them with $\frac{\cos}{\sin}$, in which either the upper or the lower may be taken; but in one equation upper must be taken with upper and lower with lower.

Equations (7) and (8) may very conveniently be expressed in the form of the theorem:

$$\begin{array}{l} \text{If} \\ \text{then} \end{array} \quad \left. \begin{array}{l} \int_0^\infty f(x) \frac{\cos}{\sin} mx \, dx = \phi(m), \\ \int_0^\infty \phi(x) \frac{\cos}{\sin} mx \, dx = \frac{\pi}{2} f(m). \end{array} \right\} \quad \cdot \cdot \cdot \cdot \quad (9)$$

In this form the equations are useful for finding certain definite integrals; for instance, from the result

$$\int_0^\infty e^{-ax} \cos mx \, dx = \frac{a}{m^2 + a^2},$$

we can at once deduce

$$\int_0^\infty \frac{\cos mx}{x^2 + a^2} \, dx = \frac{\pi}{2a} e^{-am}.$$

Further examples will not be given here, as the author hopes to discuss some results which may be obtained from equations (9) in another paper.

Equations (7) and (8) enable us to solve at once (3) and (4). Multiplying each side of (3) by $\cos ax \, dx$, and integrating from 0 to ∞ , we obtain by (7)

$$f(x) = \frac{2}{\pi} \int_0^\infty F(\alpha)^{\frac{1}{2}} \cos ax \, d\alpha, \quad \cdot \cdot \cdot \cdot \quad (10)$$

which gives the form of the even pulse; similarly, the odd one is given by

$$f(x) = \frac{2}{\pi} \int_0^\infty F(\alpha)^{\frac{1}{2}} \sin ax \, d\alpha. \quad \cdot \cdot \cdot \cdot \quad (11)$$

We will now determine the form of the pulses for some of the different formulæ that have been proposed for representing the distribution of energy in full radiation.

Both Lord Rayleigh's and Wien's formulæ are included under

$$E_\lambda \, d\lambda = C\theta^n \lambda^{n-5} e^{-\frac{c}{\lambda\theta}} \, d\lambda,$$

the former being obtained by putting $n=1$ and the latter by

putting $n=0$. Transforming this so as to obtain the energy between α and $\alpha + d\alpha$ where $\alpha = \frac{2\pi}{\lambda}$, we get

$$F(\alpha) d\alpha = A\theta^n \alpha^{3-n} e^{-2b\alpha} d\alpha,$$

where $2b = \frac{c}{2\pi\theta}$.

Hence, dropping factors outside the integral sign, the forms of the pulses are given by

$$\begin{aligned} f(x) &= \int_0^\infty \alpha^{\frac{3-n}{2}} e^{-b\alpha} \frac{\cos \alpha x}{\sin \alpha} d\alpha \quad \dots \quad (12) \\ &= \frac{\Gamma\left(\frac{5-n}{2}\right) \cos \left\{ \frac{5-n}{2} \tan^{-1} \frac{x}{b} \right\}}{(b^2 + x^2)^{\frac{5-n}{4}}}. \end{aligned}$$

Putting $n=1$ for Lord Rayleigh's formula, the expressions reduce to

$$\frac{b^2 - x^2}{(b^2 + x^2)^2} \quad \text{and} \quad \frac{2bx}{(b^2 + x^2)^2}.$$

Taking Planck's formula for the distribution of energy,

$$E_\lambda d\lambda = \frac{C\lambda^{-5} d\lambda}{\frac{c}{e\lambda\theta} - 1},$$

this transforms into

$$F(\alpha) d\alpha = \frac{A\alpha^3 d\alpha}{e^{2b\alpha} - 1}, \text{ where } 2b = \frac{c}{2\pi\theta} \text{ as before.}$$

Extracting the square root of this expression by expanding the denominator in ascending powers of $e^{-b\alpha}$, substituting the result in (10) and (11), and integrating term by term, we get for the form of the pulses

$$\begin{aligned} f(x) &= \frac{\cos \left\{ \frac{5}{2} \tan^{-1} \frac{x}{b} \right\}}{(b^2 + x^2)^{\frac{5}{4}}} + \frac{1}{2} \frac{\cos \left\{ \frac{5}{2} \tan^{-1} \frac{x}{3b} \right\}}{(3^2 b^2 + x^2)^{\frac{5}{4}}} \\ &\quad + \frac{3}{8} \frac{\cos \left\{ \frac{5}{2} \tan^{-1} \frac{x}{5b} \right\}}{(5^2 b^2 + x^2)^{\frac{5}{4}}} + \dots \quad (13) \end{aligned}$$

To the eye these pulses have much the same form as the simple ones obtained from Lord Rayleigh's formula.

It is interesting to observe that all the pulses we have found satisfy the condition $\int_{-\infty}^{\infty} f(x) dx = 0$. This is obvious for the odd pulses; for the even ones, we have only to show that $\int_0^{\infty} f(x) dx = 0$.

Now $f(x)$ consists of one or more terms of the form

$$\int_0^{\infty} x^p e^{-qx} \cos ax \, dx.$$

This integrated with respect to x gives

$$\int_0^{\infty} x^{p-1} e^{-qx} \sin ax \, dx = \frac{\Gamma(p) \sin \left\{ p \tan^{-1} \frac{x}{q} \right\}}{(q^2 + x^2)^{\frac{p}{2}}} \quad (14)$$

which vanishes when taken between 0 and ∞ if p be positive.

On the electron theory these pulses are due to the radiation from moving electrons, and the function giving the form of the pulse as a function of t is the same as the function giving the acceleration of the electron at time t . Hence, to find the displacement at any time, we must integrate the function giving the form of the pulse twice. By integrating the left-hand side of (14) twice with respect to x , we observe that we merely change p into $p-2$. Hence, to integrate the right-hand side twice we have only to change p into $p-2$. Applying this to equations (12) and (13), we get the motion of an electron which will give rise to these pulses.

For the pulses $\frac{b^2 - x^2}{(b^2 + x^2)^2}$ and $\frac{2bx}{(b^2 + x^2)^2}$, obtained from Lord Rayleigh's formula, the motion of the electron is given by

$$y = A \log(a^2 + t^2) \quad \text{and} \quad y = B \tan^{-1} t/a,$$

where $a = \frac{b}{V}$.

Equations (10) and (11) show that the pulses may be regarded as resulting from a superposition of a series of sine curves of all wave-lengths, each with a suitable amplitude so as to give the required energy distribution. This being so, we may expect that we shall be able to obtain a more general form of pulse by assigning an arbitrary phase to the

component sine curves, without thereby altering the distribution of energy in any manner. That is, we may expect

$$f'(x) = \frac{2}{\pi} \int_0^{\infty} F(\alpha)^{\frac{1}{2}} \cos \{ \alpha x - \phi(\alpha) \} d\alpha. \quad (15)$$

to be a more general solution of (2) than (10) or (11). This in fact is so, as may be verified by splitting up (2) into the two equations

$$F(\alpha) \cos \phi(\alpha) = \int_{-\infty}^{\infty} f'(\mu) \cos \alpha \mu d\mu$$

and

$$F(\alpha)^{\frac{1}{2}} \sin \phi(\alpha) = \int_{-\infty}^{\infty} f'(\mu) \sin \alpha \mu d\mu,$$

and trying to find a solution of these—in the same manner in which (3) and (4) were solved—which at the same time satisfies both of them. Equation (15) will be the result.

It is no longer, of course, believed that white light consists of a succession of pulses all of exactly the same form. It consists rather of a succession of pulses capable of being represented by an equation with one or more arbitrary parameters, the number of pulses emitted per second which have their parameters lying within definite limits being given by some law analogous to the Maxwellian distribution law. But the form we have obtained may probably be looked upon as some mean or average form of the pulses, and may perhaps be of some value in determining, to a first approximation at least, by what intermolecular forces the free electrons in a substance must be acted upon in order to give the observed distribution of energy in the spectrum. Although, as we have seen, the problem does not admit of a definite solution, yet it is not impossible that physically all the pulses in white light may be (say) even ones. For instance, if an electron moves in a straight line against an opposing force which is a function of the distance, and which is insensible at the beginning of the motion but becomes sufficiently powerful to bring it to rest and reverse its motion, the pulse produced will obviously be an even one, under which conditions the form of the pulse for a given distribution of energy is unique.

LXXXIII. *On the Measurement of Wave Length for High Frequency Electrical Oscillations.* By ALBERT CAMPBELL, B.A.*

(From the National Physical Laboratory.)

[Plate XXVIII.]

§ 1. *Introduction.*

IN all work with high frequency electrical oscillations, such as for example in wave telegraphy, it is of the utmost importance to be able to determine with accuracy the wave-length actually employed, and for this purpose several types of wavemeter are now in common use. In order to ensure accuracy in such measurements, it was suggested some time ago by the Post Office Authorities that arrangements should be made for the calibration of wavemeters at the National Physical Laboratory. As the results of our first series of investigations in the matter appeared to be of general interest, we publish them here by the kind permission of Major O'Meara, C.M.G., Engineer-in-Chief to the Post Office. Our experiments comprised the construction and testing of a standard wavemeter, and the verification of an ordinary commercial wavemeter sent to us by the Post Office. I shall designate this instrument (A) and our standard wavemeter (B) respectively. While it is unnecessary here to go into the history of the subject, I should like to give one or two references to earlier work by other observers which gave us much assistance, namely the experiments of Pierce † and those of Gehroke‡.

While our work was in progress an important paper by Diesselhorst § appeared, and the results there published were in ample confirmation of those we were obtaining, as also were the earlier results of Pierce.

§ 2. *Description of Wavemeter (A).*

Wavemeter (A) is of the Dönitz type and consists of a variable air condenser, with a range from about 100 to 1070 mmfd. (micromicrofarads), a thermo-junction and galvanometer, and a series of coils (A 1, A 2, ... to A 10) of self inductances ranging from 0.76 to 2313 microhenries.

* Communicated by the Physical Society : read June 25, 1909.

† Phys. Review, vol. xxiv. p. 152, 1907.

‡ *Elektrotech. Zeitschr.* (26), p. 697, 1905.

§ *Jahrbuch der drahtlosen Telegr. u. Teleph.* vol. i. (1908).

The combination is capable of measuring oscillation frequencies over a range extending from about 20,000,000 down to 100,000 $\sim$ per sec., or wave-lengths from $\lambda=15$ metres up to $\lambda=3000$ metres. It must be kept in mind, however, that the accuracy obtainable depends on the part of the condenser scale at which the reading is taken. For example, at a reading of 20 the frequency can barely be read to 0.5 per cent.

As the coils (A 1, A 2, ...) of (A) are of solid (not stranded) wire of diameters from 0.32 up to 3.2 mm., the values of their self inductances at the high frequencies are, for most of the coils, considerably lower than those obtained at ordinary frequencies of 0 to 1000 $\sim$ per sec.

With a view to checking the results of the direct experiments by calculation from the measured inductance and capacity of a wavemeter in which the inductances would be much less affected by frequency, and thus to obtain a standard instrument for future use, we constructed a wavemeter (B) in which the coils are all of highly stranded wire.

§ 3. *Description of Standard Wavemeter (B).*

The general arrangement of (B) was similar to that of (A). The variable condenser (from 100 to 900 mmfd.) was of special design, with amberite washers to give very high insulation. It was kindly presented to the Laboratory by Dr. Alexander Muirhead, F.R.S. We added to it a direct reading scale, which can be read to 1 in 2000 at the upper end and to 1 in 200 at the lower. The scale was constructed by a very careful series of tests by Maxwell's Commutator Method and was found to be very uniform.

For the inductances, three coils (Q b, Q 1, and Q 2) were used. They were all wound on ebonite tubes with stranded wire (7/36^s), *i. e.* containing seven insulated strands, each of diameter 0.19 mm. In each coil the terminals were brought to a considerable distance (18.5 cm.) from the centre of the coil by fixed leads run parallel to one another 1 cm. apart. In this way too close proximity between the coil and the condenser plates was avoided. A Duddell thermo-ammeter of 1.5 ohms resistance was used to complete the circuit of the coil and the condenser, and by observing its maximum deflexion the point of resonance was obtained. The sensitivity of course varies with the coil used and with the nature of the oscillatory circuit; it was found to be sufficient for the purpose in the experiments described below. Fig. 1 (Pl. XXVIII.) shows a photograph of the wavemeter and of one of the coils separately.

§ 4. *Measurement of the Self Inductances of the Coils.*

The self inductances of all the coils were measured at ordinary frequencies (0 to 1000 $\sim$ per sec.) by a method specially designed for the accurate measurement of such low values (A. Campbell, *Phil. Mag.* Jan. 1908). The comparisons were made against a standard variable mutual inductance with ranges of 0 to 200 and 0 to 2000 microhenries, the lower range being readable to 0.01 microhenry. The absolute value of this was measured in terms of the new Laboratory Standard of mutual inductance, whose value has been calculated to a very high degree of accuracy (see *Proc. Roy. Soc. A.* vol. 79, 1907). The subdivision of the scale of the variable mutual inductance was effected by the help of Maxwell's method of comparing two mutual inductances. Its accuracy was verified by an independent method as follows. A mutual inductance coil was constructed with the primary and secondary circuits both of well stranded wire (10 and 20 strands respectively), and, with all the strands in each circuit in series, the value was adjusted to be equal to the 10 microhenry reading on the scale. Then, by taking p strands of the primary and q strands of the secondary, the value would be $\frac{pq}{200} \times 10$, and thus inductances of $\frac{1}{200}$, $\frac{2}{200}$, $\frac{3}{200}$, ... of the full scale reading were obtained. On testing these against the actual scale, the subdivision was found to be perfectly satisfactory.

The details of winding and values of the self inductances of the coils of Wavemeter (B) are given in Table I. In actual working these values had to be increased by the addition of the measured self inductance of the rest of the circuit, consisting of the leads, ammeter, and condenser; the total addition was about 0.5 microhenry, of which 0.1 microhenry was due to the condenser.

TABLE I.

Coil.	Coil Diameter. cm.	Axial Length. cm.	Turns.	Self Inductance. Microhenries.
QB.....	13.5	3.6	40	360.3
Q1	7.4	3.9	48	170.7
Q2	7.6	1.7	23	57.8 ₂

§ 5. *Tests of Standard Wavemeter (B) by Photographing Sparks.*

In order to obtain an absolute calibration of the standard wavemeter, the frequencies of the oscillations with which it was tested were determined by including a spark gap in the main oscillation circuit to which the wavemeter was loosely coupled and photographing the spark trains by help of a rotating mirror.

The apparatus was arranged as follows:—

The source of current was a small alternator whose frequency could be varied from 50 to 200 ~ per second; this was connected through a small high voltage transformer to a spark gap with cadmium electrodes shunted by a glass plate condenser and a bare wire inductance in oil. A large variable air condenser consisting of six aluminium disks, each 100 cms. in diameter, could be put in parallel with the glass plate condenser, and, by suitable variation of the capacity and inductance in circuit, oscillations of frequencies from 300,000 to 1,200,000 ~ per sec. could be obtained. A special camera (fig. 2) was constructed and mounted for rigidity on a long slab of sandstone. At one end of the camera was the rotating mirror, which was concave and of 200 cms. radius of curvature. It turned on a vertical axis and was driven by a small motor, the speed being kept constant by an arrangement described below. The other end of the camera had two branches which carried respectively the spark electrodes and a pair of guides allowing the plate carrier to be smoothly raised or lowered while the rotating mirror threw the images of the spark trains as streaks upon the plate. The distance from the mirror to the plate was 200 cms. The speed of rotation of the mirror was kept constant (usually at about 60 revs. per second) as follows. On the axis was mounted a commutator and this was connected with a condenser, a bridge, battery, and galvanometer, for Maxwell's Commutator method of measuring capacity. By applying a slight variable brake to the rotating axis, the galvanometer light-spot could be kept at zero. When this was the case the speed was steady, and could be measured with a counter or deduced from the value of the condenser and the resistance in the bridge. From measurements of a number of the spark train photographs on each plate, the average displacement from spark to spark was found; and the frequency was calculated from this displacement, the distance from the mirror to the plate, and the speed of rotation of the mirror.

While each photograph was being taken the reading of the standard wavemeter was also observed, care being taken to keep the coupling to the spark circuit very loose.

The value (n) of the frequency obtained from the spark photographs was in each case compared with the values (n_1) calculated from the measured values K and L of the capacity and inductance in the wavemeter circuit. Since the wavemeter coils have a certain amount of distributed capacity, it was necessary to take account of this. The required correction was made by Glazebrook and Lodge's formula (Cambridge Phil. Trans. p. 171, vol. xviii. 1899),

$$p^2 L K \doteq 1 - \frac{N-2}{N^2} \cdot \frac{k}{K},$$

where $p = 2\pi n$, N = number of turns in coil, k = capacity from turn to turn, and the capacities and inductances are in absolute measure. The value of k was found by testing the capacities of coils with bifilar windings of wire similar to that in the coils used. The correction is small, being in no case more than 1 part in 1000. The results are given in Table II.

TABLE II.

Plates Nos.	n By Sparks ~ per sec.	n_1 From K & L of Standard Wavemeter. ~ per sec.
61 & 62	290,300	290,500
47	516,800	516,800
57 & 58	818,300	821,200
55	1,042,000	1,039,000

We may remark that the small differences between n and n_1 are quite within the limits of probable experimental error. On the same plate the value of n deduced from the various spark trains sometimes showed an extreme variation of 1.2 per cent. from the mean; in the best experiments the variation was about 0.5 per cent. from the mean. In general the mean of 5 to 10 spark trains was used and the average variation from the mean was from 0.2 to 0.6 per cent. in the value of n .

§ 6. Comparison of Wavemeter (A) with Standard (B).

The condenser of (A) was tested throughout its range by Maxwell's Commutator Method and gave the results shown in Table III. which shows the scale readings to be very nearly proportional to the capacity. At the higher readings the accuracy of measurement of K is here of the order of 2 or 3 parts in 1000.

TABLE III.

Reading. Degrees.	K mmfd.	K/Reading.
20	126	6.1
40	245	6.0 ₆
60	367	6.1 ₂
80	486	6.0 ₈
100	607	6.0 ₇
120	724	6.0 ₃
140	851	6.0 ₈
160	966	6.0 ₄
180	1075	5.9 ₈

The coils (A4, A5, ... to A9) were tested for self inductance (L_0) at ordinary frequencies. In Table IV. are given the values found (including the working circuit in each case). Approximate dimensions of the winding are also given. In the last column are given the values of L_∞ (*i. e.* the value of the self inductance for infinite frequency,

TABLE IV.

Coil.	2a Coil Diameter. cm.	l Axial Length. cm.	d Wire Diameter. cm.	N. Turns.	Self inductance. microhenries.	
					L_0 .	L_∞ .
A4	5.2	10.7	0.32	32	21.6 ₂	19.5 ₅
A5	8.0	11.3	0.32	33	44.9 ₆	42.0
A6	5.1	5.6	0.09 ₁	55	100.5	97.5
A7	5.1	9.6	0.09 ₁	95	195.0	189.8
A8	7.6	5.3	0.05 ₆	90	531.8	524
A9	7.6	8.9	0.05 ₆	150	1039	1025

assuming that the current in that case is practically confined to the skin of the wire). These values have been calculated by the approximate formula (due to Heaviside):*

$$L_0 - L_\infty = \frac{26N^2ad}{1000l} \text{ in microhenries,}$$

where N = number of turns,

a = radius of coil,

l = axial length of coil,

d = diameter of wire,

all the dimensions being in centimetres.

The application of the formula to such short coils is not quite appropriate, however, as it has been deduced on the assumption that the solenoid is long.

The two wavemeters were then loosely coupled to the same oscillation circuit and simultaneous readings taken at various frequencies. From the calibration of (B) already established and the results in Table III. the effective values of the self inductances of the coils of (A) were deduced; they are given in Table V.

TABLE V.

Coil.	Comparison Frequency ~ per sec.	Effective L (deduced). Microhenries.	L. (Mean).
A4	1,125,000 1,338,000	20.1 19.8	19.9
A5	837,400 975,500	42.6 42.5	42.5 ₅
A6	665,100	99.1	99.1
A7	366,700 394,800 466,800 473,400 665,100	190.5 190.5 191.1 192.0 191.8	191.2
A8	284,500 290,300 322,800	530 530 532.5	53 ₁
A9	290,300	104 ₈	104 ₈

* Collected Papers, vol. i., p. 356.

From Table VI. it will be seen that the observed effective values of L (to be used in working the instrument) lie between L_0 and L_∞ in all cases but the last. The discrepancy here may be due to an error of 0.5 per cent. in determining n which might occur in consequence of the reduced sensitivity when using a coil of such high resistance as A 9.

The column headed L_n gives the values of L calculated from L_∞ for the actual values of n by means of Cohen's formula *. It will be noticed that this brings the observed and the calculated values of the inductances (at the high frequencies) considerably closer.

TABLE VI.

Coil.	Effective L .	L_n .	L_0 .	L_∞ .
A4	19.9	19.6	21.6 ₂	19.5 ₅
A5	42.5 ₅	42.1	44.9 ₆	42.0
A6	99.1	98.1	100.5	97.5
A7	191.2	190.8	195.0	189.8
A8	531	526	531.8	524
A9	104 ₈	1029	1039	1025

§ 7. Conclusion.

Thus it appears that within the limits of wave-length used, the wavemeter with coils of well stranded wire gave results in close agreement with theory, while in the case of the instrument with coils of solid wire the agreement was as close as could be expected, as the correcting formulas are only strictly applicable to long solenoids.

In conclusion I would express my best thanks for kind assistance to Major O'Meara and his staff; to Prof. R. Ll. Jones, Messrs. H. C. Booth and T. L. Eckersley, who skilfully aided in the experiments; and to Dr. Glazebrook for valued help and advice throughout the work.

* Bulletin, Bureau of Standards, vol. iv., no. 1, p. 177 (1907).

LXXXIV. *An Electromagnetic Method of Studying the Theory of and Solving Algebraical Equations of any Degree.* By ALEXANDER RUSSELL, M.A., D.Sc., and J. N. ALTY, A.I.E.E., Faraday House, London *.

CONTENTS.

1. Introduction.
2. The Electromagnetic Method.
3. Quadratic Equations.
4. The Equation to Curves passing through the Neutral Points.
5. Cubic Equations.
6. Finding the Roots of an Equation.
7. Description of Apparatus.

1. *Introduction.*

THE electrical device recently invented by Mr. Arthur Wright enables us to find approximate values of the real roots of an equation at once by simple mechanical and electrical operations. In order, however, to find approximate values of the imaginary roots it is necessary to perform certain analytical operations, and then apply the device to find the roots of an equation of a higher degree. The method described in this paper has the great merit of giving approximate values of all the imaginary roots as well as all the real roots. It is not capable of such high accuracy as the Arthur Wright device, and it cannot be directly applied when the indices of the powers of the unknown quantity are fractional. On the other hand, it is exceedingly instructive, as it shows how the numerical values of both the real and imaginary roots vary as the coefficient of any power of the unknown quantity in the equation is varied. The apparatus required is exceedingly simple, and is to be found in practically every physical laboratory. We have found it quite a suitable experiment to include in a laboratory course for first year students.

The method suggested itself to one of the authors when studying a series of papers by Mr. F. Lucas which are published in the *Comptes Rendus*, t. 106 (1888). The final electrical method (p. 1072) devised by Mr. Lucas is a practical one. A sheet of tinfoil is spread over a large flat plate of glass or other insulating material. If the equation is of the n th degree $n+2$ sources and sinks for electrical current are provided. These are arranged on a line at equal distances apart. The currents in $n+2$ wires touching at the sources and sinks are adjusted so that they have certain definite values. The method of calculating these values is practically

* Communicated by the Physical Society : read June 25, 1909.

the same as in the method described below. The equipotential lines are then traced out either by an electrochemical method or by the Kirchhoff and Carey Foster method. If the line of sources and sinks be taken as the axis of x , and the origin be chosen midway between the outer wires, the coordinates of the nodal points, that is, the points where an equipotential line intersects itself, enable us to write down all the roots, real or imaginary, of the given equation at once. If (x_1, y_1) be the coordinates of a nodal point, we can see from the symmetry of the arrangement that $(x_1, -y_1)$ will be the coordinates of another nodal point. It follows from the method of adjusting the currents that $x_1 \pm y_1 \sqrt{-1}$ gives a pair of conjugate roots of the given equation. The real roots, therefore, are all given by the abscissæ of the nodal points lying on the axis of X . The method is not rigorous, as Mr. Lucas has not considered the magnitude of the error introduced owing to the finite size of the conducting sheet. It would be very laborious to apply in practice. In solving a biquadratic, for instance, the currents in five wires would have to be adjusted to given values before the equipotential lines could be mapped out.

We shall now describe an electromagnetic method, in which the horizontal field due to the earth's magnetism is used in an analogous manner to the conducting sheet in Mr. Lucas's method. A drawing-board with a slit cut in it, a few pieces of bell-wire, any form of "charm" compass, ordinary ammeters and rheostats or lamp-resistance boards such as are found in every physical laboratory can be utilized at once for the experiment (see § 7).

2. *The Electromagnetic Method.*

Let us suppose that we have to find the roots for the equation

$$f(x) \equiv a_n x^n + a_{n-1} x^{n-1} + \dots + a_0 = 0. \quad (1)$$

We first, by the ordinary methods given in books on algebra, resolve the expression

$$\frac{f(x)}{(x-b_1)(x-b_2) \dots (x-b_n)}$$

into partial fractions. The numbers $b_1, b_2, \dots, b_n$ are any convenient numbers so chosen, however, that

$$b_1 + b_2 + \dots + b_n = -a_{n-1}/a_n. \quad (2)$$

For example, if the second term of $f(x)$ be missing we must choose $b_1, b_2, \dots$ so that $\Sigma b = 0$.

We thus obtain

$$\frac{f(x)}{(x-b_1)(x-b_2) \dots (x-b_n)} = a_n + \frac{A_1}{x-b_1} + \frac{A_2}{x-b_2} + \dots + \frac{A_n}{x-b_n}, \quad (3)$$

where, by (2),

$$A_1 + A_2 + \dots + A_n = 0, \quad (4)$$

and

$$\left. \begin{aligned} A_1 &= \frac{f(b_1)}{(b_1-b_2)(b_1-b_3) \dots}, & A_2 &= \frac{f(b_2)}{(b_2-b_1)(b_2-b_3) \dots} \\ \text{and} & & A_n &= \frac{f(b_n)}{(b_n-b_1)(b_n-b_2) \dots (b_n-b_{n-1})} \end{aligned} \right\}, \quad (5)$$

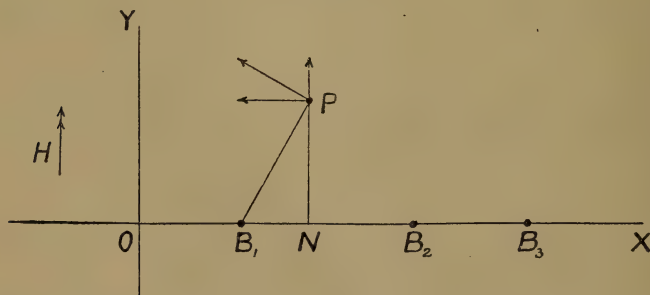
Let us now consider the magnetic field round a long vertical wire carrying a current of C amperes, and let us suppose that the earth's horizontal field in the neighbourhood is uniform, and that its horizontal intensity in c.g.s. units is H . The magnetic force at any point P at a perpendicular distance of r centimetres from the axis of the wire will be the resultant of a force $C/5r$ acting at right angles to the plane containing r and the axis of the wire and a force H directed to the magnetic pole. There is always a neutral point* on the line through the axis of the wire perpendicular to the magnetic meridian. If x be the distance of this point from the axis

$$x = C/(5H). \quad (6)$$

This formula is utilized in a well-known rough laboratory method of measuring H when C is known or *vice versâ*.

Let us now suppose that we have n vertical wires arranged in a plane perpendicular to the magnetic meridian, and let

Fig. 1.



them cut the plane of the paper perpendicularly at $B_1, B_2, \dots B_n$ (fig. 1) which are at distances $b_1, b_2, \dots b_n$ from O .

* A. Russell, 'The Electrician,' vol. xxxi. p. 282 (1893).

If $C_1, C_2, \dots C_n$ be the values in amperes of the currents in the wires and H the horizontal intensity of the earth's magnetic field, the components X and Y of the resultant magnetic force at $P(x_1, y_1)$ will be given by

$$-X = \frac{C_1}{5r_1} \cdot \frac{y_1}{r_1} + \frac{C_2}{5r_2} \cdot \frac{y_1}{r_2} + \frac{C_3}{5r_3} \cdot \frac{y_1}{r_3} + \dots$$

and
$$Y = H + \frac{C_1}{5r_1} \cdot \frac{x_1 - b_1}{r_1} + \frac{C_2}{5r_2} \cdot \frac{x_1 - b_2}{r_2} + \dots,$$

where $r_m^2 = (x_1 - b_m)^2 + y_1^2$.

Hence multiplying X by ι and subtracting we get

$$\begin{aligned} Y + X\iota &= H + \frac{C_1}{5r_1^2} (x_1 - b_1 - y_1 \iota) + \frac{C_2}{5r_2^2} (x_1 - b_2 - y_1 \iota) + \dots \\ &= H + \frac{C_1/5}{x_1 + y_1 \iota - b_1} + \frac{C_2/5}{x_1 + y_1 \iota - b_2} + \dots \end{aligned}$$

At a neutral point the resultant magnetic force is zero, and therefore both X and Y are zero. Hence, if x_1 and y_1 are the coordinates of a neutral point, $x_1 + y_1 \iota$ is a root of the equation

$$0 = H + \frac{C_1/5}{x - b_1} + \frac{C_2/5}{x - b_2} + \dots \quad (7)$$

Comparing this with equation (3) we see that if we adjust the values of the currents so that

$$\begin{aligned} C_1 &= 5H \cdot A_1/a_n, \quad C_2 = 5H \cdot A_2/a_n, \dots \\ C_n &= 5H \cdot A_n/a_n, \end{aligned}$$

then $x_1 + y_1 \iota$ is a root of the equation $f'(x) = 0$, and therefore $x_1 - y_1 \iota$ is also a root.

It follows from (4) that $\Sigma C = 0$, and therefore only $n-1$ ammeters and only $n-1$ rheostats are required.

As an introduction to the method let us consider the theory of quadratic equations.

3. Quadratic Equations.

Let us suppose that the equation is

$$x^2 + bx + c = 0.$$

In this case it is convenient to write

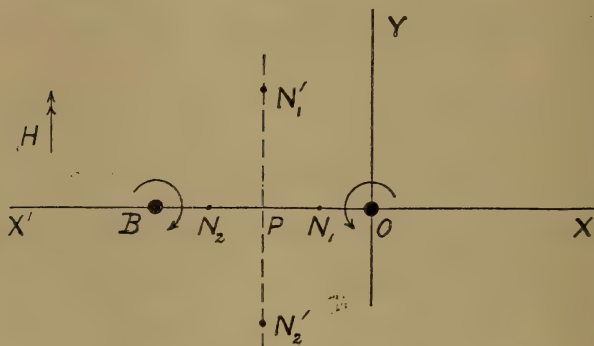
$$\frac{x^2 + bx + c}{x(x+b)} = 1 + \frac{c/b}{x} - \frac{c/b}{x+b},$$

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and, therefore,

$$H \frac{x^2 + bx + c}{x(x+b)} = H + \frac{(5Hc/b)}{5x} - \frac{(5Hc/b)}{5(x+b)} \quad (8)$$

In fig. 2 let us suppose that the plane of the paper represents a horizontal plane, and that H is the direction of the earth's magnetic force. Let us also suppose that two long

Fig. 2.



vertical wires cut the plane of the paper perpendicularly at O and B respectively, the length of BO being b centimetres and the magnitude of the current C , in amperes, in the O wire being $5Hc/b$, and the B wire carrying the return current $-5Hc/b$. If the direction of the current C be into the paper at B and out of it at O, the circular lines of force due to the currents in each wire will act in the directions of the curved arrow-heads shown in the figure.

We shall now consider how the numerical values of the roots of the quadratic equation alter as c , and therefore also C , increases from zero to infinity, b which we suppose to be positive remaining constant.

We have already shown that the coordinates of the points at which the resultant magnetic force is zero, that is, the neutral points, determine completely the numerical values of both the real and imaginary roots of the equation.

When c is very small and positive the neutral points lie between B and O. For a particular value of c , for instance, they are at N_1 and N_2 on the axis of X. We see at once from symmetry that $ON_1 = N_2B$. If the length of ON_1 on the same scale that BO is b be $-x_1$, the roots of the equation for this value of c are both real and equal to $-x_1$ and $-b + x_1$,

respectively. As we increase the value of c , and therefore of the current in the wires, the neutral points N_1 and N_2 approach one another and coincide at P . In this case the roots are each equal to $-b/2$. For greater values of c the neutral points cannot possibly lie on BO as the resultant magnetic force due to the currents at all points of this line is greater than H . From symmetry the neutral points lie on the line N_1PN_2 bisecting OB at right angles. For a particular value of the current they are at the points N_1 and N_2 on this line. If $PN_1=y_1$ then $PN_2=-y_1$ and the corresponding roots of the equation are $-b/2+y_1\sqrt{-1}$ and $-b/2-y_1\sqrt{-1}$. The real part of the imaginary roots is therefore independent of c , but the coefficient of $\sqrt{-1}$ increases as c increases.

When c is negative the curved arrow-heads in fig. 2 must be drawn in the opposite directions. N_1 will now be at a distance x_1 from O along OX , and N_2 will be at a distance $-b-x_1$ from O along OX' . The roots in this case, therefore, are always real and of opposite sign, and continually increase numerically as c increases.

When b is negative the point B in fig. 2 will be to the right of OY , and the discussion of the roots in this case is equally simple.

4. The Equation to Curves passing through the Neutral Points.

From the preceding discussion it will be seen that the locus of the real roots of an equation determined in this way is the axis of x . Fig. 2 shows that for a quadratic equation when b is positive the locus of the points giving the imaginary roots is the straight line $N_1'N_2'$. Similarly when b is negative it is the parallel line given by equation $x=b/2$. It is important to know the equations to curves on which the neutral points must lie in the general case.

Let us suppose that $x+y\iota$ is a root of the equation $f(\xi)=0$. In this case we have

$$f(x+y\iota)=0,$$

and, therefore, by Taylor's theorem,

$$\begin{aligned} f(x) - \frac{y^2}{2} f''(x) + \frac{y^4}{4} f^{iv}(x) - \dots \\ + \iota y \left\{ f'(x) - \frac{y^2}{3} f'''(x) + \dots \right\} = 0. \end{aligned}$$

This equation can be satisfied either by

$$f(x)=0, \text{ and } y=0, \dots \dots \dots (9)$$

or by

$$f(x) - \frac{y^2}{2} f''(x) + \frac{y^4}{4} f^{iv}(x) - \dots = 0 \dots (10) \left\{ \right.$$

and

$$f'(x) - \frac{y^2}{3} f'''(x) + \frac{y^4}{5} f^{v}(x) - \dots = 0 \dots (11) \left\{ \right.$$

The equations (9) obviously give the real roots of the equation which must all lie on the line $y=0$, that is, on the axis of X . The points of intersection of the equations (10) and (11) determine the imaginary roots.

As equation (11) does not contain the constant term of the equation $f(\xi)=0$, it follows that it gives the curve locus of a series of imaginary roots of the equations formed by varying c . Where the curve represented by equation (11) cuts the axis of x we have $f'(x)=0$. At these points there are at least two equal roots.

For the quadratic equation $x^2+bx+c=0$, (11) becomes $2x+b=0$, and this is the equation to the line of neutral points $N_1'PN_2'$ shown in fig. 2.

5. Cubic Equation.

Let us suppose that the equation has been reduced to the form

$$x^3 - b^2x + c = 0. \dots \dots \dots (12)$$

We easily find that

$$H \frac{x^3 - b^2x + c}{x(x^2 - b^2)} = H + \frac{C}{5(x-b)} - \frac{2C}{5x} + \frac{C}{5(x+b)}, \quad (13)$$

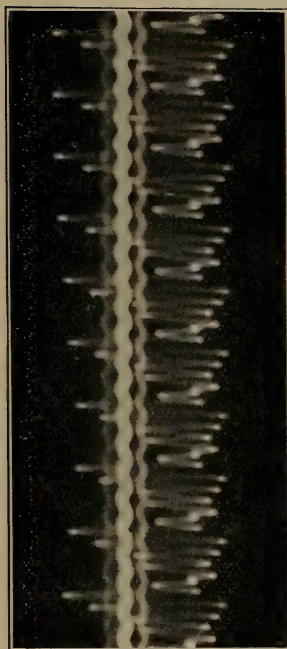
where $C=5Hc/2b^2$.

In this case (fig. 3) three long vertical wires are used. Let them cut the plane of the paper at right angles at B_1 , O , and B_3 respectively, and let $B_1O=OB_3=b$. The currents in the wires passing through B_1 and B_3 must each be made equal to C , and the current in the wire passing through O should be the return current $2C$. The arrow-heads indicate that the direction of the current flow is out of the paper at B_1 and B_3 and into it at O . H denotes the direction of the earth's magnetic field.

Let us now suppose that c increases uniformly from zero to infinity. When c is small, and therefore the currents are

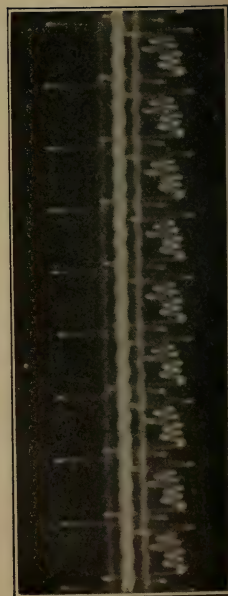


FIG. 1.



$$n_2 = 3 n_1.$$

FIG. 2.



$$n_2 = 4 n_1.$$

FIG. 3.



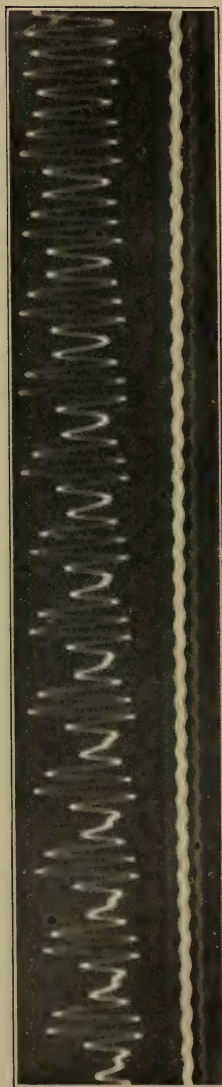
$$n_2 = 5 n_1.$$

FIG. 4.



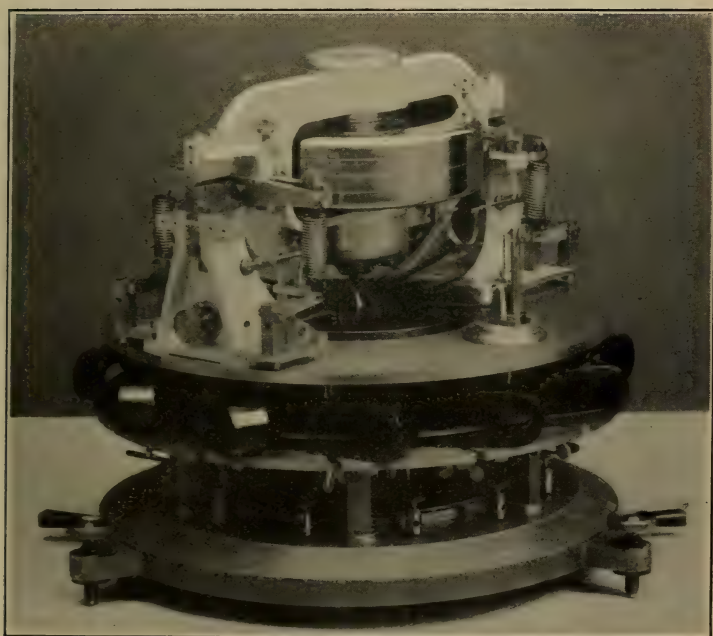
← Difference tone.
 $n_2 = \frac{3}{2} n_1.$ $n_2 - n_1 = 237.5.$

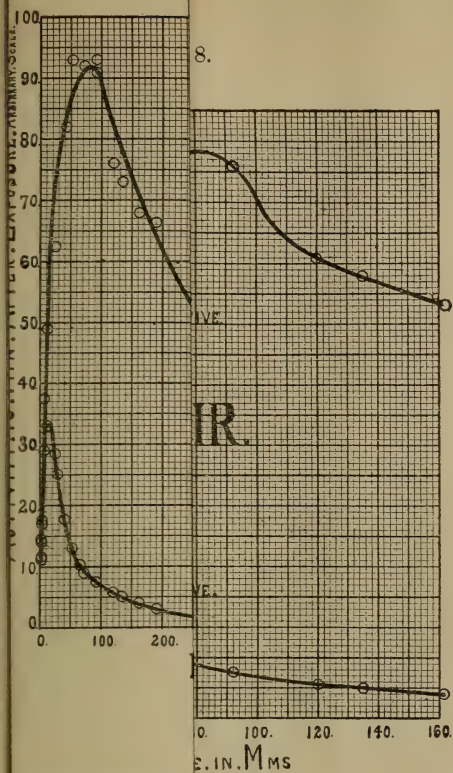
FIG. 5.



← Showing change from lower primary to difference tone.

FIG. 2.





WAVELENGTH IN MICRONS

15.

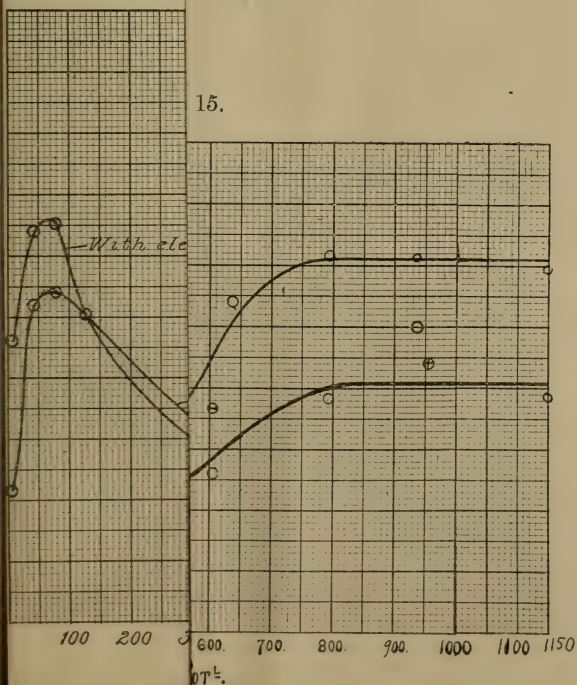


FIG. 2.

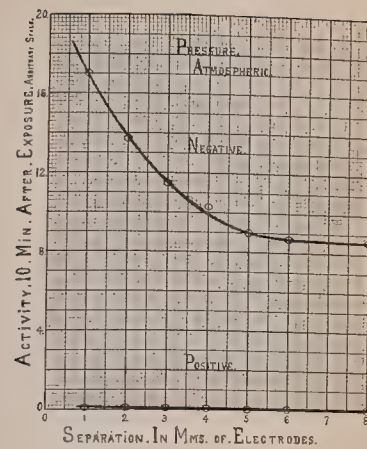


FIG. 3.

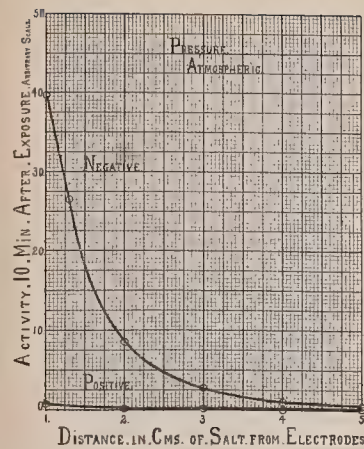


FIG. 4.

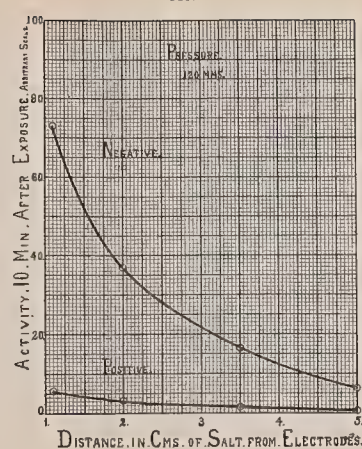


FIG. 5.

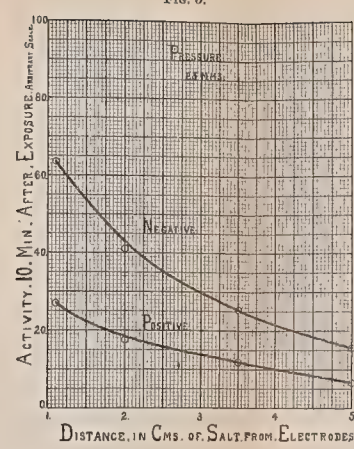


FIG. 6.

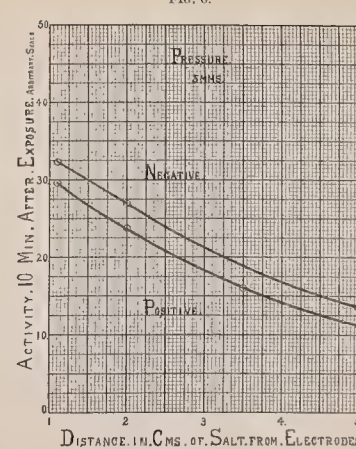


FIG. 7.

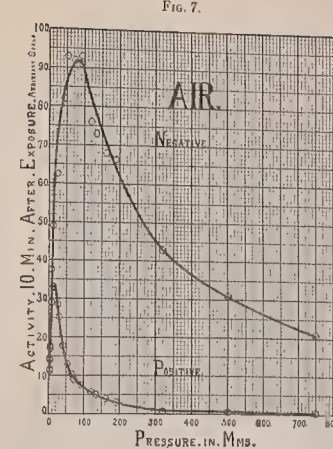


FIG. 8.

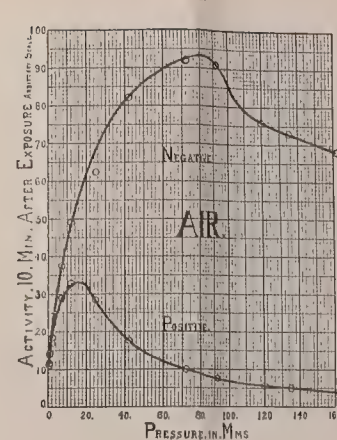


FIG. 9.

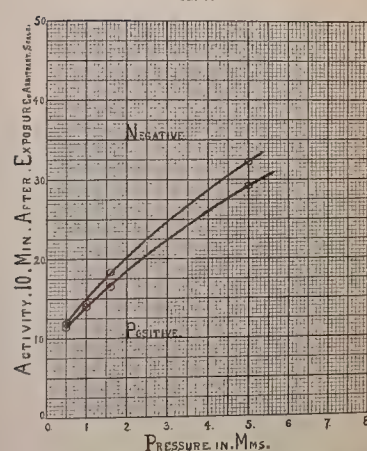


FIG. 10.

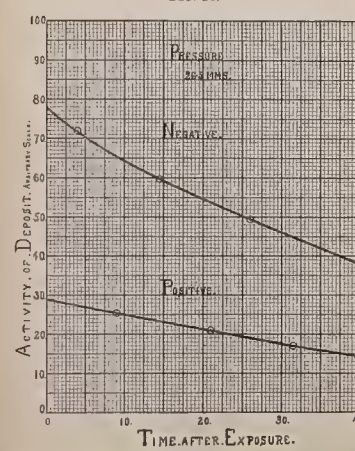


FIG. 11.

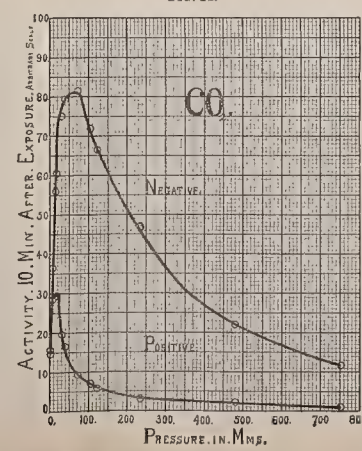


FIG. 12.

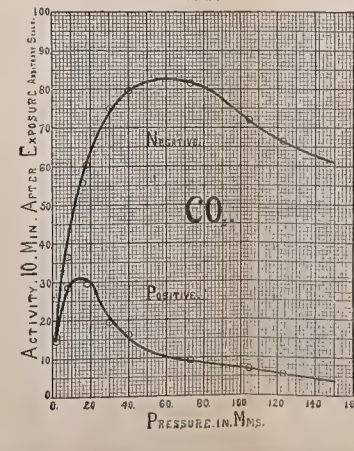


FIG. 13.

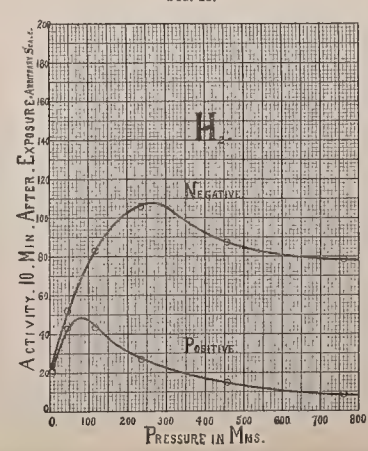


FIG. 14.

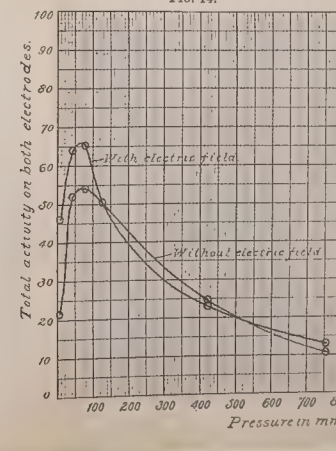


FIG. 15.

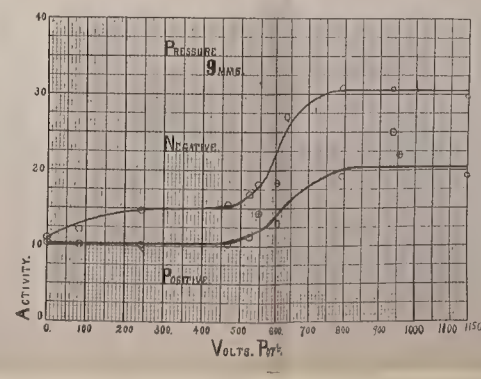


FIG. 1.

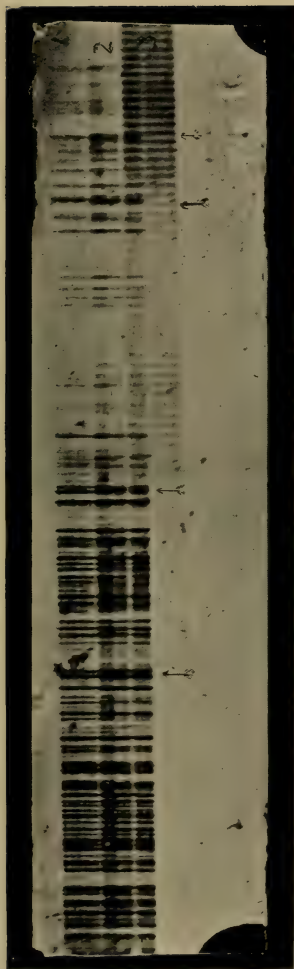
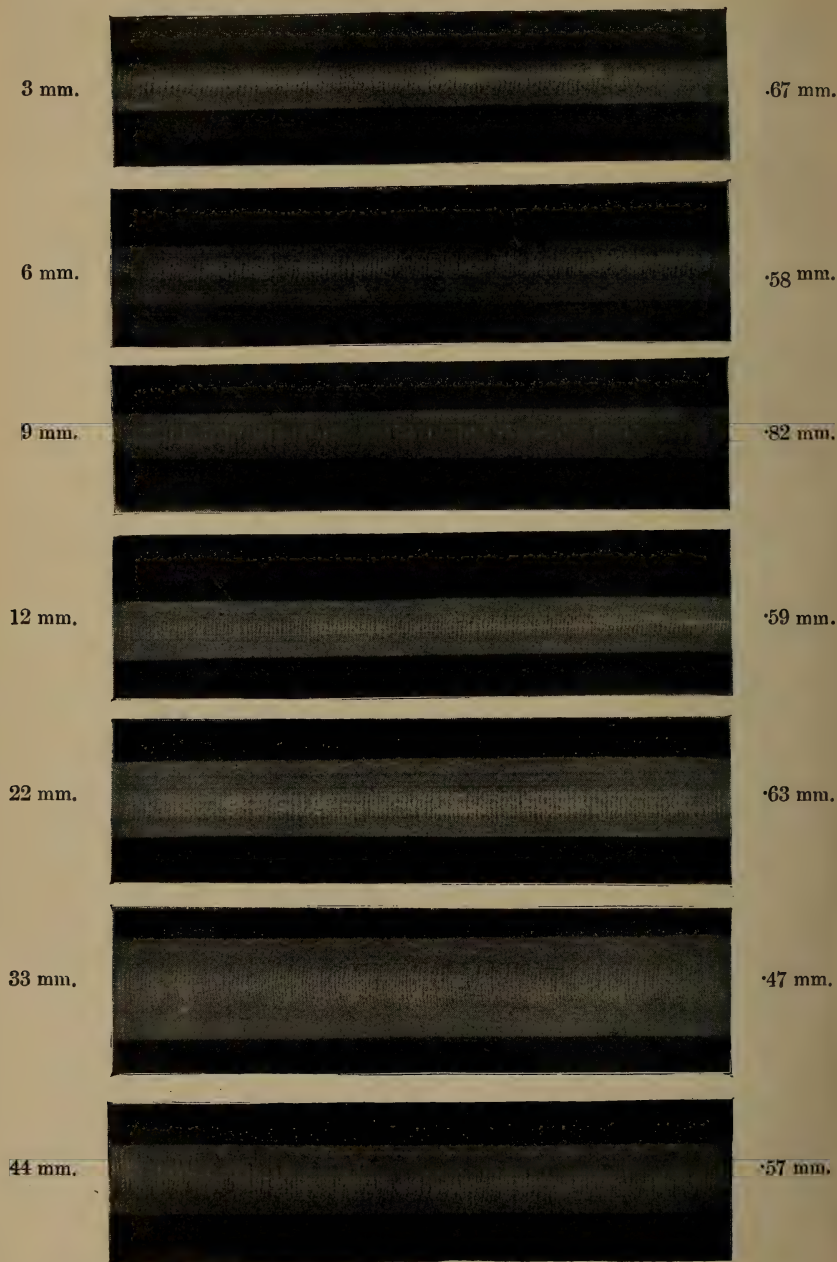


FIG. 2.



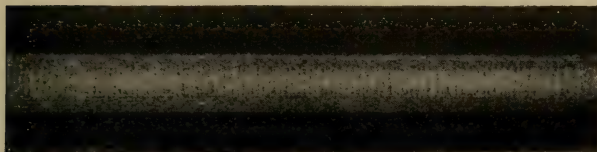
FIG. I.—Showing the variation of the striation with diameter of tube.



The diameters of the tubes are written on the left and the striæ-distances on the right.

FIG. II.—Variation of the striation for the same spark with various powders.

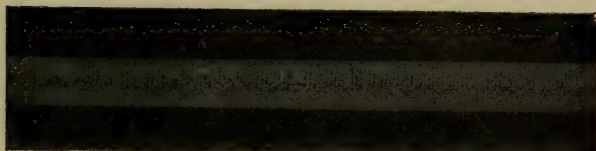
(a) Lycopodium ·76 mm.



(b) Pepper ·76 mm.



(c) Starch ·58 mm.



(d) Egg-powder ·71 mm.

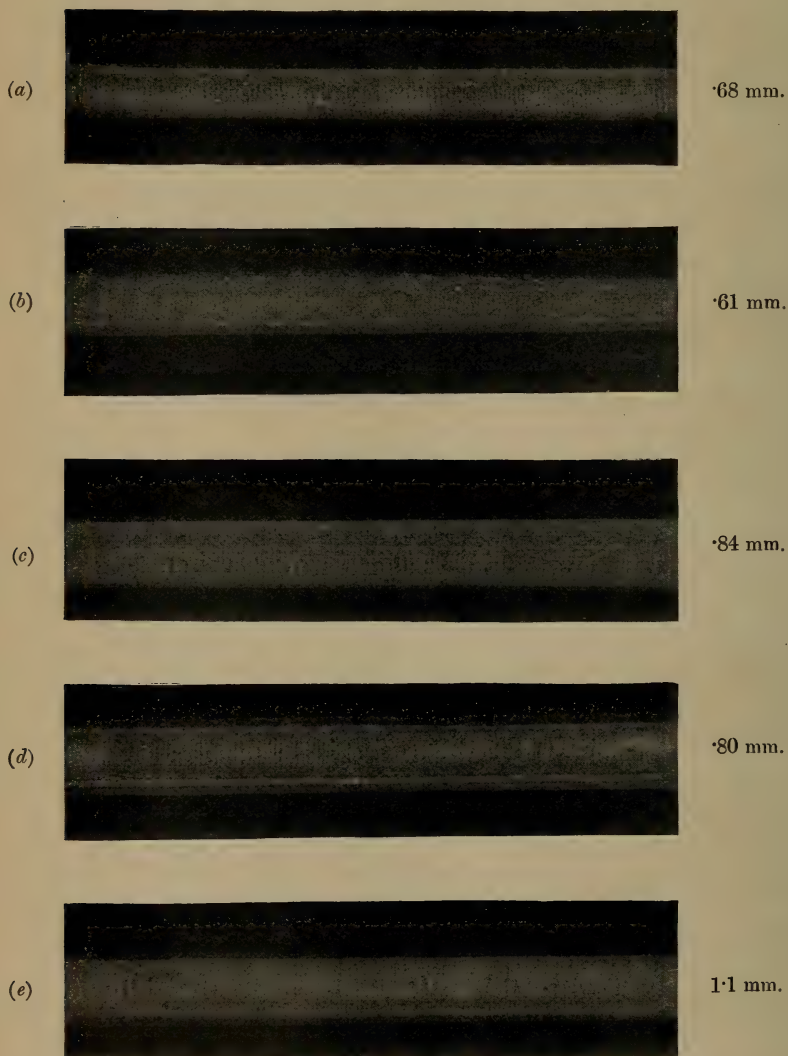


(e) Boric Acid ·76 mm.



The striæ-distances are given along with the name of the powder.

FIG. III.—Variation of the striation with variation of electrical conditions.



Figures (a) to (e) were obtained from circuits the frequencies of the oscillations from which are in descending order of magnitude. The mean striæ-distances are written on the right,

FIG. IV.—Variation of the striation with electrical conditions.



Figs. (a) to (e) refer to circuits giving oscillations in descending order of frequency.

Figs. (f) to (k) refer to a series of circuits for which the frequencies were measured. These are shown to the right of the corresponding figures.

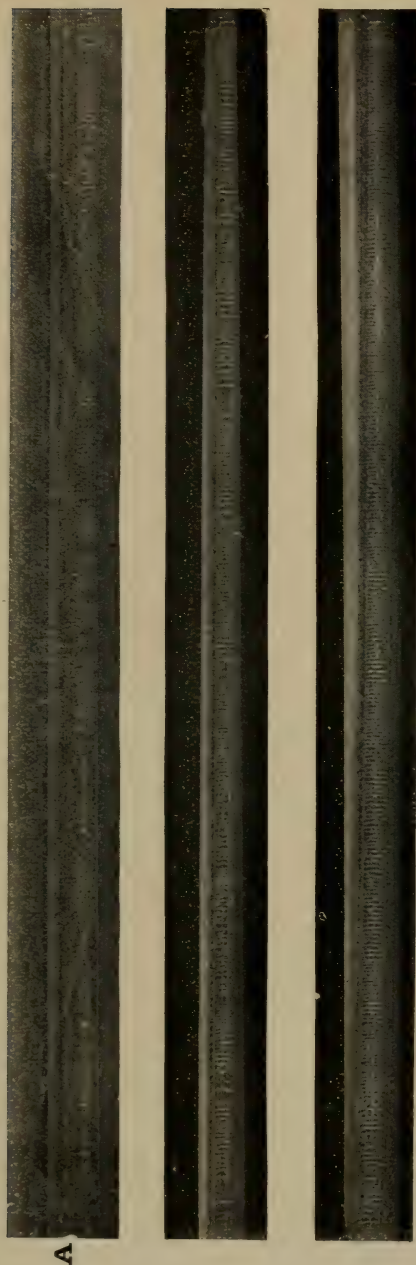


The striæ-distances are shown on the left.

FIG. V *a*.—Variation of the striae-distance along the same tube.



FIG. V *b*.—Variation of the striæ-distance along the same tube.



In Fig. V *a* the whole length of a tube 40 cms. long and 1 cm. diam. is shown beginning at A the end near the spark. The spark was from machine and various batteries of jars. Fig. V *b* similarly shows the variation for the spark from the machine only.

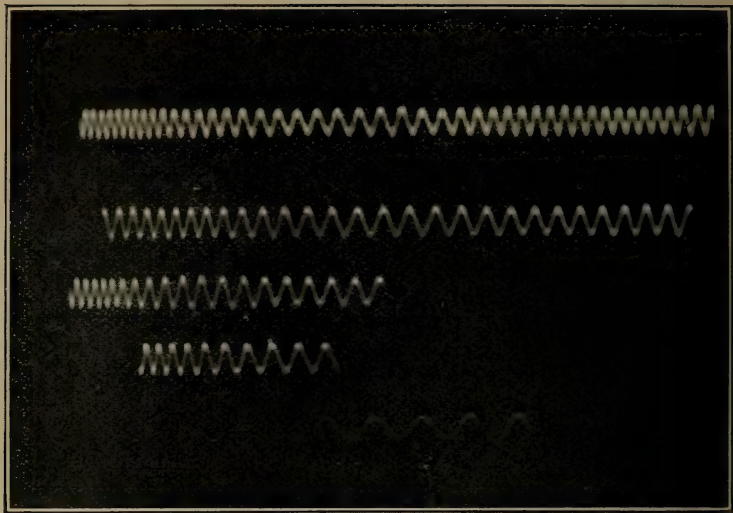


FIG. 7.

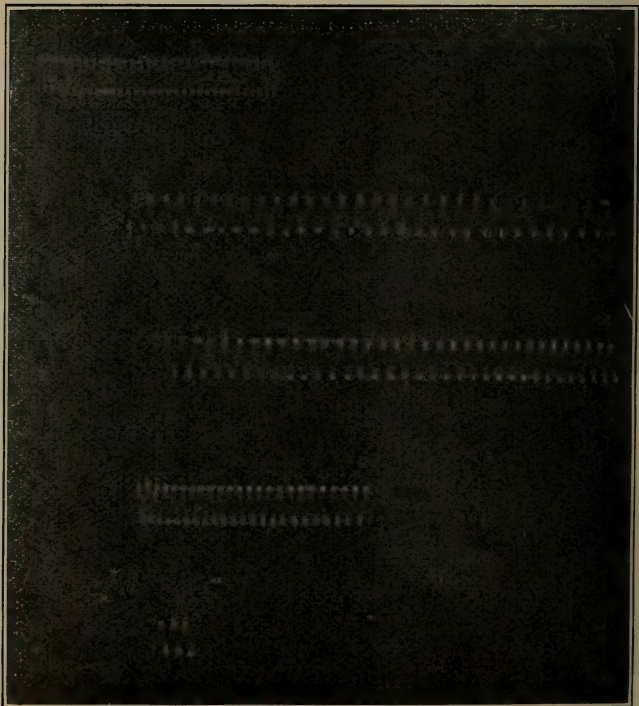


FIG. 8.

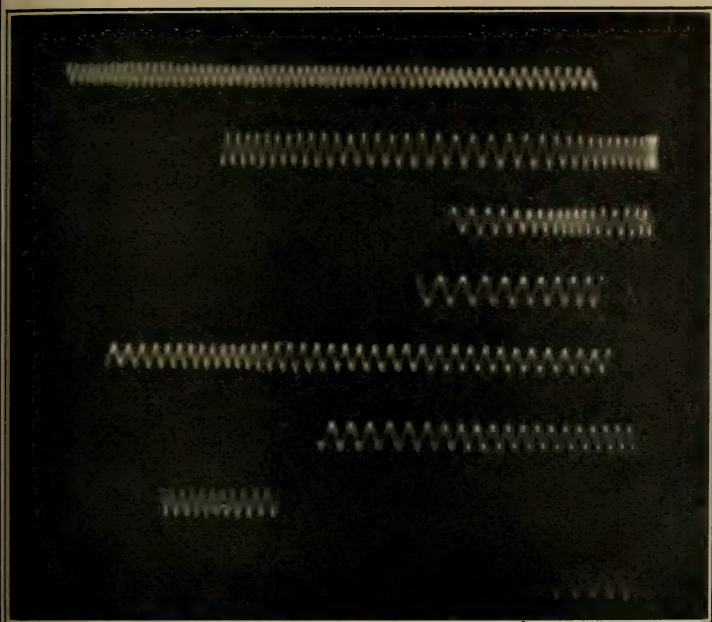


FIG. 7 a.

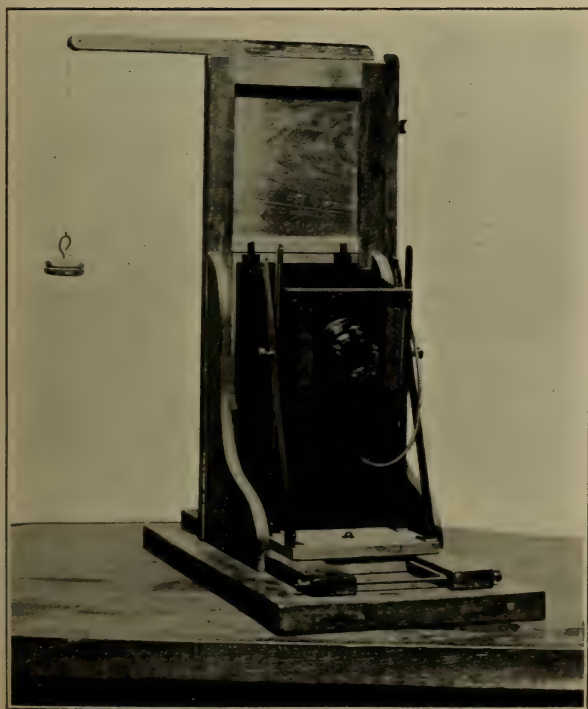


FIG. 9.

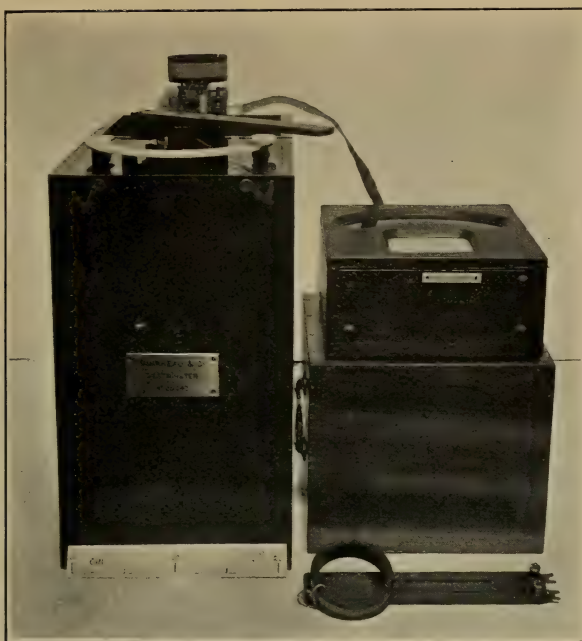


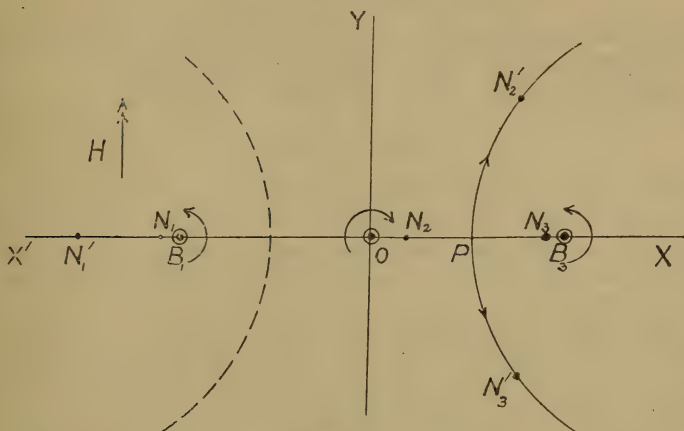
FIG. 1.



FIG. 2.

small, two neutral points N_2 and N_3 obviously lie between O and B_3 and a neutral point N_1 lies along B_1X' .

Fig. 3.



As the current increases the neutral points N_2 and N_3 approach one another and the neutral point N_1' moves along B_1X' . For a particular value of c , N_2 and N_3 coincide at P , and we have two equal roots. For greater values of c the neutral points N_2 and N_3 move uniformly along the curve $N_2'PN_3'$, one being as much above the line XX' as the other is below it. But the neutral point N_1 always moves along the axis of X .

The equation (12), therefore, has always three roots. When c is small they are all real, two of them being positive and one negative. For a certain value of c the two positive roots become equal, and for greater values of c we have two conjugate imaginary roots and one real negative root. Both the real and imaginary parts of the conjugate roots continually increase as c increases.

The equation to the curve $N_2'PN_3'$ in fig. 3 can be found at once from (11); it is

$$3x^2 - y^2 = b^2. \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The negative branch of this hyperbola indicated by the dotted curve in fig. 3 gives the locus of the neutral points when c is negative. In this case there is obviously always one real positive root. Increasing the value of b in (12) is equivalent to putting the wires B_1 and B_3 further apart. This obviously increases the limits of the values of the

positive real roots. When b is negligibly small we see by (14) that the locus of the neutral points is the two straight lines represented by

$$(y - x\sqrt{3})(y + x\sqrt{3}) = 0.$$

Hence the imaginary roots are of the form $x_1 \pm x_1\sqrt{3}\sqrt{-1}$ where $-2x_1$ is the value of the real negative root.

Similarly we can discuss the roots of the equation

$$x^3 + b^2x + c = 0.$$

In this case it will be found that the currents in the wires through B_1 and B_3 are unequal, and that the equation has always two imaginary roots, the neutral points lying on the hyperbola $y^2 - 3x^2 = b^2$, which is conjugate to the hyperbola shown in fig. 3.

Equations of the fourth and higher degrees can be discussed in like manner. To get the most instructive results care has to be taken to choose the distances between the wires so that the analytical expressions for the required currents may be as simple as possible. If this be not done analytical difficulties will often be encountered in interpreting the results.

6. *Finding the Roots of an Equation.*

The great and so far as we know the unique advantage of this method is that it enables us to find the imaginary as well as the real roots of an equation almost at once. In equations occurring in many physical problems it is the latter roots which we desire to find, and this method enables the physicist to find quickly approximate values of these roots.

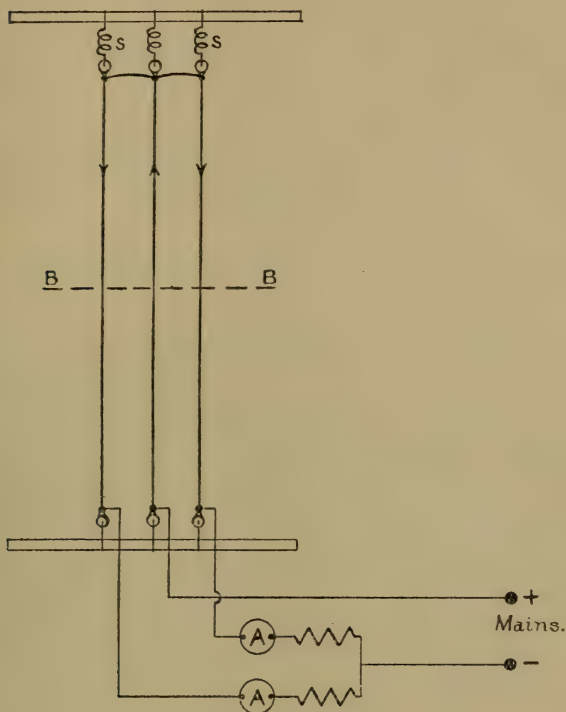
If due precautions are taken the maximum inaccuracy of this method need not exceed one per cent. This is the accuracy obtainable by careful students, who need have no previous experimental training, in finding H by measuring the distance of the neutral point from a long vertical wire carrying a known current.

7. *Description of Apparatus.*

We shall now describe the simple apparatus we use for teaching purposes. In fig. 4 the arrangement of the apparatus for solving a cubic equation is shown. S, S represent springs

attached to insulators. These are necessary in order to keep the wires tight. The wires are No. 16 single cotton-covered

Fig. 4.



wire, and pass through a long slit cut in a drawing-board BB. The ammeters A are any of the ordinary ammeters used in the laboratory reading up to 10 amperes, and are in series with lamp-boards and rheostats. The currents are adjusted to the required values, and a sheet of sectional paper with a slit in it is put on the board. By means of an ordinary charm-compass the neutral points can then be readily found. It is advisable to make a little sketch of the lines of force near the neutral points, as this is a help in indicating their true position. The coordinates of these points read off from the sectional paper give the real and imaginary roots of the equation.

LXXXV. *On the Radium Content of certain Igneous Rocks from the Sub-Antarctic Islands of New Zealand.* By C. COLERIDGE FARR, D.Sc., and D. C. H. FLORANCE, M.A., M.Sc.*

RUTHERFORD ('Radioactive Transformations') has calculated that 4.6×10^{-14} gramme of radium per unit mass of the Earth would generate an amount of heat equal to that lost by the Earth by conduction through its crust. Strutt (Proc. Roy. Soc. 1907) has examined the radium content of a number of igneous and sedimentary rocks, and his results, corrected by Eve and McIntosh for an error of standardization, give for mean values

Igneous rocks.....	1.7×10^{-12}	gramme radium
Sedimentary rocks...	1.1×10^{-12}	" "

a result which is about 28 times as large as the theoretical quantity. Eve and McIntosh (Phil. Mag. Aug. 1907) find a mean for Igneous and Sedimentary rocks of 1.1×10^{-12} per gramme of rock. Joly ('Nature,' Sept. 1908) finds values much higher than these, his mean for igneous rocks being 6.1×10^{-12} , whilst the mean content of the Basalts examined by him was 5.0×10^{-12} , which is much higher than the value found by other investigators for similar rocks.

Method of Testing.—The rocks examined by us were ground to a very fine powder, and 20 grammes were fused in a platinum crucible with ordinary fusion mixture, one gramme of the powdered rock being mixed with six grammes of the mixed carbonates which had been previously tested for radium. The fused mass was dissolved partly in distilled water and partly in HCl, both of which were separately examined. The acid and alkaline solutions were then separately corked with glass tubes and clipped rubber connexions in position, and the cork coated with sealing-wax to prevent possible leakage of the emanation, and the flasks were then set aside for three weeks to mature. The apparatus was similar to that described by Strutt (*loc. cit.*). The solution was boiled for one hour, the steam being condensed in a Liebig condenser and the gases evolved being collected over fresh distilled water. Before the boiling was stopped, the water in the jacket of the condenser was run off and steam was passed through as far as the collecting vessel to drive over the last traces of emanation. The electroscope was similar to that used by Boltwood (Amer. Journal of Science,

* Communicated by Professor E. Rutherford, F.R.S.

1904), and it was exhausted to half an atmosphere by means of an oil-pump before any of the collected gases were passed in. These gases were allowed to enter very slowly through CaCl_2 and H_2SO_4 drying-tubes, and the pressure was then brought up to atmospheric by the admission of air, the time occupied over this transference through the drying-tubes from the stoppage of the boiling to the commencement of observations in the electroscope being in every case 15 minutes. Readings of the position of the leaf were taken every two minutes for half an hour, and the instrument was then re-charged and the leak for the next half hour similarly determined. This was sufficient indication that the effect was that due to the radium emanation. In some cases, however, the gas was studied for three or four hours, but although the presence of radium was shown clearly by the increase of the rate of leak, yet, owing to the active deposit, so much time was lost in bringing the electroscope back to a normal condition of natural leak, that it was decided to pump the gases out after an hour.

The electroscope was charged to a potential of 220 volts and the movement of the leaf was read by a microscope and a micrometer eyepiece, so that the large scale-divisions could be subdivided into hundredths. For the means of its standardization we are indebted to Professor Rutherford, who kindly sent us a solution containing 3.14×10^{-9} gramme of radium. Two separate eighths of this, each containing 3.925×10^{-10} gramme of radium, were used as standards. Either of these treated in the same way as the rock solutions produced an initial leak of 64 micrometer divisions per minute, hence one micrometer division per minute corresponded to 6.13×10^{-12} gramme of radium.

“*Natural*” *Leak*.—The determination of this is an important matter in an attempt to evaluate with accuracy the amount of radium contained in a rock. In our case the electroscope was always charged over night—the insulation was sufficiently good for it to retain a considerable portion of its charge for forty-eight hours—and the leak found. In general it decreased from the time of its initial charging the night before to the introduction of the rock gases. From a value of 1.5 micrometer divisions per minute the night before, it became about 0.8 per minute at the time of introduction of the emanation. We consider that this latter number cannot be more than .2 from the actual value during the examination of any of the rocks. Such an error would lead to a wrong estimation of the amount of radium in 20 grammes of rock of 1.23×10^{-12} , or an error of 0.10×10^{-12}

per gramme of rock. We therefore consider that for the specimens we have examined the results may be relied upon to about this extent.

Results.

Granite.	Bounty Is.	2.50×10^{-12}
Trachyte.	Auckland Is., Musgrave Penin.	2.10
Granite.	Auckland Is., Musgrave Penin.	1.01
Pitchstone.	Auckland Is., Musgrave Penin.	1.9
Basic Porphyrite.	Auckland Is., Adams Island.	0.99
Basalt.	Auckland Is., Adams Is. (sea-level).	0.81
Dolerite.	Auckland Is., Adams Is. (top).	0.52
Diabase.	Auckland Is., Musgrave Penin.	0.43
Gabbro.	Auckland Is., McClure Head, Carnley.	0.33

Campbell Island Rocks :—

Porphyry.	Perseverance Harbour	2.8
Trachyte.	Filial Point	2.64
Trachyte.	Mount Lyell	1.58
Melilite.	Mount Lyell	1.61
Basalt.		
Dolerite.	Mount Honey	0.79
Limestone.	Garden Cove	0.46
Gabbro.	N.W. Bay	0.34
Marble.	N.W. Bay	0.28

The three following rocks are of special interest:—

- (1) Andesite from the latest flow of Ngauruhoe, about sixty years old. Radium content .41.
- (2) A lava from the lip of the crater of Mount Erebus. This is fairly recent, and was given us by Mr. D. Mawson, of the Nimrod Expedition. Radium content 2.38.

This rock has had to be tested rather hurriedly, and is perhaps under true value.

- (3) The Meteorite is a stony meteorite which fell at Mokoia in New Zealand, on November 26th, 1908, and was sent to us by Mr. G. R. Marriner, Curator of the Wanganui Museum. It contains 37 per cent. of iron oxide, and 37.5 per cent. of silica. Radium content .52. This value is nearly equal to that obtained by Strutt for a stony meteorite.

Conclusions.—Since all but two of the rocks examined are derived from islands to the south of New Zealand which

are mostly igneous in character, the indication appears to be that the distribution of radium in the rocks of this region is in excess of that required to maintain the constancy of the heat of the Earth; and the necessity of an examination of rocks from all places and of all ages is emphasized. The mean radium content of these rocks approximates very closely to that found for others by Strutt and by Eve and McIntosh, but differs considerably from Joly's values. Further investigations are being made on the rocks of these regions; and it is hoped to examine the distribution of radium in the material encountered in piercing the Lyttleton Tunnel, which will give a range of time from the earliest sedimentary rocks to the last flow of the volcano, through the wall of whose crater the tunnel is bored.

The following notes on the geological properties of these rocks by Mr. R. Speight, F.G.S., may be of interest.

The two granites are both biotite granites of ordinary type and of uncertain age. The other plutonic rocks—the gabbros—though coming from widely separated localities of the same region, exhibit marked similarities: both are olivine gabbros, but that from Auckland Island is very coarse-grained at times. The coincidence of their radium content is very striking.

The porphyry from Campbell Island contains large numbers of zircon phenocrysts.

The trachytes are all of highly alkaline type. Those from Auckland Island contain a very high percentage of soda and a very low percentage of alumina (11 per cent.) and a small quantity of free quartz; they are decidedly acid. Anorthoclase is a common constituent as well as an alkaline hornblende. The pitchstone belongs to the same group but is somewhat more basic in composition, and contains abundant ægerine-augite microlites. The Campbell Island trachytes, according to Dr. Marshall, are slightly more alkaline and contain less iron. They are all augite trachyte with anorthoclase and occasional riebeckite and cossyrite. These rocks together with the melilite basalt all appear to have similar amounts of radium present.

The other rocks from Auckland Island are basic in character and include flows and dykes. They are generally marked by a high percentage of titaniferous magnetite. No. 8 is a diabase of greater age than the rest, and No. 5 a porphyrite approaching an augite-camptonite in character and basic composition, found penetrating the more recent basalts. The age of these basalts is probably from middle to late Tertiary. There is no marked difference between the earlier

and later basaltic flows, but the one from sea-level at Adams Island is fine-grained and contains hornblende, while the other is very coarse-grained with large phenocrysts of olivine and augite.

The Campbell Island limestone is of Tertiary age and markedly foraminiferal; the marble is derived from it as a result of basic intrusion.

A general conspectus of these results shows that the radium content corresponds roughly with the basicity, and not with the age of the rocks.

Canterbury College, New Zealand,
May 12, 1909.

LXXXVI. *The Ultimate Product of the Uranium Disintegration Series.* By J. A. GRAY, B.Sc.*

IN Amer. Journ. Sc. (vol. xx. p. 253, 1905, and vol. xxii. p. 77, 1907) Boltwood gives considerable evidence which goes towards proving lead the ultimate product of this series. He finds, *e. g.*, that in primary uranium minerals the ratio of the lead to the uranium is practically the same for minerals of the same age, and greater the greater the age of the mineral. These are conditions that the ultimate or any inactive product must fulfil, and lead was found to be the only substance fulfilling them, except perhaps helium, which has been definitely proved to come from the α particles (Rutherford and Royds, Phil. Mag. vol. xvii. p. 281, 1909). However, in all the minerals of which analyses were obtained, there was a great deal of contamination from other substances; so that it is not unlikely that the lead was deposited in some other way than as a product of the uranium series. Certain mineral substances exist which do not appear to be open to this objection. These are the hydrated phosphates of uranium with (1) calcium, (2) copper. These substances exist in characteristic crystals having somewhat the appearance of artificially purified substances, and are undoubtedly derived from the comparatively recent recrystallization from water of the salts derived from pitchblende lodes. A spectroscopic examination was made of these salts, to see what light they would throw on the question.

A spectroscopic analysis was first made of mineral autunite, $\{\text{Ca}(\text{UO}_2)_2\text{P}_2\text{O}_8 + 8\text{H}_2\text{O}\}$ from Portugal. This contains approximately 50 per cent. of uranium. If old enough the mineral would contain the products of uranium and enough of the final product to be detectable by spectroscopic analysis.

* Communicated by Prof. R. J. Strutt, F.R.S.

Therefore any line in the spectrum obtained, other than those due to calcium or uranium, might possibly belong to a final product. To simplify the examination, a comparison was made with a carefully purified specimen of uranium oxide. To obtain wave-lengths, an iron reference-spectrum was used. The arc with carbon poles was used to burn the substances. As the mineral did not burn very well, calcium chloride was used as a flux. The three spectra were obtained on the photographic plate, one above the other, that of the mineral being in the centre. In this way we could see what lines were in the mineral spectrum and not in the uranium, and by means of the iron spectrum find the wave-lengths of such lines. In this way it was found that the only lines in the spectrum of the mineral not belonging to calcium or uranium belonged to iron, lead, barium, and strontium. Barium and strontium may certainly be discarded as most calcium salts contain a little of them. Iron is an almost universal constituent of minerals, being present even in clear white salt and Iceland spar: thus not much weight can be laid on its appearance. Other analyses carried out on (1) another specimen of autunite, locality not known, (2) torbernite from Cornwall, the double phosphate of copper and uranium $\{\text{Cu}(\text{UO}_2)_2\text{P}_2\text{O}_8 + 8\text{H}_2\text{O}\}$.

Except that in the first of these there was more barium and in the second that the calcium was replaced by copper, the analyses give the same result.

We see, therefore, by the analyses of three minerals from different localities, that lead is the only substance not accounted for. We all know that if the final product is metallic and the mineral old enough, lines belonging to that product ought to appear in the spectrum. The spectroscopic analysis of these minerals therefore strongly points to lead being the ultimate product of the uranium series.

Certain phosphatized bones have been shown by Strutt (*Proc. Roy. Soc.* Aug. 1908) to be moderately rich in radioactive constituents. It was thought worth while to test these for lead. The result was positive. No connexion could, however, be traced between the geological age of the specimens and the quantity of lead. It must be supposed that here the lead is not wholly of radioactive origin.

Summing up, we find that lead is the only substance which by spectroscopic analysis can be considered as the ultimate product of the uranium series. It has often been pointed out on theoretic grounds that lead was probably the ultimate product. Boltwood's experiments lead to the same conclusion.

A rough estimate was made of the amount of lead in the *Phil. Mag.* S. 6. Vol. 18. No. 107. Nov. 1909. 3 I

first specimen of autunite, and from that a determination of its age. A tenth of a gramme of autunite was burnt in the arc with a little calcium chloride, the image of the arc being exposed on the slit for two minutes, which was practically sufficient to burn the lead out. The same thing was done for similar amounts of two specimens of calcium chloride, the first containing $\frac{1}{10,000}$ gramme of lead per gramme, the second $\frac{1}{50,000}$ gramme of lead. There was less lead in the mineral than in the first specimen of calcium chloride and more than in the second. It was judged that the mineral contained $\frac{1}{20,000}$ gramme of lead per gramme. As it contains 50 per cent. uranium, this means that for one gramme of uranium the autunite contains $\frac{1}{10,000}$ gramme of lead. In radioactive equilibrium, in one gramme of uranium there is 3.8×10^{-7} gramme of radium. Taking the period of radium as 2500 years, the fraction of radium undergoing transformation per year is 2.8×10^{-4} . This means that $3.8 \times 10^{-7} \times 2.8 \times 10^{-4} \times \frac{207}{238}$ gramme of lead or 9×10^{-9} would be formed per year for one gramme of uranium in radioactive equilibrium. Therefore the time taken for 10^{-4} gramme of lead to form would be $\frac{10^{-4}}{9 \times 10^{-9}}$ years, or about a million years*.

In conclusion, I wish to record my thanks to Professor Strutt, who suggested these experiments, for his interest and help throughout, and also to Professor Fowler for his help in overcoming the difficulties of spectrum analysis.

LXXXVII. *Notices respecting New Books.*

Report of a Magnetic Survey of South Africa. By J. C. BEATTIE, D.Sc. Published for the Royal Society and Sold by the Cambridge University Press. London, 1909. 20s. net. 4to., pp. x+126, with six Appendices, pp. 1-235, and nine Charts at the end.

THIS volume deals with observations made at fully 400 places in S. Africa, ranging from the extreme South of Cape Colony to Victoria Falls in the North. The stations are mostly in British territory, but a few are in Portuguese East Africa. The observations were made by Prof. Beattie with the assistance of some friends, especially Prof. J. T. Morrison, between 1898 and 1906. The

* On p. 204 of his book, 'The Interpretation of Radium,' Soddy mentions that autunite contains no detectable quantity of lead. This is probably due to the small amount of lead, which would be very hard to detect by ordinary chemical methods, and which would also depend on the age of the mineral.

results are reduced to July 1, 1903, by means of secular change data derived from the repetition of the observations at a selected number of the stations. Details as to the geographical position of the stations and the observed values of the magnetic elements are reserved for Appendix E (215 pages). In the body of the work are a number of tables showing the values of the magnetic elements for July 1, 1903, as observed and as calculated by methods intended to eliminate local disturbances. Deductions are made as to the disturbances, "ridge" and "valley" lines, &c., after the method of Rücker and Thorpe, and these are illustrated by a number of small maps in the text. The large charts at the end of the volume show the position of the stations, isogonal lines, isoclinal lines, and isomagnetics for the total intensity and a variety of its components. They also include a geological map prepared by Mr. A. W. Rogers.

LXXXVIII. *Proceedings of Learned Societies.*

GEOLOGICAL SOCIETY.

[Continued from p. 680.]

March 24th, 1909.—Prof. W. J. Sollas, LL.D., Sc.D., F.R.S.,
President, in the Chair.

THE following communication was read:—

‘Glacial Erosion in North Wales.’ By Prof. William Morris Davis, For. Corr. G.S.

An excursion around Snowdon in September 1907, followed by a further visit in 1908, led the author to the conclusion that a large-featured, round-shouldered, full-bodied mountain of pre-Glacial time, had been converted by erosion during the Glacial Period—and chiefly by glacial erosion—into the sharp-featured, hollow-chested, narrow-spurred mountain of to-day. The peculiar indifference of topographic form to the trend of formation-boundaries and the insequent stream-arrangement, are what might be expected as the result of prolonged erosion upon a mass of complicated and resistant structure. The author discusses Ramsay's theory of a plain of marine denudation, and is of opinion that the upland seems to deserve classification rather with peneplains; he suggests for it a Tertiary date, which would not be inconsistent with the erosion of open valleys in the uplifted peneplain after its elevation, and argues that Snowdon and its high neighbours had a relief of some 2000 feet above the plain. As the result of a comparison with the non-glaciated regions of Devon, it is considered that the dissection of North Wales must have been somewhat less developed in pre-Glacial times than in Devon to-day. On this assumption it is possible to make a tentative restoration of the pre-Glacial form of Snowdon: subdued mountains, with dome-like summits and rounded spurs, drained by prevailingly graded streams of accordant levels at their junctions. In fact, the characters were

those of 'moels' such as Moel Tryfaen, or like the well-worn Appalachian Mountains of North Carolina, or the Cévennes.

The chief abnormal features of Snowdon are the following:—Alongside the graded summit and slopes of a 'moel' stand the head-cliffs of a rock-walled cwm, in the floor of which talus is now accumulating. The cwm-floors are generally stepped, sometimes more than once, and the streams cascade down into the valleys. The cross-profile of the valleys is often a fine catenary curve, down the sides of which streams fall from hanging valleys. The slope of the main valleys occasionally decreases even to the point of reversal, as where lakes occur; and in the immediate neighbourhood of smoothly-graded, waste-covered slopes knobby or craggy ledges and bars of rock often appear. After pointing out that such features are generally associated with glaciation, the author proceeds to discuss two out of four possible hypotheses put forward: 'that glaciers are essentially protective agencies,' or that they 'are active destructive agencies.' The consequences deduced from the hypotheses are confronted with the actual facts, and it is found that these, and especially those relating to rock-steps, cannot be explained on the protection-theory, while the theory of a destructive agency seems to explain most of the facts.

These facts are dealt with under the following heads:—Valley-head curves; valley-floors, lakes; valley-floors, rock-steps; valley-sides; hanging lateral valleys; and glacial overflows. With regard to the first, it is shown that there is no systematic relationship between the height of the cwm-cliffs and the distance of the front rock-step; the serration of 'cribs' or arêtes cannot be explained by pre-Glacial or post-Glacial weathering according to the protection-theory. It is suggested that it might be possible on the erosion-theory to work out and classify cwms according to their age and growth, and as a contribution to this enquiry the cwms of Mynydd Mawr are dealt with. The valley-lakes are likely to be only a small part of the glacial erosion. No consistent explanation of the valley-steps can be found under the theory of ice-protection, whereas they are explicable on the assumption of glacial erosion. They may have originated in resistant beds, and have then retreated up the valleys. The catenary curve of the cross-section of such valleys as those containing Llyn Gwynant and Llyn Cwellin might be expected to result from long-continued ice-erosion; and the occurrence of great cliffs on the sides of these valleys is not inconsistent with such an origin. Several good examples of hanging valleys occur, and they seem to show that the deepening of the main valleys by glacial erosion may be from 200 to 400 feet, and in some cases as much as 500 or 600 feet. The lateral erosion may easily amount to 1000 or 1500 feet. The most striking case of a glacial overflow is that at the head of the Nantlle valley, which appears to have carried much of the West Snowdon ice. The head of the pass would seem to have been farther westward and higher in pre-Glacial times.

THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[SIXTH SERIES.]

DECEMBER 1909.

LXXXIX. *Positive Electricity.* By Sir J. J. THOMSON,
Professor of Experimental Physics, Cambridge.*

THE most important questions to be settled as to the nature of positive electricity are :—(1) Does a definite unit of positive electricity exist? (2) If so, what is the size of the unit?

Question (1) may perhaps be made clearer by considering a definite case. Suppose we could get a pure gas, say oxygen or nitrogen, would it be possible to get in such a gas, positively charged particles smaller than the residue left, when a corpuscle is removed from an atom of oxygen or nitrogen?

Again, we know that we can get negative particles of the same kind whether we extract them from oxygen, hydrogen, or nitrogen. Is there anything analogous to this in the case of positive electricity? Can we in short get positively electrified particles of the same kind from different gases?

Our knowledge of charged positive particles (other than α particles) has been derived from two sources: (1) the study of positive particles in systems of positive rays found in vacuum-tubes; (2) the study of the velocity of the positive ions in ionized gas.

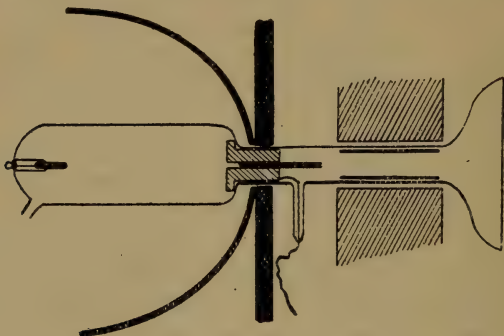
To begin with the first method. The two sets of positive rays which have been most closely studied are, firstly, the Canalstrahlen which are found passing through a perforated

* A paper introducing a discussion on this subject read on August 30, 1909, at the Winnipeg meeting of the British Association. Communicated by the Author.

cathode, and, secondly, the rays shot out from the cathode in the same direction as the cathode rays but which are deflected much less than those rays by magnetic forces, and which instead of carrying a negative charge are some of them positively charged while others are electrically neutral.

To take the Canalstrahlen first. These can be studied by an apparatus like that represented by fig. 1. It consists of a

Fig. 1.



vacuum-tube with a perforated cathode through which a beam of Canalstrahlen is streaming. The rays strike against a willemite screen at the end of the discharge-tube and produce phosphorescence when they reach the screen. In their path to the screen the rays pass between the poles of an electromagnet which deflects them in a vertical direction and simultaneously between two parallel metal plates connected with the terminals of a large number of storage-cells; the electrostatic field deflects the spot horizontally. The magnetic deflexion is equal to Be/mv , where e , m , v are respectively the charge, mass, and velocity of the particles, and B a constant depending upon the strength of the magnetic field and the distance of the screen from the cathode. The electrostatic deflexion is equal to Ae/mv^2 , where A depends on the strength of the electric field and the size of the apparatus. When both these fields are in activity, the appearance presented by the willemite screen when the rays strike against it is, when the pressure is not very low, somewhat like fig. 2. The small spot which marks the place of incidence of the pencil when the magnetic and electrostatic fields are not on, is drawn out into a ribbon the edges of which are often very approximately straight. In some cases the original position of the spot, even when the magnet is on, is more brightly

illuminated than any other part of the ribbon ; in others the illumination is fairly uniform throughout.

Fig. 2.



Fig. 3.



On the other side of the undeflected position there is a luminous tail, formed, as the direction of the deflexions shows, of negatively electrified particles. The brightness of this in comparison with the positive part changes very considerably : in some cases it is too faint to be easily perceived, in others it is sometimes almost as bright as the positive part of the ribbon ; in general, however, it is much feebler and there is distinct discontinuity in the brightness of the light in the tail and in the other portions.

If y is the vertical, x the horizontal deflexion of the rays,

$$v \propto y/x$$

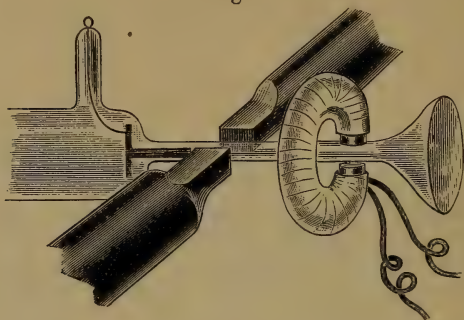
$$e/m \propto y^2/x.$$

Thus when the ribbon is straight, the values of y/x and therefore the velocity of the particles are approximately constant, while y^2/x ranges from zero up to a maximum value at the end of the strip. This maximum value, in all the cases I have tried, gives a value of e/m equal to 10^4 .

When the pressure of the gas is reduced, the luminosity of the band concentrates into a spot or spots and presents the appearance shown in fig. 3. There is a bright spot at the end and a faint band which at low pressures becomes almost invisible connecting this spot with the position of the undeflected spot ; at this place there is not infrequently a considerable amount of luminosity even when the magnet is on. This luminosity is often indistinct and sometimes vanishes. When it is present it indicates the presence of a considerable number of rays which are not deflected in passing through the magnetic and electric fields, and which are therefore not charged when they pass through these fields. Neglecting for a moment the faint luminosity connecting the spots, the bundle of rays in this case consists of a mixture of positively electrified particles and electrically neutral systems, uncharged rays we may call them ; these uncharged rays, however, in their course through the gas may gradually get broken up

and become positively charged rays. The breaking up of these uncharged rays may be shown and the rate at which they break up measured by the following method. A tube (fig. 4) in which the distance between the perforated cathode

Fig. 4.



and the willemite screen was considerably greater than in the tube fig. 1, was made, and two electromagnets arranged, one close to the cathode, and the other much nearer the screen. The magnets were arranged so that the magnetic force in the first was horizontal, and the deflexion of the spot due to the field vertical, while in the second magnet the magnetic force was vertical and the deflexion horizontal. The deflexions due to each magnet were thus separated and could be measured independently.

The effects observed when the magnets were applied in succession and then simultaneously are interesting; and a typical case is represented in figs. 5 & 6. Fig. 5 represents the appearance of the screen when only the electromagnet next the cathode is in action; fig. 6 the appearance when both magnets are on.

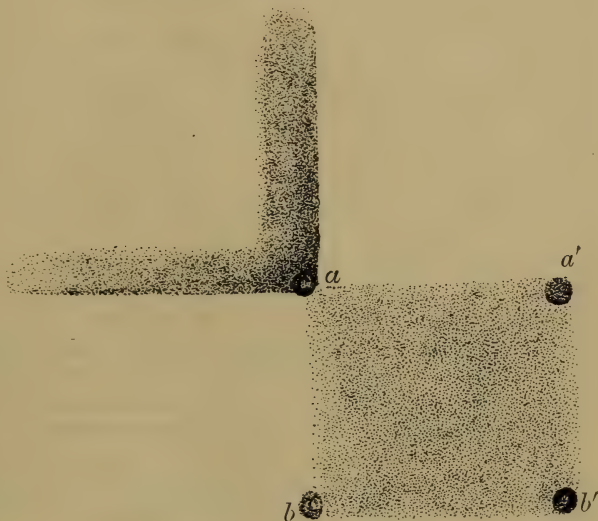
In fig. 6 the undeflected spot is much fainter than in fig. 5; part of it is deflected to another spot a' , while the spot b is deflected to b' . b' is vertically under a' , so that the deflexion produced by the second electromagnet on the rays which were uncharged when they went through the first electromagnet is the same as that produced by the second magnet on the rays which were charged when they went through the first one. It is evident that the latter have for the most part kept their charge from the cathode right through the second magnet; the formation of the spot a' shows that some of the neutral rays have in their journey from the first magnet to the second split up into a positive and negative part (the negative part being deflected far out of the field and not

reaching the screen), while the positive has been affected by the magnet to just the same extent as those which were positively electrified in the field of the first magnet; this shows that the rays which break up in the journey move with the same velocity and possess the same charge as those which break up at an earlier stage.

Fig. 5.



Fig. 6.



The relative brightness of the spots a and b varies a good deal with the pressure. I have found that when the pressure is reduced the spot a gets fainter than b , though at higher pressures it might have been brighter. This is what we should expect, since at the lower pressure the neutral doublets make fewer collisions in their course between the two magnets, and so fewer of these are broken up into ions. Again, as the pressure increases, the spots become less well-defined, there is more luminosity between the original and the displaced positions of the spots; for example, there may be considerable luminosity between the position of b when the vertical magnetic field is off and b' , the luminosity stretching right up to b' .

This shows that at this pressure some of the particles which were positively charged when they were passing through the first electromagnet have got neutralized before reaching the second and so are not deflected by it. The breaking up and re-formation of the doublets become more

frequent as the pressure increases, and more and more particles are positively charged for only part of the time they are in the magnetic field. Since they are only deflected when charged, the deflexion will vary with the time they have been uncharged while passing through the magnetic field; and if this time varies continuously the spot will be drawn out by the magnet into a continuous band such as is represented in fig. 2, and which is always observed when the pressure is not too low.

An important point as to the determination of e/m occurs in connexion with this effect: the values of e/m got from the magnetic and electric deflexions are the average values of e/m whilst the system is in the magnetic and electric fields.

Since some of the systems are at times electrically neutral, the average value of e/m cannot be greater than the true value, and it may be considerably less. It may be asked, have we any security when we deduce the value of e/m for the positive ions from the measurement of the tip of a drawn out band such as that in fig. 2, that the particle under consideration has been charged the whole time it was passing through the field; if this is not the case, the true value of e/m may be much greater than the value calculated.

This consideration does not apply to the case when the bright spot is deflected as a spot and not as a band; but a great many determinations of e/m have been made when the pressure was too high to give spots, and the ends of the bands have been used for this purpose.

I have made some experiments to see if in my measurements any errors have been made from this cause. The principle of these experiments was as follows. If the particle at the tip of the beam has not been charged the whole time, *i. e.* if it has united with a corpuscle before leaving the magnetic field, then the value of e/m determined by this method will increase as the length of the path travelled in the magnetic field is reduced until it is comparable with the length of the path described by the particle whilst it is positively charged; and when the magnetic field is so short that the particle can retain its charge for the whole of the distance between the poles of the magnet, the value of e/m given by this method would rise to a maximum.

Now take two magnetic fields, one short but strong, the other long but weak, and adjust the strengths of the fields so that if the positive particle retained its charge the whole of the time it was in either field, the deflexion of the particle would be the same in both cases. If the particles whose

deflexion we are measuring become neutralized in passing through the longer field, the deflexion produced by the short field will be greater than that by the long, provided the length of path of the particle when uncharged is comparable with the length of the shorter field.

For this experiment, chisel-shaped pole-pieces whose ends were rectangular were used, the longer side of the rectangle was 4.5 cm., the shorter side .7 cm. The distance between the pole-pieces was 6 mm. The poles were arranged (1) so that the longer side of the rectangle was parallel to the path of the positive particle, so that this was exposed to a magnetic field for about 5 cms. of path, and (2) with the shorter side parallel to the path so that the particle was exposed to the magnetic field for less than a centimetre of path. The currents through the electromagnets were adjusted so that the deflexion would be the same for a particle which retained its charge the whole of the time it remained in either field. It was found under these circumstances that when the pressure was too high to give well-defined spots the deviations of the tips of the luminous bands into which the phosphorescence was drawn out were the same in both cases; showing that the maximum value of e/m got by this method is the ratio of the charge on the particle to its mass, and not the average extending over a time during part of which it is uncharged.

Though the maximum deflexion is the same with the two fields, there are minor differences in the appearance of the phosphorescent patch; with the short field the portion of the band which is not deflected at all is noticeably brighter than in the longer field. This is what we should expect if the rays contain large quantities of neutral doublets which break up into positively and negatively electrified parts as they pass through the tube and collide against the molecules of the residual gas.

The behaviour of the portion of the luminosity due to negatively charged portions of the Canalstrahlen is interesting and throws light on their origin; the determination of e/m for these shows that they are not corpuscles but that their mass is much the same as that of the positive particles, they are in fact neutral doublets which have acquired a negative charge.

The brightness of the negative part of the phosphorescence depends very much on the intensity of the magnetic field; as the intensity of the field increases these negative rays at a certain stage increase so quickly in brightness as to give the impression that they suddenly spring into existence. Again,

with a constant magnetic field the negative rays are much brighter when the magnetic force is at some distance away from the cathode and towards the screen than when it is applied nearer to the cathode.

The action of the magnetic field in increasing the number of these negatively electrified particles is easily understood, for in a strong field the negative corpuscles can only move along the lines of force and thus get concentrated into a cloud ; an unelectrified doublet passing through this would be very liable to get electrified.

There is in general an abrupt falling off in the intensity of the luminosity under electric and magnetic forces as we pass from the position of the spot corresponding to uncharged rays into the position corresponding to negatively charged rays. This shows that the uncharged rays are not rays which start by being positively electrified, then take up a corpuscle and get neutralized, then another corpuscle and become negatively electrified, then lose a corpuscle and so on, repeating this process so that on the average the charge is as often positive as negative ; for if this were the case, the number of these would be very nearly the same as that of those which had slightly overshot the mark and become on the average just a little more negative than positive, and these would be found amongst the negative rays, and the transition from the one to the other would be gradual.

Everything points, I think, to the conclusion that even at the start from the cathode the Canalstrahlen include a large number of neutral doublets, if indeed they do not wholly consist of them.

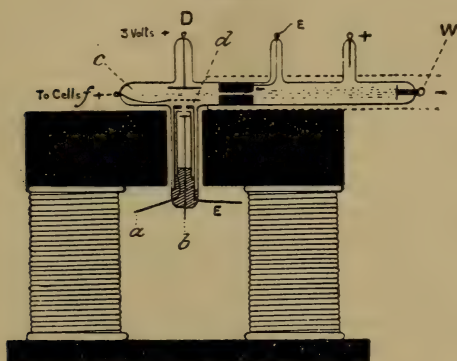
These neutral doublets are found not only in Canalstrahlen, the rays at the back of the cathode which have passed through it, they are also found in front of the cathode travelling away from it, mixed up with rays which carry a positive charge of electricity and which are thus travelling against the electric field.

The values of e/m for these positively charged rays in front of the cathode are, as I have shown (see *Phil. Mag.* Oct. 1908), the same as for the Canalstrahlen ; it is convenient to call the positively charged rays which travel away from the cathode in the same direction as the cathode rays, "Retrograde rays." The presence of neutral doublets is even more marked among the retrograde rays than among the Canalstrahlen.

These neutral doublets are very interesting, as they form an intermediate stage between the ion and the neutral molecule. They are found not only in these high tension

discharges, but also when the electric field is much less intense than when the discharge is produced by the induction-coil. This can be shown with an apparatus like that shown in fig. 7, where *W* is a hot Wehnelt cathode, connected up

Fig. 7.



with a battery of small storage-cells, causing it to emit slow cathode rays which ionize the gas in the tube *C*, *a* and *b* are two carefully insulated Faraday cylinders; there is a very small hole, in some cases less than $\frac{1}{2}$ mm. in diameter, in the plate *d* which separates the Faraday cylinders from the ionized gas; through this hole the ions produced in *C* can diffuse and charge up the Faraday cylinders.

We can, however, by suitable means prevent the ions passing through the hole: the negative ones which are mainly corpuscles, and easily deflected by a magnet, are prevented by applying a strong magnetic field which bends them back before they reach the cylinder; the positive ions, which are not easily deflected by a magnet, can be stopped by a strong electric field between the wire gauze *f* and the top of the cylinder *d*, the gauze being negative to the cylinder. When the ions are stopped by these means we find that though no perceptible electric charge reaches either cylinder, the gas between them is a conductor of electricity, and if either cylinder is charged up the other will slowly acquire the same potential, whether that potential be positive or negative.

That the conductivity of the gas is not due to ultra-violet light coming from the luminous discharge in the upper tube is shown by the fact that it is destroyed by putting a thin quartz plate over the hole in *d*; and since it is also destroyed when a piece of the thinnest aluminium foil obtainable is placed

over the hole, it cannot be due to any ordinary form of Röntgen radiation.

The conductivity can easily be explained if we suppose that there are neutral doublets in the ionized gas in the discharge-tube, and that since these are not deflected by either electric or magnetic forces they can pass through the hole though the ions are stopped. Then by collision with the molecules of the gas in the Faraday cylinder, they break up into ions and thus make the gas a conductor.

The velocity of the neutral doublets in the discharge produced by an induction-coil is approximately the same as that of the positively charged particles which accompany them. This follows from the fact that the deflexion of the spot *b* (fig. 6) is the same as that of spot *a*, though the former spot is produced by positively charged particles which have been detached from the neutral doublets after passing the first magnetic field, and the second by positively electrified particles which were free before the first field was reached.

The doublets seem to travel in directions approximately normal to the surface of the cathode. Experiments described in my paper on "Positive Rays" (Phil. Mag. Oct. 1908) show that the retrograde rays travel in this direction, and it is well known that Canalstrahlen are not obtained unless the aperture in the cathode is approximately normal to the face of the cathode facing the anode. The direction of the doublet is in short approximately that of the lines of force close to the cathode; they may move in either direction along the lines.

The doublets seem to start from beyond the dark space next the cathode, for if a vertical obstacle such as a thin glass rod or a metal wire be placed inside the discharge-tube in such a position that its projection on the plane of the cathode passes across the hole in the cathode through which the Canalstrahlen pass, then when the pressure is so low that the dark space extends beyond the obstacle, the dark shadow of the obstacle is apparent on the willemite screen, stretching across the phosphorescent patch even when this is spread out by the electric and magnetic fields. If the pressure is increased so that the obstacle is outside the dark space and in the negative glow, no shadow can be perceived.

In this form of the experiment the shadows cast by an obstacle are much more easily perceived and much more definite than in the original form in which it was tried by Schuster and Webnelt. I have got quite sharp shadows with a fine wire placed several centimetres away from the cathode.

Some very interesting questions arise in connexion with

these doublets. Is the doublet the thing set free from the atom when it is ionized? and are the ions produced by the splitting up of the doublets, or are the ions first produced and the doublet produced by a combination of the positive and negative ions?

There are arguments to be urged in favour of either view. The fact that the direction of projection of these doublets is parallel to the direction of the lines of force at the cathode, points to the projection being due to electrical forces.

We can easily understand how these might set the doublets in rapid motion.

Thus doublets which accompany the retrograde rays may have had a corpuscle attached to them when they were in the neighbourhood of the cathode, and before losing this corpuscle have been projected away from the cathode in the same direction as the cathode rays; thus these doublets acquire their velocity when they are doublets. The doublets which accompany the Canalstrahlen and which move in the opposite direction to the preceding, may be supposed to have acquired their velocity when they were in a dissociated state; *i. e.*, we may suppose that a positive ion in front of the cathode acquires in its fall to the cathode a high velocity, and then after passing through the cathode unites with a corpuscle and becomes a doublet retaining the velocity acquired by its positive constituent.

We have already had examples of the formation of doublets in the Canalstrahlen group. Thus, for example, in the case represented in fig. 6, we saw that when the pressure was not very low, some of the positive ions which had retained their charge whilst passing through the first magnetic field, and which form the spot *b*, had lost their charge, *i. e.* had become doublets before entering the second field and were not deflected by it, so that instead of the spot being entirely removed when the second magnetic field was put on, some of it was left behind.

Though we can hardly doubt that there must be positively charged particles passing through the hole in the cathode which owe their velocity to this cause, yet, as the experiments I shall now proceed to describe show, Canalstrahlen such as those used when their electric and magnetic deflexions are measured in the way I have described and which travel perhaps 15 cms. before they reach the screen, seem to owe their velocity to a different cause, for I have found that the velocity of these is but slightly affected by the potential-difference between the electrodes of the discharge-tube or the pressure of the gas.

I have made many determinations of the velocity of Canalstrahlen at different pressures using tubes of very different shapes, and the results were so nearly constant in spite of the wide variations in the conditions, that I was led to suspect that the velocity of the positive rays was not influenced to any great extent by the strength of the electric field in the tube. To test this matter in as direct a way as possible I used the following method.

Fig. 8.



This discharge was produced by a large induction-coil; the discharge-tube, fig. 8, had a perforated cathode faced with calcium, so that the discharge would pass at very low pressures without sparking through the glass walls of the tube and breaking the apparatus. The anode A was an aluminium cylinder perforated with a very small hole. When the discharge was passing through the tube a pencil of cathode rays passed through the hole and produced a small well-defined spot on the fluorescent willemite screen S, placed at the anode end of the tube.

During their path up to the screen after leaving the anode they could be exposed to a known magnetic field produced by an electromagnet, and the distance through which the spot moved when the magnet was put on enabled us to calculate the velocity of the cathode rays.

The Canalstrahlen after passing through the aperture in the cathode pass along a metal tube of very fine bore (one of the perforated needles used for hypodermic injections) and emerge between two parallel vertical metallic plates (4 mm. apart in one set of experiments, 3 mm. in another) which can be connected with the terminals of a battery of small storage-cells; the rays are thus exposed to a strong electric field and are deflected horizontally.

Two poles (about the same length as these plates) of a powerful electromagnet of the Du Bois pattern, are placed on either side of the tube, and the horizontal magnetic field due to this magnet deflects the rays in a vertical direction. The deflexion of the rays is measured by the deflexion of the

phosphorescent patch they produce on the willemite screen T; from the magnetic and electric deflexions the values of e/m and v can be deduced in the usual way. Observations were made simultaneously of the deflexions of the spots on the screens S and T when the pressure was varied from the largest value at which both rays produced phosphorescence on the screen to a very high vacuum. The deflexion of the cathode rays under a constant magnetic field was 8 mm. at the highest pressure and 2 mm. at the lowest; thus the velocity of the cathode rays was 4 times greater at the lower pressure than at the higher, but in spite of this large change in the cathode rays, there was no appreciable change in the deflexion of the Canalstrahlen over the whole range of pressure.

The results of two sets of experiments with different tubes are shown in the following table.

By equivalent spark-gap is meant the distance between two large electrodes placed in parallel with the tube and adjusted so that the sparks passed about equally readily through the tube or across the air-gap.

Deflexion of Canalstrahlen.		Deflexion of Cathode Rays.	Equivalent Spark-Gap.
Magnetic.	Electrostatic.		
mm.	mm.	mm.	cm.
6	6	6	·5
6	6	4·5	
5·5	6·5	2·5	
6·5	6	2	3·8
mm.	mm.	mm.	
6	4	6·5	
6	4·5	5	
6	4	3	
5·5	3·5	2	

The numbers in one horizontal line were observed simultaneously, one observer reading the deflexion of the cathode rays, another those of the Canalstrahlen. At the very highest pressures at which Canalstrahlen appear their deflexions seem greater than those at lower pressures, though the difference is on quite another scale to that of the cathode

rays. The phosphorescence at these pressures is very diffuse and faint and the rays are evidently much scattered in their journey to the screen. I think the increased deflexion is due more to this and their loss of velocity during the journey, than to any considerable falling off in their initial velocity of projection.

Though the equivalent spark-length has changed in the ratio of nearly 8:1 and the deflexion of the cathode rays as 3:1, there has been hardly any change in the deflexions of the Canalstrahlen. This makes it very improbable that they owe any considerable part of their velocity to the action of the electric field upon them whilst they approach the cathode.

We must remember that the bundle of Canalstrahlen when it passes through the hole is a mixture of rays of different kinds. With such an apparatus as that just described the only rays investigated are those which can traverse the very considerable distance between the cathode and the screen and yet retain their power of producing phosphorescence. Coming through the hole there may be, in fact in certain cases we know there are, positive particles which have been set in motion by the electric field, with velocities depending on those of the molecules of the gas in the tube.

These, in the experiment I am describing, have disappeared before travelling as far as the screen, their "range," to use an expression familiar to those who study the α particles, is less than the distance between the screen and cathode. The existence of this range in the case of the α particles may be explained by supposing that when the velocity of the particle falls below a certain value, the particle is no longer able to escape from the negative corpuscles which it has to pass through on its travels. The same argument will apply to Canalstrahlen; if they move with less than a certain speed they may not be able to escape from corpuscles near which they pass. Since the α particle has twice the charge of a particle in the Canalstrahlen, the velocity, other circumstances being the same, required for the escape of an α particle will be greater than for one in the Canalstrahlen. The velocity of the latter, in round numbers 2×10^8 cm. per sec., is about that required for a particle of unit charge and 10^{-8} cm. in radius to escape from a corpuscle.

In the preceding experiments the velocity of the cathode rays at the lowest pressure was 6×10^9 cm./sec., taking $e/m = 1.7 \times 10^7$. The velocity of the Canalstrahlen particle is thus considerably less than that of a cathode ray particle; the energy of a cathode particle is, however, less than that of a particle in the Canalstrahlen.

Experiments were also made with the retrograde rays, and it was found that the velocity of these was also practically independent of the strength of the field and very nearly the same as that of the Canalstrahlen.

The small range of velocities in the Canalstrahlen is explicable if we suppose that the rays we are measuring exist originally as neutral doublets which are formed by the combination of a positive unit and a negative corpuscle; these doublets moving towards or away from the cathode, the direction of motion being approximately at right angles to the cathode. These doublets could not have more than a certain amount of energy; for if the relative velocity of the carriers of the positive and negative charges when uncombined exceeded a certain value, the charges would fly past each other without entering into combination and they would remain dissociated: it is only when the kinetic energy of the two falls below a certain value that combination will take place; thus there will be a superior limit to the energy of the doublets. The doublets which we investigate when we measure the velocity of the cathode rays are those which dissociate again into positive and negative charges. Now, as the kinetic energy of the doublet decreases it gets more and more stable and less likely to dissociate, and when it ceases to dissociate we cannot measure the velocity. Thus doublets are not formed possessing energy greater than a certain value; on the other hand, they are not split up if the energy is less than a certain value. Thus the doublets whose velocity we can determine will have velocities between definite limits which are independent of the strength of the electric field. The case is analogous to that of a gas which dissociates as the temperature is raised: there is a certain range of temperature in which there is appreciable, but not complete dissociation; beyond the upper limit of this range of temperature the undissociated molecules are too few to be detected; beneath the lower limit the products of dissociation are not numerous enough to be appreciable.

The properties both of the Canalstrahlen and the retrograde rays could also be explained on the view that neutral doublets moving with high velocities are given out by the molecules of the gas in the discharge-tube, that these break up into the Canalstrahlen and the retrograde rays; these doublets are supposed to be of the same character from whatever kind of gas they may originate. We should, however, expect that these doublets shot out from the molecules by explosions would be found moving in other directions than just along the lines of force in the tube, which seems to

be the only direction in which an appreciable number of Canalstrahlen or retrograde rays move.

If doublets form an intermediate stage between the ions and the atoms, the doublets being the systems formed from the molecules of the gas by the ionizing agent, say Röntgen rays, then the number of doublets liberated may be enormously greater than the number of ions set free, so that this number may give no indication of the percentage of molecules affected by the rays.

Thus, suppose q doublets are liberated per second per cubic centimetre in a gas at a pressure p , let D be the number of doublets per cubic centimetre, let βpD of these reunite with the molecules per sec., and γpD split up into ions, let m be the number of positive or negative ions per cc., α the coefficient of recombination ; then we have

$$\frac{dD}{dt} = q - \beta pD - \gamma pD,$$

$$\frac{dm}{dt} = \gamma pD - \alpha m^2.$$

When things are in a steady state

$$m = \frac{1}{\sqrt{\alpha}} \sqrt{\frac{\gamma}{\beta + \gamma}} q.$$

The saturation current is proportional to the value of dm/dt when m is zero ; hence, when things have settled into a steady state, the saturation current is proportional to

$$\frac{\gamma}{\beta + \gamma} q.$$

Thus, if γ is very small compared with β , the number of ions taken from the gas per second may be very small compared with the number of molecules affected by the rays. This point is an important one in connexion with the structure of the Röntgen rays, the small percentage of the molecules decomposed has always been difficult to understand.

If we suppose that the ions are the first product of the Röntgen rays on the molecules, this difficulty disappears on the assumption that there is a stage intermediate between the molecules and the ions.

The values of e/m for the Canalstrahlen and for the retrograde rays are the same when the pressure is low whatever be the gas in the tube. In a former paper I described experiments which showed that if a vessel was exhausted until the pressure was so low that the discharge would not pass, and small quantities of hydrogen, helium, air, oxygen,

carbonic acid, or argon, were introduced so as to raise the pressure sufficiently to produce the discharge, the values of e/m and the velocity of the particles were the same for all gases. Since then I have repeated the experiments and have tried in addition to the gases I have mentioned, sulphur dioxide, methyl iodide, carbon tetrachloride, and also the vapour of a radioactive substance UrCl_4 . With all these gases the values of e/m for the Canalstrahlen were the same and equal to 10^4 . This seems strong evidence that there is a definite unit of positive electricity as well as of negative, the carrier of the positive unit being much larger than the negative. This view seems to me to be very much strengthened by some results recently obtained by Mr. Wellisch at the Cavendish Laboratory. The experiments were on the mobility of the positive and negative ions in mixtures of different gases. If we take the view that the positive ion is but the residue left after a corpuscle has been abstracted from the molecule, then it would seem probable that in the case of positive ions there would, in a mixture of two gases A and B, be two different types of positive ions moving with different velocities. In one type the positive ion would be the molecules of A minus a corpuscle, or an aggregate formed round it; in the other the nucleus would be the molecule B. Mr. Wellisch found, however, as M. Blanc had previously found for a mixture of air and carbonic acid, that in the mixture all the positive ions moved with the same velocity.

We might, however, without assuming an independent unit of positive electricity, explain this result by supposing that the ions were aggregates continually breaking up and reforming, and that the velocity measured is the velocity of the average aggregate. This explanation is, however, refuted by some of the cases examined by Mr. Wellisch. The ionization under Röntgen rays of methyl iodide, mercury methyl, or carbon tetrachloride, is so vastly greater than that of hydrogen, that in mixtures of these gases with hydrogen, even when there is only one or two per cent. of these gases in the mixture, practically the whole of the ionization is due to the methyl iodide. Wellisch found that in these mixtures the velocity of the positive ion is nearly the same as in pure hydrogen, although practically all the ions came from the heavy gases. If the positive ion was merely the residue left when the corpuscle was taken out, the positive ion would be at least as large as a molecule of methyl iodide. We can show, however, that the velocity with which the charged molecule would move through the

mixture, would be less than the observed velocity of the positive ion, which was about that of a charged hydrogen atom. It is clear, then, that the positive ion is not the molecule of methyl iodide. Thus the ionization of the methyl iodide has produced positive ions which are not molecules of methyl iodide. It might perhaps be urged that the molecules of methyl iodide which were positively charged to begin with, have, by the electric force they exert, pulled a corpuscle out of the neighbouring hydrogen molecule, and that this corpuscle neutralizes the methyl iodide molecule and leaves the hydrogen molecule, from which it has escaped, with the positive charge. There are several objections to this view of which I will only mention one. The result of this process is, by the electrical forces between the molecules of methyl iodide and hydrogen, to transfer the charge from the larger system (the methyl iodide molecule) to the smaller (the hydrogen molecule).

Since the potential energy of a given charge is greater when the charge is on a small system than when it is on a large one, the change under consideration would involve an increase in the potential energy, and would therefore not be brought about by the action of the electrical forces.

The results obtained by Wellisch are exactly what we should expect if there were a definite positive unit of electricity which could, like the corpuscle, be detached from the molecule of the gas. I have elsewhere suggested that the process of ionization consists in the detachment of a neutral doublet from the molecule, the doublet consisting of a positive unit and a corpuscle, and that this breaks up into its components, which subsequently attach themselves to the molecules by which they are surrounded.

Slowly moving Positive Rays and the Size of the Carriers.

The slowly moving rays were studied in the following way:—The gas at a very low pressure was ionized by the cathode rays emitted from a Wehnelt cathode W (fig. 7, p. 829). By means of a feeble electric field between the plate D, called the collector, and a piece of wire gauze f , the positive ions were then driven through this gauze, which was distant about 5 mm. from the top of the plate d .

Between this gauze and the top of the plate was the potential relied upon to give the positive ions their velocity.

The difference of potential between the collector plate and the gauze was 10 volts, that between the gauze and d varied from 40 to 200 volts. There was a very small hole in the

plate *d*, and the positive ions, after passing through the hole, travelled through the strong magnetic field produced by a Du Bois magnet. The magnetic deflexion of the rays was measured by the ratio between the charges received by the disk *b* and the cylinder *a*: these were connected with Wilson tilted electroscopes, and the ratio of the charges measured in the way described in my paper on the Positive Rays (Phil Mag. Oct. 1908).

If *V* is the fall of potential between the gauze and the top of the plate,

$$eV = \frac{1}{2}mv^2,$$

while the magnetic deflexion gave the value of mv/e .

Thus from these results m/e and v can be determined.

We find that for these slowly moving positive rays the velocity depends upon the strength of the electric field.

The point that I wish to dwell upon here is, that in some cases the method seemed to give values of e/m for the positive ions much greater than 10^4 . I think these high values are not genuine, but I will first describe the effects which seem to indicate them. We may get, perhaps, the clearest idea of these by describing what happens when the arrangement is altered so as to send negative ions and not positive through the same hole in the top plate.

In this case the corpuscles and negative ions pass through the plate.

If we compare the charges on the disk and the cylinder when there is no magnetic field on, we find that in general there is more negative charge on the cylinder than on the disk, and the effect of a small magnetic field, say 200 units, is greatly to diminish the proportion of the negative charge on the cylinder to that on the disk: when the magnetic force is greatly increased this proportion again increases, due to the deflexion by the magnet of the heavy negative ions from the disk to the cylinder.

It will be noticed that the effect of putting on the small magnetic field is to diminish the scattering. This is what we should expect if these were negative corpuscles getting through the hole when there was no magnetic field. If these rays were rather diffuse they would reach the sides of the cylinder.

Now when a magnetic field of say 100 units is put on, it twists up the path of the particles into very small circles, so that they are prevented by it from even getting as far as either of the Faraday cylinders.

The magnetic field is not sufficient to affect appreciably

the heavy negative ions which move on to the disk, and are not deflected until the magnetic force is strong enough to deflect ions whose mass is comparable with that of atoms.

I have occasionally observed a similar effect when the positive instead of the negative ions were sent through the cylinder.

This effect has only been observed when the potential-difference between the gauze and the top of the plate exceeded about 120 volts, and then only occasionally; still, on several occasions it was found that the amount of positive electricity going to the cylinder when there was no magnetic field on, was a larger multiple of that going to the disk than when the magnetic field was increased to about 100 units: this effect is similar to that observed with negative electricity and which could be explained, as we saw, by supposing that there were particles of exceedingly small mass along with the others. Are we to conclude that there are similar small positive particles, particles whose mass is very small compared with that of the atom? I do not think that the effect I am describing is due to this. In the first place, while the effect is invariably observed with negative electricity, it is quite exceptional with positive.

I had worked with the apparatus described for months, making observations almost daily, before I found an example of it. I think that it arises from the presence among the stream of positive ions which pass through the hole in the plate, of some ions which are moving with much smaller velocity than that which they would acquire by a fall through a potential-difference equal to that between the gauze and the plate.

We have seen that these positive ions are continually getting neutralized in their journey through the gas, and that some of them are hardly deflected at all, that is, do not acquire an appreciable velocity whilst they are passing through the electric field.

Thus we might expect that some of the ions would be neutralized for part of their journey between the gauze and the plate, and would thus only acquire a small fraction of the velocity due to a fall through the whole potential-difference between the plates.

These slowly moving ions would be easily deflected, and a comparatively weak magnetic field will be sufficient to prevent them entering the Faraday cylinders. If the effect I am describing is due to the slowly moving ions, it ought

to disappear if the Faraday cylinders are insulated and maintained at a potential a few volts above the plate A. The ions, which have acquired an energy due to the fall through the whole potential-difference of say 160 volts, will be but little affected by having to go against the difference of say 20 volts, but those ions which have acquired only a fraction of this velocity may be wholly stopped by it. On raising the potential of the cylinders to about 20 volts, I found that the effect in nearly every case disappears.

There were, however, one or two cases in which it still remained ; if the slow moving particles remain neutralized for part of their journey between the plate and the cylinder, they could not be much affected by the field, and the positive particles might still force their way in.

As the question of the existence of small positive ions is a very vital one, I have made some other investigations with other cases of easily deflected positive ions with the object of seeing if the deflexions were in these cases also due to the smallness of the velocity rather than to that of the mass.

The appearance of the discharge in the neighbourhood of the hole in a cathode offers several features of interest ; on some of these, light is thrown by the consideration of the distribution of the lines of force near the hole before the discharge starts.

Let A and B be two parallel metallic plates, B having a small hole in it. Let us suppose that A is at a higher potential than B, then the negative electricity is not uniformly distributed over B but is very much concentrated near the edge of the hole, the density being infinite at the boundary of the hole if the edge is sharp. The electric force is proportional to the density, so that up to a distance from the hole comparable with the radius of the hole the electric intensity is far in excess of its average value.

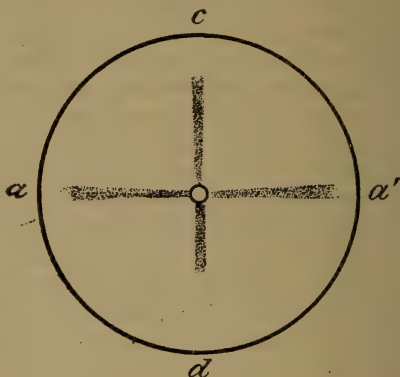
Again, this accumulation of electricity is not confined to the part of the plate facing A. If the plate B is thin in comparison with the radius of the hole there will, close to the hole, be practically as much electricity on the face of B turned away from A as on the side next to A : in fact the field of force in the neighbourhood of the hole is much as if a fine ring of the same radius as the hole were highly electrified and placed by itself in the field. The appearance of the discharge through the hole when the pressure is not very low illustrates this very well. One of the stages of the discharge is shown in fig. 9, and it will be seen that this is

symmetrical on the two sides of the hole. If a strong magnetic field is applied to the neighbourhood of the hole, the lines of force being parallel to the surface of the cathode, some very interesting effects are observed. Those in the

Fig. 9.



Fig. 10.



most developed form are represented in fig. 10; unless the pressure is within certain limits one or other of these parts may be absent. *a* and *a'* are two streamers coming from the hole and following the directions of the lines of magnetic force. In this respect they resemble the magneto-cathodic rays of Villard and Righi, but whatever may be the nature of the magneto-cathodic rays in front of the cathode, there can be no doubt, I think, that the rays we are describing consist of slowly moving corpuscles describing spirals of very small radius around the lines of force: for, as the magnetic force is very gradually reduced, we reach the stage when the widening of the radii of these spirals becomes so large that the spiral nature of their path becomes quite obvious. That they are carriers of negative electricity can easily be shown by sending them into a carefully insulated Faraday cylinder surrounded by another cylinder with a hole in it.

The outer cylinder was put to earth, the inner one was connected with a sensitive galvanometer (the one I used gave a deflexion of 1 mm. at one metre for 3×10^{-10} ampere). When one of the streams passed through the hole in the outer cylinder there was a deflexion of 70 or 80 scale-divisions in the direction corresponding to a flow of negative electricity into the Faraday cylinder.

The other two luminous portions *c* and *d* are bent at right angles to the direction of the magnetic forces, just

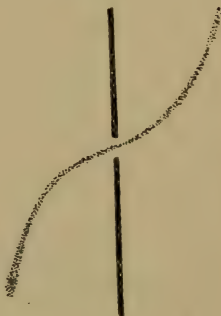
as they would be if c were a stream of negative and d of easily deflected positive particles. That there were streams of particles of this kind was proved by again directing the streamers into Faraday cylinders connected with a galvanometer. This showed that when the stream c entered the cylinder it carried with it a negative charge, while d carried a positive. c is in most cases much more developed than d .

As the pressure is reduced and the dark cathode space gets well developed, the appearances presented by the rays through the hole are very interesting ; they are represented in fig. 11.

Fig. 11.



Fig. 12.



The rays in front of the hole may be traced as a fine well-defined pencil passing through the dark space and often visible for some distance from the negative glow ; while it can be traced backwards through the hole for considerable distances, producing, when it strikes the glass walls, the phosphorescence characteristic of cathode rays. Under the action of the magnet the two parts of the pencil are bent in opposite directions, and their appearance shows that they consist of pencils of cathode rays travelling in both directions from the hole (fig. 12), those between the anode and the cathode travelling in the directions of the ordinary cathode rays, whilst those going through the hole travel in the opposite direction, the hole acting as if it sent out cathode rays backwards as well as forwards. When the pressure falls below a certain value this pencil disappears, the two portions vanishing simultaneously, and we are left with Canalstrahlen passing through the hole and without any of the other rays. These other rays can also be made to disappear by the action of a transverse magnetic force, even

though the pressure is such that they are well developed when the magnetic force is absent.

The value of the magnetic force required to get rid of these rays is very well defined, a change of a few per cent. in the value of this force will entirely alter the appearance of the discharge.

It will be seen that when the Canalstrahlen are accompanied by the other portions of the discharge, there are two sets of positive rays present which suffer very different deflexions in the magnetic field, the Canalstrahlen being very little deflected, while the portion d is very largely deflected. The comparison with the behaviour of the negatively electrified particles shows I think, however, that this difference is due to a difference in the velocity of the particles and not to a change in their mass, for we have seen that there are two streams of negatively electrified particles which are very differently deflected, viz., the rays which constitute the pencil c and those forming the pencils a, a' which are bent up into small spirals following the lines of magnetic force. In the case of the two sets of negative rays, we have clearly to do with rays moving with very different velocities; we may regard the more rapidly moving ones as primary rays, the slower ones as secondary rays arising from the ionization of the gas by the primary in weaker parts of the electric field, so that these secondary rays never acquire the velocity of the primary, and are therefore more easily deflected. A similar explanation applies to the two sets of positive rays, the more easily deflected ones arising from the ionization of the gas in the weaker parts of the electric field, where they have no opportunities of acquiring large velocities.

Note on a Method of Measuring the Effective Magnetic Field in the Magnetic Deflexion of the Canalstrahlen.

If O represents the mouth of the tube from which the Canalstrahlen emerge, B the screen supposed to be at right angles to the undeflected path of the rays which we take as the axis of x , H the magnetic force at any point parallel to z , y the deflexion of the rays due to the magnetic field, then

$$\frac{md^2y}{dt^2} = Hev$$

if v is the initial velocity of the rays. As the deflexion is

small this equation is approximately

$$\frac{md^2y}{dx^2} = \frac{He}{v},$$

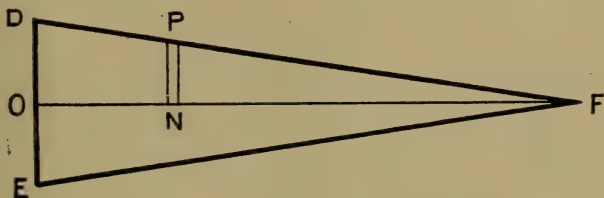
so that if l is the distance of the screen from O, H the value of the magnetic force at a distance x from O,

$$\begin{aligned} y &= \frac{e}{mv} \int_0^l \left[\int_0^x H dx \right] dx \\ &= \frac{e}{mv} \int_0^l (l-x) H dx. \end{aligned}$$

Hence the quantity we have to determine is

$$\int_0^l (l-x) H dx;$$

let us call it M .



Now suppose we have a coil of triangular section DEF ($DF=EF$) placed with its vertex at the screen and its base ED at the hole through which the rays emerge, the plane of the coil being at right angles to the magnetic field. The magnetic induction through the coil

$$\begin{aligned} &= \int_0^l 2H PN, dx = \frac{DE}{OF} \int_0^l H FN dx = \frac{DE}{OF} \int_0^l (l-x) H dx \\ &= \frac{DE}{OF} M. \end{aligned}$$

Hence if we measure the magnetic induction through the coil, which can be done in a few minutes by a Grassot fluxmeter, we can at once determine M .

This method takes into account the stray field and is applicable whether the field is uniform or variable.

I have much pleasure in thanking Mr. Kaye and Mr. Everett for the help they have given me in this investigation.

XC. *The Relation between Uranium and Radium.*—IV.*

By FREDERICK SODDY, M.A.†

THE measurements of the amount of radium in the uranium solutions purified and prepared by Mr. T. D. Mackenzie have now been in progress for nearly four years, and during the past year have all begun to yield certain evidence of the growth of radium. It may be recalled that the quantity of radium in the oldest preparation, which contained 255 grams of uranium, remained almost constant within the error of measurement for the first two years, while in the next year a very slight increase was noticed. It was deduced from this result that the period of average life of the intermediate parent of radium (ionium) must not be less than 16,500 years, on the assumption that, neglecting uranium X, it was the *only* intermediate body in the disintegration series. During the fourth year, now nearly passed, the measurements have been taken to a higher degree of accuracy than formerly, various improvements having been effected, and these have established that the production of radium within this period has been going on at a rate proportional to the square of the time, within the error of measurement. As the methods employed are the outcome of considerable experience, and as the details are of importance in securing accuracy, it may be useful to give a short account of them as they are now carried out, though they do not involve anything new in general principle. Throughout the term "unit leak" will be used to denote a leak of 1 division a minute in the electroscope, and the term "unit of radium" the quantity 10^{-12} gram. One division of the eyepiece-scale of the microscope equals .023 mm.

II. *Method of Testing for Radium.*

The uranium preparations at the commencement of the fourth year were removed from the mercury-pump, to which

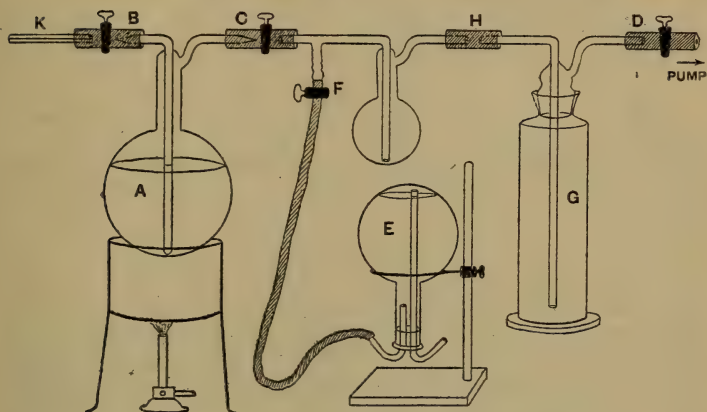
* I. Phil. Mag. June 1905, p. 768; II. (in conjunction with T. D. Mackenzie, B.Sc.), *ibid.* Aug. 1907, p. 272; III. *ibid.* Oct. 1908, p. 634.

A preliminary account of some of the results in this paper appeared in a letter to 'Nature,' May 13th, 1909, p. 308, and the *Physikalische Zeitschrift*, 1909, p. 396. The paper was read on Oct. 22nd, 1909, before the Physical Society of London, under the title "The Production of Radium from Uranium."

† Communicated by the Author.

they had previously been connected, and transferred to large flasks, as shown at A, fig. 1.

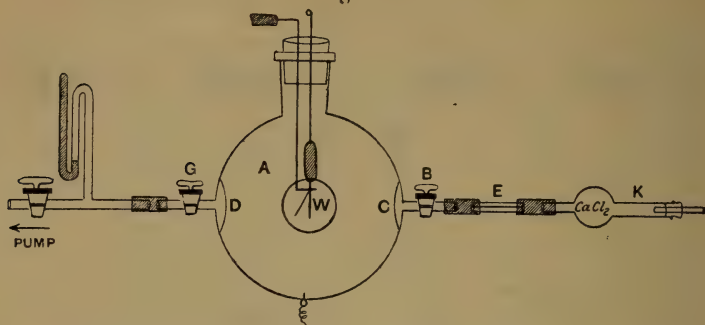
Fig. 1



In the new method of testing the principle is the same as was first employed by Boltwood. After each measurement the flask A is sealed at B and C, after being exhausted through C, and is left for at least a month for the equilibrium amount of emanation to accumulate. The apparatus as shown in fig. 1 does not need detailed description. Before the measurement it is exhausted through D, and then water in the flask E is admitted through F, by opening the clamp. This is repeated several times to remove any emanation in the water. The flask A is heated gradually by steam on the water-bath for a short time before the seal is broken to avoid percussive boiling. Violent irregular boiling is necessary thoroughly to agitate the liquid and to remove the emanation. If the flask is immersed in warm water boiling tends to be confined to the upper layer, and the extraction of emanation is imperfect. The seals of the flask are broken under thick rubber tubing, a little fresh air being allowed to enter occasionally as required through the thermometer-tube K. After half an hour of vigorous boiling, the clamp C is closed and F opened, admitting water until all but a small definite volume of G is filled. H is then disconnected, and some air thus allowed to enter, removing nitric oxide if present. The tube F is then connected to G at H, and the arrangement is ready for the transference of the emanation into the electroscope. The latter is shown in fig. 2 (p. 848). It consists of a glass

globe A of about half a litre capacity, provided with a circular plane parallel plate W to serve as a window, and silvered

Fig. 2.



internally except opposite the window. The window greatly improves the sharpness of the image in the microscope, and the accuracy of the readings. C, D are circles of aluminium-foil to act as baffle-plates, which are pasted over the inside at the inlet and outlet taps B and G. The fine thermometer-tube E also assists in moderating the disturbance of the leaf by the entering air. The electroscope is kept exhausted and charged when not in use during any series of successive measurements. Its normal natural leak has been constant within 10 per cent. at 1.1 units since it was constructed. This is first determined accurately. Several small quantities of air are admitted and pumped out, before finally filling the instrument with air. The natural leak is then taken over several hours, prior to the admission of the emanation. When this has been done the instrument is exhausted and the calcium-chloride tube K connected to the tube D of fig. 1. When the whole of the gas in G has been drawn in, D is disconnected and fresh air allowed to enter to atmospheric pressure. No trace of emanation is thus lost. The leak is taken over the period from 8 to 16 minutes after the admission of the emanation, and again after the lapse of three hours. The leaf is kept continuously charged (negatively) with the charging-rod earthed over the whole interval. This is important, as otherwise a considerable error is introduced, which does not appear to have been previously pointed out. The error just referred to applies particularly to the calibration of the electroscope. With the minute quantities of radium present in the uranium solutions, the leaf if fully charged initially remains charged over the three-hour interval, and the practice throughout since this method of measurement was adopted has been to keep it charged during the whole interval. With the radium standards, however,

the charge has to be renewed every few minutes. At first this was not done, and the active deposit, instead of being concentrated on the leaf system in the most favourable position to produce ionization, is distributed over the surface of the globe. The leak under these circumstances after the three-hour interval is from ten to twenty per cent. too low, and rises gradually with successive measurements for a further period of three hours.

III. Calibration of the Electroscope.

The two standards used in previous work had been made by directly weighing 2.9 and 3.5 milligrams of pitchblende of 53 per cent. uranium content, and differed from each other by about ten per cent., due no doubt to the error of weighing directly such small quantities on an ordinary balance. For the present electroscope an entirely new set of six standards were prepared with the utmost care, from a different sample of pitchblende. Three separate quantities weighing 59.45, 133.7, and 58.32 milligrams were separately dissolved in nitric acid which had recently been distilled over barium nitrate to remove any sulphuric acid, and diluted in 100 c.c. graduated flasks. Out of each of the three solutions two standards were made, by taking a known fraction of the liquid, determined by weighing, and sealing it up after dilution in a flask similar to A (fig. 1) of about 100 c.c. capacity. These six new standards agree extremely well among themselves. On the remainders of the solutions used in preparing the standards three analyses of uranium were performed, which agreed in showing that the proportion of uranium in the pitchblende used was 49 per cent. The results are shown in Table I.

TABLE I.

Standard.	I. PITCHBLENDE. (Milligrams).	II. URANIUM. (Milligrams).	III. RADIUM. ($\times 10^{-12}$ gram.)	IV. LEAK. (Divisions per minute).	V. CONSTANT. (III. $\div$ IV.).
I....	0.628	0.308	104.6	18.0 } 18.1 }	5.80
II....	3.040	1.490	506.4	86.5	5.85
III....	1.420	0.696	236.6	41.6	5.69
IV....	13.59	6.660	2226.6	(370)	(6.02)
V....	0.305	0.149	50.8	8.6	5.91
VI....	1.510	0.740	251.6	44.5	5.66

MEAN (omitting IV.) ... 5.78

The factor used to calculate the quantity of radium from the quantity of uranium is 3.4×10^{-7} , instead of 3.8×10^{-7} , as previously used. It will be seen that the standards differ among themselves in content of radium in the ratio of over 40 to 1, but the constant of the instrument as determined by the individual standards does not vary from the main value more than two per cent. It must be mentioned that the value for standards IV. is not comparable with the others. The quantity of radium was too high to permit of the charge on the leaf being maintained as it leaked away, so the leaf was kept permanently connected through the charging rod to the negative pole of the 250 volts mains. Under these conditions the charging-rod as well as the leaf-system would attract the active deposit, and the part deposited near the cork in the neck of the flask would not contribute its full effect. For this reason the constant, as is to be expected, is too high, and the value has been omitted in calculating the mean. The latter is 5.78, that is 5.78×10^{-12} g. of radium produces a leak of 1 division a minute. The two old standards gave a mean value of 7.3 for the constant, the leaf not being kept charged during the three-hours interval. This difference is not much, if any, greater than can be accounted for by the effect of keeping the leaf charged.

IV. *Results.*

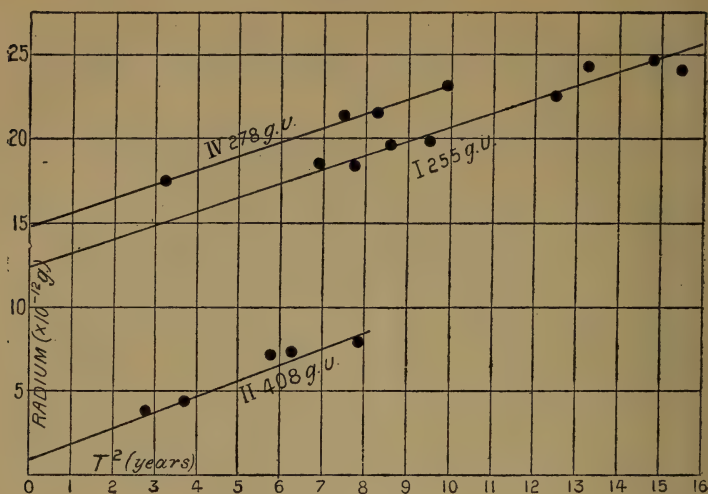
The results are shown in Table II. for the three uranium preparations described under Experiments I., II., and IV. (II. pp. 285–289 and 290). The later tests in each preparation have been performed with the electroscope in its existing condition, for which the constant has been accurately determined as described. The first tests were done with another leaf-system, which was destroyed after one test owing to nitric oxide having been present in the gases boiled out of the solution. The factor necessary to reduce the readings with this system to the same value as those with the present system was found to be 0.96. In the initial test shown both for Experiments I. and II., to which not much weight can be attached, the charging-rod was left after charging close to the system as in all previous measurements, whereas the present practice is to turn it as far as possible away, which was found to increase the sensitiveness 1.15 times. The three tests indicated by a * in the last column of the table were done by the mercury-pump before the solutions had been transferred to the sealed flasks they are now kept in. The table shows that in all three preparations the quantity

TABLE II.

Date.	Time (years)	(Time) ² .	Observed Leak.	Reduction Factor.	Reduced Leak.	Radium ($\times 10^{-12}$ gram).
EXPERIMENT I. 255 grams Uranium purified, 24/10/05.						
9/6/08	2.62	6.86	2.89	$\times .96 \times 1.15$	3.2	18.5*
7/8/08	2.78	7.70	3.3	$\times .96$	3.17	18.4*
25/9/08	2.95	8.57	3.53	$\times .96$	3.39	19.6
20/11/08	3.08	9.49	3.44	...	3.44	19.8
5/5/09	3.53	12.46	3.90	...	3.90	22.5
14/6/09	3.64	13.24	4.20	...	4.20	24.3
27/8/09	3.85	14.80	4.28	...	4.28	24.7
29/9/09	3.93	15.43	4.16	...	4.16	24.1
EXPERIMENT IV. 278 grams Uranium purified, 14/8/06.						
30/5/08	1.80	3.24	2.73	$\times .96 \times 1.15$	3.02	17.5
7/5/09	2.73	7.45	3.68	...	3.68	21.3
25/6/09	2.87	8.22	3.72	...	3.72	21.5
4/10/09	3.15	9.90	4.00	...	4.00	23.1
EXPERIMENT II. 408 grams Uranium purified, 13/12/06.						
8/8/08	1.66	2.75	0.70	$\times .96$	0.67	3.87*
19/11/08	1.93	3.73	0.80	$\times .96$	0.77	4.45
6/5/09	2.40	5.76	1.25	...	1.25	7.20
15/6/09	2.50	6.25	1.26	...	1.26	7.30
1/10/09	2.80	7.83	1.37	...	1.37	7.90

of radium has increased with time. The measurements are plotted against the square of the time in fig. 3 (p. 852). It will be seen that all the points lie close to straight lines. The maximum departure is in the case of the last test performed in Experiment I., which lies about 2 units too low. The ordinary error is not more than 1 unit, or 10^{-12} gram. Considering the nature of the measurement the agreement is perhaps closer than might have been expected, as it is doubtful if any previous measurements of these small quantities of radium have been made to anything approaching this degree of accuracy.

Fig. 3.



V. Period of the Direct Parent of Radium.

As detailed in the last communication (III. p. 635) the growth of radium from uranium should proceed according to the same power of the time as there are products in the series (counting radium itself) between the two elements, provided all these products have periods long compared with the time of observation. Rutherford, assuming only one intermediate product—the direct parent of radium—to exist, first showed that the production of radium should proceed according to the formula

$$R = 1/2 \lambda_2 \lambda_3 R_0 t^2 \quad \text{or} \quad 1/2 \lambda_1 \lambda_2 U t^2,$$

where R represents the quantity of radium, U that of uranium, R_0 the quantity of radium in radioactive equilibrium with the uranium, λ_1 , λ_2 , λ_3 the constants of uranium, the direct parent of radium, and radium respectively, and t is the time. This may be written

$$R = 6 \times 10^{-8} \lambda_2 t^2,$$

where R represents the radium formed per kilogram of uranium and λ_2 and t are expressed in years. It is of interest to calculate the value of λ_2 from this equation, although, as will be shown later, the calculation is seriously in error if other intermediate bodies of period comparable with the time of observation exist in the series. From the

curve (fig. 3) representing Experiment I. the growth in 4 years ($t^2=16$) is about 13.3 units, or 52 units per kilogram of uranium. The value of λ_2 is therefore 5.4×10^{-5} , and the value of $1/\lambda_2$, the period of average life, is **18,500 years**. The lower value (10,000 years) given in the preliminary communications is solely due to a value having been used for the constant of the instrument, which is subject to three corrections all operating in the same direction: (1) the effect of keeping the leaf charged, (2) the effect of altering the position of the charging rod, and (3) the alteration in the value of the ratio between U and Ra in pitchblende.

In Experiment IV. the value depends entirely on the initial observation to which, as already mentioned, not much weight can be attached. The experiment shows for the first three years a growth of 7.65 units, or 27.5 units per kilogram of uranium. This gives for λ_2 a value 5.1×10^{-5} , and for $1/\lambda_2$ 19,600 years.

Experiment III. is very interesting on account of the very small total amount of radium present, though the quantity of uranium is considerably greater than in the other two. It has not been in progress long enough to give the slope of the curve very accurately. As drawn in fig. 3 the growth in $\sqrt{8}$ years is 7.5 units, or 18.4 units per kilogram. The calculated value of λ_2 is 3.8×10^{-5} , and of $1/\lambda_2$ 26,000 years.

The experiment is of interest in this way. If the growth of radium had been proceeding at the same rate as in Experiment I., in $\sqrt{8}$ years 10.6 units of radium should have been produced, whereas the *total* amount of radium now present is less than this, being so far as the measurements indicate only 8.5 units. Even if it is assumed that this difference is due to errors of experiment, it is difficult to believe that the solution as initially prepared contained absolutely no radium. The results of the three experiments taken together thus indicate that the rate of growth of radium in terms of the square of the time is less for the first three years than for the subsequent year. This points to the existence of one or more intermediate bodies of relatively short period in the series, which would retard the rate of production of radium initially.

The actual measurements of the initial quantity of radium in Experiment I. bear this view out, although it must be remembered that the errors of measurement initially were very much greater than now, and this evidence thus only has a doubtful value. For Experiment II. the initial measurements may be rejected, as the quantity of radium

was almost too small to be within the range of the older methods. In Experiment IV. no initial measurements were taken. In the older tests a brass electroscope had been used, and this was calibrated at every test with reference to a γ -ray radium standard. As was pointed out (II. p. 286) the measurements so corrected were not nearly so regular as the actual observations. In light of recent results on the uncertainty attaching to γ -ray measurements, it is best to reject these calibration tests altogether and to assume that the sensitiveness of the electroscope did not vary. Recalculating from the table (II. p. 282) gives, according to present data, the value 12 for the constant of the instrument. The mean of the first nine tests over the first 309 days given in the table (II. p. 286) recalculated gives the initial quantity of radium as 15.8 units. The initial quantity indicated by the curve on the assumption that the rate of production in terms of the square of the time has been constant from the start, is about 12.5 units. This evidence so far as it goes thus bears out the view that new short-lived intermediate bodies exist.

VI. *The Effects of Short-lived Intermediate Products.*

The general effects of short-lived intermediate bodies may now be considered. Let A, B, C, D, E ... and $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \dots$ denote the quantities and radioactive constants respectively of a parent element and its successive products. It is assumed that initially the products are absent, and that the change of the parent is so slow that its quantity may be considered constant. The differential equations connecting the terms are

$$\frac{dB}{dt} = \lambda_1 A - \lambda_2 B; \quad \frac{dC}{dt} = \lambda_2 B - \lambda_3 C;$$

$$\frac{dD}{dt} = \lambda_3 C - [\lambda_4 D]; \quad \frac{dE}{dt} = \lambda_4 D - [\lambda_5 E].$$

If we assume that the periods of the first two products, B and C, are short and those of the others long the terms in square brackets may be neglected. The case then corresponds to the disintegration series Uranium $\rightarrow$ Uranium X $\rightarrow$ "Uranium A" $\rightarrow$ Parent of Radium $\rightarrow$ Radium, where B denotes uranium X, for which λ_2 is about 11.4 (year)^{-1} , C the suspected new intermediate body "uranium A," for which the period is probably of the order of unity, D the direct parent of radium, and E radium.

Solving the equations, the growth of radium is given by

$$E = \lambda_1 \lambda_4 A \left\{ \frac{t^2}{2} - \left(\frac{\lambda_2 + \lambda_3}{\lambda_2 \lambda_3} \right) t + \frac{\lambda_2^2 + \lambda_2 \lambda_3 + \lambda_3^2}{\lambda_3^2 \lambda_2^2} + \left(\frac{\lambda_2}{(\lambda_3 - \lambda_2) \lambda_3^2} \right) e^{-\lambda_3 t} - \left(\frac{\lambda_3}{(\lambda_3 - \lambda_2) \lambda_2^2} \right) e^{-\lambda_2 t} \right\}. \quad (I.)$$

The last term is always very small, and may be neglected at once. The penultimate term approaches to zero as t increases. After a period several times that of uranium A it may be neglected. The expression then becomes

$$E = \lambda_1 \lambda_4 A \left\{ \frac{t^2}{2} - \left(\frac{\lambda_2 + \lambda_3}{\lambda_2 \lambda_3} \right) t + \frac{\lambda_2^2 + \lambda_2 \lambda_3 + \lambda_3^2}{\lambda_3^2 \lambda_2^2} \right\}.$$

If instead of reckoning the time from the start we reckon it from a date later by the sum of the periods of average life of the two short-lived bodies, that is, for t in the equation we substitute T , where

$$t = T + \frac{1}{\lambda_2} + \frac{1}{\lambda_3},$$

then

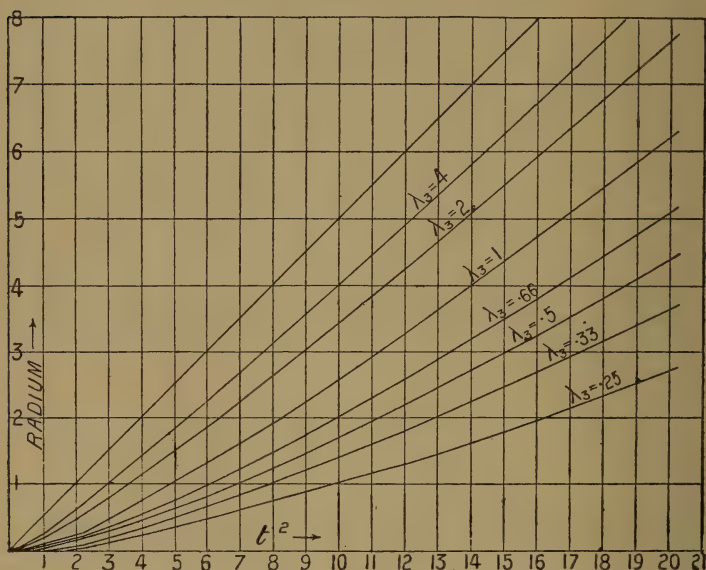
$$E = 1/2 \lambda_1 \lambda_4 A \left\{ T^2 + \frac{1}{\lambda_2^2} + \frac{1}{\lambda_3^2} \right\} = 1/2 \lambda_1 \lambda_4 A T^2 + \text{constant}.$$

That is to say, when the short-lived intermediate bodies come into equilibrium the production of radium proceeds strictly according to the square of the time reckoned, not from the start, but from a date later by the sum of the periods of the intermediate bodies.

To show the effect of the intermediate bodies initially the graphs of equation (I.) have been plotted against the square of the time, for the following values of λ_3 :—4, 2, 1, .66, .5, .33, and .25 (fig. 4, p. 856). The curves therefore show the production of radium according as the hypothetical body uranium A has the period of three months, six months, 1, 1.5, 2, 3, and 4 years. The straight line uppermost on the diagram shows the production of radium on the same scale if no short-lived intermediate products intervened. It will be seen at once from these curves that if the portion between the third and fourth years, that is between $t^2=9$ and $t^2=16$, is examined, even for the lowest curve corresponding to the four-year period, the curve departs but little

from the straight line. It is doubtful whether the departure would be experimentally detectable over this period. This shows that the straightness of the experimental curve over the period available is no argument against the existence of

Fig. 4.



intermediate bodies. On the other hand, the figure at once reveals the serious error introduced into the calculated period of the parent of radium by the formula used in Section V. if intermediate bodies exist. The true period of this body is obtained from the slope of the straight line uppermost in the diagram, which represents the rate of production of radium if no intermediate bodies intervened. If an intermediate body with a period of 1 year intervened, the period of the parent of radium instead of being 18,500 years as calculated in Section V. would really be only 0.72 times this or 13,300 years. If the period of the intermediate body were 4 years, the true period of the parent of radium would be only 0.32 of the apparent period, or 6000 years. Yet even a four-year period body might exist without producing in the curve a departure from the straight line enough to be detected experimentally with certainty as far as the measurements have yet gone.

VII. *New Experiments.*

Two new experiments are being commenced which it is hoped will yield in the course of time further evidence on the existence or otherwise of short-lived intermediate bodies in the series. In the first the purest fraction of the 50 kilograms of uranyl nitrate which has been repeatedly crystallized from fresh water in the work on the γ -rays of uranium (Soddy and Russell, *Phil. Mag.* Oct. 1909, p. 620) has been withdrawn from the fractionation and sealed up in a flask. It contains 3 kilograms of uranium (element), which is 12 times the quantity in the oldest preparation. It is being tested at monthly intervals for growth of radium. The initial quantity present has been determined in two agreeing consecutive experiments to be 42 units. Considering the mass of the material this is gratifyingly low. In the course of a year or two the growth of radium from this solution should settle the question whether new intermediate products exist in the series.

Secondly, now that the rest of the 50 kilograms of uranyl nitrate has been repeatedly purified, it is proposed to seal up one of the preparations of uranium X to be separated from it, as in the work with Mr. Russell, and to measure from it the rate of production of radium. Unless intermediate bodies exist this would hardly be worth while, as it would be practically impossible to distinguish the parent of radium formed from uranium X—if it is so formed—from that initially separated with the uranium X. Keetmann (*Inaug.-Dissert. d. Verf., Berlin, 1909*) has stated that uranium X and the parent of radium both have the same chemical properties as thorium and cannot be separated from this element or from one another. But if an intermediate body exists of period of the order of a year it should be easy to distinguish between the parent of radium initially present with the uranium X and that subsequently produced from it. If U represents the quantity of uranium initially in equilibrium with the uranium X separated from it, and R represents the quantity of radium formed from the uranium X, it can be shown that

$$\frac{R}{U} = \frac{\lambda_1 \lambda_4}{\lambda_3 - \lambda_2} \left\{ t \left(1 - \frac{\lambda_3}{\lambda_2} \right) + \frac{\lambda_3}{\lambda_2^2} (1 - e^{-\lambda_2 t}) - \frac{1}{\lambda_3} (1 - e^{-\lambda_3 t}) \right\},$$

assuming the same disintegration series as before. Whereas if all the parent of radium were present initially, the production of radium would of course proceed proportionally to the time.

VIII. *Conclusions.*

The measurements on the growth of radium in the three uranium solutions purified by Mr. T. D. Mackenzie between three and four years ago, have shown that in all of them the growth of radium is proceeding according to the square of the time within the error of measurement. The period of the direct parent of radium calculated from these results, on the assumption that no other intermediate bodies intervene, is 18,500 years in the case of the oldest solution for which the most complete data are available. But in the solution prepared last, the total quantity of radium now present is less than what would have been produced from the radium, assuming the rate of production to have been the same as in the first solution. This suggests the existence of at least one new product "uranium A" intermediate between uranium X and the parent of radium, with the period of the order of one year. From a mathematical investigation of the effect of such a body on the growth of radium it is concluded that it would not, if it existed, appreciably alter the production of radium according to the square of the time over the period observations have been made, but it would vitiate the calculations of the period of the average life of the parent of radium according to Rutherford's formula. In conclusion, it may be mentioned that a good deal of additional evidence bearing directly on this question of the existence of new intermediate products has been accumulated in an investigation on the rays and product of uranium X, with which it is convenient to deal in a separate communication.

Physical Chemistry Laboratory,
University of Glasgow,
October 1909.

XCI. *The Rays and Product of Uranium X.*

By FREDERICK SODDY, M.A.*

THE preceding paper has dealt with the evidence from the rate of production of radium from uranium, which suggests the possibility of the existence of at least one new intermediate product in the disintegration series. Although the experiments are not yet complete, it is advisable also to publish a short account of the results obtained in a different

* Preliminary accounts of part of this work appeared in two letters to 'Nature,' Jan. 28th, 1909, 79, p. 366, and March 11th, 1909, 80, p. 37. The paper was communicated to the Physical Society of London on Oct. 22nd, 1909.

field of investigation which bear also upon the same problem. Attempts have been made to determine whether, during the decay of the β -radiation of the intensely active preparations of uranium X, separated with the help of Mr. Russell (Phil. Mag. Oct. 1909, p. 620) from 50 kilograms of pure uranyl nitrate, there occurred the growth of a feeble α -radiation increasing concomitantly as the other decayed. This is to be expected from the work of Boltwood, Keetmann and others, who have shown that the direct parent of radium gives an α -radiation, provided that it is the direct product of uranium X. The work was started in the hope that intermediate bodies intervened, and that the growth of α -rays would occur long enough after the β -rays of the uranium X had decayed to be detectable by ordinary measurements. But through the generosity of the friend who provided the uranium a powerful electromagnet of the Du Bois type was acquired, and with this it was attempted to deviate the β -rays of the uranium X, so that measurements of the α -rays of the preparations could be taken from the start. The problem proved a difficult one, but gradually the experimental methods have been perfected, so that now it is claimed that they are sufficiently delicate to detect such a growth of α -rays if it occurred in the intensely active uranium X preparations investigated to the extent to be expected from theory. Much remains unaccountable in connexion with these experiments, but they appear to have definitely answered the main question in the negative. No production of α -rays, such as is to be expected from theory, has occurred, and therefore it does not seem possible that the direct parent of radium can be the direct product of uranium X. The experiments indeed raise the further question whether the parent of radium can be a product of uranium X at all, but are not yet far enough advanced to be conclusive.

Were the parent of radium a product of uranium X the number of α -particles it emits should either be equal to, or one-half of, the number emitted by the uranium in equilibrium with it, according as one or two α -particles are given off from uranium. The latter assumption will be made as it gives the smaller growth of α -rays theoretically to be expected from uranium X. Assuming that the α -activity is proportional also to the ranges of the respective α -particles, which are 2.7 and 3.5 cm., the α -activity of the parent of radium will be $2.7 \div 3.5 \times 0.5$, or .38 of that of the uranium. The period of the parent of radium as deduced in the last paper on the assumption that no new intermediate bodies exist in the series, is 18,500 years, while that of uranium X

is about 32 days, or 5×10^{-6} in terms of the other as unity. The amount of uranium X in equilibrium with a definite amount of uranium should produce therefore upon complete disintegration about 5×10^{-6} of the amount of the parent of radium in equilibrium with the same quantity of uranium. The α -activity of this product would therefore be about 2×10^{-6} of that of the uranium. In other words, the uranium X in equilibrium with 1 kilogram of uranium should thus produce in its complete disintegration a product having the α -activity of 2 milligrams of uranium.

The uranium X, after concentration to the greatest possible extent, was finally prepared in the form of films in rectangular trays having an area of 10 sq. cm., which could be slipped between the poles of the electromagnet. The undeviated radiation entered an electroscope placed above the poles provided with an opening in its base covered with very thin aluminium-foil. The first attempts were frustrated and the first preparations decayed without result, owing to the totally unexpected difficulty in deviating the β -rays. The latter previously had been supposed to consist of homogeneous rays having a value for $H\rho=2000$. But it was found that a field twice as strong as was necessary completely to deviate such rays failed to deviate about 5 per cent. of the total β -radiation. In the later dispositions the width of the gap was reduced to one-half the former width, and the field was sufficient to deviate completely all rays with a value for $H\rho$ below 8640; but the leak due to the still undeviated β -radiation, although now proportionately very small, was still several times that due to the γ -radiation. It was still too great in the earlier measurements for the small growth of α -radiation, theoretically to be expected, to be within the range of certain detection. In the later experiments the electroscope was filled with hydrogen instead of with air, and this constituted an enormous advance. Not only is the ionization due to the β - and γ -radiation greatly reduced and that due to the α -radiation greatly increased, but the convection-currents generated in the electroscope by the warming up of the magnet are also enormously reduced. The readings are far more regular and can be continued for a longer time before the leaf begins to be disturbed. In air, after half-an-hour's work or less, the leaf commences to wave like a flag, and nothing can then be done for eight hours. A preparation which gave (in divisions per minute) a leak of 370 divisions bare, and 351 covered with thin mica (α -rays therefore 19) when the electroscope was filled with air, gave in the same apparatus filled with hydrogen, bare 105.2,

covered 66.2 (α -rays therefore 39). As the subsequent history of the preparations showed, a large proportion of the radiation reckoned as " α -" in air is due to the slight absorption of the β -rays in the mica, whereas in hydrogen this absorption is nearly negligible. So that in air, even if the theoretical growth of α -radiation occurred, it would probably be largely, if not entirely, masked by the diminution in the difference between the two leaks, covered and uncovered, due to the diminution in the absorption of the β -rays as these decayed. In hydrogen the effect is still present to a slight extent but not enough to prevent the observation of the growth of α -rays, if it occurred at the theoretical rate. Subsequent work has thrown doubt on the earlier measurements in air, and on the conclusion drawn from them in the second communication to 'Nature,' that in one preparation a rapid growth of α -rays occurred which reached a maximum in 2.5 days from the start. This observation has not been repeated either in air or in hydrogen with more perfect methods. This is not itself conclusive against it, for it has been impossible to prepare two sets of uranium X preparations in the same way. The chemical methods, of separation from uranium and of concentration from unidentified impurities, employed have varied widely, being dictated by circumstances from moment to moment during the process, owing to the perpetually changing character and amount of impurities and the totally different chemical behaviour of the uranium X accordingly. In fact the methods adopted in the last separation were often the converse of those formerly most relied upon, the uranium-free, uranium X-containing substances being *dissolved* in excess of ammonium carbonate and fractionally precipitated by boiling. Sometimes the uranium X came down in the *last* minute fraction almost entirely, and in other cases in one of the middle fractions. This new method proved an extremely valuable one in the last separation. Unfortunately it was not discovered before the whole of the uranium X had been first separated by the tedious barium sulphate process.

The earlier observations and the conclusion referred to may be rejected at least for the present. The difficultly deviable rays behave in an anomalous manner which was at first not suspected, in that they do not seem to be a constant quantity. In the case of the most recent preparation, which, as already explained, differed in the manner of its separation from the earlier ones, two successive measurements of the difficultly deviable radiation from the covered preparation in air, the first immediately after preparation, and the second

9 hours later, showed a diminution in the interval in the ratio of 4 to 3. The γ -rays of the preparation, measured in the usual way in a lead electroscope at the same times, showed no perceptible variation. It was later found that the value of the leak due to the difficultly deviable radiation varied in a completely unaccountable way. The variation is far more marked in air than in hydrogen. The behaviour suggested strongly in certain cases that the preparation was giving off a radioactive emanation, and from the manner of its preparation thorium was suspected. But this was completely disproved by special experiments. But the most striking proof that the behaviour was not in any way due to this cause or to defects in the method of measurement is that the α -radiation as measured by the difference between the leaks with the preparation covered and uncovered remained from the start perfectly constant. This shows that all the numerous errors to be guarded against in experiments where the gas in the instrument is changed have been avoided.

A good many rapidly improvised experiments, done with the first batch of preparations before their activity decayed, had indicated that the difficultly deviable radiation was entirely similar in general character, both in the direction of their deviation and in the value of their absorption coefficient, to the ordinary β -rays. The absorption coefficient for aluminium, determined by placing strips of foil directly over the preparation, placed in the magnetic field, had been found to be about 24 (cm.)^{-1} . This is much *less* than the usual value (14), given for the ordinary β -rays, and is about equal to the value after 2.5 mm. of aluminium have been traversed. The field would cause the trajectories of the rays in the metal to be lengthened, but owing to the complete scattering suffered by these rays by even thin films of metal, it is difficult to state precisely what the effect of the field on the absorption coefficient would be. If these rays are really β -rays, the value of $H\rho$ shows that they must possess a velocity practically that of light, and a kinetic energy many times that of the ordinary β -rays. In view of the great decrease in penetrating power which a very slight diminution of velocity appears to produce in this type of radiation, it might be expected that the difficultly deviable rays would have a very great penetrating power, whereas the value found was actually less than the normal value. Similar difficultly deviable radiation is given out also by radium, although here the value of $H\rho$ is even higher, and the upper

limit does not appear to be reached at 9000 to 11,000. Further study may show that they are to be connected with the scattering of the β -rays, though the experiments done rather point to the view that they may be a new kind of radiation. So far attention has been concentrated rather upon the main problem.

The electroscope used for hydrogen had a very thin mica screen, thick enough to absorb α -rays, hinged inside, which could be turned up or down from the outside, without opening the instrument, to uncover or cover the preparation at will. The first measurements, with the third set of preparations, were interfered with somewhat by difficulties encountered in making and keeping the electroscope perfectly gas-tight. Later a new instrument with properly soldered joints throughout was constructed and is now used entirely. There were three preparations in the third set, two practically identical in every particular, containing initially the uranium X in equilibrium with 5.1 kilograms of uranium, and a third containing that of 2 kilograms, as measured by means of the γ -radiation. They all showed the same behaviour, and one only of the two stronger preparations need be considered here. It weighed 200 milligrams. Measurements were done in hydrogen only. No evidence of a growth of α -rays either initially or at any subsequent time was obtained. The mean of the first five observations over the first 28 days gave 14 for the α -rays, which, corrected for the difference between the two instruments, gives 16 for the leak in terms of the present instrument. The actual observations varied a few per cent. from the mean for the reason stated. Subsequent measurements with the new electroscope 76, 99, and 137 days from the start, gave for the α -radiation 17, 17.2, and 16.6 respectively.

In the fourth separation there was one preparation only, weighing 77.6 milligrams, and containing initially the uranium X in equilibrium with 5.05 kilograms of uranium. The measurements of the α -radiation were done at first both in hydrogen and air. Those in hydrogen, at different times from preparation, are given in the following table. They are the most accurate so far done and are all comparable with one another.

Time (days).	0	0.5	1.0	2.0	3.0	4.0	6	9	18	33
α -rays.....	38.6	39.7	38.8	38.9	39.0	39.2	37.2	37.1	39.8	40.5

The initial value of the leak due to the penetrating rays was 75. It will be seen that from the start over a period of 33 days—about the period of average life of uranium X, in which 0.632 of the total disintegrates—the α -radiation has been practically constant. To obtain an idea of the growth of α -rays theoretically to be expected, the α -radiation of a film containing 24 mg. of uranyl nitrate (= 12 mg. uranium) was measured in hydrogen under precisely similar conditions. It gave an α -ray leak of 17.8. The growth of α -rays to be expected in the last experiment in 33 days, if the change of uranium X into the parent of radium occurred directly, is equal to that of $0.632 \times 5.05 \times 2 = 6.5$ mg. of uranium, or about 9.6 divisions. Even if a considerable reduction is made for the fact that the weight of the uranium-X film was about three times the weight of the uranyl nitrate film, it is difficult to believe that the growth would not easily have been detected in the experiment if it had occurred.

In the preceding experiment with the preparations of the third separation, the initial α -activity was less than half the value in the last experiment. Any rise could therefore have been still more easily detected. But up to the period of 137 days from preparation, when less than 2 per cent. of the original uranium X remains undisintegrated, not the slightest growth has been recorded.

The most active preparation of the second separation tested recently in the new hydrogen electroscope in hydrogen on two occasions 200 days and 227 days from preparation, gave the same α -ray leak (about 80). The best preparation of the first separation—the best one ever prepared from the point of view of smallness of mass—tested in air (in a much weaker field than now used) has latterly, now that the β -radiation has nearly decayed, shown no change in the considerable α -radiation present. In the last tests 264 and 291 days from preparation, the α -leak has been constant at 35 (in air). This and a still earlier preparation now 16 months old, prepared from 3 kilograms of uranyl nitrate in preliminary work, have now decayed so far that their α -radiation can be examined without the aid of a magnet in an ordinary electroscope against suitable standards. Since these tests were started the α -radiation has remained constant. A still older preparation, dating from 1903/4, prepared from probably not more than 50 grams of uranyl nitrate, has also quite an appreciable α -radiation. This has been kept under careful observation for over a year and no change has been recorded.

These tests, therefore, although still incomplete, cover for different preparations the period from the start up to nearly a year in the case of the main preparations, and for considerably longer for weaker preparations. They have given no evidence of the growth of an α -radiation at any time during this period. They all show a considerable constant α -radiation, but in all it appears to be due to a body present from the start, unconnected genetically with uranium X. The statement of Keetmann, that uranium X and the parent of radium are identical in chemical behaviour and are always separated together, would explain the presence of the α -ray body in all the preparations so far kept under observation. The conclusion seems to be justified that the parent of radium is not the direct product of uranium X, and the question now arises whether it can be considered a product of uranium X at all. Even if there was a new intermediate product in the series "uranium A," with a period of a many years, provided there was but one, some indication of growth of α -rays should before now have been obtained in these experiments. For it must be remembered that its existence would greatly reduce, as explained in the last paper, the period of the parent of radium, as calculated from the measurements of the rate of growth of radium, and so would increase correspondingly the growth of α -rays ultimately to be expected. The subsequent history of the preparations may be expected to throw further light on this question.

The results so far obtained are difficult to reconcile with the experiments on the rate of production of radium from uranium, on the assumption that the parent of radium is a product of uranium X. There is no proof of this, though on account of the intense character of the β -rays of uranium, it seems natural to suppose that uranium X, the β -ray-producing body, is in the main radium series rather than in that of actinium.

Physical Chemistry Laboratory,
University of Glasgow.
October 1909.

XCII. *Effect of Temperature on the Hysteresis Loss in Iron in a Rotating Field.* By W. P. FULLER, *B.Eng.*, and H. GRACE, *B.Eng.**

IT was shown by Professor Bailly † that the hysteresis losses due to a rotating field in iron reached a maximum value with an induction density (B) of about 16,000 C.G.S. units. With a value of B equal to 20,000 the hysteresis was approximately $\frac{1}{20}$ of the value. These results were confirmed by Messrs. Beattie & Clinker ‡. The experiments described below were undertaken in order to determine to what extent the results attained by Professor Bailly were modified by variation in temperature.

In these experiments, the rotating field was produced by means of two phase-currents. Fig. 1 shows the arrangement, the two magnetizing coils carrying the two phase-currents being marked E, D. A is a slab of plaster 2 cms. thick and 22 cms. square, having a circular hole $8\frac{1}{2}$ cms. diameter in the centre. At the top and bottom of the hole the heaters B are placed, and in the chamber so formed the iron specimen is suspended. Each heater was made by winding No. 40 s.w.g. nickel wire zigzag fashion over the surface of a circular piece of mica. Its resistance cold was 13 ohms, but increased rapidly with temperature. With the two in series a current of 1.6 amps. at 180 volts produced a maximum temperature of 500° C. at a point close to the iron disk. The specimen used was a circular disk of iron 4 cms. diameter, .027 thick: it was attached by nuts to a brass spindle C, which had a weight attached to one end, and a *concave* mirror at the other for indicating the motion of the specimen. The whole was supported by a bifilar suspension, the sensitiveness of which could be varied by altering the weight or by varying the distance apart of the supporting wires. The weight of the whole moving part was 285 grams.

The two magnetizing-coils D D, belonging to one phase, are each made by winding 14 turns of 7/14 asbestos-covered cable. The two coils are arranged to slide over the plaster slab to within a short distance of each other, at the centre. The coils for the second phase, E E, are placed so as to produce a field at right angles to that of D D. They are of the same shape as D D, so that the two together produce a close approximation to a uniform rotating field at the centre of the coils where the specimen is placed. The large coil was found

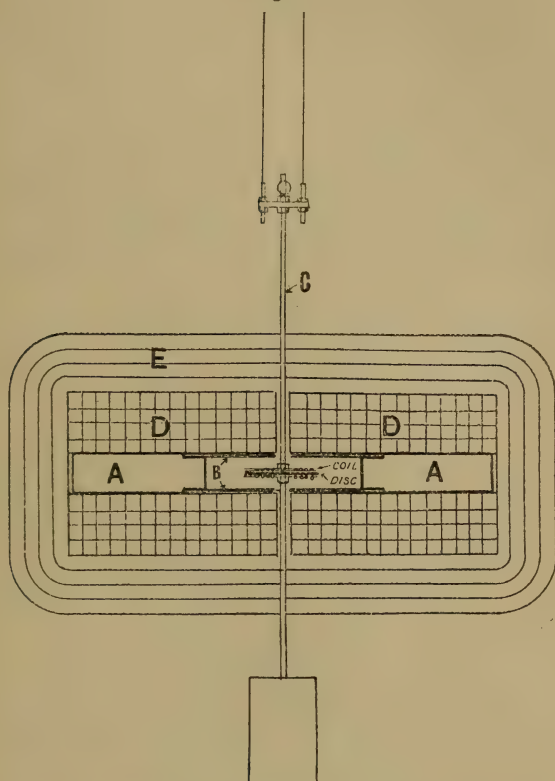
* Communicated by the Physical Society: read May 14, 1909.

† Phil. Trans. 1896.

‡ 'Electrician,' 1896.

to produce a field 10 per cent. greater than that of the smaller for the same current, but with 25 amps in one and 27.5 in the other, the maximum values of the two fields were equal,

Fig. 1.



General arrangement of the magnetizing coils and specimen.

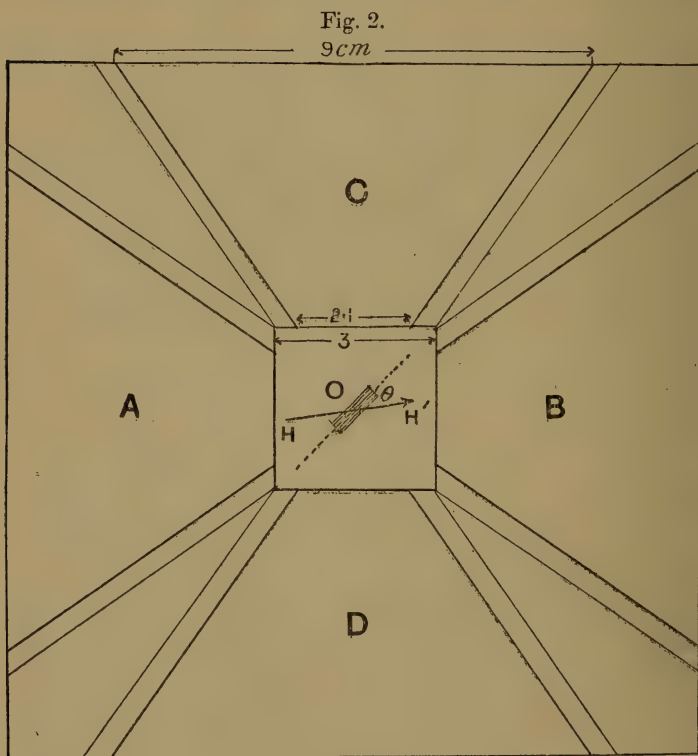
each being about 250 c.g.s. units, and the two together producing a uniform rotating field of this magnitude. At a distance of 2 cms. from the centre of the coil the field produced was $1\frac{1}{2}$ per cent. less.

The phase relationship of the currents in the two phases was indicated by means of a wattmeter, the thick coil being in one circuit and the thin coil connected between the ends of a non-inductive resistance in the circuit of the other phase. A phase-difference of 90° between the currents in the two circuits was shown by a zero reading on the wattmeter. The phase-difference was kept within .3 per cent. of

90° by using a variable choking-coil in the primary of one of the transformers supplying current.

The numerous adjustments which have to be made are a disadvantage of the alternating current method compared with the rotating magnet method used by Baily and others; but the absence of rotating parts is a distinct advantage, and this fact would render the method very suitable for high-frequency experiments.

To measure the flux in the iron, a coil of 8 turns of bare wire was wound round it and insulated therefrom with mica. As the E.M.F. induced in the coil is alternating and of the order of .04 volt, a suitable galvanometer had to be constructed in order to measure it. The instrument constructed for the purpose is shown in fig. 2. A B C D are four



coils of conical shape and wound with no. 28 wire. A and B were connected in series through a resistance to one phase, C and D to the other phase, of the machine supplying current to the magnetizing-coils of the test apparatus D E. With a

current of .31 amp. a rotating field of about 20 C.G.S. units was produced at the point O. A small coil, made up with 200 turns of no. 46 wire (140 ohms resistance) 5 cms. long and 1 cm. broad, is suspended in the central space between the coils, and is connected to the search-coil on the iron specimen. In the galvanometer-coil there are two E.M.F.'s, one, $E \cos pt$ (say) due to the flux in the iron specimen, and one due to the rotating field of the galvanometer itself. Referring to fig. 2, let H H' be the direction of the rotating field of maximum value H when $E \cos pt$ is a maximum. The angle between it and the coil at a time t will be $\theta + pt$, and the component along the coil $H \cos \overline{pt + \theta}$. The flux normal to the coil is $H \sin \overline{pt + \theta}$, and the E.M.F. induced is proportional to

$$H p \cos \overline{pt + \theta} = E_1 \cos \overline{pt + \theta}.$$

If R be the resistance of the coil and the self-induction is negligible, the resultant current is equal to

$$\frac{1}{R} (E \cos pt + E_1 \cos \overline{pt + \theta}).$$

The torque at a time t will be proportional to

$$H \cos \overline{pt + \theta} \left\{ \frac{1}{R} (E \cos pt + E_1 \cos \overline{pt + \theta}) \right\}.$$

Hence the mean torque acting on the coil is proportional to

$$\frac{H}{R} \{ E \cos \theta + E_1 \}.$$

This deflecting torque therefore consists of two parts, one constant and one which depends on the position of the four magnetizing-coils A B C D. In the apparatus used these coils were fixed to a turntable, which could be rotated until the deflexion was a maximum. The deflecting torque would then be proportional to $\frac{H}{R} (E + E_1)$. Another way of using the instrument is to get E and E_1 in opposition, in which case the minimum deflexion is proportional to $(E_1 - E)$; and this is the better method because the total deflexion is smaller, and greater sensitiveness is obtained.

The instrument was calibrated by putting a resistance of 4000 ohms in series with the coil and applying a measured pressure of 1.95 R.M.S. volts to it. The deflexion, after deducting that obtained by shortcircuiting the coil through the 4000 ohms, equalled 22.4 centimetres, consequently one

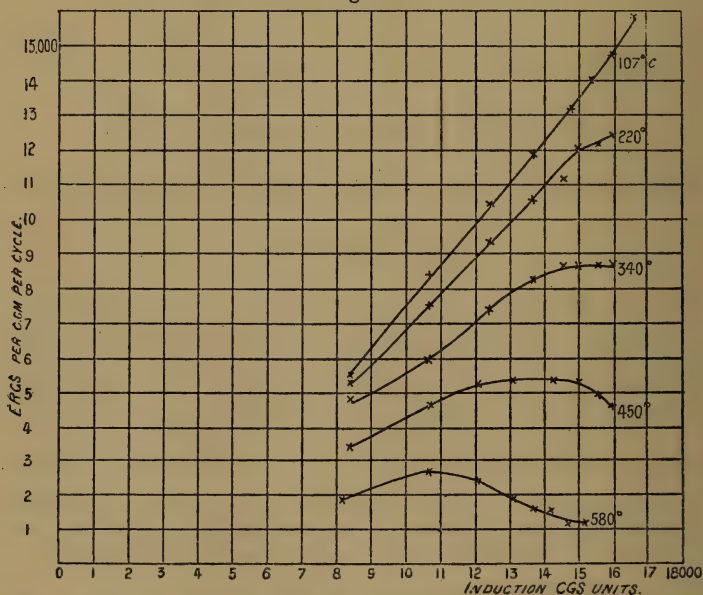
centimetre corresponds to a voltage of maximum value $\cdot 00416$ applied to the moving coil.

The induction density in the iron disk was readily calculated from the maximum voltage measured as above, the dimensions of the search-coil on the iron being known.

The disadvantage of this type of instrument is that the constant part of the deflexion depends on the cube of the frequency, so that the speed of the machine must be very exactly regulated for a constant value. The E.M.F. induced in the coil on the iron varies approximately as the frequency, and so the torque on the galvanometer due to this current depends on the square of the speed. It was found better in practice to have the two voltages E and E_1 in opposition, as mentioned above, so getting a deflexion proportionate to $E_1 - E$.

One other measurement required to be made, namely, that of temperature. This was effected by means of a Platinum-Platinum-Rhodium junction (placed close to the specimen) connected to a suitable galvanometer.

Fig. 3.

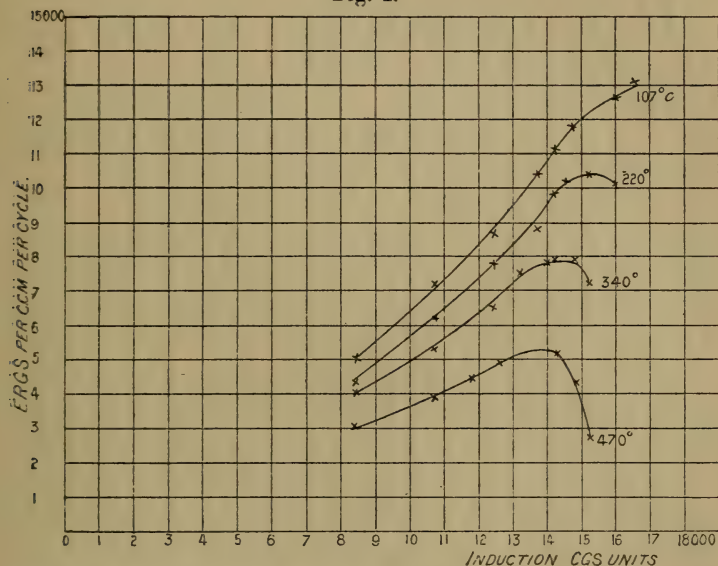


Curves showing relation between Hysteresis Loss and Induction at different Temperatures in a sample of armature iron from Messrs. Sankey.

The results obtained are shown in figs. 3 and 4. The temperature of 107°C . was the lowest at which it was found possible to make any measurements, on account of the heating

effect of the magnetizing-coils. The results in fig. 4 were obtained after the iron had been heated to 580° and cooled slowly. These experiments show that the effect of increasing the temperature of iron is to greatly reduce the hysteresis

Fig. 4.



loss at a given induction, and to cause the maximum to occur at a lower value of B . At a temperature of 580°C . the maximum hysteresis loss occurs at an induction density of 11,000 C.G.S. units instead of 16,000, while the maximum hysteresis loss is only 2600 as compared with a maximum of over 15,000 per c.c. per cycle at ordinary temperatures. The effect of heating to 580° , as shown in fig. 4, is most marked; although the actual maximum hysteresis loss is not greatly altered at the higher temperature, the reduction in the hysteresis loss at the lower temperature is considerable. The shape of the curve between ergs per cycle and B is quite different, the hysteresis loss falling off much more rapidly from the maximum.

The permeability of the iron is not greatly affected by changes in temperature within the limits of the experiment, as was shown by Morris*.

The above experiments were carried out at the Applied Electricity Laboratories of the University of Liverpool, and the authors thank Professor Marchant for his valuable advice and assistance.

* Phil. Mag. Sept. 1897.

XCIIL. *The Apparent Dispersion of Light in Space, and the Minuteness of Structure of the Æther.* By C. V. BURTON, D.Sc.*

1. **R**ECENT observations by Nordmann† and by Tikhoff‡ have been held by these authors to establish a dispersion of light in interstellar space: the velocity of propagation appearing to be a direct function of the wave-length. On the other hand, the reality of the phenomenon has been denied by Lebedew§, who attributes the effects recorded to peculiarities of the star-systems observed. It is not the purpose of this note to discuss questions of astrophysics on which the views of specialists are so widely different, but rather to direct attention to a theoretical point regarding the possible scale of structure of the æther, and the relation which might be expected to obtain between that scale of structure and differences of velocity corresponding to radiations of given wave-length.

2. Amongst the considerations advanced by Lebedew, there is one concerned with the inevitable connexion between dispersion and absorption of light in material media. He points out that any known substance (hydrogen at low pressure, for example), if present in sufficient amount to give rise to the dispersion which has been inferred, would absorb so much light that the sun and the stars would be invisible to us. This appears to be undeniable, and accordingly it would seem that any definitely proved dispersion of light in interstellar space would have to be attributed to properties of the æther itself.

3. In any homogeneous isotropic || elastic medium, which can be adequately treated as a continuum—that is to say, in which the specific properties of the medium in bulk exist unimpaired in any volume thereof, however small—any dis-

* Communicated by the Author.

† C. Nordmann, *Comptes Rendus*, cxlvi. pp. 266 & 383 (1908); cxlvii. pp. 24 & 620 (1908).

‡ G. A. Tikhoff, *Comptes Rendus*, cxlvi. p. 570 (1908); cxlvii. p. 170 (1908).

§ P. Lebedew, *Astrophys. Journ.* xxix. no. 2, p. 101 (March 1909).

|| Though isotropy is not a necessary condition for ensuring that (in any given direction) the velocity of propagation is independent of the wave-length, the statement in the text is limited to isotropic media for the sake of simplicity of expression. The term *elastic* is used in its most extended sense, denoting merely that it is possible to increase the potential energy of the medium by doing work upon it, and that the medium is capable of doing an equivalent amount of work in reverting to its normal state.

turbance of given type and of sufficiently small amplitude will be propagated with a velocity entirely independent of the wave-lengths involved. This is readily proved from general considerations, and familiar examples immediately present themselves. There is no better instance than the elementary form of electromagnetic theory, according to which an isotropic nonconducting medium is characterized simply by a definite dielectric capacity together with a definite magnetic permeability; and the failure of any such simple theory to account for dispersion,

4. But although the dispersive and other optical properties of material substances compel our attention to their *structure*, there are as yet no well-established facts of observation which forbid us to regard the free æther as an absolutely homogeneous medium, ideally divisible to any extent; in a word, structureless. It is interesting to attempt to form some idea as to how far this elementary conception would have to be modified if the velocity of light *in vacuo* were actually found to be a function of the wave-length.

5. Not knowing what kind of structure should be provisionally assumed for the æther, we are driven back on the consideration of mechanical models and analogies; these may perhaps furnish a hint as to the order of magnitude of the quantities involved; as to the sort of relation which must hold between wave-lengths and the scale of structure of the æther if certain specified dispersion effects are to be accounted for. Essentially we must aim at representing a structure which will produce dispersion without absorption; or at least in which the absorption is far less than in any material medium of comparable dispersive power.

6. In the first place, consider waves of transverse disturbance which are being propagated along a loaded string of indefinite length. Suppose the string itself to be without appreciable inertia, while the loads, each equal to m , are concentrated at points which succeed one another at regular intervals (b) along the string. The transverse vibrations of a limited length of such a loaded string are fully investigated in Lord Rayleigh's 'Theory of Sound'*, and the results there obtained could be adapted to the problem now proposed. But the required relations can in this case be more simply deduced from first principles. We may confine our attention to vibrations taking place in a single plane which passes through the string in its undisturbed state.

7. Calling S the tension of the string, let y_r denote the

* Vol. i. § 120.

(lateral) displacement of the r th load. The force on this load is

$$(S/b)(y_{r-1} + y_{r+1} - 2y_r),$$

which is therefore equal to md^2y_r/dt^2 . Assume now that the motion is periodic, with period $2\pi/p$, and let the motion of the r th load be given by

$$y_r = A_r e^{ipt}. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Solutions can be obtained corresponding to the further assumptions that the motion of each of the remaining loads is periodic, and is represented by an equation of the same form as (1), while

$$A_{r+1}/A_r = A_r/A_{r-1} = \dots = k. \quad . \quad . \quad . \quad (2)$$

Then, if for brevity we write

$$\alpha^2 \equiv p^2bm/S, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

it follows that

$$(\alpha^2 - 2)k + 1 + k^2 = 0; \quad . \quad . \quad . \quad . \quad (4)$$

whence

$$k = 1 - \frac{1}{2}\alpha^2 \pm i\alpha(1 - \frac{1}{4}\alpha^2)^{\frac{1}{2}}. \quad . \quad . \quad . \quad (5)$$

8. It is here supposed that the wave-length of the disturbance in question is sufficiently great to make p^2bm/S less than 4. If, then, we put

$$k = B(\cos \beta \pm i \sin \beta) \quad . \quad . \quad . \quad . \quad (6)$$

as equivalent to (5), B and β being real; we obtain on equating real and imaginary parts

$$B = 1 \quad \text{and} \quad \tan \beta = \pm \alpha(1 - \frac{1}{4}\alpha^2)/(1 - \frac{1}{2}\alpha^2); \quad . \quad (7)$$

so that

$$\sin \frac{1}{2}\beta = \pm \frac{1}{2}\alpha = \pm \frac{1}{2}p\sqrt{(bm/S)}. \quad . \quad . \quad . \quad (8)$$

9. Rejecting now the imaginary terms in (1) and in the solution, there remain the given motion

$$y_r = A_r \cos pt \quad . \quad . \quad . \quad . \quad (9)$$

and the solutions

$$y_{r+q} = A_r \cos (pt \pm q\beta); \quad . \quad . \quad . \quad (10)$$

where q is any integer positive or negative, and β is an acute angle given by (8). The lower sign corresponds to a wave-train propagated in the direction of q -increasing, the upper sign to a train in the contrary direction. In either case the amplitude is invariable throughout the train; that is to say, there is *no absorption*. On the other hand (as

below), the velocity of propagation is a function of the frequency $p/2\pi$, or in other words, it is a function of the wave-length.

10. If x is the distance from the r th mass to the $(r+q)$ th, then $q=x/b$ and (10) may be written

$$y = A \cos (pt \pm \beta/b \cdot x); \quad . \quad . \quad . \quad (11)$$

the suffixes being suppressed. The velocity of propagation is thus

$$V = pb/\beta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$= pb/2 \sin^{-1} \left\{ \frac{1}{2} p \sqrt{(bm/S)} \right\} \\ = \sqrt{(bS/m)} \left(1 - \frac{1}{24} p^2 bm/S \dots \right). \quad . \quad . \quad (13)$$

11. For infinitely great wave-lengths (when $p=0$) (13) reduces to

$$V = \sqrt{(bS/m)}; \quad . \quad . \quad . \quad . \quad (14)$$

and for wave-lengths (λ) which are great enough to make $\frac{1}{24} p^2 bm/S$ a small fraction, p is equal to $2\pi V_\infty/\lambda$ to a first approximation. Thus (13) becomes

$$V = V_\infty \left(1 - \pi^2 b^2 / 6 \lambda^2 \dots \right). \quad . \quad . \quad . \quad (15)$$

12. Now let U be the group-velocity* for disturbances of wave-length λ . Then

$$U = V - \lambda dV/d\lambda;$$

or from (15)

$$U = V_\infty \left(1 - \pi^2 b^2 / 2 \lambda^2 \dots \right). \quad . \quad . \quad (16)$$

13. Before any numerical values are inserted in (15) or (16) it may be remarked that the law of dispersion thus established for a loaded string is readily seen to be applicable to a suitably constituted medium having extension in three dimensions. We have only to imagine a multitude of equal masses arranged in cubical order, and connected by uniformly tense massless strings running in three mutually perpendicular directions. For laminar disturbances propagated in any one of these three directions, the mode of propagation will be precisely the same as for the single loaded string already considered, and there will be the same law of dispersion. In certain other cases—longitudinal vibrations of an extensible loaded string, or transverse vibrations of a tense chain with elongated, smoothly articulated links—there

* Cf. Rayleigh, 'Theory of Sound,' 2nd ed. vol. i. Appendix. I believe it was first pointed out by Lord Rayleigh that, if there is any dispersion of light in space—that is if there is any difference between the wave-velocity and the group-velocity, it is with the group-velocity that the tidings of any recognizable astronomical event are transmitted to us.

will also be dispersion according to an identical law. Thus the general form of (15) does not appear to be due to the details of structure of the vibrating system, but rather to the circumstance that the system *has* a structure on a scale not infinitely minute compared with the wave-length λ . This conclusion, I think, is confirmed when we review the steps by which (15) was obtained.

14. Though we may conceive the electromagnetic vibrations of the æther to be executed in a manner widely different from anything occurring in these simple mechanical models, we may yet attempt to form some idea of the coarseness of structure of the æther which could suffice to account for such dispersions as Nordmann and Tikhoff have thought to be indicated by their observations. The conclusions reached are in any case quite tentative, and (as mentioned in § 17 below) they cannot be put forward without further reservations.

15. The first thing to be observed is that, in the case of our mechanical analogy, the group-velocity given by (16) is a direct function of the wave-length: the shorter the wave-length the less is the group-velocity. To this extent the indications afforded are in agreement with the observations of Nordmann and Tikhoff. In one set of measurements made by Nordmann on Algol, it appeared that the red rays (about wave-length 0.68μ) arrived 16 minutes in advance of the blue rays (about wave-length 0.43μ). Moreover, the parallax of Algol as given by Pritchard is $0''.0556$, corresponding to a distance of about 61 light-years. Thus if we assume provisionally that an equation not very different from (16) gives the relation between wave-length and group-velocity *in vacuo*, b then representing a linear magnitude which expresses the coarseness of structure of the æther, we find

$$b = 1.7 \times 10^{-8} \text{ cm.}; \quad . \quad . \quad . \quad . \quad . \quad (17)$$

or, say, $1/3000$ of a mean wave-length of visible light. From Tikhoff's observations we should obtain a smaller value for b —roughly

$$b = 3.1 \times 10^{-9} \text{ cm.}, \quad . \quad . \quad . \quad . \quad . \quad (18)$$

or $1/16,500$ of a mean wave-length of visible light.

16. The disparity between these numbers is very great, and arises from a still greater disparity between the values found for the dispersion of light in space, Nordmann's estimate exceeding Tikhoff's in the proportion of about 30 to 1. On this discrepancy Lebedew bases one of his arguments against the reality of the supposed dispersion; another point of view is gained by attempting to examine how far the value

of b given by (17) or by (18) might be reconcileable with what we know of the properties of the æther.

17. Sir J. J. Thomson's estimate* of the radius of a corpuscle (or negative electron) is 10^{-13} cm., and if we adopt this, we may hold it probable that any structure possessed by the free æther is on a scale considerably more minute; that is to say, that the linear magnitude corresponding to b , and serving to define the coarseness of structure, would be small compared with 10^{-13} cm. From such a standpoint, even the lesser value (18) deduced from Tikhoff's observations seems altogether excessive; though this conclusion is necessarily discounted by more than one source of uncertainty. In the first place the applicability of (16) depends on a rather elusive analogy, as pointed out in § 13; and, further, the value 10^{-13} cm. for the radius of an electron is unavoidably based on the assumption that, even up to an electromotive intensity of 5×10^{16} electrostatic units (1.5×10^{19} volts per cm.), the relation of electromotive intensity to electric polarization remains linear.

18. On the whole, in any medium transmitting disturbances of given type, it does not seem likely that the effect of coarseness of structure in producing dispersion would be very widely different from that expressed by (15) and (16); and even when very large allowances are made for the uncertainty regarding the actual radius of an electron, it appears that the dispersion announced by Tikhoff (to say nothing of Nordmann's much larger estimate) is of an altogether greater order of magnitude than could be accounted for by coarseness of structure of the æther. Lebedew† considers that any dispersion of light in free æther would be contrary to established principles of electromagnetism; it is not necessary to take so uncompromising a view in order to realize that there would be difficulty in explaining ætherial dispersion of anything like the degree which has been suggested. But for this reason all the more it is to be hoped that the observations will be extended, and that progress will be made in the delicate task of disentangling the relative retardations due to dispersion in space from any effects arising from the special nature of the star-systems observed. Apart from the astronomical importance of the question, any addition to our knowledge of the properties of the æther must be of the highest interest.

Kennington, near Oxford, June 5th, 1909.

* The estimate is provisional only, and is conditional on the whole inertia of the corpuscle being of electromagnetic origin. Cf. 'Conduction of Electricity through Gases,' 2nd ed. p. 655.

† *Loc. cit.*

XCIV. *On Ionization in various Gases.* By E. PARR METCALFE, B.Sc. (Lond.); 1851 Exhibition Scholar, Emmanuel College, Cambridge*.

[Plate XXIX.]

THE relative ionizations produced, under similar conditions, in a large number of elementary and compound gases, have been measured by several investigators†. The results which have been obtained seemed to suggest that the ionization per molecule may be the sum of the ionizations contributed by the atoms composing the molecules, the contribution of each atom being characteristic of that atom and not influenced by the manner of the chemical linkage or by the presence of the other atoms in the molecule. It was mainly with the object of gaining further evidence on this subject that the work now described was undertaken.

The system adopted was that of obtaining values for the ionization per molecule for members of complete sequences of three or more different compounds of the same two elements. The two sequences which first suggested themselves were O_2 , CO_2 , CO and N_2 , N_2O , NO . In these no values existed for the ionization in CO and NO . A beginning was accordingly made with these two compounds. The sequence next dealt with was that of the paraffin gases, values being obtained for the first four members.

Before proceeding with these systematic determinations, careful measurements were made of the ionization in hydrogen, by way of gaining experience of the working of the apparatus, and as a check on the accuracy of the method employed. Three values of this constant have already been recorded, two by Strutt and one by Bragg. With these the values now found are in fair agreement.

On the completion of the paraffin sequence, the apparatus was working so satisfactorily that a continuation was made with helium, hydrogen chloride, and bromine, in this order.

The ionizing agent used was the radiation from thin layers of uranium oxide; and the effect may be considered as being entirely due to the bombardment of the molecules of gas by α -particles; the influence of other radiation under these conditions is negligible in comparison with the experimental error.

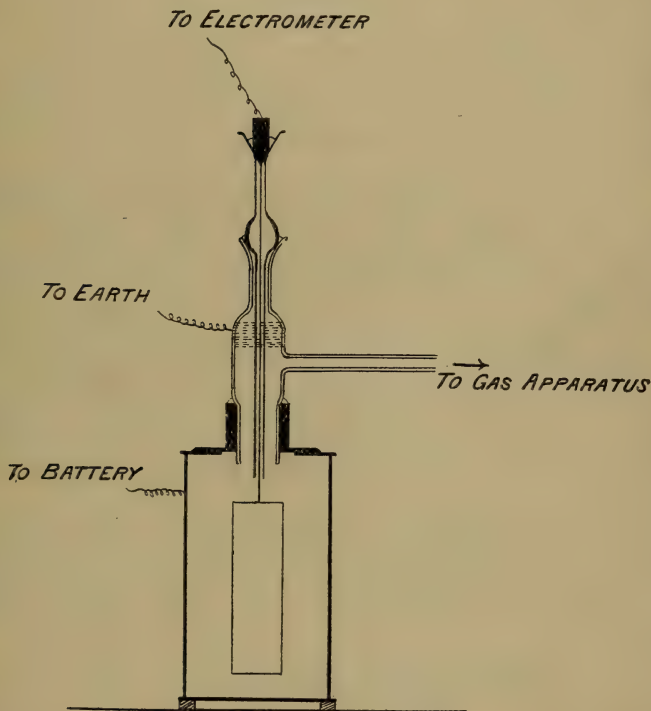
In assigning the apparatus for the work the writer's aim was to obtain as large an ionization-current as possible at low

* Communicated by Sir J. J. Thomson.

† Strutt, Phil. Trans. A. cxvi. p. 507, 1900; Roy. Soc. Proc. 1903, p. 208. Bragg, Phil. Mag. 1906, May, and 1907, March; also Trans. Roy. Soc. S. Australia, 1906. Kleeman, Roy. Soc. Proc. A. lxxix. p. 220. Laby, Roy. Soc. Proc. A. lxxix. p. 206.

pressures in a chamber of moderately small dimensions. The chamber was made of a piece of brass tube 7.5 cms. in diameter and 13 cms. long, closed at the lower end by a flat circular plate of brass. The upper end was partly closed by a similar brass plate, in which was cut a concentric circular hole 4 cms. in diameter. Both plates were soldered to the tube with silver, and the upper surface of the upper plate was "tinned" with ordinary soft solder. The hole in the upper plate permitted the entrance of the electrode, and was closed

Fig. 1.



by another smaller brass plate 6 cms. in diameter with a bevelled and "tinned" edge. This plate was also pierced with a concentric hole into which was hard-soldered a short length of iron gas-pipe. The upper end of the iron pipe was coned out and ground to fit a glass tube, furnished with a side tube through which to introduce the gas under experiment into the chamber. The upper end of the glass tube was drawn down and coned out so as to form a small cup, and into this cup was ground the bulged middle of a length of thin

silica tube. The upper end of the silica tube was blown out into a cup-shaped opening into which a small iron bung was ground to fit. These three joints were made tight with marine glue, and that between the glass and iron tubes immersed in mercury. Into the iron bung was screwed and soldered a thin steel rod which passed down the silica tube into the chamber, and supported a cylindrical electrode made of thin steel tubing 2.5 cms. in diameter and 9 cms. long (see fig. 1, p. 869).

Both the inner surface of the chamber and the outer surface of the electrode were coated with a very thin layer of uranium nitrate by spraying them while warm with a solution of the salt. A uniform coating having been deposited in this way, the temperature was raised to a red heat so as to drive off oxides of nitrogen. In this way a very thin and adherent layer of uranium oxide was formed on the curved surfaces of chamber and electrode.

This manner of supporting the radiating material possesses the advantage of subjecting the gas to bombardment from a large active surface, while the thinness of the coating tends to diminish the relative importance of the more penetrating β -radiation.

No attempt was made to separate the effects due to the α - and β -radiation respectively. The amount of ionization due to β -radiation is relatively very small; and moreover it appears from the experiments of R. Kleeman that the relative β -ionization in a gas is usually not very different from the relative α -ionization under the same conditions in the same gas.

The silica insulation proved very effective; but to reduce the chance of electric leakage to the electrode through its supports, bands of silver were deposited chemically on the glass tube, both inside and outside, and these were earthed.

The ionization-chamber was insulated on three ebonite blocks, and was connected to the positive pole of a battery of secondary cells, the other pole of the battery being earthed. Except where otherwise stated, the voltage used was 200. The electrode in the chamber was connected to an earthing-key and to one pair of quadrants of a Dolezalek electrometer; the other pair of quadrants was earthed. As usual the leads from chamber to electrometer, as well as the earthing-key, were enclosed in earthed metal sheaths. The electrometer-needle was connected to the 200 volt terminal of the battery. A low sensitiveness of the electrometer was

found to be suitable, *e. g.* about 150 mm. divisions per volt at 1 metre from the mirror.

The chamber communicated through the glass side tube, already referred to, with a manometer, a Toepler pump, and the gas-generating apparatus. All the gases dealt with were prepared by the writer from the purest materials obtainable (usually from Kahlbaum), and were carefully purified before use.

The manometer was of the U-type, one limb communicating with the chamber, while by a special arrangement the other could be periodically evacuated with the pump and then isolated. At first mercury was used in the manometer, but it was found that the ionization current in the case of air was unmanageably large except for pressures less than about 1 cm. of mercury. This did not render accurate pressure-readings easy; so the mercury was replaced by sulphuric acid. This liquid had the additional advantage of being without action on any of the gases used.

In nearly every measurement the procedure was as follows:—The chamber was evacuated to about $\frac{1}{100}$ mm. of mercury, and tests were made both of the tightness of the vessel and of the perfection of the insulation. The sensitiveness of the electrometer was ascertained by connecting to a Clark cell, and two indexes were placed on the semitransparent scale at a distance apart corresponding to the E.M.F. of the Clark cell. A little dry air was introduced into the chamber, which after a minute or so was re-evacuated. More dry air was slowly run in until the pressure rose to about 10 cms. of H_2SO_4 . The earth connexion to the electrode was then broken and the time taken for the spot of light to travel between the indexes on the electrometer-scale was measured with a chronograph watch. Several successive time-readings were taken for each pressure-reading; and when conditions were favourable a succession of three or four differing by not more than $\frac{2}{5}$ second was deemed sufficient for the accuracy desired. Such sets of measurements corresponding to each pressure were made for three or four pressures. The air was removed by the pump, any residual air being “washed out” with a little of the pure gas under experiment. The gas was then introduced and its ionization currents and corresponding pressures were measured in the same way: and in its turn it was removed and “washed out” with dry air, and a second set of air-readings was taken. Frequent observations of the temperature of the ionization-vessel were made by means of a thermometer placed in contact with its outer surface.

The relative molecular ionization in the gas is given by

$$\left(\frac{\text{Current in gas}}{\text{Pressure of gas}} \div \frac{\text{Current in air}}{\text{Pressure in air}} \right) \\ = \frac{1}{P_g T_g} \div \frac{1}{P_a T_a} \text{ or } \frac{P_a T_a}{P_g T_g},$$

where P_g , P_a are the pressures in gas and air respectively, corrected for temperature if necessary, and T_g , T_a represent the times taken for the electrometer-needle to swing through the range corresponding to the E.M.F. of a Clark cell. At the comparatively low pressures used it was unnecessary to allow for deviations from Boyle's law or for change in the capacity of the electrode.

In introducing the gas into the chamber the precaution was taken of adjusting its pressure so that its stopping power for α -radiation should be approximately the same as that of the air previously occupying it. The stopping power is calculated as being proportional to the sum of the square roots of the weights of the atoms composing the molecule of the gas (Kleeman). This precaution is well worth taking in view of the variation of the ionizing power of the α -particle at different parts of its range.

HYDROGEN.—The gas was prepared by the slow electrolysis of pure dilute H_2SO_4 . It was collected over the electrolyte and allowed to stand for some hours, being then passed in a slow stream through a train of purifiers containing successively moist caustic potash and strong H_2SO_4 : it was stored until needed (next day) over phosphorus pentoxide. Two sets of measurements were made, the sulphuric acid manometer being used for both. In these earlier experiments the closed limb of the manometer had been evacuated and sealed off from the pump. It was found that after three or four days a small quantity of dissolved gas was liberated from the H_2SO_4 , and that its pressure became sufficiently great to necessitate the application of a correction to the manometer indications. For this reason the mercury manometer was reverted to for the next gas (CO); but it was so unsatisfactory that an arrangement was made by which the turning of a double tap placed the closed limb of the manometer in communication either with the other limb, or with the pump, or isolated it altogether. In this way it was possible to remove any evolved gas which might have collected in the space above the liquid before each experiment. The results obtained for hydrogen are in very close agreement: they are also not far from the two values given by Strutt ($\cdot 213$ and $\cdot 226$) and that of Bragg ($\cdot 24$).

CARBON MONOXIDE.—Prepared by the action of strong sulphuric acid on sodium formate. The vessel containing the salt was evacuated and the sulphuric acid dropped in through a tap-funnel. The gas was passed slowly through a tube packed with moist caustic potash, and was kept for several hours in contact with phosphorus pentoxide. Two sets of determinations were made, the first with a mercury manometer, the second with the improved sulphuric acid manometer. For the reason already referred to the results of the second set are to be preferred to those of the first. Probably the number 1.00 represents the relative ionization in CO to within an error of about one per cent.

NITRIC OXIDE.—Prepared by the action of strong H_2SO_4 on a pasty mass composed of about four parts of pure crystallized FeSO_4 and one of KNO_3 with a little water. The gas was collected over a weak solution of KOH and when required was passed very slowly first through concentrated H_2SO_4 and then through a 20 cm. tube packed with P_2O_5 . Two sets of determinations were made with concordant results.

The Paraffin Group.

Methane.—Prepared by the action of the zinc-copper couple on an alcoholic solution of methyl iodide. The gas was passed in succession through an alcoholic solution of KOH and through concentrated H_2SO_4 , being finally dried over P_2O_5 . Two sets of measurements were taken. In the second an attempt to purify the gas still further by fractionation with liquid air was only partially successful, on account of the considerable vapour pressure of CH_4 at the temperature of boiling liquid air. A sufficient quantity of the fractionated gas was obtained for one determination. The number found in the second was slightly greater than in the first set of measurements.

Ethane.—Two sets of measurements were taken, and different methods were employed for preparing the gas. For the first set the ethane was obtained by the action of the zinc-copper couple on an alcoholic solution of ethyl iodide; for the second a solution of potassium acetate was electrolysed, the resulting mixture of ethane and carbon dioxide being passed through a solution of KOH to remove the latter gas. The ethane in both cases was further purified by fractionation, care being taken in the case of the second (electrolytic) sample to keep back higher hydrocarbons. There is a difference of nearly two per cent. between the mean results of the two sets. This is probably to be attributed to the presence of higher hydrocarbons in the first sample, so that the second value is to be preferred.

Propane.—Prepared by the action of nascent hydrogen (from zinc and hydrochloric acid) on isopropyl iodide. The propane mixed with hydrogen and a little air was passed in succession through alcoholic potash and strong sulphuric acid, and was collected over water. It was then drawn slowly from the reservoir through concentrated sulphuric acid and over P_2O_5 into a previously evacuated fractionating bulb immersed in liquid air. Here the propane, with impurities of higher boiling-point, was retained—the hydrogen and air being removed through the pump. Only the middle fractions of the propane were used.

Butane.—Prepared by the electrolysis of a solution of sodium propionate, the CO_2 being removed from the resulting mixed gases by passing through KOH. The gas was carefully dried and then fractionated, the middle fractions only being used.

Pentane.—The “total ionization” in this gas has been measured by Laby*. From his results the number 4.83 is deduced as the molecular ionization of pentane.

Helium.—Prepared by the writer by heating finely ground thorinite *in vacuo* in a specially made silica tube. The gas was purified in the usual way by passing through a liquid air-cooled charcoal tube. The only lines foreign to helium observed in the spectrum of the purified gas were those of mercury, which were so faint as only to be visible with a wide spectroscopic slit.

Ionization by collision has been shown† to be much greater in helium than in other gases under the same conditions. This is amply confirmed, qualitatively, by the writer’s experiments. Some little difficulty was experienced at first in determining whether the voltage used was sufficiently great to produce “saturation” without complicating matters by setting up ionizing collisions. By suitably adjusting the pressure satisfactory results were at length obtained.

It is interesting to note that the molecular α -ionization of helium is less than that of hydrogen, and is thus the lowest yet measured.

In view of the peculiar behaviour of helium, it is thought that a record of the actual measurements may be of interest. A table is accordingly appended giving them in the order in which they were taken, and the results are shown graphically in fig. 2 (Pl. XXIX.).

* Laby, *loc. cit.*

† See E. W. B. Gill and F. B. Pidduck, *Phil. Mag.* August 1908, p. 286.

TABLE I.—*Ionization in Helium.*

Gas in Chamber.	Voltage of Chamber.	Pressure Readings.	P. Pressure in cms. of H_2SO_4 .	T. Mean time of charging to E.M.F. of Clark cell.	$\frac{1}{P}$ Calculated Ionization for 1 cm. pressure.	Means of Ionization Values.	
Air	200 200	59.47 58.82	49.43 50.17	10.04 8.65	270.5 271.5	271	
Helium.....	200 200 160 120 80 80 80 120 160 200 200 160 120 160 200 200 120 80 40 200 120 200 40 80	55.21 58.29 58.29 58.29 58.30 64.33 64.34 64.34 64.34 71.57 71.57 71.57 77.78 77.78 77.78 77.78 77.78 77.78 77.78 82.63 82.63 82.63 85.74 85.74 85.74 85.74	54.25 51.00 51.00 51.00 51.00 44.62 44.62 44.62 44.62 36.92 36.92 36.92 30.37 30.37 30.37 30.37 30.37 30.37 30.37 25.10 25.10 25.10 21.87 21.87 21.87 21.87	.96 7.29 7.29 7.29 7.29 19.71 19.72 19.72 19.72 34.65 34.65 34.65 47.41 47.41 47.41 47.41 47.41 47.41 47.41 57.53 57.53 57.53 63.87 63.87 63.87 63.87	110.6 27.4 501 72.1 146.0 215.6 89.2 86.8 86.8 81.0 64.0 48.8 50.0 51.0 36.3 37.0 37.0 37.0 37.0 29.8 29.8 29.8 27.2 27.2 27.3 27.2	943 501 190 93.9 63.8 57.0* 58.2 62.6 79.2 59.0 57.7* 56.6* 58.2 57.0* 57.0* 57.0* 57.0* 57.0* 57.0* 58.3* 58.3* 58.3* 57.7* 57.7* 57.7* 57.7*	Mean of values selected (*) as being most free from influence of "collision- effect" = 57.4.
Air	200 200	61.51 60.22	46.98 48.49	14.53 11.73	25.1 31.4	275	

HYDROCHLORIC ACID.—From Strutt's experience of the behaviour of this gas in a brass ionization vessel, some difficulty was anticipated in dealing with it. When it was first run into the vessel the gas was absorbed very rapidly, and it was feared that the attempt to work with it must be abandoned. As a last resource the generating vessel, in which sulphuric acid was being dropped on to ammonium chloride, was placed in free communication with the ionization-chamber, and the gas was allowed to remain at rather more than atmospheric pressure in contact with the metal surfaces for about eighteen hours. It was found that the absorption had then ceased, and reliable readings were obtained without difficulty. The metal surfaces had presumably become coated with a protecting layer of chloride. That the uranium oxide had suffered some change is evident from the diminution of the air-constant of the chamber by some eight per cent.

BROMINE.—A specimen of the pure liquid was run through a tap-funnel into a previously evacuated vessel, in which it remained for some hours in contact with pure phosphorus pentoxide. From this the vapour was drawn as required into the ionization-chamber. The bromine seemed to be absorbed very slowly by the metal of the chamber; but after a short time it was found possible to maintain a sufficiently steady pressure to enable fairly reliable readings to be taken. The error of the determination is probably under two per cent.

The results obtained are given in the following table (p. 887).

The Additive Rule.

The relation between molecular ionization and chemical composition in the case of hydrogen and the first five paraffin gases is shown graphically in fig. 3 (p. 888). It will be seen that the correspondence between ionization and chemical composition is, in the case of this sequence, very close. The fact that hydrogen falls into line with the paraffins is somewhat remarkable, as hitherto the hydrogen atom has been regarded as exceptional in possessing a very different ionization value when in combination with other atoms to that which it has in the molecule H_2^* .

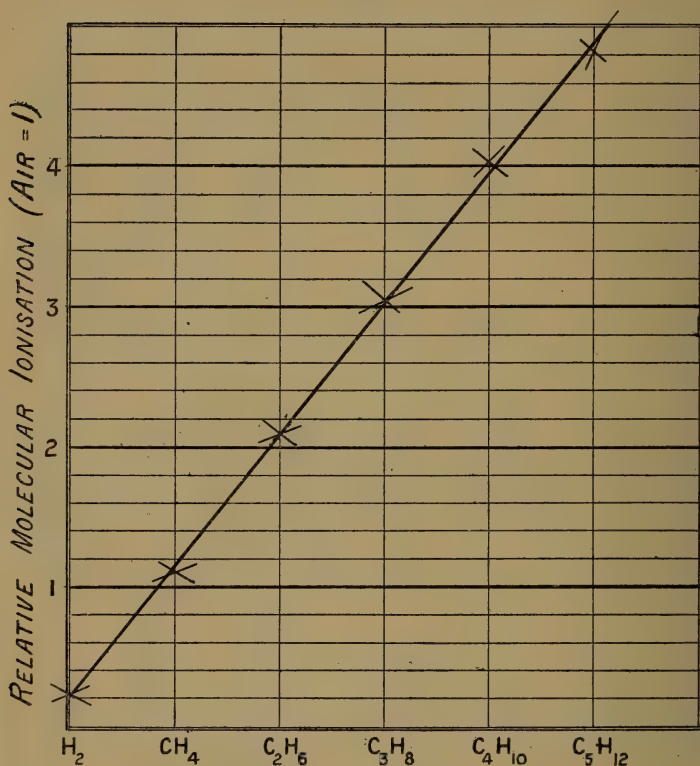
* Kleeman (Proc. Roy. Soc. A. vol. lxxix. 1907, p. 222) gives as the "combining" value of the atomic ionization of hydrogen the number 175, while its value in the molecule H_2 is 116.

TABLE II.

Gas.	Mean value found for relative molecular ionization.	Number of determinations averaged.	Range of pressures used, in cms. of H_2SO_4 .	Remarks.	Number adopted as best representing value of the relative molecular ionization
Air				Standard	1.00
Hydrogen. 1st set	.2325	3	43 to 28		.233
2nd set	.233	4	48 to 22		
Carbon monoxide 1st set	1.003	5	(4.5 to 0.7*)	*Mercury manometer ... H_2SO_4 manometer	1.00
2nd set	.995	4	13.8 to 4.2		
Nitric oxide. 1st set	1.28	5	12.3 to 4.5		1.28
2nd set	1.285	5	11.1 to 4.5		
Methane. 1st set	1.10	5	14.7 to 6.1		1.10
2nd set	1.105	1	5.07	After fractionation ...	
Ethane. 1st set	2.12	5	6.7 to 3.2		2.08
2nd set	2.08	5	8.4 to 3.7	Electrolytic.....	
Propane	3.05	4	2.9 to 3.5		3.05
Butane	4.02	3	2.6 to 3.0		4.02
Pentane				Laby's value	483
Helium	.211	6	20 to 64	See separate table	.211
Hydrogen chloride .	1.396	5	5.2 to 11.0		1.40
Bromine	3.94	6	4.4 to 2.9		3.9

In the case of the paraffins, the addition of ($C+2H$) to the molecule increases the molecular ionization by a mean amount $\cdot92$. If we deduct $\cdot23$ as being the contribution of the $2H$, we obtain as that of a single carbon atom $\cdot69$.

Fig. 3.



The values found by Kleeman and Bragg, and by the writer for the ionizations of the series O_2 , $2CO$, $2CO_2$, also indicate a linear relationship to composition.

Compound.	Ionization.	Difference.
O_2	1.15	} 44
CO_2	1.59	
$2CO$	2.00	} 41
	Mean ...	<u>.425</u>

Thus the contribution of the carbon atom to molecular ionization is, in this series of compounds, only $\cdot 42$.

The ionizations of three alcohols are given by Kleeman:—

Compound.	Ionization.	Difference.
CH_3HO	1.74	} $\cdot 72$
$\text{C}_2\text{H}_5\text{HO}$	2.46	
.....		} 1.94
$\text{C}_4\text{H}_9\text{HO}$	4.40	

Mean value of contribution of $(\text{C} + 2\text{H}) = \cdot 89$

Reference back to the number deduced for the contribution of the same group to the ionization of the paraffins (*i.e.* $\cdot 92$) shows good agreement.

But the ionization values of the two alkyl iodides CH_3I (3.43) and $\text{C}_2\text{H}_5\text{I}$ (4.00) indicate a contribution by the group $(\text{C} + 2\text{H})$ of only $\cdot 57$.

In the case of the series O_2 , 2NO , $2\text{N}_2\text{O}$, no simple additive law is followed.

The conclusions to be drawn seem to be (1) that the contribution of a constituent atom to the molecular ionization of the compound is not constant, but depends on the chemical bond or on the proximity of its companion atoms, or on both; (2) that in a series of compounds in which each member is formed by adding the same atom or group of atoms to the molecule of the preceding member, a sort of additive law is sometimes closely followed; (3) but that the contribution of the same atom or group of atoms in different series is not usually the same.

Before more definite statements can be made, still further data are required. A promising source of information would seem to be a systematic study of ionization in other simple homologous series such as those of the olefines and acetylenes. Such an investigation would prove useful in developing a subject which may add considerably to our knowledge of the mechanism of ionization.

The writer is under the greatest obligation to Professor Sir J. J. Thomson both for his interest in the work and for his kind permission to carry it out in the Cavendish Laboratory.

Bangalore, 18th Feb. 1909.

XCV. *The Reflexion of Light at an Ideal Plane Mirror moving with a Uniform Velocity of Translation.* By H. BATEMAN*.

THE question whether the motion of a mirror has any effect on the laws of reflexion and refraction was considered by Fresnel†, who came to the conclusion that no discrepancy need be expected. Stokes‡, examining the question in a more general manner, showed that the laws of reflexion and refraction are satisfied provided the directions of the rays are measured relative to the mirror and quantities of an order higher than the first, in the ratio of the velocity of the mirror to that of light, are neglected.

The question was brought into prominence in the discussions of the Michelson-Morley experiment, and is of fundamental importance in the theory of radiation§, where we have to consider the case in which the mirror is moving relatively to the observer.

An exact theory of reflexion is required for a rigorous discussion of the Michelson-Morley experiment, Wien's law, the pressure of radiation on a moving mirror, and the effect of motion on the temperature of radiation. The precise laws of reflexion have accordingly been discussed by Voigt¶, Lodge¶¶, Hicks**, Abraham††, Einstein‡‡, Planck§§, von Mosengeil||, and Kohl¶¶.

The object of this paper is to make a small addition to the results obtained by these writers. It will be convenient to use them in the form obtained by Hicks, Abraham, and Einstein.

Let c be the velocity of light in free æther, w the velocity of the mirror referred to a set of axes which are at rest relatively to the observer. Let v be the component of w along

* Communicated by the Author.

† *Annales de Chimie*, t. ix. (1818) p. 57.

‡ *Phil. Mag.* vol. xxvii. (1846) p. 76.

§ Larmor, *British Association Reports*, 1900, p. 657. *Encyclopædia Britannica*, article 'Radiation,' in vol. xxxii. (1903). Boltzmann, *Festschrift*, p. 590 (1904).

¶ *Göttinger Nachrichten*, 1887, p. 215.

¶¶ *Phil. Trans. A.* (1893) p. 793.

** *Phil. Mag.* [6] iii. (1902) p. 9.

†† Boltzmann, *Festschrift*, p. 85 (1904); *Ann. d. Phys.* [4] xiv. p. 236 (1904); *Theorie der Elektrizität*, vol. ii. p. 343.

‡‡ *Ann. d. Phys.* [4] xvii. (1905).

§§ *Theorie der Wärmestrahlung*, p. 71.

|| *Ann. d. Phys.* [4] xxii. (1907).

¶¶ *Ibid.* xxviii. p. 28 (1909).

the normal to the mirror, the positive direction of the normal being on the reflecting side of the mirror.

Take the axis of x along the normal and let the electric vector in the incident wave be given by

$$E = e \sin \nu \left[t + \frac{x \cos \phi + y \sin \phi}{c} \right],$$

where e is independent of (x, y, z, t) .

This represents a plane wave of light of frequency ν travelling in a direction opposite to that in which the mirror is moving. The normal to the wave front makes an angle $\pi + \phi$ with the normal to the mirror, so that the angle of incidence is ϕ .

The reflected wave is given by

$$E' = e' \sin \nu' \left[t - \frac{x \cos \phi' - y \sin \phi'}{c} \right],$$

where*

$$\left. \begin{aligned} \nu \cos \phi &= \nu' \left[-\frac{2cv}{c^2 - v^2} + \frac{c^2 + v^2}{c^2 - v^2} \cos \phi' \right] \\ \nu \sin \phi &= \nu' \sin \phi' \\ \nu &= \nu' \left[-\frac{2cv}{c^2 - v^2} \cos \phi' + \frac{c^2 + v^2}{c^2 - v^2} \right] \end{aligned} \right\} \quad (I.)$$

so that

$$\cos \phi = \frac{(c^2 + v^2) \cos \phi' - 2cv}{c^2 + v^2 - 2cv \cos \phi'},$$

$$\tan \frac{\phi'}{2} = \frac{c - v}{c + v} \tan \frac{\phi}{2},$$

$$\nu(c + v \cos \phi) = \nu'(c - v \cos \phi'),$$

$$\nu'' = \frac{\nu(c + v \cos \phi + u \sin \phi)}{\sqrt{c^2 - w^2}} = \frac{\nu'(c - v \cos \phi' + u \sin \phi')}{\sqrt{c^2 - w^2}} = \nu''',$$

where u is the component of w along the axis of y .

The last equation indicates that the apparent frequencies ν'' , ν''' of the incident and reflected rays, relatively to an observer moving with the same velocity as the mirror, are the same†.

* See the papers by Einstein and Hicks.

† Cf. Hicks and Einstein, *l. c.* We may show that the expression on the left-hand side represents the apparent frequency of the incident light by applying Lorentz's transformation so as to pass from a system moving with velocity w to a system at rest.

It will be noticed that the laws of reflexion depend only on the normal component of the mirror's velocity.

What I wish to point out is that these laws of reflexion can be obtained in a very simple way by supposing that the *image* of an object is derived by means of the space time transformation:

$$\begin{aligned}x' &= -\frac{c^2 + v^2}{c^2 - v^2}x + \frac{2c^2v}{c^2 - v^2}t, & y' &= y, \\t' &= \frac{c^2 + v^2}{c^2 - v^2}t - \frac{2v}{c^2 - v^2}x, & z' &= z.\end{aligned}$$

This transformation reduces to the one considered by Voigt *, Lorentz †, and Einstein ‡, if we change the sign of x' and put

$$U = \frac{2c^2v}{c^2 + v^2}.$$

Since

$$x' = x - \frac{2c^2}{c^2 - v^2}(x - vt), \quad t' = t - \frac{2v}{c^2 - v^2}(x - vt),$$

it is clear that at any time every point of the moving mirror ($x = vt$) is transformed into itself. The image of a point moving with the same velocity as the mirror is another point moving with the same velocity, for if dots denote differentiations with regard to t

$$\dot{x}' = \dot{x} - \frac{2c^2}{c^2 - v^2}(\dot{x} - v) = \dot{x},$$

$$\dot{y}' = \dot{y}, \quad z' = z,$$

$$t' = 1 - \frac{2v}{c^2 - v^2}(\dot{x} - v) = 1.$$

$$\therefore \frac{dx'}{dt'} = \dot{x}, \quad \frac{dy'}{dt'} = \dot{y}, \quad \frac{dz'}{dt'} \dot{z} = \dot{z}.$$

The image of a stationary point, on the other hand, is a point moving with the uniform velocity

$$U = \frac{2c^2v}{c^2 + v^2}.$$

* *Göttinger Nachrichten*, 1887, p. 41.

† *Amsterdam Proceedings*, 1904.

‡ *Ann. d. Phys.* xvii. (1905).

Further, since

$$x = -\frac{c^2 + v^2}{c^2 - v^2}x' - \frac{2c^2v}{c^2 - v^2}t',$$

we have for a given value of t'

$$x_1 - x_2 = -\frac{c^2 + v^2}{c^2 - v^2}(x_1' - x_2'),$$

$$y_1 - y_2 = y_1' - y_2',$$

$$z_1 - z_2 = z_1' - z_2'.$$

The image of a stationary object, therefore, suffers a contraction proportional to

$$\frac{c^2 - v^2}{c^2 + v^2} = \sqrt{1 - \frac{U^2}{c^2}}$$

times its length in a direction parallel to the axis of x . This is the proper contraction for velocity U according to the hypothesis put forward by FitzGerald* and Lorentz† to explain the negative result of the famous experiment made by Michelson and Morley.

To show that this transformation gives the correct laws of reflexion of a plane wave we introduce dashed letters into the argument of the periodic function of the reflected wave and make the necessary substitutions. The argument

$$\nu' \left[t' - \frac{x' \cos \phi' - y' \sin \phi'}{c} \right]$$

is then transformed into

$$\nu' \left[\frac{c^2 + v^2 - 2cv \cos \phi'}{c^2 - v^2} t + \frac{(c^2 + v^2) \cos \phi' - 2cv}{c^2 - v^2} \frac{x}{c} + \frac{y}{c} \sin \phi' \right],$$

and if we compare this with

$$\nu \left[t + \frac{x \cos \phi + y \sin \phi}{c} \right],$$

we obtain the relations (I).

The transformation can evidently be applied to waves of any form.

The formulæ connecting the components of the electric

* Public Lectures in Trinity College, Dublin.

† 'Versuch einer Theorie der Elektrischen Körpern' (1895). See also Larmor's 'Æther and Matter.'

and magnetic force in the incident and reflected waves are obtained by putting *

$$[+E_x' dy' dz' + E_y' dz' dx' + E_z' dx' dy' - cH_x' dx' dt' - cH_y' dy' dt' - cH_z' dz' dt'] \\ = E_x dy dz + E_y dz dx + E_z dx dy + cH_x dx dt + cH_y dy dt + cH_z dz dt,$$

where (E_x, E_y, E_z) (H_x, H_y, H_z) are the components of the electric and magnetic force respectively.

Calculating $E_x', H_x' \dots$ by means of the formulæ

$$+E_x' = E_x \frac{\partial(y, z)}{\partial(y', z')} + E_y \frac{\partial(z, x)}{\partial(y', z')} + E_z \frac{\partial(x, y)}{\partial(y', z')} - cH_x \frac{\partial(x, t)}{\partial(y', z')} \\ - cH_y \frac{\partial(y, t)}{\partial(y', z')} - cH_z \frac{\partial(z, t)}{\partial(y', z')},$$

we have

$$E_x' = +E_x, \quad H_x' = H_x, \\ E_y' = -\frac{c^2 + v^2}{c^2 - v^2} E_y + \frac{2cv}{c^2 - v^2} H_z, \quad H_y' = -\frac{c^2 + v^2}{c^2 - v^2} H_y - \frac{2cv}{c^2 - v^2} E_z, \\ E_z' = -\frac{c^2 + v^2}{c^2 - v^2} E_z - \frac{2cv}{c^2 - v^2} H_y, \quad H_z' = -\frac{c^2 + v^2}{c^2 - v^2} H_z + \frac{2cv}{c^2 - v^2} E_y.$$

These give

$$E_y' + vH_z' = -(E_y + vH_z), \\ E_z' - vH_y' = -(E_z - vH_y), \\ H_y' - vE_z' = -(H_y - vE_z), \\ H_z' + vE_y' = -(H_z + vE_y).$$

Relative to an observer moving with velocity v the components of the electric and magnetic force are

$$E_x, \quad \frac{E_y + vH_z}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \frac{E_z - vH_y}{\sqrt{1 - \frac{v^2}{c^2}}}, \\ H_x, \quad \frac{H_y - vE_z}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \frac{H_z + vE_y}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

Hence the above equations indicate that the sum of the tangential components of the two vectors in the incident and reflected waves is zero at the surface of the mirror.

* See a paper by the author, Proc. London Math. Soc. 1909.

The formulæ of transformation for the convection currents and volume density of electricity are obtained from the equation *

$$\begin{aligned} \rho' w_x' dy' dz' dt' + \rho' w_y' dz' dx' dt' + \rho' w_z' dx' dy' dt' - \rho' dx' dy' dz' \\ = \rho w_x dy dz dt + \rho w_y dz dx dt + \rho w_z dx dy dt - \rho dx dy dz, \end{aligned}$$

where $(\rho w_x, \rho w_y, \rho w_z)$ are the components of the convection currents, ρ the volume density of electricity. Thus

$$\rho' = - \frac{2v}{c^2 - v^2} \rho w_x - \frac{c^2 + v^2}{c^2 - v^2} \rho.$$

Since $(w_x) < c$ it appears that ρ' and ρ are of opposite signs.

The University, Manchester.

XCVI. *The Doppler Effect in Positive Rays in Hydrogen.*

By T. ROYDS, M.Sc., 1851 Exhibition Science Scholar†.

LITTLE is known of the cause of luminescence of the canal rays and of the origin of the discrepancy between their actual velocity and that calculated from the potential drop through which they have fallen. Some explanation of these would probably be obtained if the velocity of the positive rays at different parts of the discharge were compared with the distribution of potential, of electric force, and of the rate of change of the latter. Unfortunately there is no satisfactory method of isolating the different portions between the cathode and the negative glow for the observation of the Doppler effect. At one point of the discharge, however, a comparison is possible, namely, at the point where the cathode glow (Goldstein's "erste Kathodenschicht") commences. Since the canal rays mostly start from the negative glow, we can assume that the commencement of the cathode glow corresponds to the minimum Doppler effect when the cathode is viewed from the anode side. That the cathode glow near its commencement shows the Doppler effect was proved by placing the discharge-tube inclined to the axis of the collimator until light from only the first portions of the cathode glow fell on the slit, the portions nearer the cathode being screened off by the cylinder carrying the cathode. The

* Proc. London Math. Soc. (2), 1909.

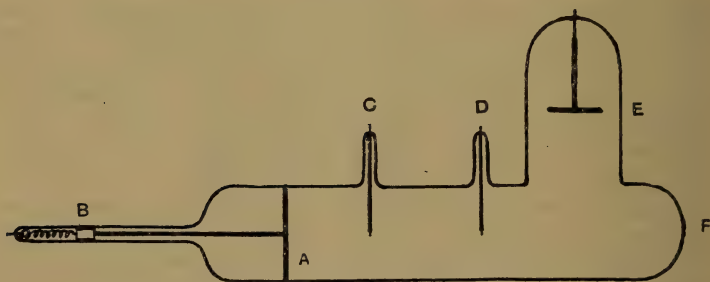
† Communicated by the Author.

length of the dark space was 5.0 cm., the distance between the negative glow and the cathode glow 2.2 cm., and light from the first centimetre of the cathode glow was focussed on the slit. After 20 hours' exposure with a cathode fall of 2800 volts, a marked Doppler effect was seen corresponding to the positive rays approaching the cathode.

When the gas (hydrogen) is pure and the pressure not too low the cathode glow commences at a rather sharply defined surface which can be fixed upon with reasonable accuracy. The potential fall from the negative glow to this surface has been compared with the minimum Doppler effect, *i. e.*, the distance between the stationary line and the nearest edge of the shifted line, the cathode (without holes) being viewed from the anode side.

The discharge-tubes had a diameter of from 4.5 to 4.8 cm. The potential in the discharge was measured by means of sondes of thin platinum wire inserted at right angles to the axis of the tube. A length of 2 mm. was exposed near the axis and the rest insulated by means of fine glass tubing.

Fig. 1.



The sondes were fixed in the tube, and the aluminium cathode A, fitting closely in the tube, could be made to approach the sondes by means of a magnet outside the tube attracting the iron piece B attached to the cathode. The sonde C gave the potential in the dark space, the sonde D remaining in the negative glow throughout which the potential is practically constant. This second sonde was introduced in order to show that the cathode fall was not altered by bringing up the cathode. The anode was placed in a side tube E in order to allow the cathode to be seen through the end F of the tube. The current was taken from a battery of accumulators with a maximum potential of 3000 volts and the potential measured by means of a calibrated Braun

electrometer. A telephone was inserted in the circuit and measurements only made when it was absolutely silent. Measurements were only possible before disintegration of the cathode set in, for as soon as a metallic film forms on the walls of the tube the discharge is altered by bringing up the cathode.

The values of the electric force and its rate of change along the discharge were obtained from the potential changes when the cathode was displaced a few millimetres on each side of the point where the value was required.

It is found that the commencement of the luminescence of the cathode glow seems to depend on the potential fall from the negative glow rather than on the values of the electric force and its rate of change at this point. The value of the potential fall to the commencement of the cathode glow is independent of the pressure but rises slowly according to a straight line law with the cathode fall. The electric force and its rate of change vary between wider limits. Some of the values obtained are given in the accompanying table.

TABLE I.

Cathode Fall.		800 volts.	1600 volts.	2400 volts.
Values at the commencement of the cathode glow.	Potential fall ...	150 volts.	260 volts.	360 volts.
	Electric force ...	100 volts/cm.	260 volts/cm.	500 volts/cm.
	Rate of change of electric force.	50 volts/cm. ²		300 volts ² /cm.

The Doppler effect was observed from the direction F (fig. 1) by focussing the cathode glow on the slit of a grating spectrograph. The grating was mounted in the manner described by Paschen*. The camera included from H_α to H_ζ , the dispersion varying from 9.90 to 9.15 Å.U. per mm. The density of the photographs was not very different for H_α , H_β , and H_γ . H_β was the strongest, H_α being sometimes weaker sometimes stronger with different batches of plates than H_γ .

The Doppler effect when the cathode without holes is viewed from the anode side has already been observed by Paschen†,

* Paschen, *Ann. d. Phys.* xxiii. 247 (1907).

† Paschen, *Phys. Zeits.* vii. 924 (1907).

and for the cathode with holes by Stark * and Trowbridge †. In the latter cases the Doppler effect is chiefly due to the canal rays behind the cathode which are seen through the holes.

It is of interest to note that the general appearance of the Doppler effect in front of the cathode is similar to that observed in the canal rays behind the cathode in that with values of the cathode fall between certain limits it consists of two strips. The extreme edge of the shifted line is not sharp as might have been expected. As noted by Paschen, the shifted line lies nearer the stationary one than in the case behind the cathode. It was observed by Paschen and by Trowbridge that there is, besides the Doppler effect of rays approaching the cathode, also one on the opposite side of the stationary line denoting a motion towards the anode. Paschen found that the latter effect was more intense than the former. These rays probably consist of the particles reflected on impact at the cathode surface, for when a holed cathode is focussed on the slit the Doppler effect of rays approaching the negative glow is weaker at the holes. The reflexion is probably diffuse, for the Doppler effect is smaller and the space between the stationary and shifted lines is not so dark; this effect may also be due to light reflected diffusely at the matted surface of the cathode.

On account of the feebleness of the light in front of the cathode, experiments were only made with high potentials and as strong a current as consistent with the safety of the discharge-tube. With a cathode fall of 2650 volts an exposure of 10 hours was found necessary to avoid under-exposure, which would cause the measurement of the minimum Doppler effect to be too large.

The photographs were measured by means of a Zeiss comparator. The cross-wires were set upon the edge of the stationary and shifted lines by eye. The densest portion was estimated photometrically with the arrangement described by Paschen ‡. Although it does not follow that the homogeneity of the light in the canal rays is the same as that in the stationary line, the minimum Doppler effect was measured from the farther edge of the stationary line, since the spectral width of the latter was small compared with the width of the slit-image. Even when the minimum was measured from the centre of the stationary line the *relative* differences in

* Stark, *Phys. Zeits.* vii. 747 Anm. (1906).

† Trowbridge, *Phil. Mag.* xvii. 250 (1909).

‡ Paschen, *Ann. d. Phys.* xxiii. 247 (1907).

the values for different spectral lines were made but a little smaller.

The observed values of the velocities are given in the following table.

TABLE II.

Cathode Fall = 2650 volts. Length of dark space 3.0 cm.

Velocity of rays approaching the cathode, cm./sec.			Velocity of reflected rays in cm./sec.	
	Minimum.	Densest portion.	Minimum.	Densest portion.
H _α ...	1.20 × 10 ⁷	1.41 × 10 ⁷	1.05 × 10 ⁷	1.21 × 10 ⁷
H _β ...	1.43	1.46	1.34	1.31
H _γ ...	1.49	1.40		

It is seen that the minimum velocity is not constant for different wavelengths, the differences being rather larger than the errors of measurement. Since H_α generally appears on the photographic plates weaker than H_β and stronger than H_γ, the result cannot be explained as a photographic effect. The minimum velocity is approximately inversely proportional to the square root of the wavelength, the product of these two quantities having the following values.

	$v\sqrt{\lambda}$ (rays approaching the cathode).	$v\sqrt{\lambda}$ (reflected rays).
for H _α	97.1	85.0
H _β	99.7	93.4
H _γ	98.1	

Stark and Steubing * have found this law to hold for the minimum velocity behind the cathode. In front of the cathode, although the light is fainter, there is the advantage that the edge of the shifted line is sharper.

The variations in the measurements of the velocity corresponding to the densest portion of the shifted line do not lie outside the limits of the errors of measurement.

* Stark and Steubing, *Ann. d. Phys.* xxviii. p. 974 (1909).

In the following table are given the ratios between the observed and calculated velocities assuming $\frac{e}{m} = 9654$. In column 1 the minimum velocity in front of the cathode is compared with the potential fall from the negative glow to the commencement of the cathode glow (400 volts for cathode fall of 2650 volts). Columns 2 and 3 refer to the canal rays behind the cathode with the same cathode fall of potential, these being less complicated than in front of the cathode since all rays have passed through the cathode fall.

TABLE III.
Ratio $\frac{\text{observed velocity}}{\text{calculated velocity}}$.

	Minimum in front of cathode.	Densest portion behind the cathode.	Maximum behind the cathode.
H α ...	·43	} ·36	} ·56
H β ...	·52		
H γ ...	·54		

It is seen that the ratio for the minimum in front of the cathode is about the same as for the maximum behind the cathode.

Since the minimum velocity in front of the cathode was found to be not constant for different wavelengths, measurements were also made for the canal rays behind the cathode. It was observed in agreement with Stark and Steubing that the minimum velocity was smaller for greater wavelengths. For the point of maximum density, however, the variations were within the limits of error, even for the outer strip observed with a cathode fall of 550 volts.

I wish to express my thanks to Prof. Dr. Paschen for his interest in these experiments and the numerous suggestions he has made.

Physikalisches Institut, Tübingen,
Aug. 4th, 1909.

XCVII. *Some Relations between the Critical Constants and certain Quantities connected with Capillarity.* By R. D. KLEEMAN, D.Sc., B.A., MacKinnon Student of the Royal Society; Emmanuel College, Cambridge*.

IN a recent paper† in the Philosophical Magazine the writer has modified Einstein's‡ formula for the potential energy of a surface of a liquid. The modified formula is

$$E = \frac{\kappa' \rho^2}{m} (\Sigma c_a)^2,$$

where E denotes the potential energy of unit surface, κ' is a constant which is the same for all liquids at corresponding states, ρ is the density, m the molecular weight, and Σc_a is the sum of a number of constants, each of which refers to an atom of the molecule and is independent of physical or chemical conditions. The writer has shown that if the above formula is true, there is a similar formula for the surface tension,

$$\lambda = \frac{\kappa'' \rho^2}{m^2} (\Sigma c_a)^2,$$

where λ is the surface tension, and κ'' a constant which is the same for all liquids at corresponding states. Both of these formulæ are shown to agree well with the experimental facts.

By means of the above and certain other relations given in the previous paper the following relations were deduced:—

$$\frac{L^{\frac{1}{2}} m^{\frac{7}{6}}}{\rho^{\frac{2}{3}}} = B \Sigma c_a, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\frac{T^{\frac{1}{2}} m^{\frac{2}{3}}}{\rho^{\frac{2}{3}}} = H \Sigma c_a. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

L denotes the internal heat of evaporation, and B and H are constants which are the same for all liquids at corresponding states.

Another relation similar in form to (1) and (2) will be deduced in this paper. $M \Sigma c_a$ will be expressed as a function of p , m , and ρ , where M is a constant similar to B and H.

The internal latent heat of evaporation of a liquid is given

* Communicated by the Author.

† Phil. Mag. Oct. 1909.

‡ *Annalen der Physik*, iv. p. 513 (1901).

by the well-known thermodynamical relation,

$$L + p(v_1 - v_2) = T(v_1 - v_2) \frac{dp}{dT},$$

where v_1 and v_2 are the volumes of one gram of the substance in the gaseous and the liquid states respectively, p and T denoting the pressure and temperature. For all liquids at corresponding states, v_1 and v_2 are both the same fraction of the critical density, or

$$v_1 = a_1/\rho_c, \quad v_2 = a_2/\rho_c,$$

and therefore

$$v_1 - v_2 = \frac{a}{\rho_c},$$

where a is the same constant for all liquids at corresponding states. Thus

$$L + p \frac{a}{\rho_c} = T \frac{a}{\rho_c} \frac{dp}{dT}.$$

Combining this with (1) we get

$$\frac{B^2(\Sigma c_a)^2 \rho^{\frac{4}{3}}}{m^{\frac{7}{3}}} + p \frac{a}{\rho_c} = T \frac{a}{\rho_c} \frac{dp}{dT},$$

or

$$\frac{\rho_c B^2}{T a} \frac{(\Sigma c_a)^2 \rho^{\frac{4}{3}}}{m^{\frac{7}{3}}} = \frac{-p}{T} + \frac{dp}{dT}.$$

Dividing each side of the equation by T and integrating, we obtain

$$C + \int \frac{\rho_c B^2}{T^2 a} \frac{(\Sigma c_a)^2 \rho^{\frac{4}{3}}}{m^{\frac{7}{3}}} dT = \frac{p}{T}.$$

Expressing the density and temperature in terms of the critical density and temperature, $\rho = \alpha \rho_c$ and $T = \beta T_c$, the integral becomes

$$\begin{aligned} & \frac{\rho_c^{\frac{7}{3}} (\Sigma c_a)^2}{m^{\frac{7}{3}}} \frac{T}{T_c^2} \int \frac{B^2 \alpha^{\frac{4}{3}}}{a \beta^2} d\beta \\ &= \frac{\rho_c^{\frac{7}{3}} (\Sigma c_a)^2}{m^{\frac{7}{3}}} \frac{1}{T} \left\{ \frac{\beta}{\alpha^{\frac{7}{3}}} \int \frac{B^2 \alpha^{\frac{4}{3}}}{a \beta^2} d\beta \right\}. \end{aligned}$$

The part within the brackets is the same for all liquids at corresponding states, it may be denoted by N .

The integrated equation becomes

$$TC + \left(\frac{\rho}{m} \right)^{\frac{7}{3}} (\Sigma c_a)^2 N = p.$$

To determine C put ρ , p , and T equal to their critical values. The equation may then be written

$$T \left\{ \frac{p_c}{T_c} - \frac{1}{T_c} \left(\frac{\rho_c}{m} \right)^{\frac{7}{3}} (\Sigma c_a)^2 N_c \right\} + \left(\frac{\rho}{m} \right)^{\frac{7}{3}} (\Sigma c_a)^2 N = p,$$

$$\text{or} \quad W (\Sigma c_a)^2 \left(\frac{\rho}{m} \right)^{\frac{7}{3}} = p - \frac{T}{T_c} p_c, \quad . \quad . \quad . \quad (3)$$

where W is the same for all liquids at corresponding states. Putting $p = \phi p_c$, which is approximately true, we have

$$p \left(\frac{\beta}{\phi} - 1 \right) = W (\Sigma c_a)^2 \left(\frac{\rho}{m} \right)^{\frac{7}{3}}.$$

If we denote by M the quantities which are the same for all liquids at corresponding temperatures, we have

$$p^{\frac{1}{3}} \left(\frac{m}{\rho} \right)^{\frac{7}{6}} = M \Sigma c_a. \quad . \quad . \quad . \quad (4)$$

At the critical state M is an indeterminate fraction, but its limiting value must be finite, because all the other quantities in the equation have perfectly definite values at the critical temperature.

This equation will now be applied to a number of liquids at their critical state. The values of m , p_c , and ρ_c are given in Table I. (p. 904). The values of c_a for the eight elements involved are obtained from the application of equation (1) * to the eight liquids, ether, methyl formate, carbon tetrachloride, benzene, fluor-benzene, bromo-benzene, iodo-benzene, and stannic chloride. It will be seen that M is approximately a constant for all liquids at their critical states. This shows that equation (4) is approximately true. The mean value of M is 116.8.

From the physical behaviour of the liquids, water, methyl alcohol, propyl alcohol, and acetic acid, it has been deduced that they are polymerized at ordinary temperatures. The surface-tension formulæ, in particular that of Eötvös, Ramsay and Shields, have not been found to apply to these liquids as the extent of polymerization, and therefore the molecular weight, is not known. In fact, the departure of the surface tension from that given by the formula of Eötvös, Ramsay and Shields, is regarded as evidence of polymerization, and the extent of it has been calculated by means of this formula.

* See previous paper, Phil. Mag. Oct. 1909.

TABLE I.

Values of c_a :—

H=1, C=5.299, O=5.939, F=5.757, Cl=8.401, Br=10.65, Sn=14.68, I=15.49.

Name and formula of liquid.	Critical pressure in atmospheres.	Critical density.	Molecular weight.	$M = \frac{pc^{\frac{1}{2}}}{\Sigma c_a} \left(\frac{m}{\rho c} \right)^{\frac{7}{6}}$.
Isopentane C_5H_{12}	32.92	.2343	72.10	119.1
Pentane C_5H_{12}	33.03	.2323	72.10	120.5
Hexane C_6H_{14}	29.62	.2344	86.11	116.9
Heptane C_6H_{16}	26.86	.2341	100.13	127.3
Octane C_8H_{18}	24.64	.2327	114.14	113.2
Di-isopropyl C_6H_{14}	30.72	.2411	76.0	127.4
Di-isobutyl C_8H_{18}	24.55	.2366	114.14	110.8
Fluor-benzene C_6H_5F	44.62	.3541	96.09	108.4
Bromo-benzene C_6H_5Br	44.62	.4853	157	119.3
Iodo-benzene C_6H_5I	44.62	.5814	203.9	119.0
Chloro-benzene C_6H_5Cl	44.62	.3654	112.5	117.7
Hexamethylene C_6H_{12}	39.8	.2735	84.09	115.0
Stannic chloride $SnCl_4$	36.95	.7419	260.8	117.6
Benzene C_6H_6	47.89	.3045	78	118.2
Ether $C_4H_{10}O$	36.28	.2604	74	118.1
Propyl formate $C_4H_8O_2$	42.7	.305	88	118.0
Ethyl acetate $C_4H_8O_2$	39.65	.2993	88	116.2
Methyl propionate ... $C_4H_8O_2$	39.88	.300	88	114.2
Propyl acetate $C_5H_{10}O_2$	33.17	.2957	102	108.7
Ethyl propionate $C_5H_{10}O_2$	34.64	.286	102	115.7
Methyl butyrate $C_5H_{10}O_2$	36.02	.291	102	115.4
Methyl isobutyrate ... $C_5H_{10}O_2$	33.88	.3012	102	107.6
Isobutyl formate $C_5H_{10}O_2$	38.29	.2879	102	120.6
Carbon tetrachloride CCl_4	44.97	.5576	154	121.4
Hydrochloric acid ... HCl	86.00	.610	36.5	116.8
Carbon dioxide CO_2	72.9	.464	44	100.7
Ethyl butyrate $C_6H_{12}O_2$	30.24	.276	116	113.6
Ethyl isobutyrate $C_6H_{12}O_2$	30.13	.276	116	113.4
Isobutyl acetate $C_6H_{12}O_2$	31.4	.281	116	113.3
Methyl valerate $C_6H_{12}O_2$	31.5	.279	116	114.5
Amyl formate $C_6H_{12}O_2$	34.12	.282	116	117.7
Monochlorinated chloride of ethyl $C_2H_4Cl_2$	53.0	.4516	99	124.7
Chloraldehyde $C_2H_4Cl_2$	50.0	.419	99	132.2
Methyl acetate $C_3H_6O_2$	47.54	.320	74	117.0
Ethyl formate $C_3H_6O_2$	48.7	.315	74	120.6
Methyl formate $C_2H_4O_2$	56.62	.3489	60	115.4
			Mean ...	116.8
Water H_2O	194.6	.2086	18	318.5
Methyl alcohol CH_4O	78.63	.2722	32	145.9
Propyl alcohol C_3H_8O	50.16	.2734	60	127.9
Acetic acid $C_2H_4O_2$	57.11	.3506	60	115.1

The writer found that formula (1), first given in the previous paper, did not apply to the first three of the above-mentioned liquids, and the discrepancy was explained in the same way. The four liquids have, therefore, been placed separate in Table I. It will be seen that the constants for water and methyl alcohol differ considerably from those obtained for the liquids in the first part of the table. The values of the constants for propyl alcohol and acetic acid are, however, approximately normal. Since polymerization decreases with increasing temperature it is probably small at the critical temperature, or perhaps zero. However, the polymerization at lower temperatures may modify the value of the critical constants, and thus produce a departure from the above formula.

The formula (2) was tested in the previous paper for different liquids at $\frac{2}{3}$ of their critical temperatures. In Table II. the same formula is tested for the critical temperatures. H appears to be approximately constant for all the liquids, and its mean value is 24.88.

The polymerized liquids are placed separately in the table. As before, they show considerable deviations from the law, and probably for the same reasons.

The equations (2) and (4) when referring to the critical state may be written

$$p_c = M^2 \left(\frac{\rho_c}{m} \right)^{\frac{7}{3}} (\Sigma c_a)^2,$$

$$T_c = H^2 \left(\frac{\rho_c}{m} \right)^{\frac{4}{3}} (\Sigma c_a)^2.$$

Combining these two we obtain

$$\frac{p_c}{T_c} = \frac{M^2}{H^2} \frac{\rho_c}{m},$$

or

$$p_c = \left(\frac{M^2}{H^2} \right) \frac{T_c \rho_c}{m},$$

which is the well-known empirical law of Young and Thomas, which states that the critical temperature, pressure, and volume are connected in the same way as are the temperature, pressure, and density for a perfect gas. Young and Thomas expressed their results in the form

$$p_c = \frac{R}{A} \frac{T_c \rho_c}{m},$$

where R is the usual gas constant and A a numerical quantity

equal to 3·70. Comparing this with the theoretical form of the equation, we see that

$$\frac{M^2}{H^2} = \frac{R}{A}.$$

On putting R equal to $82\cdot6 \times 10^6$ and the mean values for M and H from Tables I. and II., A is found to be equal to 3·66, which agrees well with the value given by Young and Thomas.

TABLE II.

Name of liquid.	Absolute critical temperature.	$H = \frac{T^{\frac{1}{2}}}{\Sigma c_a} \left(\frac{m}{\rho_c} \right)^{\frac{2}{3}}$	Name of liquid.	Absolute critical temperature.	$H = \frac{T^{\frac{1}{2}}}{\Sigma c_a} \left(\frac{m}{\rho_c} \right)^{\frac{2}{3}}$
Isopentane	460·8	25·41	Isobutyl formate	551·2	24·31
Pentane.....	470·3	25 25	Carbon tetrachloride ..	556·1	25·48
Hexane	507·8	24·82	Hydrochloric acid	325·3	29·41
Heptane	539·9	25·25	Carbon dioxide	304·3	21·13
Octane	569·2	24·56	Ethyl butyrate.....	565·8	23·97
Di-isopropyl.....	500·4	28·95	Ethyl isobutyrate.....	553·4	23·72
Di-isobutyl	543·8	23 73	Isobutyl acetate	561·3	23·59
Fluor-benzene	559·5	23·23	Methyl valerate	566·7	23·82
Bromo-benzene	670·0	25·72	Amyl formate	575·6	23·83
Iodo-benzene	721·0	25 58	Monochlorinated chloride of ethyl	561·4	27·43
Chloro-benzene	633·0	25·30	Chloraldehyde.....	523·0	27·83
Hexamethylene	553·0	23·36	Methyl acetate	505·9	23·82
Stannic chloride	591·7	25·03	Ethyl formate	503·0	25·38
Benzene	561·5	25·29	Methyl formate	487·0	25·77
Ether	467·4	25·04			
Propyl formate	533·8	24·56		Mean ...	24·88
Ethyl acetate	522·5	24·60			
Methyl propionate	528·7	24·68			
Propyl acetate	549·2	23·83	Water	637·3	62·05
Ethyl propionate	545·4	24·28	Methyl alcohol.....	513·0	34·37
Methyl butyrate	551·0	24·13	Propyl alcohol.....	536·7	28·25
Methyl isobutyrate	540·5	23·35	Acetic acid	594·6	28·40

If we apply equation (4) to liquids below their critical temperatures the agreement obtained is not so good. The reason appears to be that pressures at corresponding temperatures are only roughly equal to the same fraction of their critical values which was supposed to hold exactly in deducing equation (4) from equation (3). That the agreement is best at the critical state would then indicate that the pressures for corresponding temperatures become more nearly corresponding pressures as we approach the critical point. The equation (3) should therefore apply better to a liquid in the form in which it stands.

When the temperature is sufficiently below the critical

point, the value of p in equation (3) may be neglected in comparison with p_c . Omitting p and combining equation (3) with equation (1), we obtain

$$\frac{L\rho}{p_c} = U, \dots \dots \dots (5)$$

where U is a constant which is the same for all liquids at corresponding states. The value of $\frac{L\rho}{p_c}$ for values of L and ρ at temperatures equal to $\frac{2}{3}T_c$ is calculated for a number of liquids (Table III.). The constancy of $\frac{L\rho}{p_c}$ does not appear to be very good.

TABLE III.

Name of liquid.	p_c .	ρ .	L .	$U = \frac{L\rho}{p_c}$.
Di-isopropyl	30.72	.6237	69.39	1.41
Di-isobutyl	24.55	.6333	66.05	1.70
Pentane	33.03	.6057	75.47	1.39
I opentane.....	32.92	.6048	71.09	1.31
Hexane	29.62	.6167	72.61	1.51
Heptane	26.86	.6247	71.40	1.66
Octane	24.64	.6299	66.05	1.69
Fluor-benzene	44.62	.9233	69.71	1.44
Bromo-benzene	44.62	1.2792	48.84	1.40
Iodo-benzene.....	44.62	1.5347	40.79	1.40
Chloro-benzene.....	44.62	.9611	65.88	1.42
Hexamethylene	39.8	.7047	76.25	1.35
Stannic chloride	36.95	1.9597	27.11	1.44
Benzene	47.89	.7826	81.73	1.34
Ether	36.28	.6907	75.44	1.44
Propyl formate	42.7	.8304	78.35	1.49
Ethyl acetate.....	39.65	.8305	78.61	1.65
Methyl propionate	39.88	.8408	79.72	1.68
Propyl acetate	33.17	.8046	74.30	1.80
Ethyl propionate	34.64	.8071	74.38	1.73
Methyl butyrate	36.02	.8147	71.07	1.61
Methyl isobutyrate	33.88	.8102	69.74	1.67
Carbon tetrachloride	44.97	1.4385	39.87	1.28
Methyl acetate	47.54	.8741	89.08	1.64
Ethyl formate	48.70	.8658	86.93	1.55

The difference between the internal specific heat of a liquid and that of its saturated vapour (*i. e.* the heat capacity of unit mass less the work done in overcoming external pressure) can be thrown into a form which may be useful for some purposes by means of equation (1).

Let S' and S'' denote the internal specific heat of the

liquid and vapour respectively. When a gram of the liquid in contact with its saturated vapour is raised dT in temperature, the internal energy absorbed is $S'dT$. Let one gram of the liquid evaporate. The energy absorbed is $L+dL$, where L denotes the internal latent heat at the temperature T . Let the temperature be now lowered by dT , adjusting the pressure so that the vapour remains saturated. The internal energy given out by the vapour is $S''dT$. Let one gram of the vapour be now condensed, the heat given out is L . The substance has been taken through a complete cycle and the algebraic sum of the internal energy absorbed is therefore zero. Hence

$$S'dT + L + dL = S''dT + L,$$

$$\text{or} \quad S'' - S' = \frac{dL}{dT}.$$

Integrating,

$$\int_{T_2}^{T_1} S'' dT - \int_{T_2}^{T_1} S' dT = L_{T_1} - L_{T_2},$$

which may be written

$$S''_{T_1T_2} - S'_{T_1T_2} = L_{T_1} - L_{T_2}, \quad . \quad . \quad . \quad . \quad (6)$$

where $S'_{T_1T_2}$ and $S''_{T_1T_2}$ denote the change in the internal energy of a gram of liquid and of its saturated vapour respectively when the temperature is raised from T_1 to T_2 , L_{T_1} and L_{T_2} denote the latent heats at those temperatures.

When T_2 is the critical temperature, L_{T_2} is zero, and the equation becomes

$$L_{T_1} = S''_{T_1T_2} - S'_{T_1T_2}.$$

The values of L_{T_1} and L_{T_2} can be obtained from equation (1). Combining this with equation (6) we get the difference of the energies in terms of the quantities involved in equation (1). B can be found for any liquid, and is the same for all liquids at corresponding states, while the values of c_a for eight common elements are given at the head of Table I. This illustrates the application of equation (1).

The value of L may of course be deduced from the common thermodynamic equation given at the beginning of this paper when p , v_1 , v_2 , T , and $\frac{dp}{dT}$ are known. Cases may arise when the values of these quantities are not all known, but the values of m and ρ may be obtainable; in such cases L may be found from equation (1).

Cambridge, October 30, 1909.

XCVIII. *The Scattering of the β Rays of Radium.* By J. P. V. MADSEN, D.Sc. (Adel.), B.E. (Syd.), Lecturer in Electrical Engineering, University of Adelaide*.

[Plate XXX.]

§ I. *Introductory.*

IN a paper by the author upon the secondary γ rays† it was shown that in passing through matter the γ rays were scattered and softened. The scattered radiation showed a distinct lack of symmetry about a plane perpendicular to the direction of the original stream, more scattered radiation moving on in the direction of the original stream than was turned back. The distribution of the scattered radiation was found to depend upon the quality of the incident radiation and also upon the nature of the medium in which the scattering occurred.

As the results arrived at in that investigation were used as an argument in support of the material theory of γ rays proposed by Bragg, and as Crowther‡ has recently shown that the β rays are subject to scattering by even very thin layers of material, it became of special interest to see whether any parallel could be drawn between the effects of scattering in the case of the material β particles and the γ rays.

It will be seen from the present paper that the parallel is very close in many respects, the differences being such as might reasonably be expected on the theory that the γ ray is a neutral pair.

At the same time it is hoped that some of the results to be described may help to clear up some of the difficulties which have arisen in the study of the absorption of β rays.

§ II. *Apparatus.*

The apparatus used in these experiments is shown in fig. 1. The radium contained in a small conical hole cut in a piece of Al was covered by a sheet of Cu foil .002 cm. thick. The β rays passed up through a conical hole cut in a block of

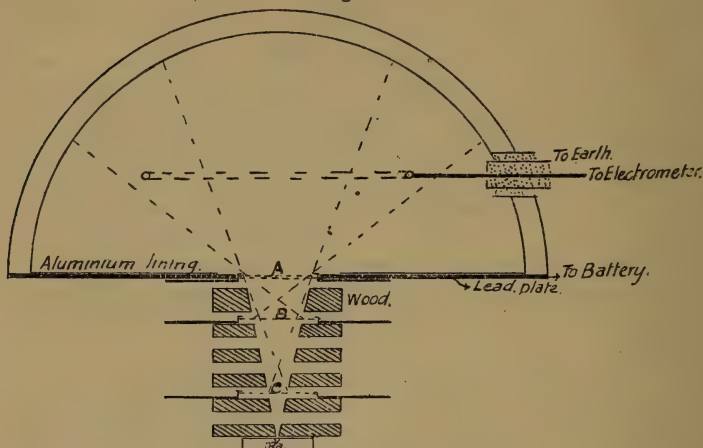
* Communicated by the Author. From 'Transactions of the Royal Society of South Australia,' vol. xxxiii. 1909. Preliminary Account read before the Australasian Association for the Advancement of Science, Brisbane, January 13, 1909.

† Trans. Roy. Soc. S.A. vol. xxxii. (1908).

‡ Proc. Roy. Soc., A, vol. lxxx. (1908).

wood, portions of the block being removed as shown to allow of the introduction of the screens in different positions as at A, B, C. The ionization-chamber was hemispherical and made of wood, with the inner surface covered with very thin

Fig. 1.



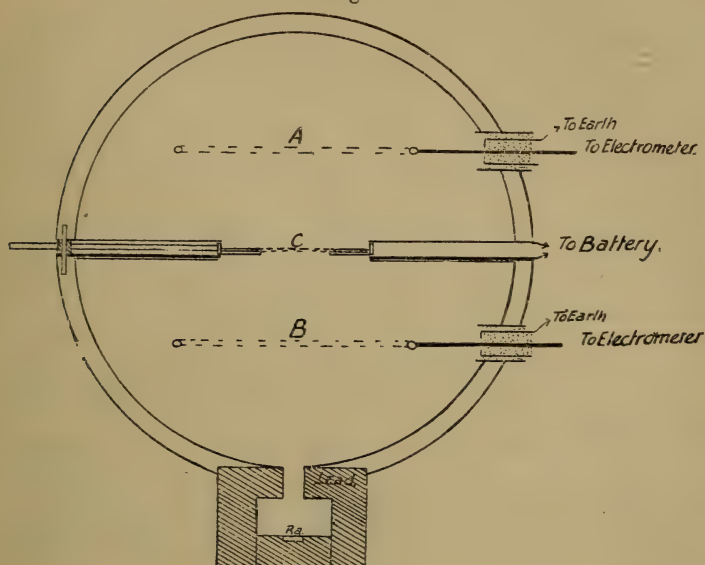
Al foil. The electrode connecting to the electrometer was in the form of a circular ring of wire, suitably protected by the usual methods. The hemispherical chamber rested upon a circular plate of Pb, above which was laid a sheet of Al. A circular hole cut centrally in the Pb and Al plates enabled the screen to be placed in the position A. In this position practically all the emergent scattered radiation was able to produce its effect to the same extent as the rays in the main stream, all rays having the same length of path in which to produce ionization, and the complications of secondary effects being reduced by having the walls of the chamber wood.

If we may for the present neglect any alteration in speed of the scattered radiation and consider the original stream of rays more or less homogeneous, the current may be taken approximately as a measure of the number of β particles which enter the chamber, no matter what their direction, proper correction being made for the effect produced by γ rays.

By subtracting the readings taken with a screen at A and at C a measure is obtained of the amount of radiation which has been turned out of its original path or scattered by that screen. Another reading with the screen at B enabled the distribution of the emergent scattered radiation to be followed out.

To obtain a measure of the returned, or incident, scattered radiation the apparatus shown in fig. 2 was used.

Fig. 2.



The top chamber, A, was the one already described, and a similar hemispherical chamber, B, was placed as shown with the Ra outside, contained in a Pb block provided with an opening through which the β rays could pass, impinging on screens placed in the position C. A stronger sample of Ra, kindly lent by Dr. Hermann Laurence, was used in these experiments, but care was taken to cover it with Cu foil, as in the first set of experiments. Either of the electrodes A or B could be connected to the electrometer, and as the chambers were made as nearly as possible alike no appreciable change in capacity was introduced, using either chamber separately. It was necessary to use a balance chamber, as the initial effect was so large compared with that which was to be measured. By placing a thin Al foil at C and then a thick Pb plate, a measure was obtained of the incident and of the maximum return radiation for that substance, from the effects measured separately in the chambers A and B. This enabled the readings for the incident scattered radiation to be reduced to their correct values relatively to those of the emergent rays, using the maximum return radiation from Pb as a standard of reference.

§ III. *Results of Experiments.*

Fig. 3 (Pl. XXX.) shows the results of experiments performed with the apparatus of fig. 1, using Al screens.

Curves D and E give the currents for different thicknesses of screen, with the screens in the positions A and C respectively. The abscissæ represent grammes per square cm. from which the thickness of screen may be immediately deduced, knowing its density.

Curve C is obtained by subtracting the values of D and E corresponding to any screen, and is a measure of the total amount of emergent scattered radiation.

It will be seen from fig. 1 that the whole of the scattered radiation is not quite included, as the effects are somewhat interfered with by geometrical conditions. When, for example, the screen was brought nearer the Ra than C a slight rise was observed in the reading. The intensity of the radiation falling on the screen was slightly increased owing to some of the more oblique rays from the Ra being now able to fall upon the screen.

Curves A and B represent the results of subtracting readings with the screen at B and C and A and B respectively (fig. 1), and are measures of the amount of radiation slightly deflected, and of that which has suffered much larger deflexion.

Curve F represents the returned radiation from aluminium screens of different thicknesses. Similar curves to the above are shown in fig. 4 for Au screens.

§ IV. *Discussion of Results.*

In fig. 3, from curve C, it is seen that the total emergent scattered radiation increases rapidly to a maximum, and then steadily decreases as the thickness of screen is increased. The maximum occurs at about $\cdot 013$ cm.

Comparing the curves C and F, it is seen that for thin screens the emergent is much greater than the incident scattered radiation. The greatest value of the ratio is about 9 : 1.

Comparing the similar curves for Au, fig. 4, it is again seen that a considerable lack of symmetry exists between emergence and incidence radiation, though not so marked. In this case the greatest value of the ratio is about 4.5 : 1. The maximum for the emergence radiation is reached at about $\cdot 0008$ cm.

The effects of scattering in the case of β rays are thus very similar to those observed for γ rays, a material of high

atomic weight being able to turn back in the process of scattering more of the original radiation than a material of smaller atomic weight.

Comparing curves A and B, it is observed that A reaches a maximum sooner than B. A more careful examination of A and B for smaller thicknesses of screen has shown that the ratio of A to B is practically constant until about one-third of the maximum reading is reached, after which the ratio gradually decreases. It would appear that while the ratio remains constant we are concerned with only a single collision of any β particle, that as the screen is further thickened it becomes possible for a β particle to suffer more than one collision before emerging, thus making the emergent beam appear to gradually swing round from its original direction, a greater thickness of screen being required to produce the maximum intensity for very oblique rays than for those corresponding more nearly with the direction of the original stream.

A fuller consideration of the effects of scattering and absorption for very thin films will be reserved for a future paper.

A theory of scattering similar to that proposed by Sir J. J. Thomson in 'Conduction of Electricity through Gases' seems capable of explaining the observed results. The nearness of approach of a β ray to a constituent of an atom will determine the amount and nature of the deflexion experienced, the speed of the β ray and the constitution of any particular atom being also necessary factors.

Until a β ray is subject to more than one collision the distribution is approximately constant for a given material, the intensity of the radiation deflected by an angle θ from the original direction being a function of that angle for any one material and with rays of a given quality.

We are to consider this function of θ as being different for the different atoms.

The lack of symmetry in the distribution of scattered X-rays has been shown by Bragg*, and assuming, as seems reasonable on many grounds, that X- and γ -rays are of the same nature, it appears from that investigation that the softer radiation shows less want of symmetry when falling on a given material than does the harder.

Now although the lack of symmetry shown by the scattered β rays is much greater than that found for γ - and X-rays, even though the former are less penetrating, the general nature of the effect has been shown to be much the same in

* Trans. Roy. Soc. S. A. vol. xxxii. (1908).

the case of all three, and the difference in magnitude may possibly be explained by the difference in distribution of the fields of the rays concerned.

Curves similar to C, figs. 3 and 4, have been obtained for Ag and paper; they show the same general characteristics. It is remarkable, however, that the maximum value of the curve C is very nearly the same for all the substances tested.

In a recent paper by McClelland* an account is given of the distribution of the returned β radiation from plates of different substances when the incident beam of radiation is inclined to the plate. The results seem capable of explanation, in view of the effects which have just been described, upon a theory of scattering without the need of introducing the idea of a true secondary radiation proceeding from the atoms affected by the incident β rays.

The general effect observed by McClelland is that the distribution of the returned radiation is more uniform for Pb than for Al. This is to be expected in view of the nature of distribution of the scattered rays from thin films of such substances as Au and Al, which has been described in the present paper.

From the results shown in figs. 3 and 4 it is at once seen that the effects of scattering may considerably modify the results obtained in the usual form of absorption experiment with β rays. The shape of the ionization-chamber and the positions of the screen and active material relatively to the chamber and to each other may produce considerable modifications in the results.

Again, in studying the absorption of β rays it would seem necessary to deal with very thin screens as is necessary in observing the effects of scattering; for thicker screens the results are likely to become considerably complicated.

It would seem almost better to replace the name of "absorption coefficient," as it is usually employed, by that of "transmission coefficient," reserving the former as a measure of effects which, as has been explained, can probably be obtained only from a study of very thin screens.

If the interpretation of the foregoing experiments be correct it seems that the β particle in traversing a thick screen may suffer many collisions and deflexions.

Now it has been shown by Allen (Phys. Review, Aug. 1906) that the secondary or reflected β radiation consists of electrons moving on the whole with a somewhat slower speed than the original radiation.

As the experiments described in the present paper indicate that in some cases these reflected electrons have suffered

* Proc. Roy. Soc., Series A, vol. lxxx.

many collisions before emerging, it would appear that the loss of energy due to a single collision is as a rule not very great, even though the effect of the collision may have produced a considerable change in the direction of motion of the electron. It is not surprising, then, that some of the returned rays have been found to have practically the same speed as some of the original rays; they would appear to be electrons which have suffered only one collision of sufficient violence to cause them to reverse their original direction of motion, or several minor collisions leading to the same result.

From the curves shown in figs. 3 and 4 (Pl. XXX.) it is seen that for small thicknesses of screen, before much actual absorption has occurred, the number of β rays turned back may be large, so that many of the original rays would appear to lose their energy gradually, rather than by a very sudden stoppage and complete absorption. Since the cathode rays behave in many respects like the β rays, it seems difficult to understand how the whole of the energy of the X-rays can be derived from the stoppage of the cathode particles, for, as pointed out by Professor Bragg*, the stoppage must be very sudden for this to be the case.

Summary.

Experiments with the β rays of radium support the results previously obtained by Crowther, using uranium, upon the scattering of the rays by thin films of materials.

The distribution of the scattered β rays is unsymmetrical, about a plane of right angles to the direction of the original stream.

A close parallel thus exists between the scattering of β rays and that of γ and X rays.

The shape of the so-called absorption curve may be modified by the shape of the ionization-chamber and the position of the screen and active material relatively to the chamber and to each other.

Absorption of a beam of β rays, combined with the effects of scattering and softening, seem sufficient to account for observed effects without the introduction of the idea of a true secondary radiation proceeding from the atoms affected by the primary stream of rays.

An electron appears to be able to suffer collisions, producing considerable change in its direction of motion, without any great loss of energy.

In conclusion, I wish to express my best thanks to Professor Bragg for the suggestions he has kindly given me from time to time during this investigation.

University of Adelaide, Jan. 5, 1909.

* Trans. Roy. Soc. S. A. vol. xxxi. (1907),

XCIX. *The Mechanism of the Adsorption ("Sorption") of Hydrogen by Carbon.* By JAMES W. McBAIN, M.A., Ph.D.*

ADSORPTION is quite variously regarded by different authorities as one of the following :—

- a. True chemical combination.
- b. True solid solution.
- c. A modified solid solution in which practically only the outer layers become saturated owing to the difficulty of diffusion in solids.
- d. Condensation on the outside of the surface of the solid.

The first two hypotheses, chemical combination or simple solid solution, are notoriously at variance with the requirements of thermodynamic theory. The third may be shown by mathematical analysis to be irreconcilable with the known laws of diffusion, as I expect to discuss in detail elsewhere, and it has further been directly refuted by my measurements below.

The fourth hypothesis is found insufficient to explain the somewhat complex time relationships studied here, which, however, point strikingly to the conclusion that both true solid solution (true diffusion) and surface-condensation occur † They are independent of each other, and their relative importance and magnitude depend upon the conditions of the experiment.

The non-committal name "*sorption*" may be coined to designate the sum of the phenomena, while "*absorption*" and "*adsorption*" should be restricted to proven cases of the solution and surface condensation respectively.

Experimental.

The experiments now to be described differ from all previous work in this subject in that the time factor has been taken into account and carefully studied. It has been found that surface-condensation or *adsorption* of hydrogen by carbon requires only a few minutes to attain equilibrium, even at the temperature of liquid air. On the other hand diffusion into the interior of carbon (*absorption*, solid solution) requires many hours for its completion.

* Communicated by the Author.

† Iodine "adsorbed" from solution by carbon behaves similarly, as was shown in this laboratory by O. C. M. Davis, Trans. Chem. Soc. (1907) xci. p. 1666. His communication was written with a knowledge of many of the results of the present paper.

Thus by suitable manipulation a sample of carbon can be prepared highly charged with hydrogen in a state of solid solution, but almost destitute of adsorbed hydrogen condensed on the surface. This is clearly attainable (if the hypothesis be correct) by suddenly exposing to a vacuum carbon which has been previously saturated by long contact with hydrogen at a constant temperature.

Such carbon, exposed to a low pressure of hydrogen and cut off from all external influences, took up hydrogen at first (surface-condensation) although already supersaturated (*i. e.* in respect to the solid solution) and then gave it off again in still greater quantity until final equilibrium was established. *Thus the manometer first fell and then rose to a higher point than the initial value (cf. Table VIII. below).*

In the converse case where the interior was free from hydrogen but the surface had become supersaturated by a very short exposure to a high pressure of gas, the hydrogen was first given off, and then taken up again by diffusion into the carbon. *Here the manometer automatically rose and then steadily fell to a lower value than previously obtained (cf. Tables II. to VII.).*

The experimental investigation consisted in modifications of these two types of experiments and included various tests systematically applied as checks. The results bear out the theory proposed in the most satisfactory manner.

The experiments were carried out in an apparatus identical with that which Travers had employed (see Proc. Roy. Soc. lxxviii. A. p. 9 (1906), where a figure is also given). The apparatus consists essentially of a bulb filled with carbon, connected with a mercury manometer by means of a capillary tube*. In addition various attachments allow measured quantities of gas to be introduced or removed by the aid of a Toepler pump.

The carbon was practically freed from gas by exposing it to a vacuum at a temperature of 440° C. (boiling sulphur), but to obtain a high vacuum in the presence of carbon requires very prolonged pumping indeed.

During the experiments the bulb was kept at the experimental temperature by means of a water-bath or more usually by immersing it in liquid air.

The mercury manometer was read by means of a telescope attached to a permanent rigid support. The eyepiece was

* A small plug of glass-wool was employed to prevent the carbon from rising into the capillary tube, in place of the asbestos used by Travers.

fitted with a micrometer adjustment by Hilger, on which 110 divisions corresponded to one millimetre. In the following tables these results, in micrometer scale-divisions, are given as hundredths of a millimetre as this does not affect the qualitative aspect of the experiments.

The gas was measured by means of a constant pressure gas burette surrounded by a water-jacket. The hydrogen, prepared by action of hydrochloric acid on zinc, was dried by phosphorus pentoxide and collected in tubes over mercury.

Carbon was prepared in large quantity by heating in a muffle the white tissues of the fruit of the coconut to a temperature just below redness for several hours, until no more fumes were evolved. The temperature was then allowed to rise to dull red heat for about thirty seconds, but without causing more vapour to be given off. Only the carbon taken from the interior of the mass was employed *. It was dried at 160° C. before weighing.

In the calculation of the results no allowances were made for change in the volume of the bulb or of the carbon, nor for the deviation of hydrogen from the gas laws. This, however, does not affect the present investigation which is mainly concerned with relative and not absolute measurements.

An idea of the quality of the carbon employed may be obtained from the amount of gas sorbed by it in actual experiment. Thus in one case, the weight of carbon was 2.65 g., the amount of hydrogen in the apparatus was 63.15 c.c. (corr.), under a pressure of 10.0 mm. Since the volume of the dead space in the connexions and capillary tubing amounted to 1.79 c.c. and that of the bulb immersed in liquid air to 18.44 c.c., the amount of unsorbed gas was 0.82 c.c. (corr.). Hence 2.65 g. of carbon used in the experiment had sorbed 62.33 c.c. (corr.) of hydrogen, or 5.607 mg., that is 0.21 per cent. by weight. This value is over three times as great as that obtained by Travers, whose specimen of carbon sorbed at a slightly higher pressure an amount equal to 0.065 per cent. at 12 mm.

Time required for Saturation.

The time required for complete equilibrium between carbon and hydrogen at any definite pressure could not be accurately measured at low temperatures, because the temperature of

* Davis, *loc. cit.* pp. 1678 and 1682, used some of this carbon in his experiments. It will be noted that my results show that it is highly sorbent towards hydrogen while Davis's measurements showed that it sorbed relatively very little iodine from solution.

liquid air employed to surround the carbon bulb was not constant. The rise in temperature is very slow, as Travers has already mentioned * ; but after about a dozen hours it begins to compensate for the fall in pressure due to the slow completion of the absorption of the gas. Thus in all cases the pressure begins to rise after ten or twelve hours, due to the evolution of gas from the carbon with rising temperature.

The two experiments quoted below give an idea of the rate of sorption of hydrogen. In both cases the gas was placed in the apparatus at room temperature, and the carbon bulb then immersed in liquid air.

TABLE I.

Time.	Temperature.	Pressure.	
		I.	II.
0 minutes.	Room temp.	600 mm.	1500 mm.
0 "	Liquid air.	—	—
2 "	" "	1·57 mm.	20·41 mm.
4 "	" "	1·47 "	19·80 "
6 "	" "	1·39 "	19·44 "
8 "	" "	1·33 "	19·32 "
11 "	" "	1·22 "	19·15 "
40 "	" "	1·20 "	18·72 "
2½ hours.	" "	1·12 "	18·35 "
9 "	" "	0·94 "	—
Next day.	" "	1·00 "	19·70 "

These experiments demonstrate the extreme rapidity with which the main amount of the gas is taken up (chiefly surface condensation), and further that the absorption of gas extends over at least a dozen hours, a fact which cannot be attributed to temperature changes. It will be noted that the carbon had been previously in contact with the gas at room temperature at a hundred-fold pressure, and that this did not prevent the slow diffusion effect (see p. 930).

Supersaturated Surface, Empty Interior.

These experiments are considered to be the most important described in the present paper, and they were repeated a dozen times with entirely consistent results. The procedure was as follows:—Carbon freed from gas by exposure to a vacuum was cooled in liquid air for at least an hour. A measured amount of hydrogen was then admitted, and after a few minutes a measured portion removed. The apparatus

* Travers, *loc. cit.* p. 14.

was then left to itself and pressure readings were taken at suitable intervals extending over the next twenty-four hours.

Now after carbon has been exposed for a few minutes to gas the surface-condensation comes into equilibrium with the pressure of the gas. Immediately following the reduction of the pressure by removal of some of the hydrogen from the apparatus, the surface was in a state of supersaturation and quickly threw off gas, and thus the pressure actually rose for a few minutes in spite of the fact that the interior of the carbon was nearly destitute of dissolved hydrogen. But after the first few minutes the diffusion into the interior which had been going on continuously, once more resumed the upper hand, and the pressure now fell for hours. Three experiments are cited in illustration of this. In Table II. the amount of liquid air was insufficient and the rise in temperature was rapid and continuous.

TABLE II.

Remarks.	Time.	Pressure.	Difference.
Put in 77.55 c.c. (corr.) H_2 ...	0- $\frac{1}{2}$ minute.	49.0 mm.	—
Took out 39.28 c.c. (corr.) ...	$4\frac{1}{2}$ -14 minutes.	—	—
	16 $\frac{1}{2}$ "	8.23 mm.	—
	18 $\frac{1}{2}$ "	8.35 "	+0.12 mm.
	20 $\frac{1}{2}$ "	8.47 "	+0.24 "
	25 $\frac{1}{2}$ "	8.50 "	+0.27 "
	31 "	8.48 "	+0.25 "
	42 "	8.46 "	+0.23 "
	70 "	8.42 "	+0.19 "
	138 "	8.41 "	+0.18 "
	19 hours.	10.14 "	+1.91 "

TABLE III.

Remarks.	Time.	Pressure.	Difference.
Put in 71.20 c.c. (corr.)	0- $\frac{1}{2}$ minute.	—	—
Took out 25.13 c.c. (corr.) ...	3-11 $\frac{1}{4}$ minutes.	—	—
	13 "	8.70 mm.	—
	14 $\frac{1}{2}$ "	8.78 "	+0.08 mm.
	16 "	8.80 "	+0.10 "
	18 "	8.82 "	+0.12 "
	20 "	8.75 "	+0.05 "
	27 "	8.64 "	-0.06 "
	42 "	8.56 "	-0.14 "
	105 "	8.43 "	-0.27 "
	285 "	8.38 "	-0.32 "
	22 hours	9.72 "	+1.02 "

TABLE IV.

Remarks.	Time.	Pressure.	Difference.
Put in 71.59 c.c. (corr.)	0-1 minute.	29 mm.	—
Took out 28.40 c.c. (corr.) ...	4-14 minutes.	—	—
	15½ "	7.51 mm.	—
	17½ "	7.57 "	+0.06 mm.
	19½ "	7.59 "	+0.08 "
	21½ "	7.61 "	+0.10 "
	23½ "	7.60 "	+0.09 "
	25½ "	7.58 "	+0.07 "
	27 "	7.57 "	+0.06 "
	50 "	7.47 "	-0.04 "
	5 hours.	7.20 "	-0.31 "
	18 "	7.28 "	-0.23 "

These results show that the initial rise of pressure and the subsequent fall are quite real. Before discussing the extent of their significance it may be well to point out under what conditions this effect may be masked.

It is clear that diffusion into the interior must set in immediately on contact of the gas with the carbon, and that it will be most active at first. For not only is diffusion quickest at first (as shown by the laws of diffusion), but in addition the pressure is at first much greater than after a fraction of the gas has been pumped out from the apparatus in the course of the next few minutes. Thus if the initial exposure at the relatively high pressure be too prolonged, or if not sufficient gas to cause a very decided change in the pressure be pumped out, or further if the gas is removed too slowly, the chief phenomenon observed will only be the steady fall of pressure due to the absorption of the hydrogen by the carbon. This is shown in the following three experiments.

TABLE V.

Remarks.	Time.	Pressure.	Difference.
Put in 77.55 c.c. (corr.)	0-1 minute.	48.0 mm.	—
Took out 24.76 c.c. (corr.) ...	2-6 minutes.	—	—
	7½ "	15.53 mm.	—
	9½ "	15.56 "	+0.03 mm.
	11½ "	15.50 "	-0.03 "
	13½ "	15.40 "	-0.13 "
	17½ "	15.26 "	-0.27 "
	21½ "	15.15 "	-0.38 "
	26½ "	15.06 "	-0.47 "
	39 "	14.83 "	-0.70 "
	83 "	14.43 "	-1.10 "
	6½ hours.	13.61 "	-1.92 "
	21 "	14.62 "	-0.91 "

TABLE VI.

Remarks.	Time.	Pressure.	Difference.
Put in 71.20 c.c. (corr.)	0 - $\frac{1}{2}$ minute.	29.0 mm.	—
Took out 12.49 c.c. (corr.) ...	$3\frac{1}{2}$ - $5\frac{1}{4}$ minutes.	—	—
	7 "	18.48 mm.	—
	9 "	18.33 "	-0.15 mm.
	11 "	18.11 "	-0.37 "
	$16\frac{1}{2}$ "	17.78 "	-0.70 "
	22 "	17.46 "	-1.02 "
	$37\frac{1}{2}$ "	17.21 "	-1.27 "
	$54\frac{1}{2}$ "	16.66 "	-1.82 "
	95 "	16.25 "	-2.23 "
	212 "	15.85 "	-2.63 "
	275 "	15.72 "	-2.76 "
	6 hours.	15.75 "	-2.73 "
	19 "	15.98 "	-2.50 "
	23 "	16.09 "	-2.39 "

TABLE VII.

Remarks.	Time.	Pressure.	Difference.
Added 75.52 c.c. (corr.)	0-1 minute.	—	—
	3 minutes.	31.0 mm.	—
	4 "	29.7 "	-1.3 mm.
Took out 5.69 c.c. (corr.).....	$4-4\frac{1}{4}$ "	—	—
	$5\frac{1}{2}$ "	25.18 mm.	—
	$7\frac{1}{2}$ "	25.05 "	-0.13 mm.
	9 "	24.61 "	-0.57 "
	13 "	24.17 "	-1.01 "
	$18\frac{1}{2}$ "	23.76 "	-1.42 "
	$33\frac{1}{2}$ "	22.84 "	-2.34 "
	73 "	21.52 "	-3.66 "
	117 "	20.74 "	-4.44 "
	4 hours.	20.56 "	-4.62 "
	7 "	19.93 "	-5.25 "
	8 "	19.20 "	-5.98 "
	$8\frac{1}{2}$ "	19.09 "	-6.09 "
	$10\frac{1}{2}$ "	18.98 "	-6.20 "
	$12\frac{1}{2}$ "	19.01 "	-6.17 "
	$28\frac{1}{2}$ "	20.43 "	-4.75 "
	45 "	24.01 "	-1.17 "

The importance of these pressure changes may now be discussed, for at first sight their magnitude appears to be insignificant. Thus in Table VII. the highest pressure recorded is that at $5\frac{1}{2}$ minutes, namely 25.2 mm., and the lowest at $11\frac{1}{2}$ hours, 19.0 mm. The high pressure indicated that out of the total 69.83 c.c. (corr.) of hydrogen in the apparatus 66.97 c.c. (corr.) was taken up by the carbon, and

at the lowest pressure 67.70 c.c. (corr.); that is a difference of 0.73 c.c., or just over 1 per cent.

But attention must be drawn to the one feature of great significance, viz. that the higher pressure at $5\frac{1}{2}$ minutes was even less than corresponded to the gas condensed on the surface of the carbon, and yet after a dozen hours had elapsed a much lower pressure was attained, a pressure which then actually did correspond to the condensed gas in equilibrium with it. *Thus a considerable body of hydrogen has been transferred from the surface to the interior of the carbon.*

The extent of this transfer can be at least approximately calculated. The calculation may be based upon the assumption that at the beginning all the gas is on the surface, and that only after many hours does the interior solid solution reach its normal concentration.

The pressure at $5\frac{1}{2}$ minutes (*e. g.* Table VII.) is certainly too low to correspond to the surface condensation alone (although it must be taken for the calculation), for there is considerable opportunity for diffusion to take place, even under the best circumstances. The approach to the value it would have were all the gas condensed in the surface and none in the interior, is clearly greater the sooner the pressure reading is taken after the removal of the gas, and the less amount removed from the apparatus and the shorter the initial exposure of the carbon to the gas at high pressure.

The minimum pressure observed later on in the experiment requires two corrections. In the first place the value is higher than it would have been had no new gas passed into the carbon out of the bulb and manometer. This can be allowed for by using the equilibrium formula found by Travers *, $\sqrt[3]{p} = kx$, where p is the pressure, and k a constant, and x the percentage of hydrogen sorbed by the carbon. In the second place the liquid air of the thermostat has been evaporating, its composition varying, and consequently the temperature slowly rising; a correction for this was made by blank experiments.

If the calculation be carried out for the experiment recorded in Table VII. an observed minimum pressure of 19.0 mm, is found. The temperature correction leaves approximately 17.9 mm. The total amount of sorbed gas is 67.70 c.c. (corr.), of which 0.73 c.c. passed into the carbon from the bulb and manometer spaces while the diffusion was taking place. Hence the true minimum pressure should

* Travers, *loc. cit.*

have been $17.9 \left(\frac{67.70}{69.83} \right)^3 = 16.3$ mm. The maximum pressure was 25.2 mm. If the (equilibrium!) formula $\sqrt[3]{p} = kx$ be employed a value is obtained for the ratio of the amount of gas on the surface at the end of the exponent to that on the surface at the beginning $\sqrt[3]{\frac{16.3}{25.2}} = 86.5$ per cent. approximately.

Hence 13.5 per cent. of the final sorbed gas is in a state of solid solution. 13.5 per cent. $\times$ 67.70 c.c. (corr.) = 9.1 c.c. (corr.) for 2.293 g. carbon. That is, the solubility of hydrogen in coconut carbon is roughly 4.0 c.c. (corr.) per gram at a pressure of 19 mm. and at the temperature of liquid air. These values are certainly too small, for the maximum pressure is not entirely due to surface-gas, and the minimum pressure would have still further decreased with time at a truly constant temperature.

Table VI. gives the total amount of gas sorbed by the carbon at 7 and 275 minutes respectively, as 56.70 c.c. (corr.) and 57.02 c.c. (corr.). The minimum pressure is 15.7 mm., and after temperature corrections 15.5 mm., and this corrected for the new gas taken up becomes 15.2 mm. The ratio between the amounts of surface hydrogen at the end and the beginning is therefore 93.7 per cent. Since 2.293 g. carbon was used the dissolved hydrogen is 1.6 c.c. (corr.) per gram under a pressure of 15.5 mm. If the dissolved hydrogen be proportional to the square root of the pressure (see later) the value calculated for 19 mm. from the above results amounts to 1.8 c.c. (corr.). From the conditions of the experiments the result obtained from Table VII. must be very much nearer the true value. Consequently, it appears that the order of magnitude of the solubility of hydrogen at 19 mm. at the temperature of liquid air is 4 c.c. (corr.) per gram of coconut carbon.

Supersaturated Interior. Bare Surface.

It appeared to the author that the best check upon the explanation of the qualitative phenomena studied above would be to attempt to produce the pressure changes in the inverse sense. This is possible if the carbon by long contact is allowed to become saturated with hydrogen. Now when this carbon is quickly exposed to a much lower pressure the surface at once gives up its hydrogen, while the diffusion of the dissolved gas requires time.

Thus we have carbon with empty surface and supersaturated

interior. If a small amount of gas be then quickly introduced before this very unstable condition has disappeared, the interior is still surcharged with respect even to the new (intermediate) pressure, and it will continue to give off gas. But this effect is masked for a few moments because the surface is not saturated with gas at the new pressure, and consequently the hydrogen very rapidly condenses to make up the deficiency.

It is clear that the cycle of changes must be rapidly carried through, and the following experiment shows how well the predictions are confirmed. The carbon had previously stood in liquid air, and exposed to 62.84 c.c. (corr.) of hydrogen, for twenty-four hours. After this the liquid-air vessel was filled up to the top once more, about an hour before the experiment began.

TABLE VIII.

Remarks.	Time.	Pressure.	Difference.
Pumped out 14.72 c.c. (corr.).	0 minute.	22.0 mm.	—
Added 4.70 c.c. (corr.).	0-4 $\frac{1}{4}$ minutes.	—	—
	4 $\frac{1}{2}$ "	—	—
	6 $\frac{1}{2}$ "	14.59 mm.	—
	8 "	14.54 "	-0.05 mm.
	10 "	14.58 "	-0.01 "
	12 "	14.63 "	+0.04 "
	14 "	14.66 "	+0.07 "
	25 "	14.78 "	+0.19 "
	47 "	14.87 "	+0.28 "
	2 hours.	15.03 "	+0.44 "
	5 $\frac{1}{2}$ "	15.42 "	+0.83 "
	20 "	16.51 "	+1.92 "

The characteristic phenomena of the previous experiments have here been reversed (in the most thoroughgoing manner), and yet the results are entirely in accordance with the hypothesis that has been advanced.

Blank Experiment.

The complexity of the pressure changes described in the foregoing paragraphs makes it desirable to be certain that these complications do not occur where not predicted by the theory of the process which I have advanced. In the following experiment a portion of the gas was removed from the apparatus after the carbon had been standing for a day in contact with it at the temperature of liquid air. The total

amount of hydrogen was 71 c.c. (corr.) of which 19 c.c. (corr.) was removed.

TABLE IX.

Remarks.	Time.	Pressure.	Difference.
Removed gas	0-7½ minutes.	—	—
	8 "	8.43 mm.	—
	10 "	8.51 "	+ 0.08 mm.
	12 "	8.55 "	+ 0.12 "
	14 "	8.60 "	+ 0.17 "
	19 "	8.68 "	+ 0.25 "
	26 "	8.76 "	+ 0.33 "
	50 "	8.88 "	+ 0.45 "
	5 hours.	9.22 "	+ 0.79 "
	21 "	10.16 "	+ 1.73 "

As expected, the sign of the pressure-change is constant.

Surface Equilibrium.

It is clear that adsorption does not take much time, for in one experiment quoted later (Table XV.) carbon was exposed to gas at a pressure of 1200 mm. at room temperature. It took only 65 seconds to plunge the whole of the carbon bulb into liquid air, but meanwhile the pressure had fallen to 35 mm.

But while the initial stages of the surface condensation may consequently be enormously rapid, it does not follow that the final equilibrium value is fully attained in such a short time. The following experiment was carried out in order to test this point. The carbon bulb was allowed to stand for a day in liquid air with 59 c.c. (corr.) hydrogen. Then a measured amount of gas was added and almost immediately removed again.

TABLE X.

Remarks.	Time.	Pressure.	Difference.
Put in 12.49 c.c. (corr.)	0 minute.	—	—
	1 "	35.6 mm.	—
	2½ minutes.	34.5 "	—
Removed 13.14 c.c. (corr.) ...	3-5 "	—	—
	7½ "	20.26 mm.	—
	9½ "	20.35 "	+ 0.09 mm.
	13 "	20.40 "	+ 0.14 "
	19 "	20.47 "	+ 0.21 "
	47 "	20.47 "	+ 0.21 "
	88 "	20.49 "	+ 0.23 "

The effects of this considerable disturbance had vanished in less than 14 minutes, which observation affords a maximum value for the time required for surface equilibrium and for thermal effects. Indeed even this must be mainly attributed to the diffusion simultaneously involved. Part of the diffusion is further due to the fact that rather more gas was taken out than was added.

Surface Action at Higher Pressure.

Since the amount of solid solution is proportional to the square root of the pressure (see later), whereas the amount of adsorption varies only with the cube root of the pressure, diffusion must be much more prominent at higher pressure. For this reason the isolation of the time required for surface action is much more difficult. No attempt was made in the following experiment to minimise this disturbing diffusion factor. The procedure was as before, except that the hydrogen was removed by quickly opening the connexion to the Toepler pump and then the same amount of gas was replaced a few minutes later.

TABLE XI.

Remarks.	Time.	Pressure.	Difference.
Removed gas	0 minute.	161.3 mm.	—
	0 "	—	—
	3 minutes.	120 mm.	—
Put gas in again	3 "	—	—
	4 "	175.3 mm.	—
	6 "	165.7 "	— 9.6 mm.
	11 "	163.6 "	— 11.7 "
	24 "	162.5 "	— 12.8 "
	70 "	160.9 "	— 14.4 "
	4 hours.	160.6 "	— 14.7 "

Solubility at Higher Pressure.

As would be expected from the relatively great importance of diffusion at higher pressure no alternation was here observed in the sign of the pressure changes when carrying out experiments of the type of those in Table VII.

TABLE XII.

Remarks.	Time.	Pressure.	Difference.
Added 35.39 c. c. (corr.)	0.10 second.	—	—
	1 minute.	380 mm.	—
	2 minutes.	280 "	— 100 mm.
	3 "	255 "	— 125 "
Took out 5.60 c. c. (corr.) ...	3½ "	—	—
	5 "	139.1 mm.	—
	6 "	134.8 "	— 4.3 mm.
	9 "	127.9 "	— 11.2 "
	39 "	108.5 "	— 30.6 "
	227 "	95.7 "	— 43.4 "
	8 hours.	89.0 "	— 50.1 "
	9 "	88.0 "	— 51.1 "
	10 "	87.7 "	— 51.4 "
	19 "	89.4 "	— 49.7 "
	23 "	90.8 "	— 48.3 "

TABLE XIII.

Remarks.	Time.	Pressure.	Difference.
Put in 42.17 c. c. (corr.)	0 minute.	—	—
	1 "	495 mm.	—
	3 minutes.	405 "	—
Took out 6.34 c. c. (corr.) ...	3½-3¾ "	—	—
	5 "	231.0 mm.	—
	7½ "	220.5 "	— 10.5 mm.
	12 "	200.6 "	— 30.4 "
	32 "	178.7 "	— 52.3 "
	100 "	165.9 "	— 65.1 "
	4½ hours.	161.5 "	— 69.5 "
	10¼ "	160.5 "	— 70.5 "
	11 "	160.7 "	— 70.3 "
	23 "	165.0 "	— 66.0 "

Less than ever can these experiments realize the full value of the diffusion effect. Table XII. yields the following computation:—Volume of bulb 7.40 c.c. Total gas taken up by the carbon at 5 minutes and 10 hours, 25.10 c.c. (corr.) and 26.84 c. c. (corr.). Minimum pressure 87.7 mm.; corrected for temperature drift 84.2 mm., and for extra gas sorbed 68.9 mm. The ratio between the final and initial surface hydrogen is therefore 79 per cent. The carbon weighed 0.626 g. The dissolved hydrogen at a pressure of 88 mm. is therefore 8.9 c.c. (corr.) per gram of carbon.

The corresponding calculation for Table XIII.:—Bulb

7.40 c.c. ; gas in carbon at 5 minutes and 10 hours, 28.04 c.c. (corr.) and 30.40 c.c. (corr.). Minimum pressure 160.5 mm., corrected for temperature drift 155.5 mm., and for extra gas 122.0 mm. Therefore the ratio between final and initial surface condensation is 81 per cent. The dissolved hydrogen under a pressure of 160 mm. is thus 9.3 c.c. (corr.) per gram of carbon.

Inaccurate as the *absolute* values must be, yet the mathematical treatment has shown that all the values (apart from experimental error) are affected by a relative error of the same order of magnitude for each case. Thus the *relative* values may be subsequently trustworthy to give an insight into the variation of the solubility with the pressure, and hence to furnish information with regard to the molecular weight of the hydrogen dissolved in the carbon. A large part of the inaccuracy must be ascribed to the inconstancy of the temperature of the thermostat and to the difference of 3 to 4 degrees in absolute temperature depending upon the liquid air employed in different experiments. Again, the formula here employed is not the true absorption formula but the total sorption formula $\sqrt[3]{p} = k.x$.

The hypotheses between which it is necessary to discriminate are that the dissolved hydrogen : (a) is polymerized, (b) has the same molecular weight as in the gaseous state, or (c) is dissociated into atoms. In the first case the solubility will be proportional to the square (or higher power) of the pressure, in the second directly proportional, and in the third proportional to the square root of the pressure. In the following Table a comparison is made between these assumptions.

TABLE XIV.

Pressure.	Solubility observed.	Calculated for		
		H.	H ₂ .	H ₄ .
19 mm.	4.0 c. c.	4.1 c. c.	1.9 c. c.	0.4 c. c.
88 mm.	8.9 c. c.	8.9 c. c.	8.9 c. c.	8.9 c. c.
160 mm.	9.3 c. c.	12.0 c. c.	16.2 c. c.	29.4 c. c.

The results thus lead to the deduction that dissolved hydrogen is split up into atoms, and it further follows that

the ratio between the absorbed and adsorbed hydrogen is proportional to the pressure.

It is very important to point out that these experiments at a high pressure constitute a direct disproof of the fundamental assumption of the clogged diffusion column hypothesis. For they show that just the same length of time is taken for hydrogen to diffuse at 160 mm. pressure as is taken at 20 mm.; whereas Zacharias's hypothesis had predicted that it would take sixty-four times as long.

Solubility at Room Temperature.

There is no reason why the methods presented in the previous section should not be applied in the demonstration and measurement of the solubility of hydrogen in carbon at room temperature (compare also page 919). Instead of this, carbon was exposed to hydrogen for a day at a pressure between one or two atmospheres at room temperature. The carbon bulb was then quickly plunged into liquid air, and experiments of the type of Table VII. were carried out with it. The apparatus contained for the first day 58.06 c. c. (corr.) hydrogen.

TABLE XV.

Remarks.	Time.	Pressure.	Difference.
Cooled in liquid air.....	0 minute.	1200 mm.	—
	0-1 "	35 "	—
	1 $\frac{2}{3}$ "	31 "	- 4 mm.
	5 minutes.	23.7 "	-11.3 "
	29 "	21.7 "	-13.3 "
Removed 4.68 c.c. (corr.) ...	31-32 "	—	—
	34 "	18.32 mm.	—
	36 "	18.39 "	+0.07 mm.
	38 "	18.38 "	+0.06 "
	56 "	18.24 "	-0.08 "
	142 "	17.41 "	-0.91 "
	4 hours.	16.54 "	-1.78 "
	5 "	16.35 "	-1.97 "
	19 "	16.59 "	-1.73 "

The amount of gas which had passed into solution subsequent to the slight removal at 31-32 minutes is therefore 3.5 c.c. (corr.). From this the solubility at 19 mm. would be 3.9 c.c. (corr.). But it was shown (page 924) that the

total solubility at 19 mm. was 4.0 c.c. (corr.). The difference between the two numbers (which in these particular experiments were merely semiquantitative) indicates the excess of solubility of hydrogen in carbon at the temperature of liquid air (at 19 mm.), over its solubility at room temperature (under 1200 mm. pressure).

It follows that *the solubility of hydrogen in carbon at room temperature is nothing like as great as that at the temperature of liquid air, even though the gas pressure be 60-fold greater in the former case.* Of course the pressure difference may be allowed for on the assumption that the solubility at room temperature also varies as the square root of the pressure, and the two solubilities may then be compared directly. Here we get the result that the solubility is at least a hundredfold greater at the temperature of liquid air. For accurate work a thermostat of fixed and constant temperature (*e. g.* liquid oxygen) would be required, and comparative experiments would have to be carried out under strictly comparable conditions.

Probably the most accurate method of all, which was not investigated, would consist in exposing the carbon at room temperature to greater and greater pressures of hydrogen, until on plunging the carbon bulb into liquid air, the pressure becomes quite constant within say fifteen minutes (time for the carbon to assume the temperature of the liquid air). It is easily seen that this phenomenon will only be observed when the actual amount of dissolved hydrogen at room temperature under very high pressure is the same as it is at very low temperature and pressure. Thus, the relationship between solubility and pressure at ordinary temperature might be studied.

It is interesting to note that this method measures directly the difference in the amount of surface condensation at the two temperatures. This follows from the fact that owing to the conditions of the experiment, the solid solution is constant in amount.

Sorption at Room Temperature.

The diffusion of hydrogen into carbon is not quite instantaneous even at ordinary temperatures, as shown by the following experiment (among others):—

TABLE XVI.

Remarks.	Time.	Pressure.	Difference.
Added hydrogen	0 minute.	—	—
	2 minutes.	142.9 mm.	—
	9 "	142.5 "	-0.4 mm.
	14 "	142.3 "	-0.6 "
	23 "	142.1 "	-0.8 "
	45 "	141.9 "	-1.0 "
	4 hours.	141.9 "	-1.0 "

Further investigation of sorption phenomena at room temperature was not undertaken. The question as to whether the smallness of the effect measured here is due to the rapidity of diffusion at this temperature or to the relative unimportance of the absorption factor was left untouched.

Discussion of the Results.

The main thesis of the present paper is the demonstration that the so-called adsorption of hydrogen by carbon is dual in its nature. The striking fact upon which this is based is that hydrogen is temporarily evolved from unsaturated carbon, and that such gas is taken up by carbon, which is known to be already supersaturated.

It is considered that the experiments disprove the simple surface action theory. Their weakness consists in the fact that they do not in the same way exclude a clogged diffusion hypothesis*. The only refutation of the latter that it seems possible to bring forward appears to be, first, that it is irreconcilable with the known laws of diffusion, and secondly that the fundamental assumption of the clogged diffusion hypothesis has admitted of direct experimental disproof (page 930), for this assumption was that diffusion would be more difficult and slower the higher the pressure or concentration.

In Davis's paper† the method adopted for showing the co-existence of surface action and solid solution, when iodine solutions are exposed to carbon, was as follows:—It was first demonstrated that an equilibrium of some type

* See, for example, Zacharias, *Zeitschr. physikal. Chem.* xxxix. p. 468 (1902); Travers, *loc. cit.*; compare *Zeitschr. physikal. Chem.* lxi. p. 241 (1907).

† Davis, *loc. cit.*

could be attained in a short time ; for the amount of sorption after a day was the same when approached from both directions (supersaturation and unsaturation), a phenomenon which is characteristic for all equilibria.

This equilibrium point lay, however, appreciably below the final saturation value attained after several weeks. Hence the first equilibrium value was referred to surface condensation, and the after increase was ascribed to diffusion. It should be mentioned that in these experiments with iodine as in those with hydrogen the diffusion effect is greater than it at first seems ; for it is partly masked by the necessary simultaneous transfer of some of the surface accumulation into the interior, as the concentration of the liquid iodine solution falls off.

This method was not employed in the experiments with hydrogen, for it would demand a thermostat of accurately reproducible and sensibly constant temperature, such as furnished by liquid oxygen.

The weak point in Davis's proof lay in the fact that the after diffusion effect could be ascribed to pores and fine cavities in the carbon employed. Consider a tiny cavern in the carbon, having a relatively very small entrance. As soon as the carbon was wetted, the cavern would probably become filled with solution. Practically at once the walls of this cavity would become permeated with surface condensation of iodine. But the amount of carbon surface which would have to be supplied with iodine from the small amount of enclosed solution might easily be *relatively* much greater than the total carbon surface which was supplied from the total iodine solution in the vessel. Thus the minute amount of solution in the enclosure would become very much weaker than the main bulk of the solution. Since the only communication with the body of the liquid outside would be through the small opening of the pore or cavity it follows that a slow diffusion would take place, but it would be diffusion through the liquid enclosed in the pores.

But the whole purpose of the method is to ascribe this after diffusion effect to diffusion in the *solid* carbon phase. Consequently the existence of this possible objection must be taken into account. It is true that experiments might be carried out on finely pulverised carbon, and where the pores were *excessively* small (molecular dimensions) the objection would fall to the ground, as diffusion would then become nearly instantaneous.

It is, however, satisfactory that such objection cannot be

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urged against the experiments with hydrogen, nor against the methods employed in the present paper.

No mention has been made in the preceding pages of the large evolution of heat which accompanies sorption. It will certainly influence some of the methods employed, as sorption is very sensitive to temperature changes. Fortunately, however, the experimental results cannot be ascribed to such effects. Most probably the effects of these heat changes would be in the very opposite direction to those observed in each case; this is practically certain in Table VIII. where the manipulation was very rapid. However, Table II. may be discussed; immediately after the final removal of gas the carbon must be either warmer or colder than the liquid-air bath. In the first case the above sentence holds good, the temperature influence would be opposed to the observed pressure changes. The second case is obscure unless it is clearly kept in mind that throughout the whole period from "14 minutes" to a couple of hours later the carbon is *not saturated* with hydrogen even at the higher final temperature; hence if sorption be simple and not dual in its nature the observed intermediate *rise* of pressure is *a priori* impossible. A division of the total surface condensation into more accessible and less accessible regions does not suffice to meet this difficulty, for it ultimately leads to the untenable alternative explanation of my experimental results—that a considerable fraction of the *surface* of coconut carbon, at the very lowest estimate one-seventh, is so inaccessible that a dozen hours are necessary for hydrogen to come fully into contact with it. Similar reasoning holds for Table VIII.

Finally there remains but one objection to be discussed. The apparent after diffusion effect cannot be caused by slow chemical action; for it always ceased after some hours, and further no difference could be detected in the amount of hydrogen before and after the experiment.

Summary.

1. It is considered that the "adsorption" of hydrogen by carbon has been definitely proven to consist of a surface condensation and a diffusion (solid solution) into the interior of the carbon.
2. The surface condensation is nearly instantaneous at the temperature of liquid air, requiring only a few minutes *in maximo*; whereas the diffusion requires about twelve hours.
3. Various experimental methods have been elaborated for the isolation and study of these phenomena.

4. An approximate measurement was made of the true solubility (as distinguished from the surface condensation) of hydrogen in the coconut carbon. At the temperature of liquid air it varies with the square root of the pressure. Hence hydrogen dissolved in carbon is split up into simple atoms. The solubility amounted to 4 c.c. (corr.) hydrogen per gram of carbon at a pressure of 19 mm., equivalent to one seventh of the total gas taken up by the carbon. The true solubility at room temperature is less than one hundredth as great.

5. Since "adsorption" in the only cases hitherto investigated has been shown to be of a dual nature, the general non-hypothetical term *sorption* is proposed to embrace all adsorption and occlusion phenomena.

University of Bristol.

July 27, 1908.

C. *Proceedings of Learned Societies.*

GEOLOGICAL SOCIETY.

[Continued from p. 820.]

April 7th, 1909.—Prof. W. J. Sollas, LL.D., Sc.D., F.R.S., President, and afterwards H. W. Monckton, Treas.L.S., Vice-President, in the Chair.

THE following communications were read:—

1. 'On Overthrusts at Tintagel (North Cornwall).' By Henry Dewey, F.G.S.

In this paper the author deals with the geological structure of the Tintagel area. After brief reference to the stratigraphy north of Bodmin Moor, mention is made of the apparent difference in order of superposition of the beds near Tintagel.

The several types into which the Upper Devonian rocks are divided are next described. The beds in descending order are:—

- (6) Tredorn Phyllites.
- (5) Trambley Cove Gritty Slates.
- (4) Volcanic Series.
- (3) Barras Nose Beds.
- (2) Woolgarden Phyllites.
- (1) Delabole Slates.

The above order is preserved for many miles, between the Boscastle coast and Lewannick on the eastern side of Bodmin Moor. A change of strike at Tintagel reveals the anticlinal structure of the district. To the south of the nose of this great fold, minor folds cross the strike. These folds increase westwards, until they are

replaced by overthrusts. Four sections from east to west show the increased folding and overthrusting towards the northern part of the area.

The paper concludes with a reference to the age of the folding.

2. 'The Lahat "Pipe": A Description of a Tin-Ore Deposit in Perak (Federated Malay States).' By John Brooke Scrivenor, M.A., F.G.S. (Geologist to the Federated Malay States Government).

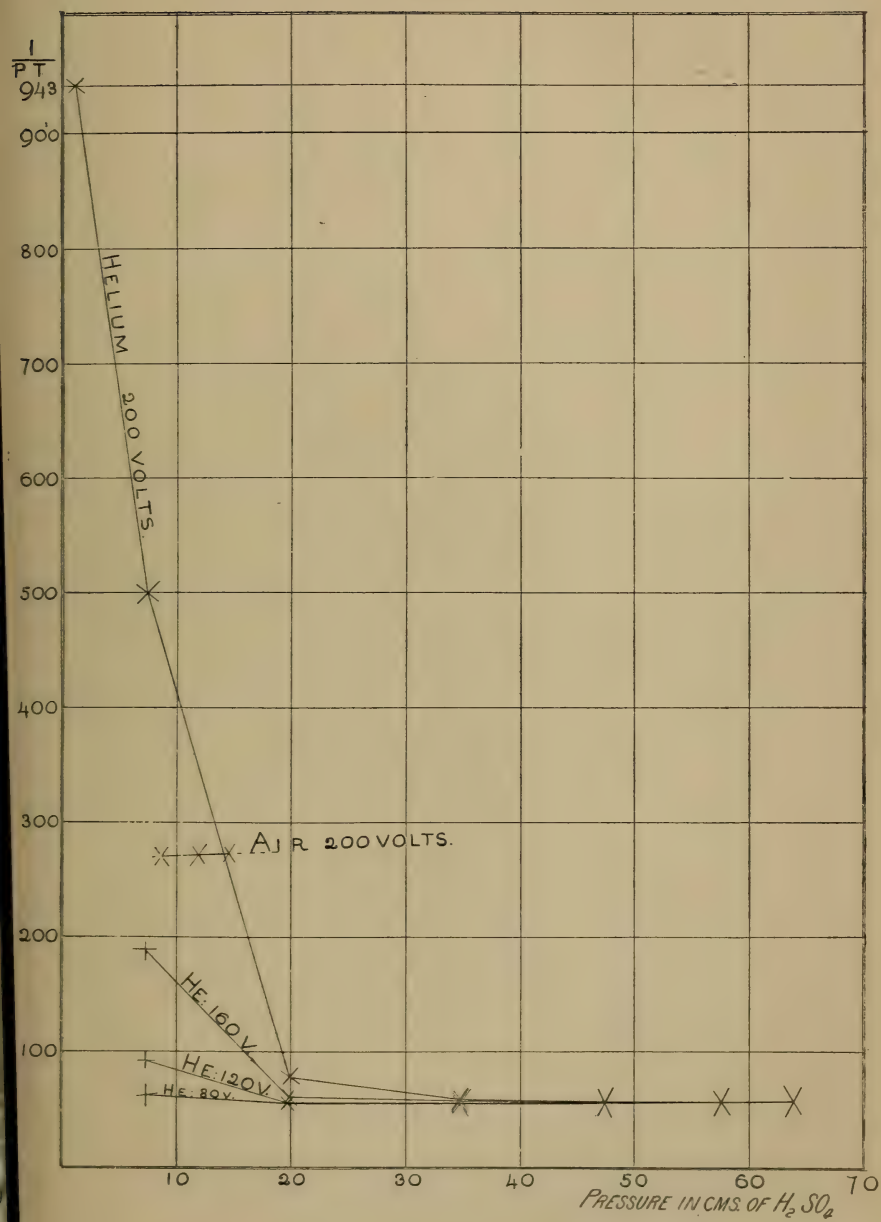
Large quantities of tin-ore have been obtained during recent years in the Kinta district of Perak, principally from detrital deposits, but also in some cases from the limestone which forms the floor of the Kinta Valley. From 1903 till 1907 the Société des Étains de Kinta secured over 1000 tons of dressed tin-ore from a peculiar deposit which had the form of a pipe in the limestone, measuring only 7 feet by 2 at the surface, but widening when followed downwards. It was worked to a depth of 314 feet. The vein-stone was a deep red mixture of calcite and iron-oxide with some quartz, chalybite and chalcopryrite, but no tourmaline was found in it. In this the cassiterite occurred in irregular pieces and broken fragments, some of which consisted of radiating needles.

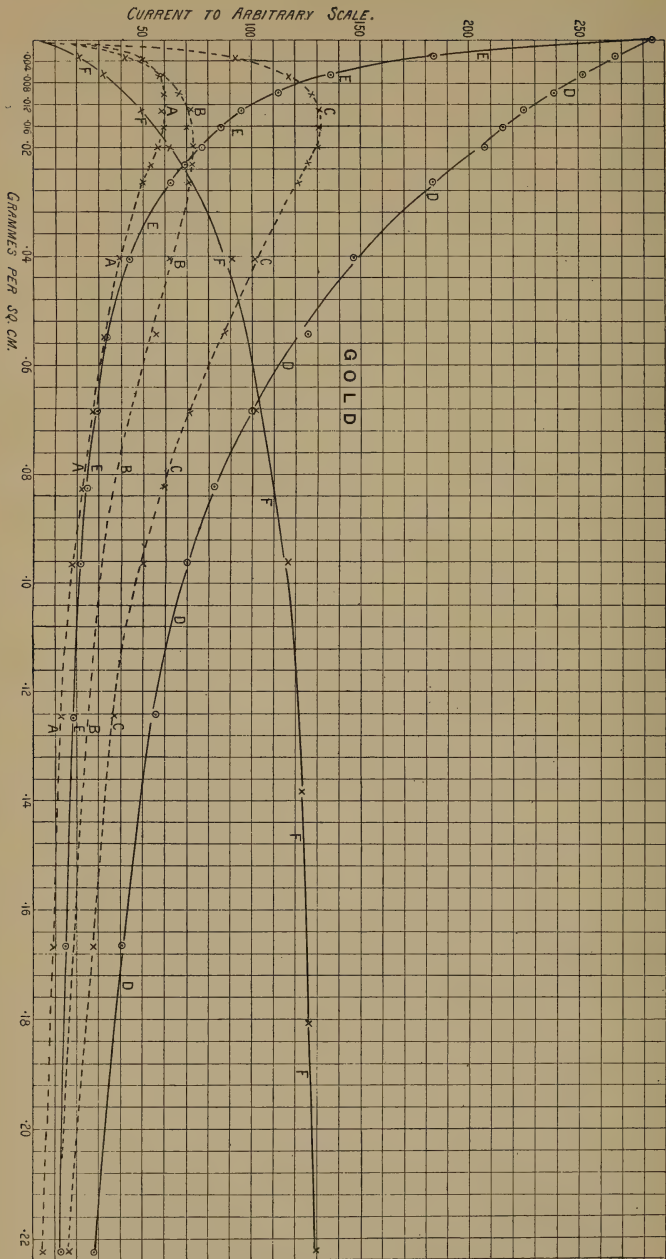
In Kinta the tin-ores occur in the limestone in two different ways:—(1) As lodes or veins with fresh sulphides but not iron-oxides. The tin-oxide crystals have a definite arrangement. (2) As transformed masses, deposited in fissures. The cassiterite is in rounded grains, and quartz, tourmaline, and other minerals, also well rounded, accompany it. For a long time, it has been doubtful to which of these classes the Lahat pipe should be assigned, as it presents some features of each class. Recently, however, specimens have been obtained showing veins of arsenopyrite and cassiterite in limestone, and from these the author concludes that the Lahat pipe was originally a vein or lode-deposit in the limestone, which subsequently afforded a course for surface-waters; these dissolved away the calcite and oxidized the sulphides, caves being formed into which the insoluble ores were carried by the water. Finally, the brecciated mass was recemented. Some foreign material has been introduced, but the bulk of the contents of the pipe is of local origin and consequently not rounded by transport. In other words, the Lahat pipe is a lode-deposit which has been converted into a detrital deposit *in situ*.

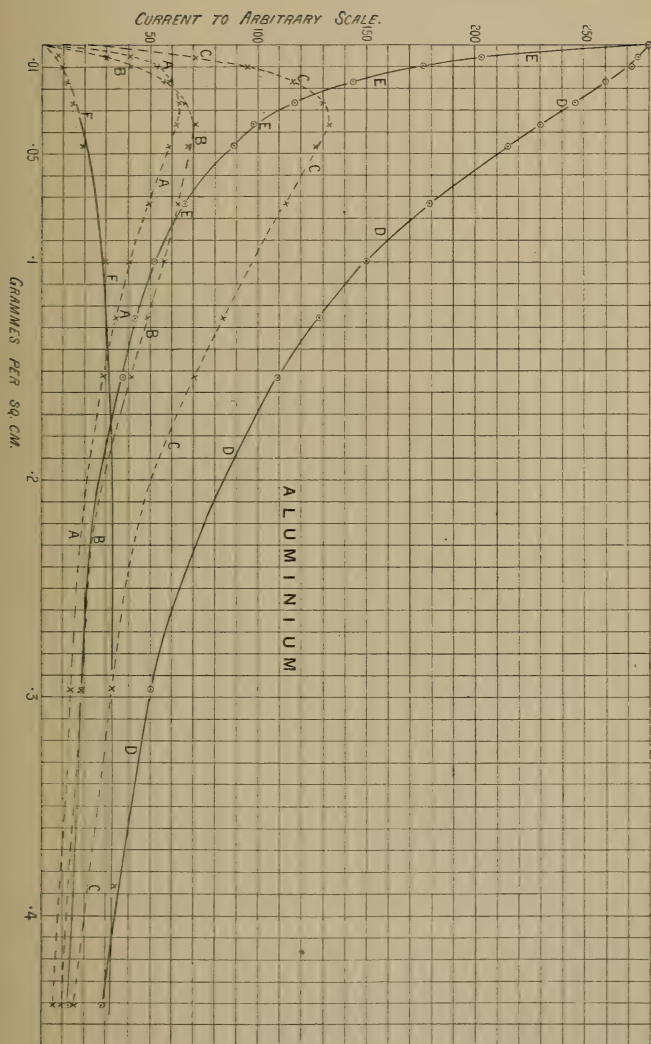
3. 'On the Sculptures of the Chalk Downs in Kent, Surrey, and Sussex.' By George Clinch, F.G.S., F.S.A. Scot.

The author classifies the various forms of sculpture of the Chalk Downs under three heads, namely, (1) dry valleys of simple form, (2) dry valleys of complex form, and (3) wet valleys. He draws attention to the relatively small catchment-areas of the dry valleys, and to the large number of tributary valleys found in some districts, two points which he considers have not received hitherto entirely satisfactory explanation.

FIG. 2.







While accepting the view that frozen conditions in former times altered the drainage-system of the Chalk, he argues that the most potent excavating force was the frost itself acting on Chalk saturated or highly charged with water. He propounds a theory to account for (1) the great size and breadth of the valleys in relation to their catchment-basins; (2) the ramifications of some of the valley-systems; and (3) the remarkable fact that many dry valleys die out just before the crest of the Chalk Downs is reached.

C.I. Intelligence and Miscellaneous Articles.

INDUCTANCE AND RESISTANCE IN TELEPHONE AND OTHER
CIRCUITS.

IN the paper published by me, under this title, in the Phil. Mag. for September last, there is an error to which I desire to call attention. The magnitude h in the investigation should be replaced by h^2c , so that, in the results, ρ stands not for $\log_e ha$, but for $\log_e h^2ac$.

Trinity College, Cambridge,
November 17, 1909.

J. W. NICHOLSON.

THE ULTIMATE PRODUCT OF THE URANIUM DISINTEGRATION
SERIES.

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

Manchester, Nov. 24, 1909.

I would like to correct a mistake which occurred in a paper of mine, published in this month's Philosophical Magazine. Taking lead as the final product of the uranium series, I find the age of some autunite to be $\frac{10^{-4}}{9 \times 10^{-9}}$ years, or nearly 10,000 years, not a million years, as appears in the paper. As in the calculation no account was made of the fact that radium takes some time to reach equilibrium with the uranium, a greater age must be given to the mineral.

I am,

Yours faithfully,

J. A. GRAY.

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